Nucleon electromagnetic form factors from Twisted Mass QCD

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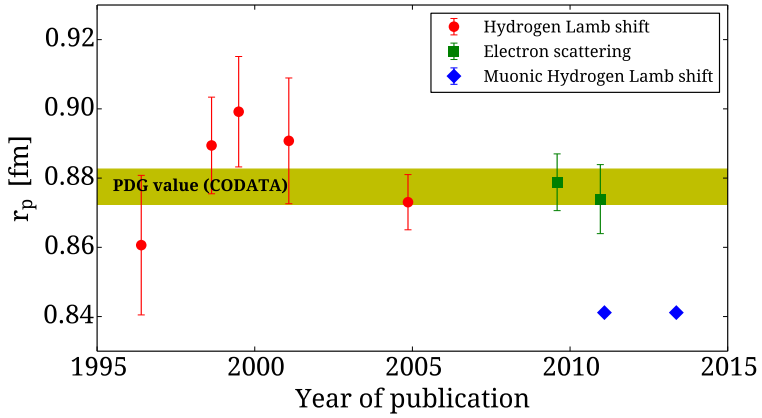
Outline

- **Motivation**
  - The proton radius and more
- **Lattice Setup**
  - Form factor extraction
  - Twisted mass ensembles used
  - Lattice spacing determination
  - Excited state effects
- **Results**
  - Electric, magnetic, Dirac and Pauli form factors
  - Associated radii
  - Comparison with other discretizations
- **Outlook**
  - Noise reduction techniques
- **Summary**

Motivation

- **Nucleon EM form-factors (FFs): insight on internal structure of the proton and neutron**
  - Slope of FFs at $Q^2 = 0$ defines radius
  - Contact to perturbative QCD at large Q^2
- **Timely, due to proton "radius puzzle"**



  - Discrepancy in experiments when comparing muonic hydrogen Lamb shift to hydrogen Lamb shift and electron scattering
  - $\sim$2% accuracy lattice measurement could give a QCD prediction of radius

Decomposition

- Dirac (F_1) and Pauli (F_2) FFs :

$$\langle N(p',s')|j^\mu|N(p,s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\bar{u}(p',s')\mathcal{O}^\mu u(p,s)$$

$$\mathcal{O}^\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N}F_2(q^2), \quad q = p' - p$$

- Alternatively, the Electric (G_E) and (G_M) Sachs form factors:

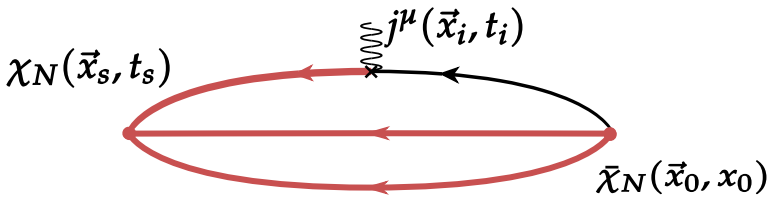
$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2}F_2(q^2), \quad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

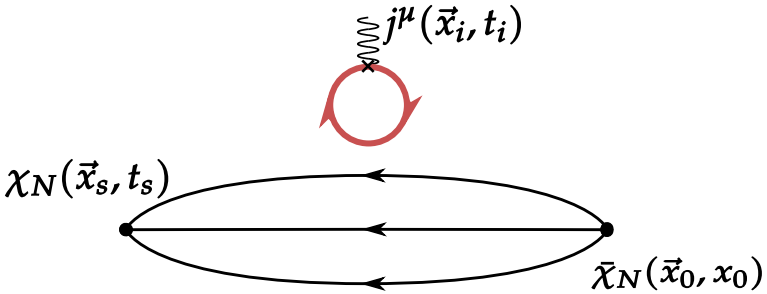
- Radii defined as slope at $Q^2 = 0$:

