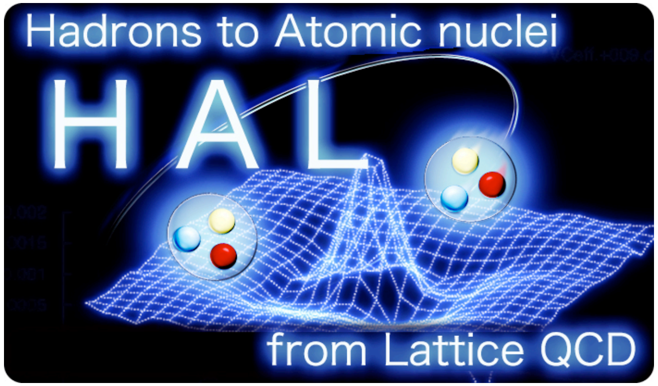
Quark mass dependence of Three-nucleon forces in Lattice QCD

Takumi Doi

(Nishina Center, RIKEN)

for HAL QCD Collaboration



S. Aoki (YITP)

B. Charron (Univ. of Tokyo)

T. Doi, T. Hatsuda , Y. Ikeda (RIKEN)

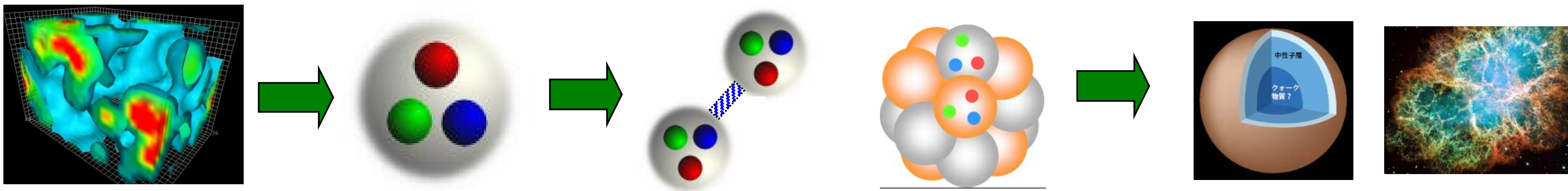
F. Etminan (Univ. of Birjand)

T. Inoue (Nihon Univ.)

N. Ishii, K. Murano (RCNP)

H. Nemura, K. Sasaki, M. Yamada (Univ. of Tsukuba)

Motivation:
Nuclear Physics and Astrophysics from Lat QCD



QCD Vacuum | Baryon | Few-Body | Nuclei | Neutron Star / Supernova

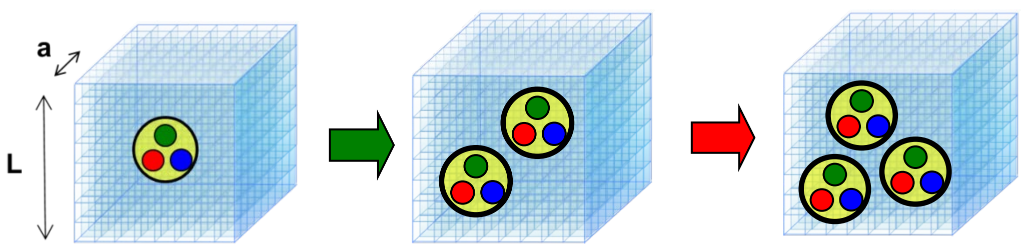
Standard Model

1st-principle lat calc.

Nuclear Forces

Ab-initio nuclear calc.

Lattice QCD predictions play a crucial role

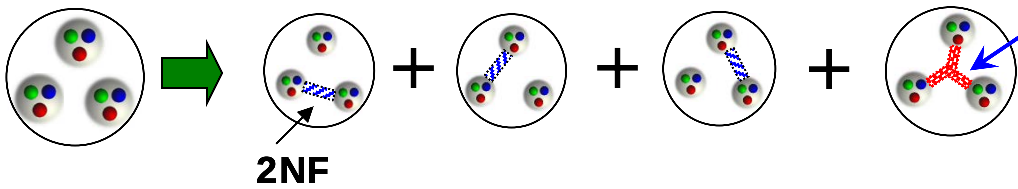


1st-principle lat calc.

