Lepton anomalous magnetic moments from twisted mass fermions

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Introduction

The muon anomalous magnetic moment constitutes a prime candidate to possibly detect new physics beyond the standard model, since a 3 to 4 standard deviations discrepancy has been observed between its experimental and theoretical values. The dominant uncertainty originates from its hadronic contributions. Here, we show our results for the four-flavour leading hadronic contributions to the anomalous magnetic moments of all three leptons, being sensitive to very different momentum regions. Additionally, we confirm the results of our earlier chiral extrapolations by computing the light-quark contributions at the physical point. Furthermore, our attempts to reduce remaining systematic as well as statistical errors are described by showing the status of investigations of disonnected contributions, different fitting strategies, and the all-mode-averaging technique.

Basic Equations

leading hadronic contribution:

$$a_{\text{lepton}}^{\text{hvp}} = \alpha^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{H^2}\frac{H_{\text{phys}}^2}{m_{\text{lepton}}^2}\right) \Pi_{\text{R}}(Q^2)$$

$H = H_{\text{phys}} = 1$ standard definition
$H = m_V$ improved definition

vacuum polarisation determined from correlator of point-split vector currents at source and sink

MNBC fit function:

$$\Pi(Q^2) = (1-\Theta(Q^2-Q^2_{\text{match}}))\,\Pi_{\text{low}}(Q^2) + \Theta(Q^2-Q^2_{\text{match}})\,\Pi_{\text{high}}(Q^2)$$

with

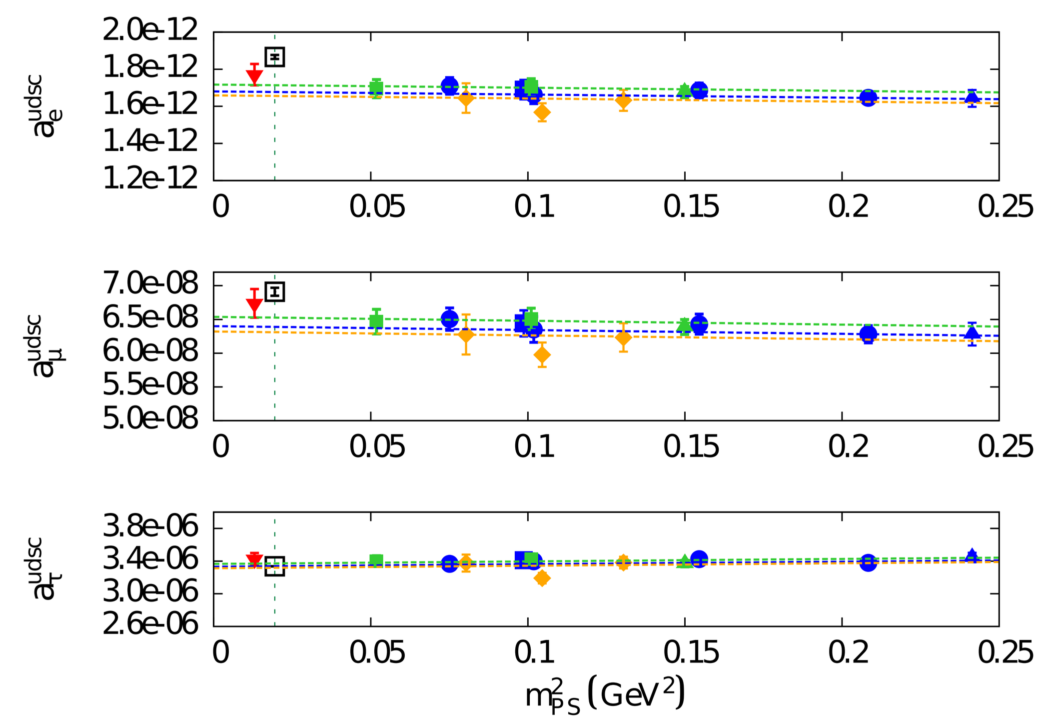
$$\Pi_{\text{low}}(Q^2) = \sum_{i=1}^{M} \frac{f_i^2}{m_i^2+Q^2} + \sum_{j=0}^{N-1} a_j (Q^2)^j$$

$$\Pi_{\text{high}}(Q^2) = \log(Q^2) \sum_{k=0}^{B-1} b_k (Q^2)^k + \sum_{l=0}^{C-1} c_l (Q^2)^l$$

f_i and m_i are determined consistently from vector meson correlators

$\mathrm{N_f} = 2+1+1$ fermions

continuum limit performed
chiral extrapolation with improved definition
analysis as described in [1] using M1N2B4C1 fits