$$\langle r_i^2\rangle = -\frac{6}{F_i}\frac{dF_i}{dQ^2}|_{Q^2=0}$$

similarly for $\langle r_E^2\rangle$, $\langle r_M^2\rangle$

Lattice Setup





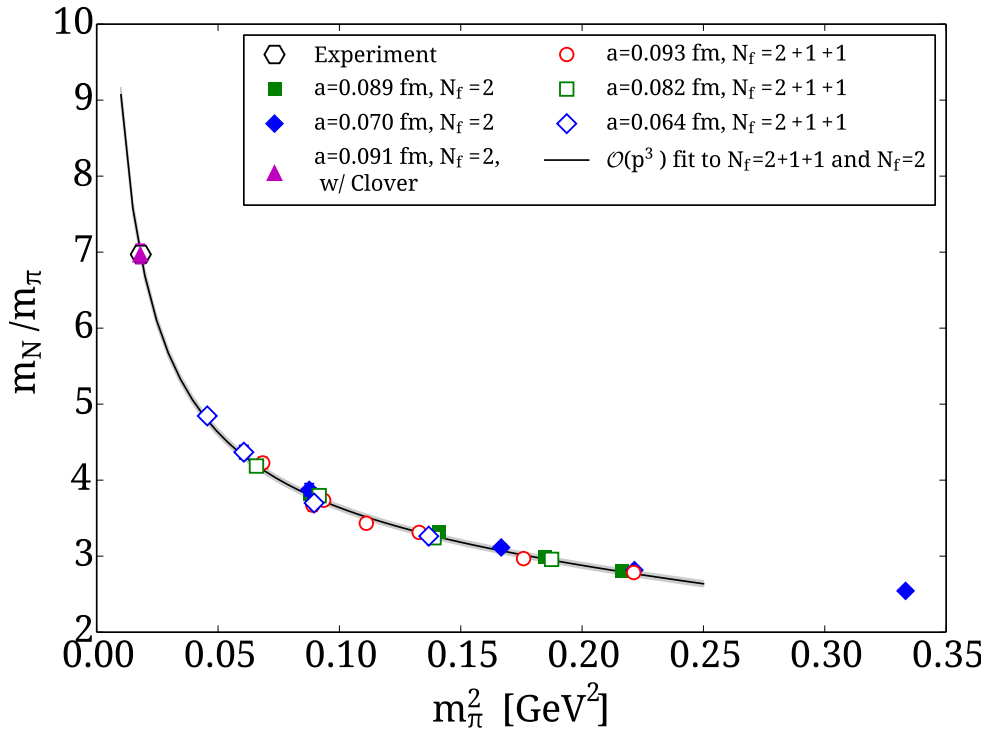
- The three point correlation function

$$G^\mu(\Gamma; \mathbf{q}; t_s, t_i) = \sum_{\mathbf{x}_s \mathbf{x}_i} e^{-i\mathbf{p}'\mathbf{x}_s} e^{-i(\mathbf{p}'-\mathbf{p})\mathbf{x}_i} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^\beta(\mathbf{x}_s; t_s) | j^\mu(\mathbf{x}_i; t_i) | \chi_N^\alpha(\mathbf{x}_0; t_0) \rangle$$

  obtained with a sequential inversion through the sink (x_s)
- Two sequential inversions:
  - Unpolarized $\Gamma_0 = \frac{1}{4}(1+\gamma_0) \to G_E$
  - Polarized $\Gamma = \sum_k i\Gamma_0\gamma_k \to G_M$
- Isovector and isoscalar combinations
  - $F^p - F^n = F^u - F^d$
  - $F^p + F^n = \frac{1}{3}(F^u + F^d)$

  assuming flavor SU(2) isospin symmetry, i.e. $p \leftrightarrow n$ when $u \leftrightarrow d$
- Fixed sink momentum $\mathbf{p}'$=0

Lattice Setup



$N_f = 2+1+1$

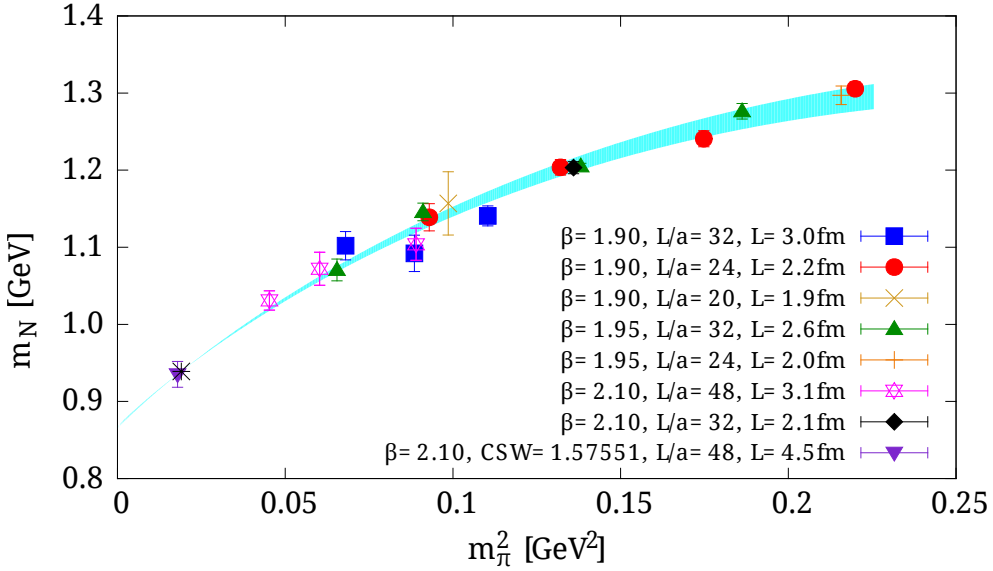
- $\beta = 1.95$, $m_\pi \simeq 375$ MeV
  - $a \simeq 0.082$ fm
  - $32^3 \times 64$
  - Multiple sink separations: $(t_s - t_0)/a$=4, 6, 8, 10, 12, 14, 16, 18
- $\beta = 2.1$, $m_\pi \simeq 210$ MeV
  - $a \simeq 0.065$ fm
  - $48^3 \times 96$
  - Single sink separation: $(t_s - t_0)/a$=18

$N_f = 2$

- $\beta = 2.1$, $c_{SW} = 1.57551$, $m_\pi \simeq 135$ MeV
  - $a \simeq 0.091$ fm
  - $48^3 \times 96$
  - Four sink separations: $(t_s - t_0)/a$=10, 12, 14, 16

Lattice Setup

Lattice spacing determined from nucleon mass



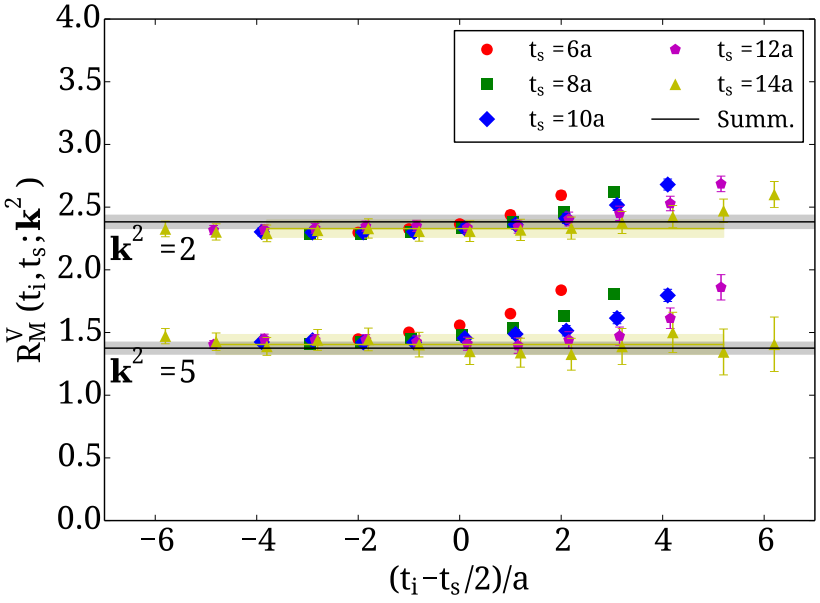
- $\mathcal{O}(p^3)$ form:

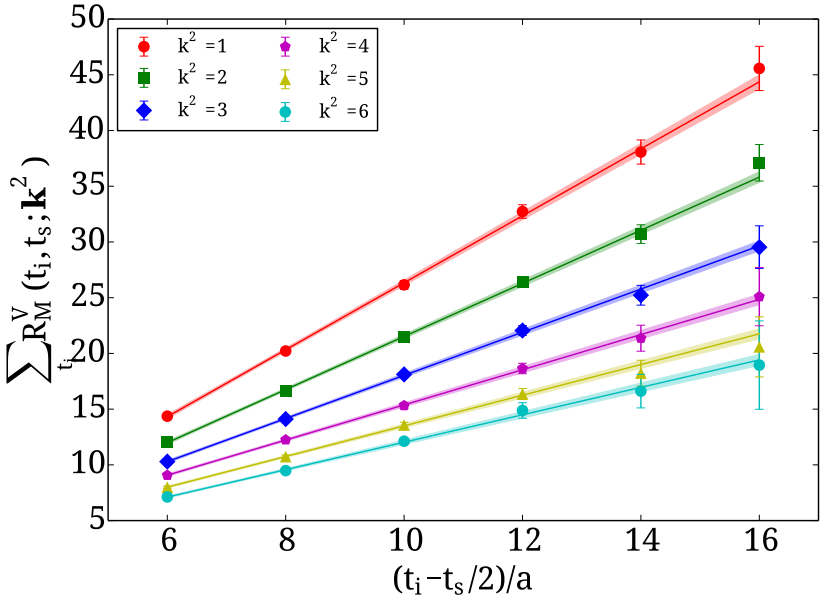
$$m_N = m_N^0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2} m_\pi^3$$

- Fit for: c_1, m_N^0 and spacings
- Demand m_N to reproduce m_N^{phys}
- Systematic error from:
  - Next order HBχPT
  - Pion mass range
  - Allow $O(m_\pi^3)$ coefficient to vary

- Spacings determined via the nucleon mass
  - $\beta = [1.90, 1.95, 2.10]$, $a = [0.0936(13)(32), 0.0823(11)(35), 0.0646(7)(25)]$ fm
  - $\beta = 2.10$, $c_{\text{SW}} = 1.57551$, $a = 0.091(2)(1)$ fm
  - $r_0 \simeq 0.479(4)$ fm at $a = 0$
  - σ-term from $\mathcal{O}(p^3)$: $\sigma_{\pi N} = 65(2)(20)$ MeV

Form factor extraction





- Plateau method

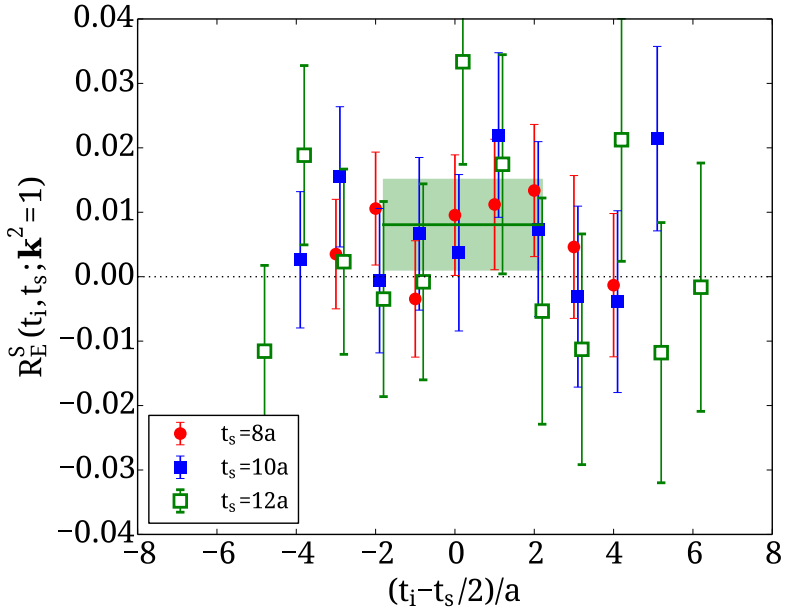
$$R_M^V(t_i, t_s; \mathbf{k}^2) \xrightarrow[t_i - t_9 \gg]{t_s - t_i \gg} G_M^V(\mathbf{k}^2)[1 + O(e^{-\Delta M(t_s - t_i)}, e^{-\Delta E(\mathbf{k})(t_i - t_0)})]$$

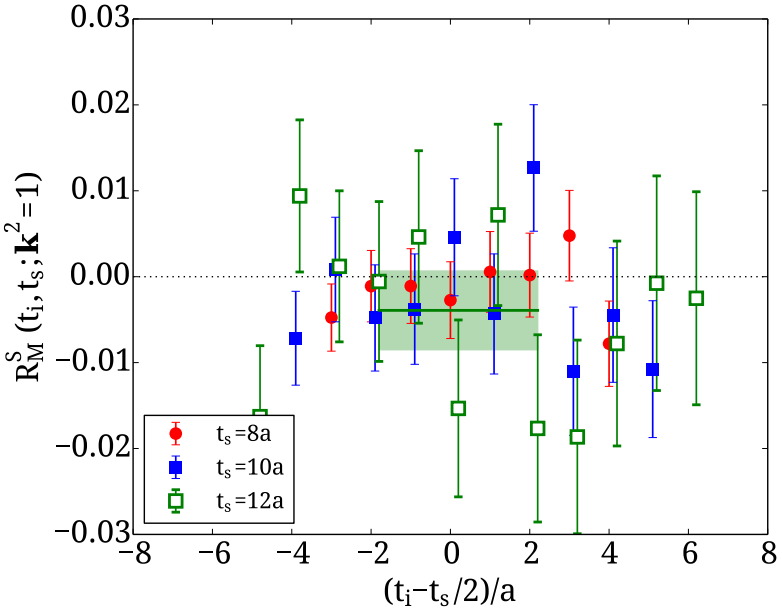
- Summation method

$$\sum_{t_i} R_M^V(t_i, t_s; \mathbf{k}^2) \xrightarrow{t_s \gg} C + G_M^V(\mathbf{k}^2) t_s [1 + O(e^{-\Delta M(t_s - t_0)}, e^{-\Delta E(\mathbf{k})(t_i - t_0)})]$$