Talk by Yamazaki

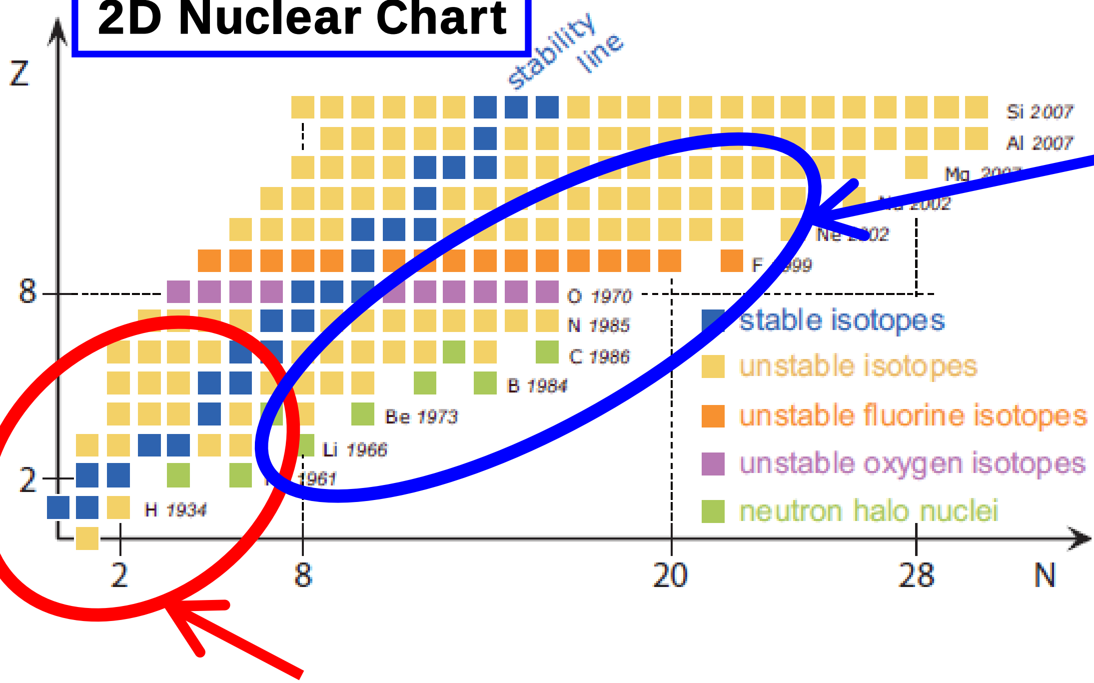
Three-nucleon forces (3NF)

What is 3NF ?



3NF: Forces which cannot be explained by pair-wise 2NF

2D Nuclear Chart



Paradigm Shift in
Unstable Nuclei
(New Magic Numbers !)

← Important role of 3NF

T.Otsuka et al., PRL105(2010)032501

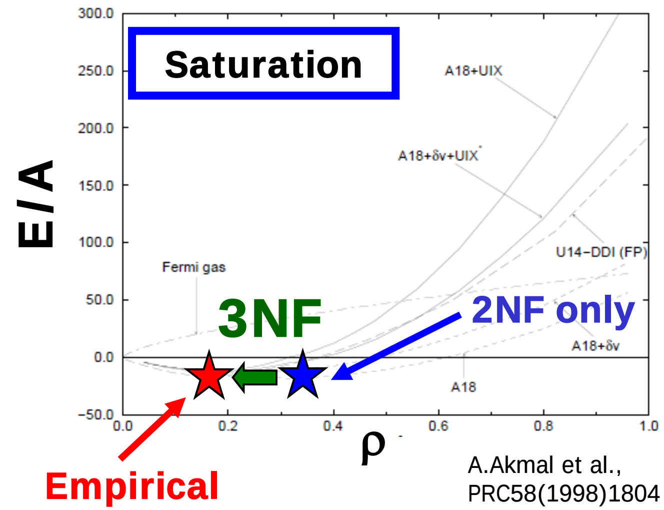
Nucleosynthesis
@ Neutron star Merger
SuperNova

Precise ab initio calculations show 3NF is indispensable

New Horizons w/ **Three-Nucleon Forces (3NF)**

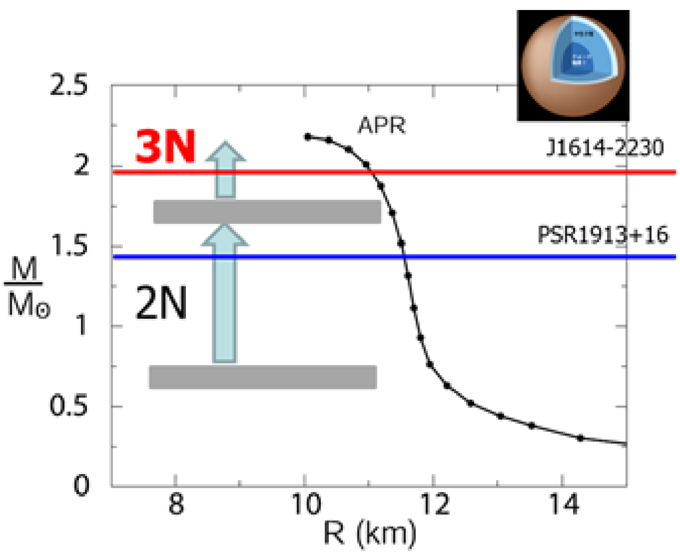
- 3NF is crucial to understand EoS at high density matter

Saturation point of nuclear matter



A.Akmal et al., PRC58(1998)1804

Neutron Star / Supernova



Short-range repulsive 3NF is phenomenologically required

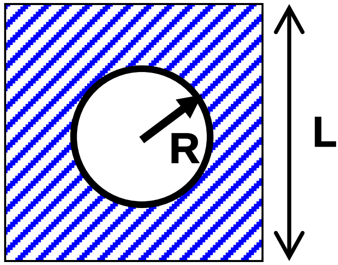
Nuclear Forces from Lattice QCD [HAL QCD method]