- electron

$a_e^{\text{hvp}} = 1.77(06)(04)\cdot 10^{-12}$

dispersive analysis [2]:
$a_e^{\text{hvp}} = 1.87(01)(01)\cdot 10^{-12}$

- muon

$a_\mu^{\text{hvp}} = 6.74(21)(18)\cdot 10^{-8}$

dispersive analysis [3]:
$a_\mu^{\text{hvp}} = 6.91(01)(05)\cdot 10^{-8}$

- tau

$a_\tau^{\text{hvp}} = 3.42(08)(05)\cdot 10^{-6}$

dispersive analysis [4]:
$a_\tau^{\text{hvp}} = 3.38(4)\cdot 10^{-6}$

$\mathrm{N_f} = 2$ fermions at the physical point

Comparison of MN fits and Padé fits in low-momentum region

perform correlated Padé fits & uncorrelated MN fits (strong curvature of vacuum polarisation towards zero momentum) up to $Q^2_{\max} = 0.75\,\text{GeV}^2$

compare fit results by pull with respect to [1,1] Padé fit: $\text{pull} = \frac{|\text{value}([1,1]) - \text{fitvalue}|}{\sigma_{\text{diff}}}$

	M1N2	M1N3 (standard)	[0,1] Padé fit	χ^2/dof	[1,1] Padé fit	χ^2/dof
$a^2\times$ pole	0.155(20)	0.155(20)	0.198(07)	-	0.136(26)	-
a_μ^{hvp} $(H=1)$	$5.43(40)\cdot 10^{-8}$	$5.73(31)\cdot 10^{-8}$	$4.99(16)\cdot 10^{-8}$	1.17	$5.96(56)\cdot 10^{-8}$	1.08
pull	1.02	0.85	1.57	-	-	-

Systematic uncertainty due to different fit functions comparable to statistical uncertainty.

Result obtained with continuation method as in [6]

$$a_\mu^{\text{hvp}} = 4.84(43)\cdot 10^{-8}$$

same $Q^2_{\max} = 0.75\,\text{GeV}^2$
summation up to $T_{\max} = 14$
$T_{\max} > 14$:use ground state dominance

Notation of Padé Fits as in [5]

[N,N] Padé fit function:

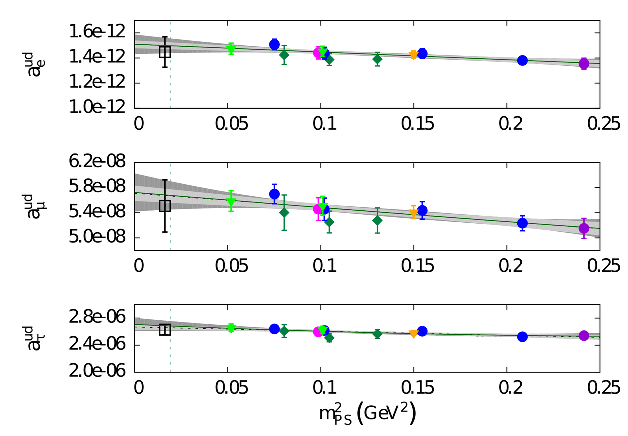
$$\Pi(Q^2) = \Pi(0) - Q^2\left(a_0 + \sum_{n=1}^{N}\frac{a_n}{b_n+Q^2}\right)$$

[N-1,N] Padé fit function:

$$\Pi(Q^2) = \Pi(0) - Q^2 \sum_{n=1}^{N}\frac{a_n}{b_n+Q^2}$$

constraints: $a_n > 0 \quad \forall n \in \{0,1,\ldots,N\}$
$b_n > 0 \quad \forall n \in \{1,2,\ldots,N\}$

Comparison of light-quark contributions



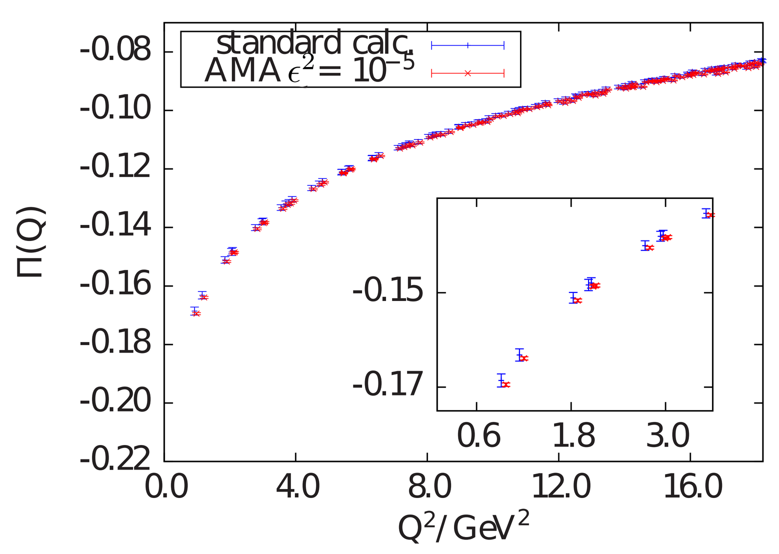
results at physical point obtained with standard definition and same analsis as in [1] using M1N3B4C1 fits due to higher statistics