Disconnected contributions to isoscalar

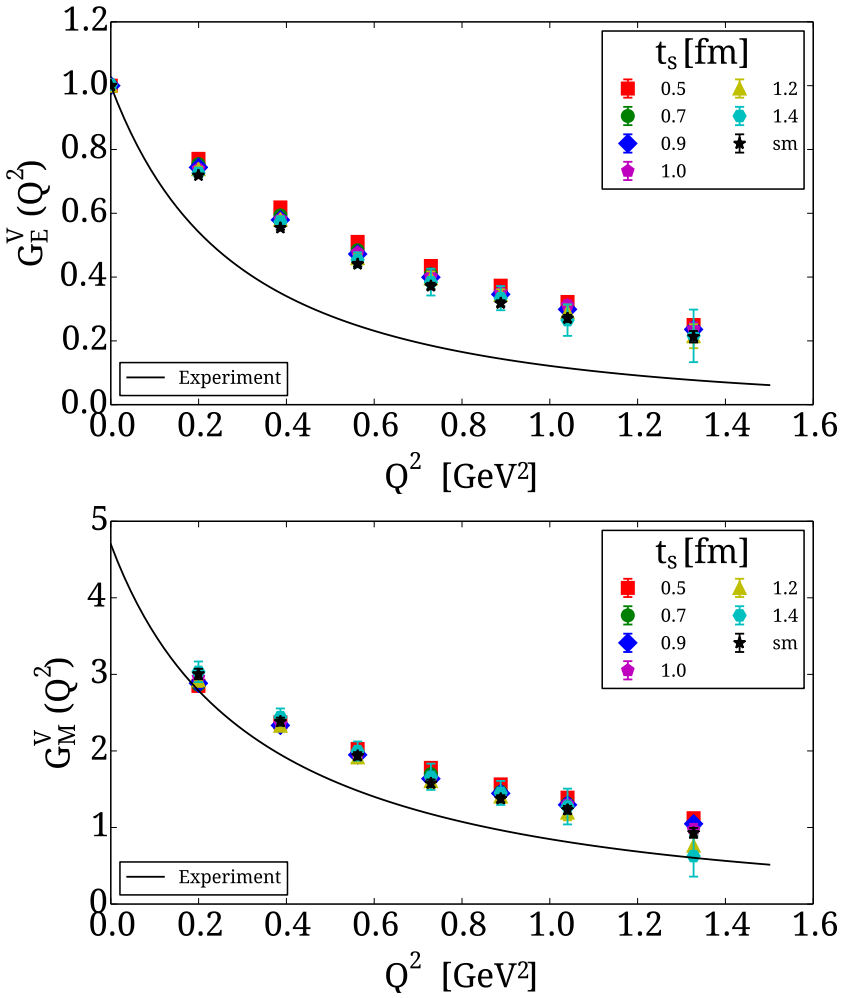
- $N_f = 2 + 1 + 1$, $a \simeq$0.085 fm, $m_\pi \simeq$375 MeV
- $\sim$150,000 statistics





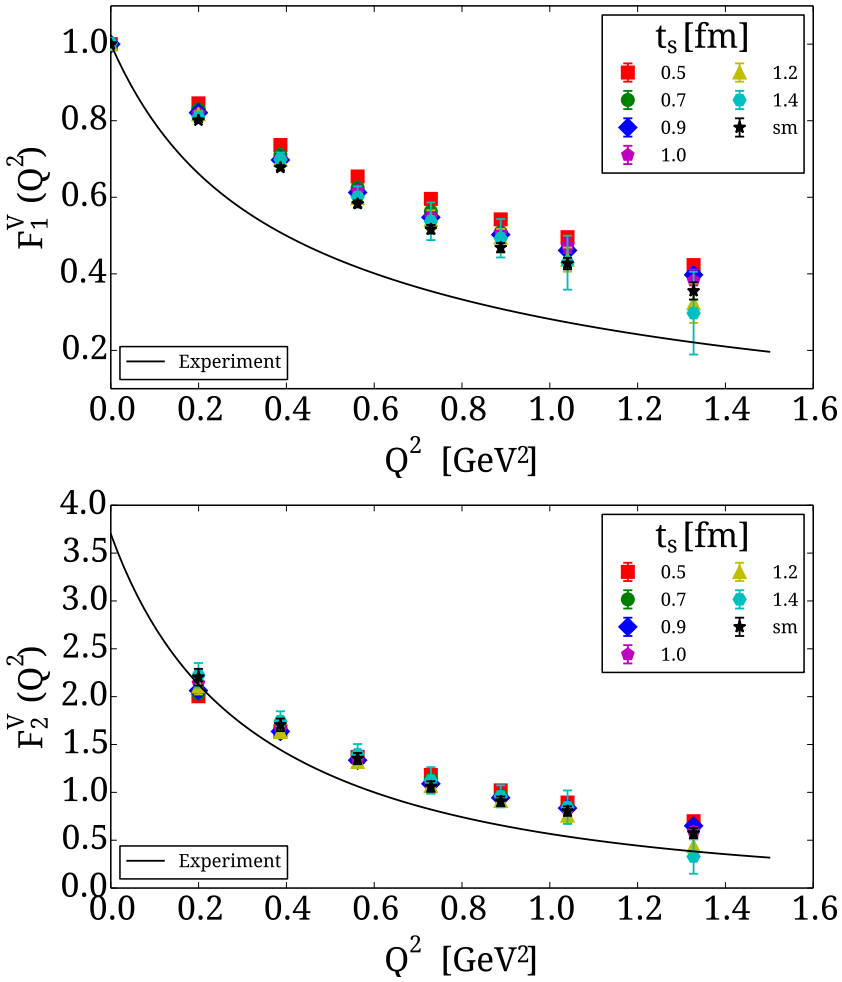
- Connected contribution $O(1) \Rightarrow$ disconnected bound to $\sim$1%
- Details by A. Vaquero