- Potential is constructed so as to reproduce the NN phase shifts (or, S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0|N(\vec{x}+\vec{r};t)N(\vec{x};t)|2N\rangle$$

$$\psi_l(r) \simeq e^{i\delta(k)}\frac{\sin(kr - l\pi/2 + \delta(k))}{kr} \qquad (r > R)$$



M.Luscher (1991), C.-J.Lin eta l. (2001), CP-PACS Coll. (2005), Ishizuka (2009)

  – Multi-particle systems

S.Aoki et al. (HAL Coll.), PRD88(2013)014036

$$\psi(\vec{x}_1,\vec{x}_2,\cdots,\vec{x}_n) = \langle 0|N(\vec{x}_1)N(\vec{x}_2)\cdots N(\vec{x}_n)|n\,N\rangle$$

$$\psi_{[L],[K]}(R,Q_A) \simeq \sum_{[N]} U_{[L][N]}(Q_A)\boxed{e^{i\delta_{[N]}(Q_A)}\frac{\sin(Q_A R - \Delta_L + \delta_{[N]}(Q_A))}{(Q_A R)^{(D-1)/2}}}U^{\dagger}_{[N][K]}(Q_A)$$

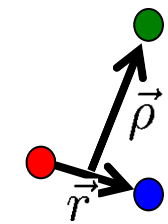
(non-rela limit)

c.f. Finite V spectrum: Talks by Briceno, Sharpe

3NF from NBS wave function [HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r},\vec{\rho}) = \langle 0|\boxed{N(\vec{x}+\vec{r})}\boxed{N(\vec{x})}\boxed{N(\vec{x}+\vec{r}/2+\vec{\rho})}|3N\rangle$$



- Obtain 3NF through

$$(E-H_0^r-H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\underbrace{\textstyle\sum_{i<j} V_{ij}(\vec{r}_{ij})}_{\text{by 2N calc}} + V_{3NF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

- NBS is obtained by 6pt. correlator

$$G(\vec{r},\vec{\rho},t-t_0) = \textstyle\sum_{\vec{x}}\langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)N(\vec{x}+\vec{r}/2+\vec{\rho},t)\overline{NNN}(t_0)|0\rangle$$

$$\left(-H_0 - \frac{\partial}{\partial t}\right)R(\boldsymbol{r},t) = V(r)R(\boldsymbol{r},t)$$

In practical calculation,
we employ **time-dependent HAL QCD method**

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

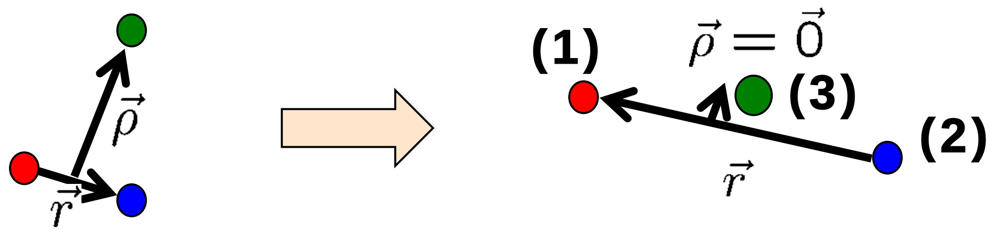
➔ Ground state saturation is NOT necessary !

3NF calculation in Lat QCD

- We fix the geometry of 3N (🡐 this is not an approximation)

- We study **linear setup**

We consider
Triton channel

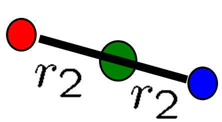


$(\vec{r}_2 \equiv \vec{r}/2)$

  - ➔ $L^{(1,2)\text{-pair}} = L^{\text{total}} = 0$ or 2 only
  - ➔ **Bases are only three**, labeled by **$^1S_0$, $^3S_1$, $^3D_1$ for (1,2)-pair**

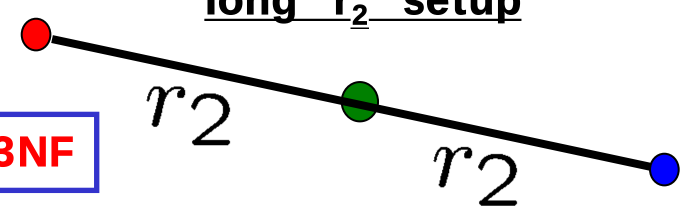
- **Linear setup** with various distance **"r_2"**

short "r_2" setup



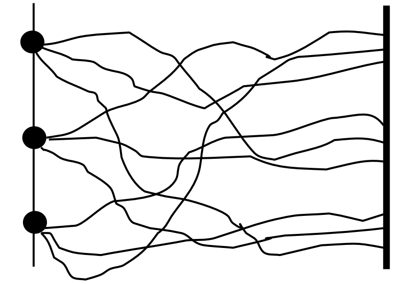
Study r_2-dependence of 3NF

long "r_2" setup



Challenge in multi-baryons on the lattice

- Enormous computational cost for correlators
  - # of Wick contraction (permutations) $\sim [\,(\frac{3}{2}A)!\,]^2$
  - # of color/spinor contractions $\sim 6^A \cdot 4^A$ or $6^A \cdot 2^A$
    - **Total cost:**
      - ^{2}H : 9 x 144 = 1 x 10^3
      - ^{3}H : 360 x 1728 = 6 x 10^5
      - ^{4}He : 32400 x 20736 = 7 x 10^8



