Nucleon electromagnetic form factors from Twisted Mass QCD

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Outline

- Motivation
  - The proton radius and more

- Lattice Setup
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  - Twisted mass ensembles used
  - Lattice spacing determination
  - Excited state effects

- Results
  - Electric, magnetic, Dirac and Pauli form factors
  - Associated radii
  - Comparison with other discretizations

- Outlook
  - Noise reduction techniques

- Summary
Motivation

- **Nucleon EM form-factors (FFs): insight on internal structure of the proton and neutron**
  - Slope of FFs at $Q^2 = 0$ defines radius
  - Contact to perturbative QCD at large $Q^2$

- **Timely, due to proton ”radius puzzle”**
  - Discrepancy in experiments when comparing muonic hydrogen Lamb shift to hydrogen Lamb shift and electron scattering
  - $\sim 2\%$ accuracy lattice measurement could give a QCD prediction of radius
Decomposition

— Dirac \((F_1)\) and Pauli \((F_2)\) FFs:

\[
\langle N(p', s')|j^\mu|N(p, s)\rangle = \sqrt{\frac{M_N^2}{E_N(p')E_N(p)}} \bar{u}(p', s') \mathcal{O}^\mu u(p, s)
\]

\[
\mathcal{O}^\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N} F_2(q^2), \quad q = p' - p
\]

— Alternatively, the Electric \((G_E)\) and \((G_M)\) Sachs form factors:

\[
G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2} F_2(q^2), \quad G_M(q^2) = F_1(q^2) + F_2(q^2)
\]

— Radii defined as slope at \(Q^2 = 0\):

\[
\langle r_i^2 \rangle = -\frac{6}{F_i} \frac{dF_i}{dQ^2} \bigg|_{Q^2=0}
\]

similarly for \(\langle r_E^2 \rangle, \langle r_M^2 \rangle\)
**Lattice Setup**

The three point correlation function

\[
G^\mu(\Gamma; q; t_s, t_i) = \sum_{x_s x_i} e^{-i p' x_s} e^{-i (p' - p) x_i} \Gamma^{\alpha \beta} \langle \bar{\chi}_N(x_s; t_s) | j^\mu(x_i; t_i) | \chi_N(x_0; t_0) \rangle
\]

obtained with a sequential inversion through the sink \((x_s)\)

- Two sequential inversions:
  - Unpolarized \(\Gamma_0 = \frac{1}{4} (1 + \gamma_0) \rightarrow G_E\)
  - Polarized \(\Gamma = \sum_k i \Gamma_0 \gamma_k \rightarrow G_M\)

- Isovector and isoscalar combinations
  - \(F^p - F^n = F^u - F^d\)
  - \(F^p + F^n = \frac{1}{3} (F^u + F^d)\)
  - Assuming flavor SU(2) isospin symmetry, i.e. \(p \leftrightarrow n\) when \(u \leftrightarrow d\)

- Fixed sink momentum \(p' = 0\)
Lattice Setup

\[ N_f = 2 + 1 + 1 \]

- \( \beta = 1.95, \ m_\pi \simeq 375 \ \text{MeV} \)
  - \( a \simeq 0.082 \ \text{fm} \)
  - \( 32^3 \times 64 \)
  - Multiple sink separations:
    \( (t_s - t_0)/a = 4, 6, 8, 10, 12, 14, 16, 18 \)

- \( \beta = 2.1, \ m_\pi \simeq 210 \ \text{MeV} \)
  - \( a \simeq 0.065 \ \text{fm} \)
  - \( 48^3 \times 96 \)
  - Single sink separation:
    \( (t_s - t_0)/a = 18 \)

\[ N_f = 2 \]

- \( \beta = 2.1, \ c_{SW} = 1.57551, \ m_\pi \simeq 135 \ \text{MeV} \)
  - \( a \simeq 0.091 \ \text{fm} \)
  - \( 48^3 \times 96 \)
  - Four sink separations:
    \( (t_s - t_0)/a = 10, 12, 14, 16 \)
Lattice Setup

Lattice spacing determined from nucleon mass

\[ m_N = m_N^0 - 4c_1 m^2_\pi - \frac{3g_A^2}{16\pi f_\pi^2} m_\pi^3 \]

- \( O(p^3) \) form:

- Fit for: \( c_1, m_N^0 \) and spacings

- Demand \( m_N \) to reproduce \( m_N^{\text{phys}} \)

- Systematic error from:
  - Next order HB\( \chi \)PT
  - Pion mass range
  - Allow \( O(m_\pi^3) \) coefficient to vary