Dependence on source-sink separation



- $N_f = 2 + 1 + 1$, $a \simeq 0.085$ fm, $m_\pi \simeq 375$ MeV
- 1,200 statistics
- 10 source-sink separations (6 shown here)
- Mild dependence on source-sink separation
- Consistency between summation and $t_s - t_0 \geq 1.2$ fm (at least for this pion mass)

Dependence on source-sink separation



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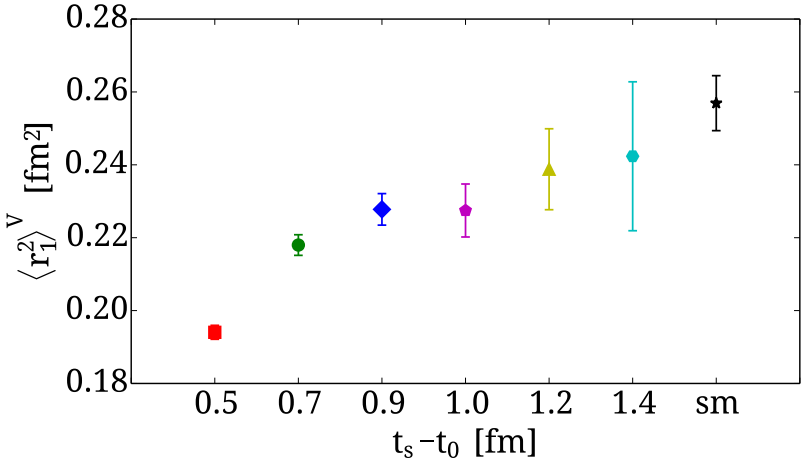
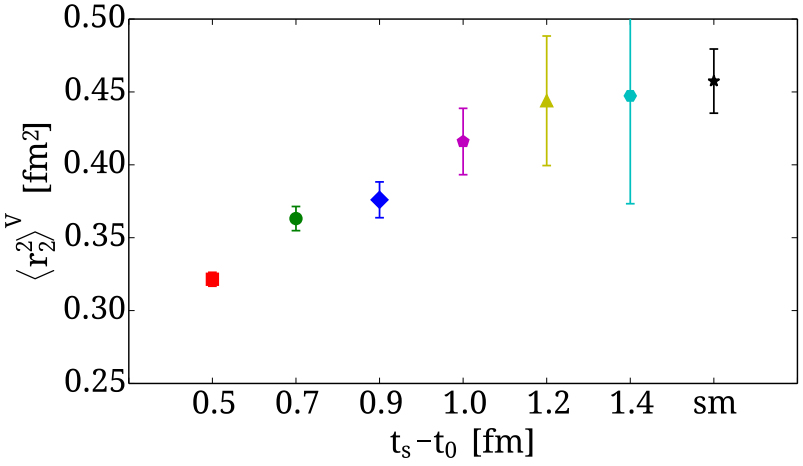
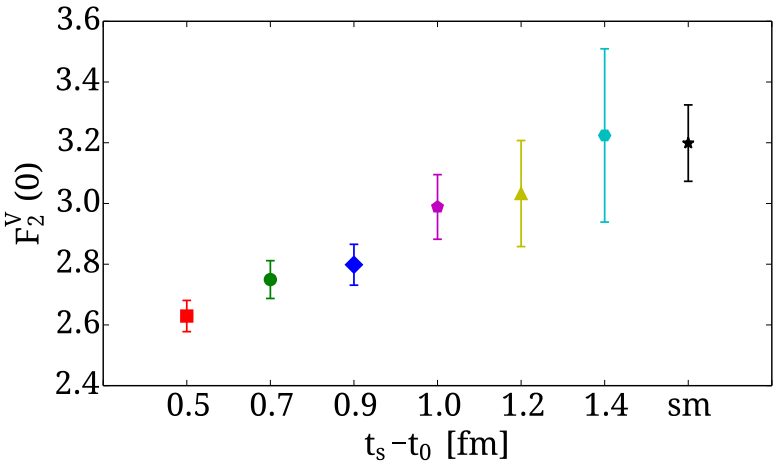






- $N_f = 2 + 1 + 1$, $a \simeq$0.085 fm, $m_\pi \simeq$375 MeV
- 1,200 statistics, 10 source-sink separations
- From dipole fit:

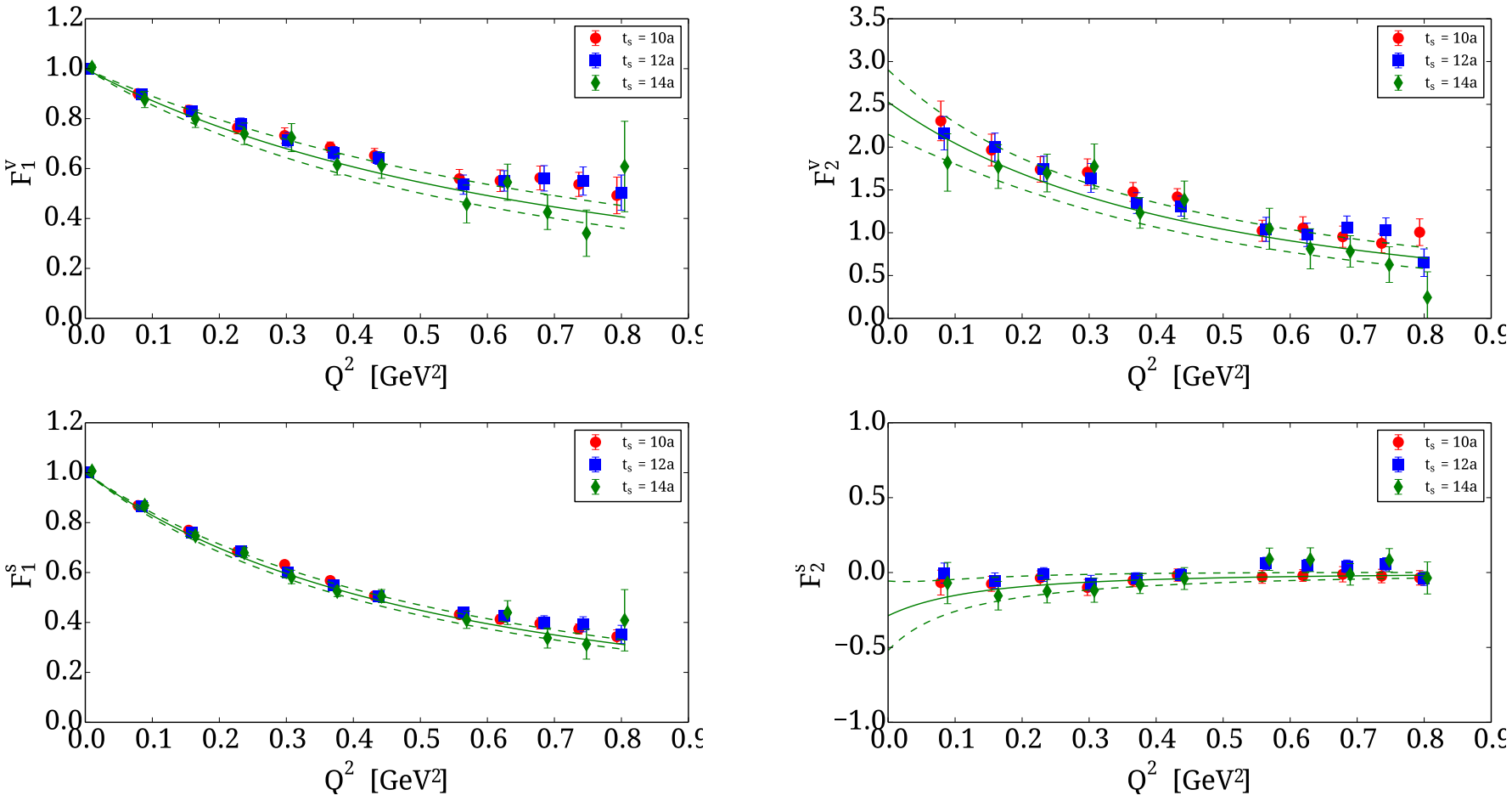
$$F_1(Q^2) = \frac{1}{(1 + Q^2/M_1^2)^2},$$

$$F_2(Q^2) = \frac{F_2(0)}{(1 + Q^2/M_2^2)^2}$$

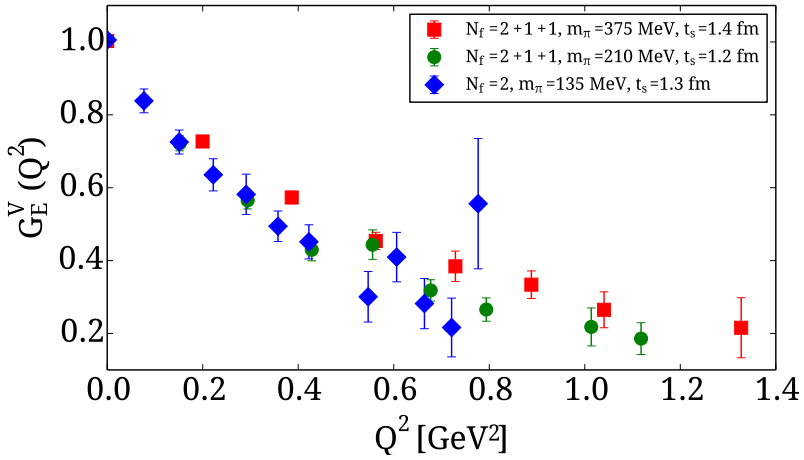
$$\langle r_i^2 \rangle = \frac{12}{M_i^2}$$

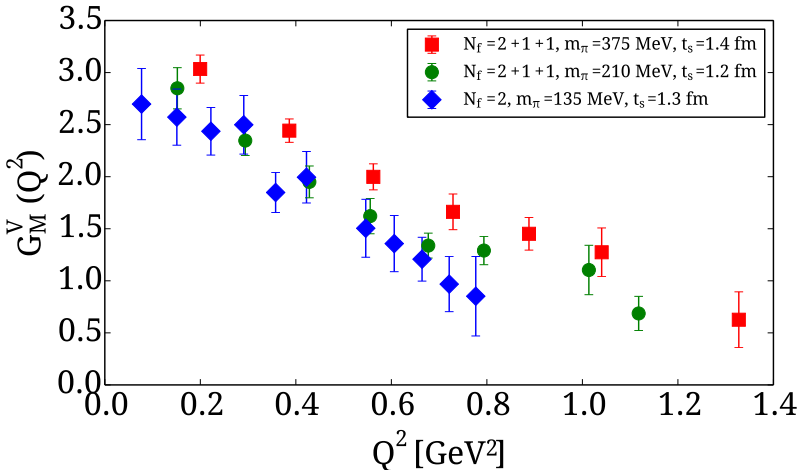
Results at the physical point

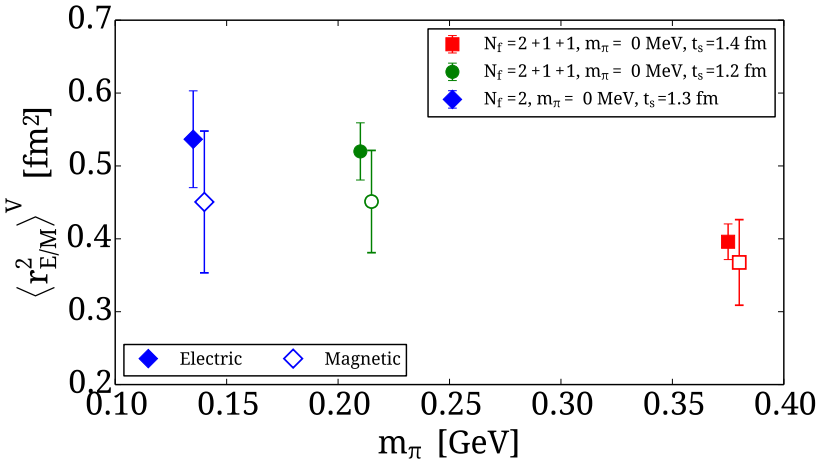
- $N_f = 2$, $a \simeq 0.091$ fm, $m_\pi \simeq 135$ MeV
- $\sim$1,000 statistics for $t_s - t_0 = 1.1$ and 1.3 fm, $\sim$300 for 0.9 fm