Improvement:
T.Yamazaki et al., PRD81(2010)111504

- **[Unified contraction algorithm (UCA)]** TD, M.Endres, CPC184(2013)117
  - Treat Wick/color/spinor contractions in a unified index space
    - ➔ huge redundancies can be eliminated systematically
    - **Significant improvement**

$\boxed{\times 192 \text{ for } {}^3\mathrm{H}/{}^3\mathrm{He},\ \times 20736 \text{ for } {}^4\mathrm{He},\ \times 10^{11} \text{ for } {}^8\mathrm{Be}}$ (x add'l. speedup)

See also subsequent works: Detmold et al., PRD87(2013)114512
Gunther et al., PRD87(2013)094513
Nemura @ Lat2013

w/ other tunings etc. ➔ ~ x1000 speedup for 3NF !

Lattice simulation setup

- Nf=2 dynamical clover fermion + RG improved gauge action
  - a^{-1}=1.269GeV, a=0.1555fm (beta=1.95)
  - 16^3 X 32 lattice, L=2.5fm

CP-PACS Coll. S. Aoki et al.,
Phys. Rev. D65 (2002) 054505

- Masses: (π, N, Δ) = (1.13, 2.15, 2.31) GeV
  - Kappa(ud)=0.13750
  - 599 configs x 32 measurements, t+1=[5,12]

← T.D. et al. (HAL Coll.)
PTP127(2012)723,
PoS LATT2012,009

- Masses: (π, N, Δ) = (0.925, 1.85, 2.02) GeV
  - Kappa(ud)=0.13900
  - 686 configs x 32 measurements, t+1=[5,12]
- Masses: (π, N, Δ) = (0.757, 1.61, 1.81) GeV
  - Kappa(ud)=0.14000
  - 686 configs x 32 measurements, t+1=[5,12]