agree with extrapolated values on two- and four-flavour ensembles

rho mass determined from correlated fit of vector meson correlator: $m_V = 810(40)\,\text{MeV}$ $(\chi^2/\text{dof} = 1.11)$

	physical point	extrapolated $N_f=2$	extrapolated $N_f=2+1+1$
a_e^{hvp}	$1.45(12)\cdot 10^{-12}$	$1.51(04)\cdot 10^{-12}$	$1.50(03)\cdot 10^{-12}$
a_μ^{hvp}	$5.51(42)\cdot 10^{-8}$	$5.72(16)\cdot 10^{-8}$	$5.67(11)\cdot 10^{-8}$
a_τ^{hvp}	$2.63(08)\cdot 10^{-6}$	$2.65(05)\cdot 10^{-6}$	$2.66(02)\cdot 10^{-6}$

All-mode-averaging



$\mathrm{N_f} = 2$ test ensemble

$16^3\times 32$, $m_{\text{PS}} \approx 450\,\text{MeV}$

all-mode-averaging as described in [7]
$\mathrm{N_G} = 16$, stopping condition $\epsilon = 3.16\cdot 10^{-3}$

All-mode-averaging provides an error reduction by a factor 4 on test ensemble.

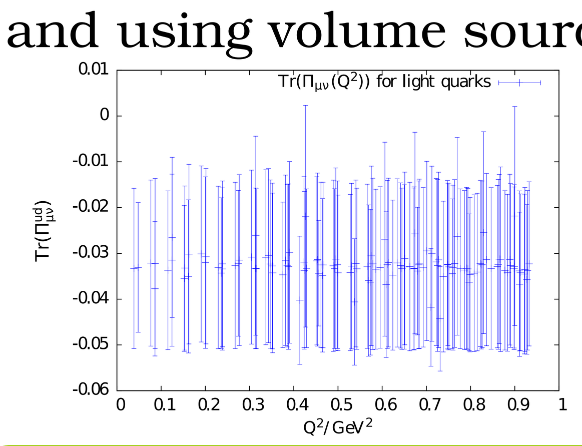
Tuning AMA parameters at physical point not yet completed

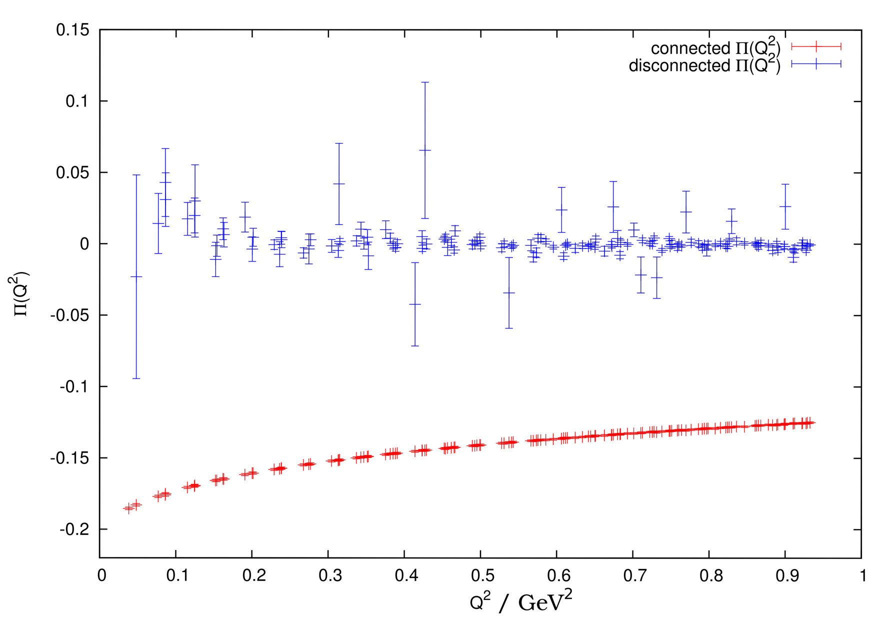
Disconnected contributions

obtained with point-split vector currents at source and sink

tried: estimating both loops stochastically with volume sources
computing source loop exactly and using volume sources only for the other one

due to large fluctuations no Q^2 dependence visible





Disconnected contributions smaller in magnitude than contributions from quark - connected diagrams.

$N_f = 2+1+1$

$a \approx 0.08\,\text{fm}$, $m_{\text{PS}} \approx 390\,\text{MeV}$,
strange & charm quark masses close to physical
200 configurations, 12 samples

next steps: local current correlator, time diluted sources, spin-colour dilution, all-mode-averaging

Conclusions

- anomalous magnetic moments of all three leptons compatible with phenomenological values
- no signal for disconnected contributions found so far
- computations at physical point confirm results from chiral extrapolations with improved definition
- different analysis strategies and all-mode-averaging at physical point under examination

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