- Spacings determined via the nucleon mass
  - \( \beta = [1.90, 1.95, 2.10], a = [0.0936(13)(32), 0.0823(11)(35), 0.0646(7)(25)] \) fm
  - \( \beta = 2.10, c_{SW} = 1.57551, a = 0.091(2)(1) \) fm
  - \( r_0 \approx 0.479(4) \) fm at \( a = 0 \)
  - \( \sigma \)-term from \( O(p^3) \): \( \sigma_{\pi N} = 65(2)(20) \) MeV
Form factor extraction

- Plateau method

\[ R_M^V(t_i, t_s; k^2) \xrightarrow{t_s \rightarrow t_i \gg} G_M^V(k^2) \left[ 1 + O(e^{-\Delta M(t_s-t_i)}, e^{-\Delta E(k)(t_i-t_0)}) \right] \]

- Summation method

\[ \sum_{t_i} R_M^V(t_i, t_s; k^2) \xrightarrow{t_s \gg} C + G_M^V(k^2)t_s \left[ 1 + O(e^{-\Delta M(t_s-t_0)}, e^{-\Delta E(k)(t_i-t_0)}) \right] \]
Disconnected contributions to isoscalar

- \( N_f = 2 + 1 + 1, \ a \simeq 0.085 \text{ fm}, \ m_{\pi} \simeq 375 \text{ MeV} \)
- \( \sim 150,000 \) statistics

- Connected contribution \( O(1) \Rightarrow \) disconnected bound to \( \sim 1\% \)
- Details by A. Vaquero
Dependence on source-sink separation

- \( N_f = 2 + 1 + 1, \ a \simeq 0.085 \ \text{fm}, \ m_\pi \simeq 375 \ \text{MeV} \)
- 1,200 statistics
- 10 source-sink separations (6 shown here)
- Mild dependence on source-sink separation
- Consistency between summation and \( t_s - t_0 \geq 1.2 \ \text{fm} \) (at least for this pion mass)
Dependence on source-sink separation

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Dependence on source-sink separation

\[ \langle r_1^2 \rangle \]

\[ \langle r_2^2 \rangle \]

\[ F_{1}(Q^2) = \frac{1}{(1 + Q^2/M_1^2)^2}, \]

\[ F_{2}(Q^2) = \frac{F_2(0)}{(1 + Q^2/M_2^2)^2} \]

\[ \langle r_i^2 \rangle = \frac{12}{M_i^2} \]

- \( N_f = 2 + 1 + 1, \ a \simeq 0.085 \text{ fm}, \)
- \( m_\pi \simeq 375 \text{ MeV} \)
- 1,200 statistics, 10 source-sink separations
- From dipole fit:
Results at the physical point

- $N_f = 2$, $a \approx 0.091$ fm, $m_\pi \approx 135$ MeV
- $\sim 1,000$ statistics for $t_s - t_0 = 1.1$ and 1.3 fm, $\sim 300$ for 0.9 fm
\(G_E\) and \(G_M\) from Twisted Mass

- \(t_s - t_0 > 1.2\) fm
- Tendency for steeper \(G_E\) and \(G_M\) as \(m_\pi \to 135\) MeV \(\Rightarrow\) larger radii
Comparison with other formulations

- LHPC arXiv:1404.4029
- Clover improved, $a = 0.116$ fm
- $m_\pi = 149$ MeV
- Consistency between the two discretizations
Comparison with other formulations

- Confirmed curvature towards physical pion mass
- Increasing trend for enlarging source-sink separation at near physical pion masses
- Need $\sim 1\%$ error to contact experiment, or a multiple-fold increase in statistics.
Outlook

- Nucleon mass for $m_\pi \sim 210$ MeV
- 250 configs. $\times$ 80 low-precision per config.
- With EigCG, low-precision $\sim 10 \times$ cheaper

$\Gamma t \approx (N_{\text{lp}})^{\frac{3}{4}}$

relative error: $a M^\text{eff}_N(t)$

Fit ($N_{\text{lp}}$)

$= 0.0626745$

$r = 0.9994$

If $r \approx 1$ scale as $\sqrt{2(1 - r)} + \frac{1}{N_{\text{lp}}}$

Reasonable scaling with low-precision vectors

Low precision tuned for $r \approx 0.99$
Summary

■ **Excited state effects**
  — EigCG or similar multipl-rhs methods allow multiple source-sink separations per configuration
  — Summation method useful for assessing excited state contamination
  — Consistency with plateau at \( t_s \geq 1.3 \) fm

■ **Results now at the physical point**
  — Slope of form factors towards right direction
  — Broader nuclei towards physical point
  — Consistency between Twisted Mass and Clover at similar volumes and near physical pion mass

■ **Towards precision form factors and radii**
  — Disconnected diagrams are now possible to bound, if not compute directly
  — As expected, physical point is especially noisy
  — Compare 375 MeV with 1,200 statistics with 135 MeV with 1,000 statistics
  — Still need multiple increases in statistics to compare with experiment
  — Methods for noise reduction being investigated, such as CAA with EigCG
Thank you!

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