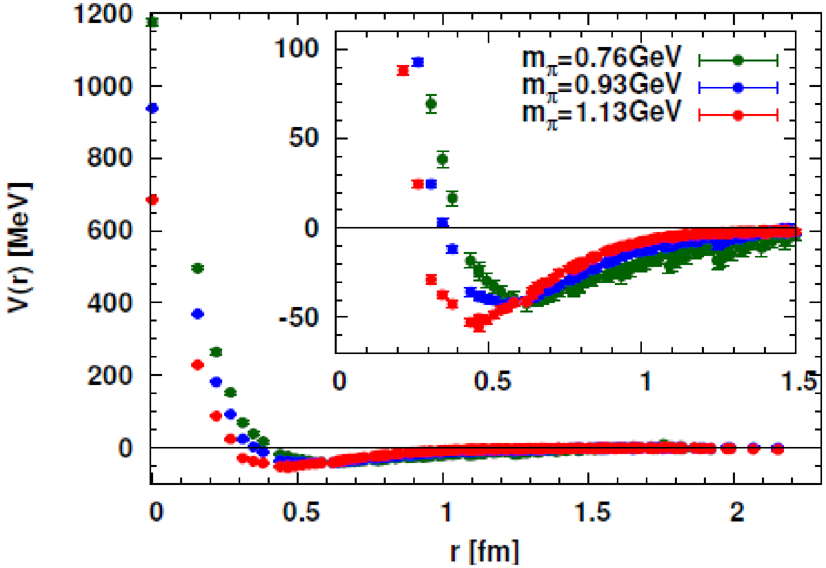
Calc @ two lighter masses

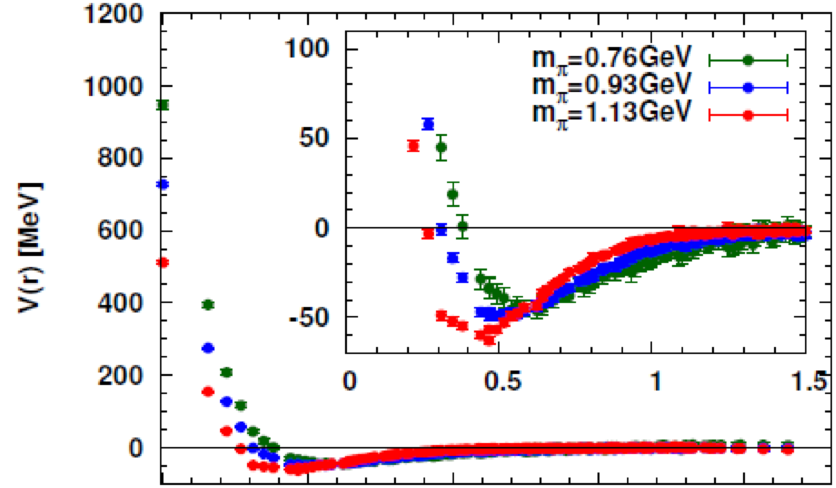
Wall source w/ Coulomb gauge

Quark mass dependence on 2NF

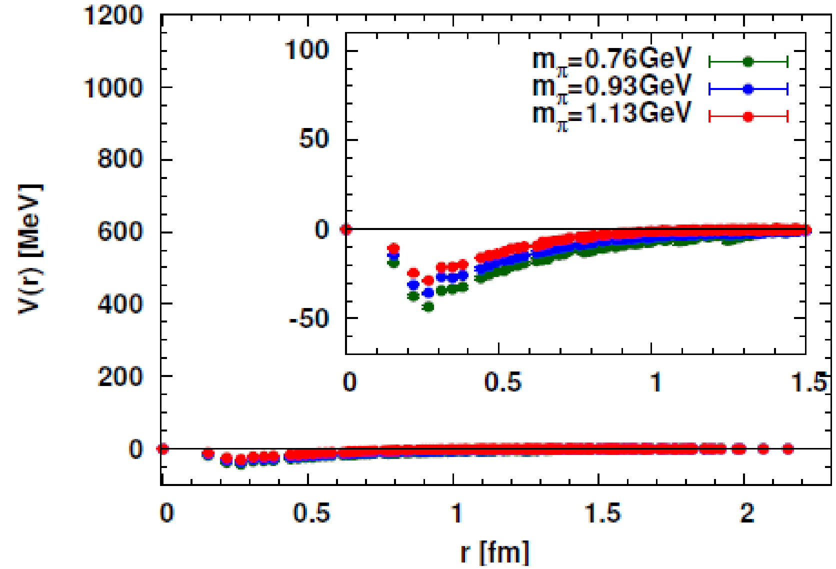
Central in $^1S_0$

$^3S_1$-$^3D_1$ channel





Central



Tensor

Lighter mass corresponds to…

- **Longer interaction range**
- **Stronger Tensor Force**
- **Larger Repulsive Core**

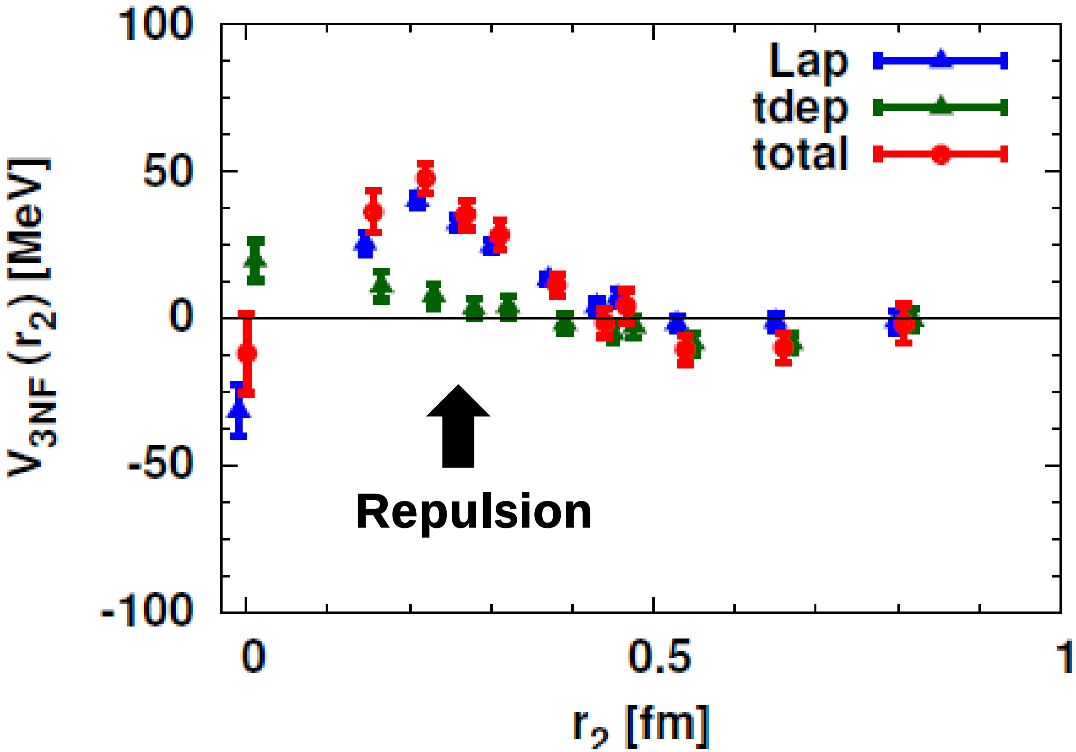
($t-t_0=7.5$)

3N-forces (3NF) on the lattice

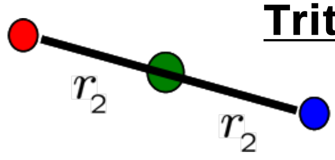
T.D. et al. (HAL QCD Coll.) PTP127(2012)723