G_E and G_M from Twisted Mass

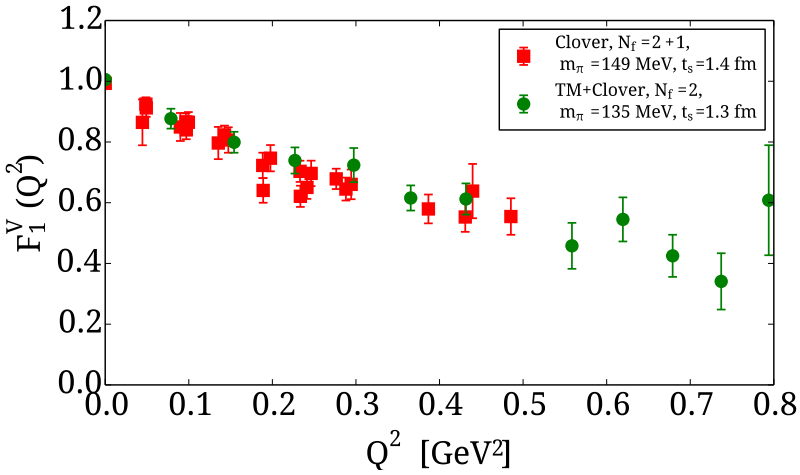


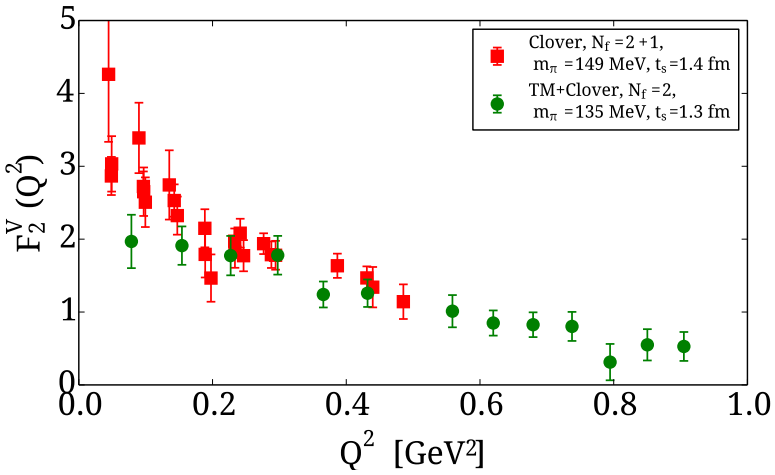


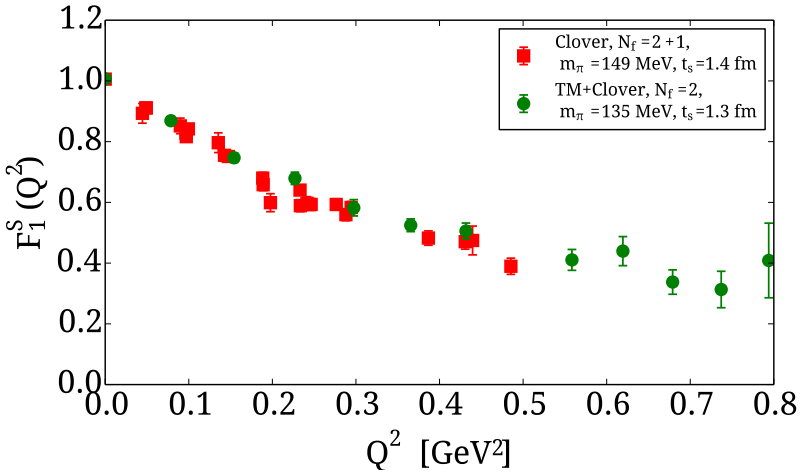


- $t_s - t_0 > 1.2$ fm
- Tendency for steeper G_E and G_M as $m_\pi \to 135$ MeV $\Rightarrow$ larger radii