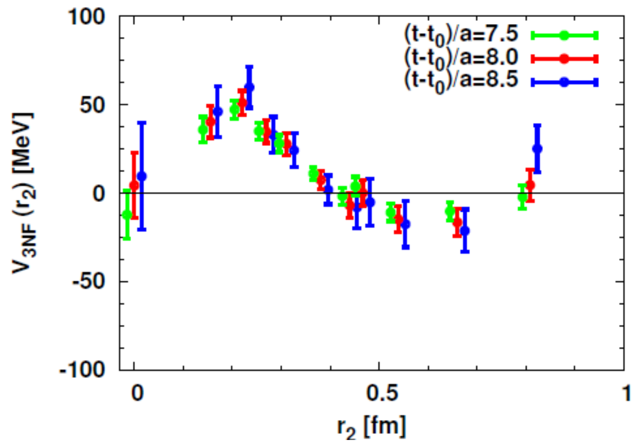
+ t-dep method updates etc.

m_π=1.1GeV



(breakup at $t-t_0=7.5$)





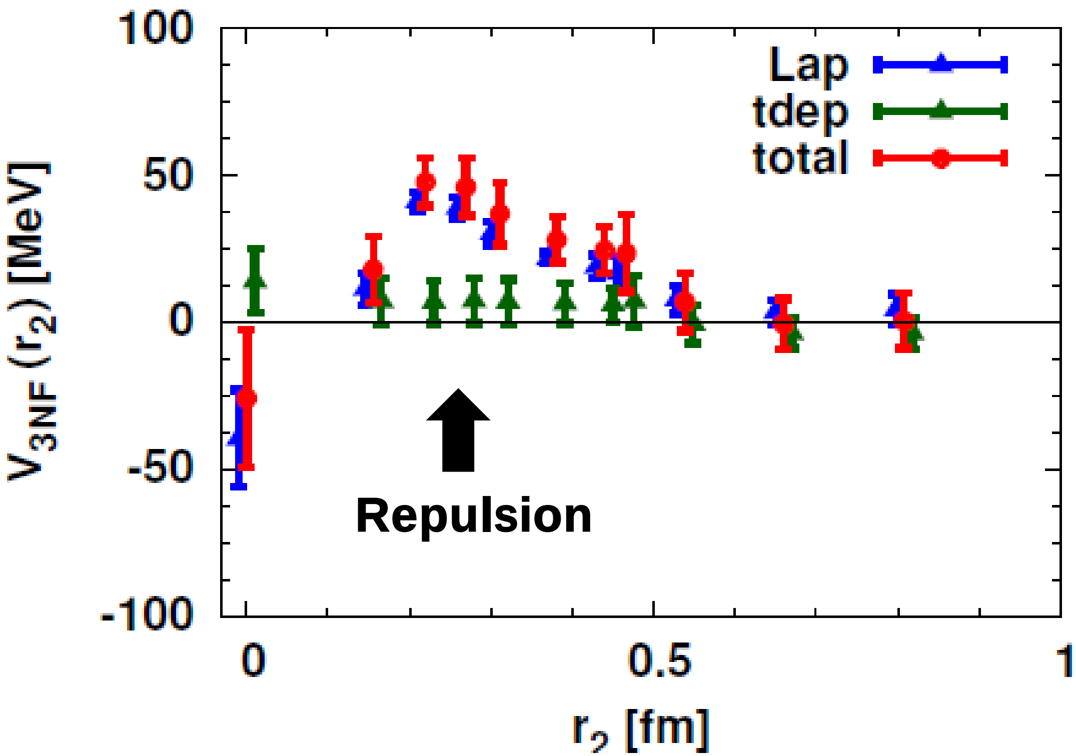
Sink time dependence is small

Lap tdep

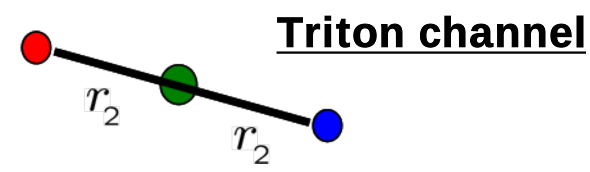
$$\left(-H_0 - \frac{\partial}{\partial t}\right) R(\boldsymbol{r}, t) = V(r) R(\boldsymbol{r}, t)$$

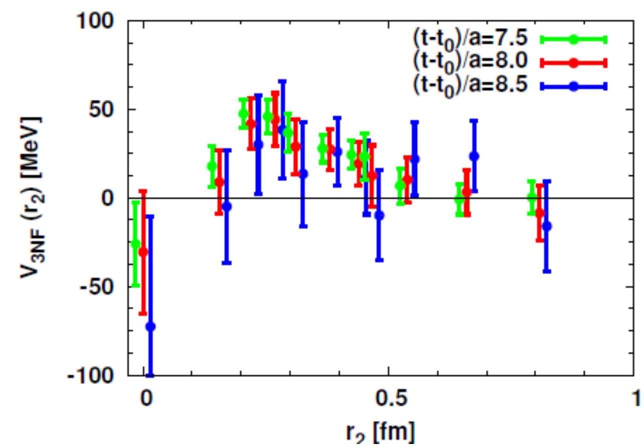
3N-forces (3NF) on the lattice

$m_\pi = 0.93$GeV



(breakup at $t\text{-}t_0=7.5$)