Comparison with other formulations

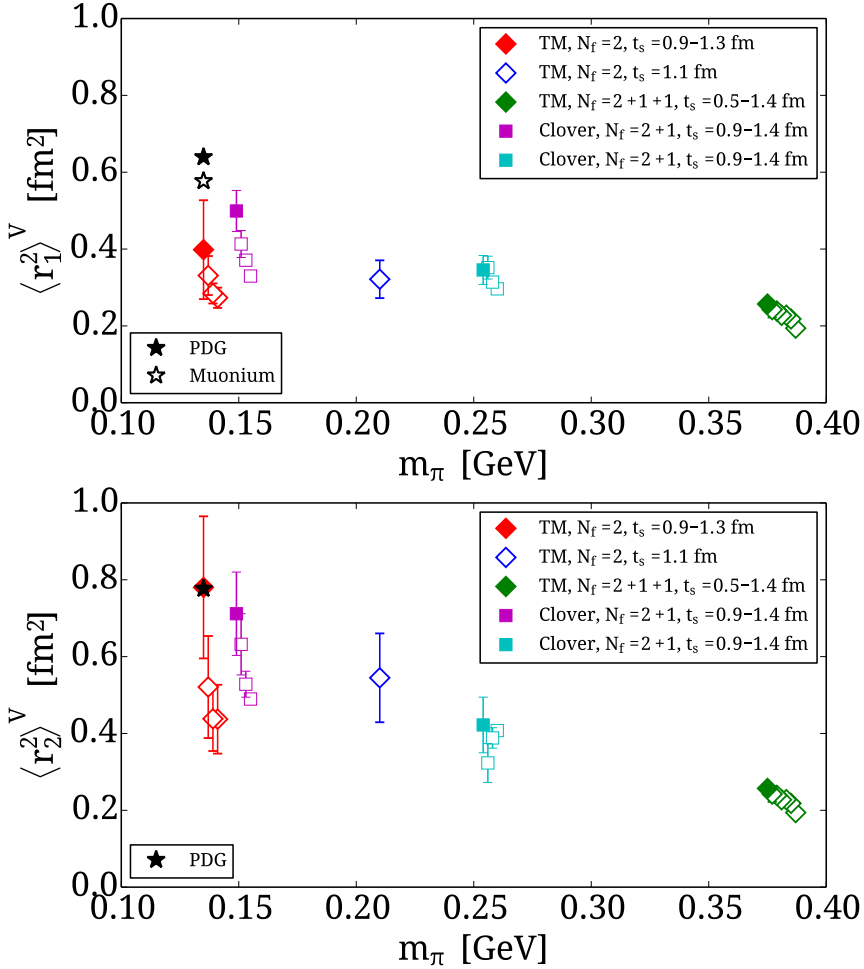






- LHPC `arXiv:1404.4029`
- Clover improved, $a = 0.116$ fm
- $m_\pi = 149$ MeV
- Consistency between the two discretizations

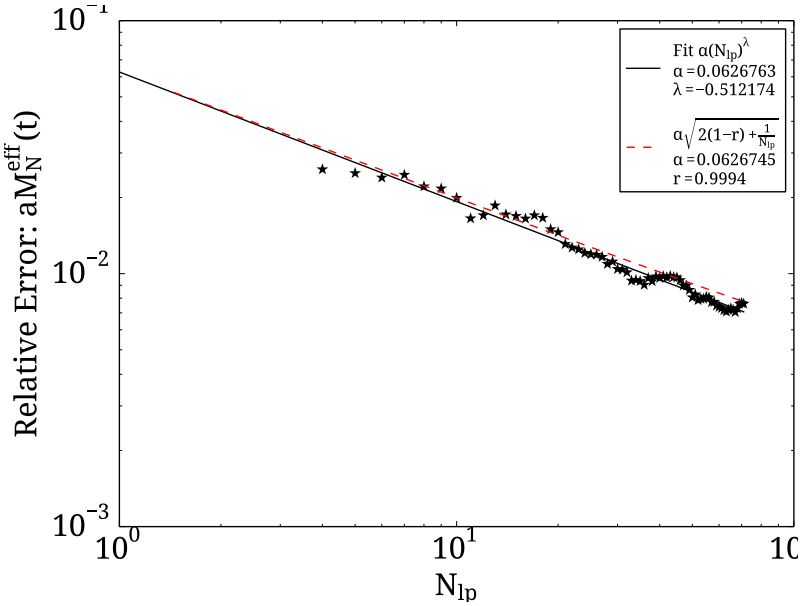
Comparison with other formulations

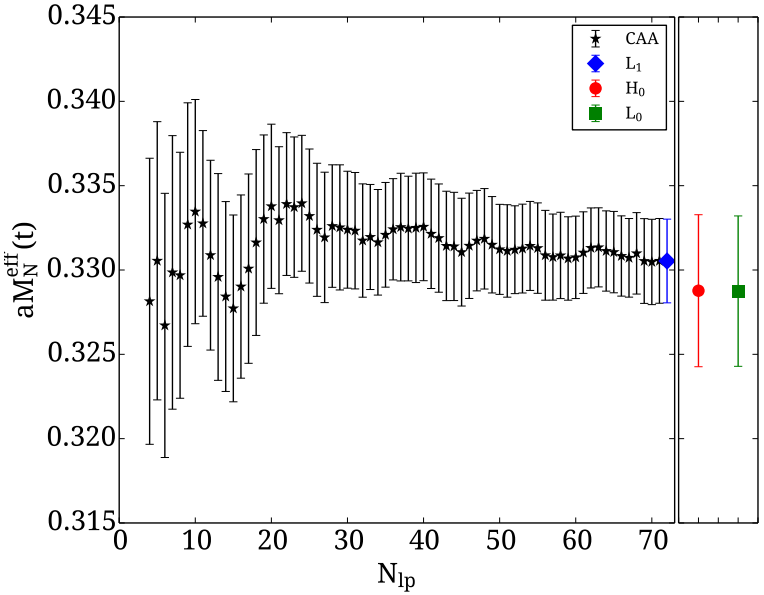


- Confirmed curvature towards physical pion mass
- Increasing trend for enlarging source-sink separation at near physical pion masses
- Need ∼1% error to contact experiment, or a multiple-fold increase in statistics.

Outlook

- Nucleon mass for $m_\pi \simeq 210$ MeV
- 250 configs. $\times$ 80 low-precision per config.
- With EigCG, low-precision $\sim$10$\times$ cheaper





- If $r \simeq 1$ scale as $\sqrt{2(1-r)+\frac{1}{N_{lp}}}$
- Reasonable scaling with low-precision vectors
- Low precision tuned for $r \simeq$0.99

Summary

- **Excited state effects**
  - EigCG or similar multipl-rhs methods allow multiple source-sink separations per configuration
  - Summation method useful for assessing excited state contamination
  - Consistency with plateau at $t_s \geq 1.3$ fm
- **Results now at the physical point**
  - Slope of form factors towards right direction
  - Broader nuclei towards physical point
  - Consistency between Twisted Mass and Clover at similar volumes and near physical pion mass
- **Towards precision form factors and radii**
  - Disconnected diagrams are now possible to bound, if not compute directly
  - As expected, physical point is especially noisy
  - Compare 375 MeV with 1,200 statistics with 135 MeV with 1,000 statistics
  - Still need multiple increases in statistics to compare with experiment
  - Methods for noise reduction being investigated, such as CAA with EigCG

Thank you!

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