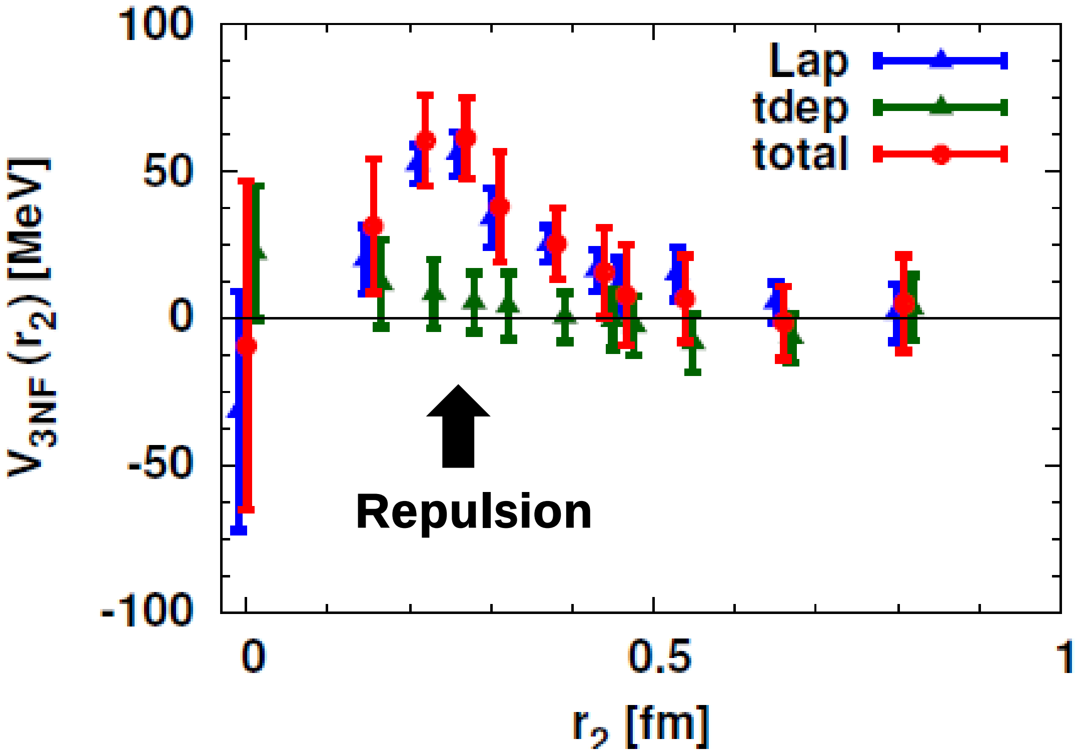
Sink time dependence is small

Lap tdep

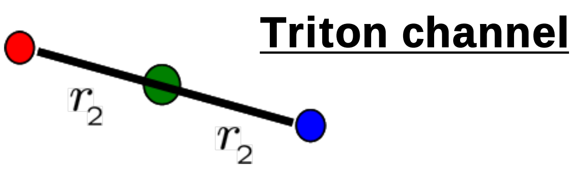
$$\left(-H_0 - \frac{\partial}{\partial t}\right) R(\boldsymbol{r}, t) = V(r) R(\boldsymbol{r}, t)$$

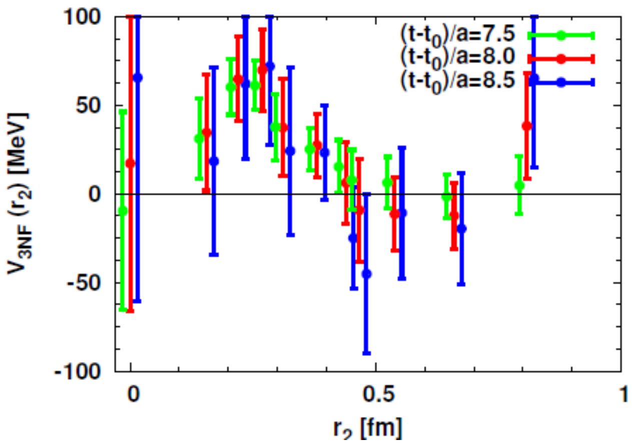
3N-forces (3NF) on the lattice

$m_\pi = 0.76$GeV



(breakup at $t-t_0=7.5$)





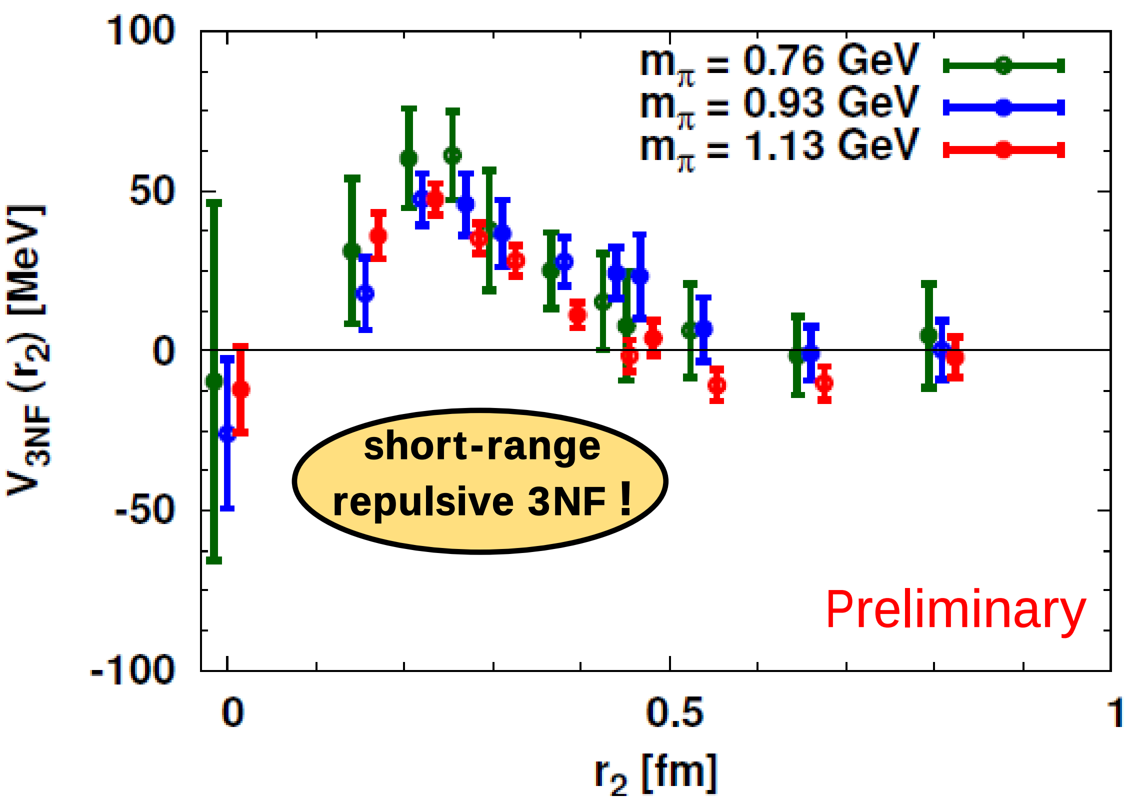
(Sink time dependence is small ?)

Lap tdep

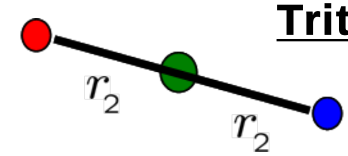
$$\left(-H_0 - \frac{\partial}{\partial t}\right) R(\boldsymbol{r}, t) = V(r) R(\boldsymbol{r}, t)$$

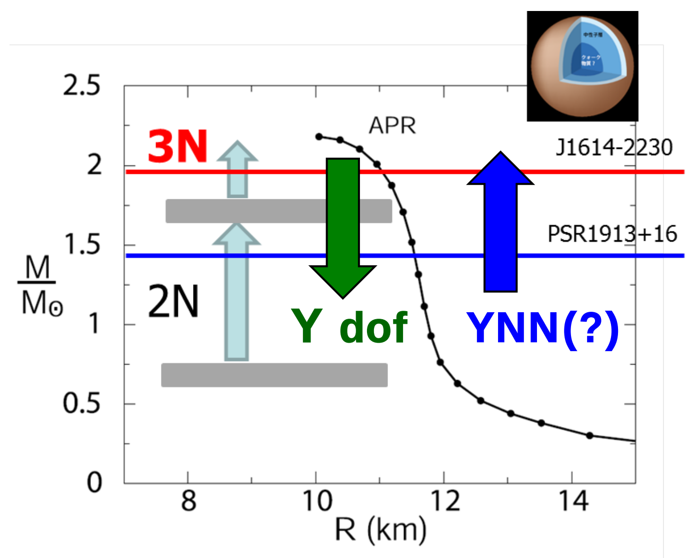
3N-forces (3NF) on the lattice

Quark mass dependence



Triton channel



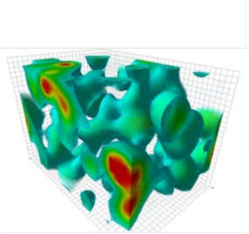
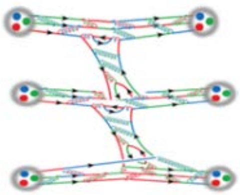
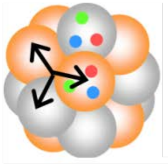


Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=0.76-1.1GeV, m_N=1.6-2.1GeV

How about other geometries ?

How about YNN, YYN, YYY ?

Summary and Prospects

- We have studied Three-nucleon forces on the lattice
  - NBS wave func. carries proper phase shifts
  - Compt. cost drastically reduced [unified contraction algorithm]

- Nf=2 dynamical clover fermion
  - Three quark masses: m_π=0.76, 0.93, 1.1GeV
  - Repulsive 3NF at short distance observed
  - Quark mass dependence is weak so far

- **Outlook**
  - Lighter quark masses
  - Study systematics, e.g., cutoff dependence
  - Other geometries, Three-baryon forces (YNN, YYN, YYY)