Fluctuations of the electric charge in theory and experiment

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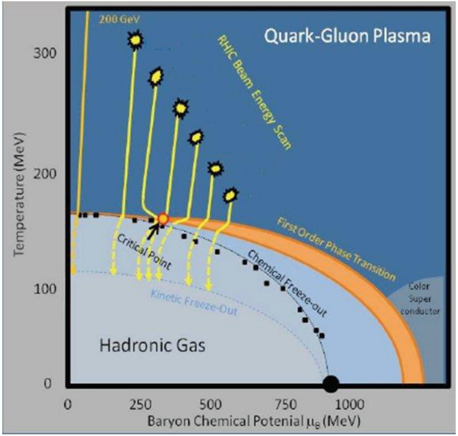
Beam energy scan and freeze-out curve



Chiral crossover region from lattice:
$T_c = 147 \ldots 157$

[BW hep-lat/0611014,hep-lat/0609068,0903.4155,1005.3508]

Curvature: $\kappa = 0.0066(2)(4)$ [WB: 1102.1356]

At RHIC a broad energy range $\sqrt{s_{\mathrm{NN}}} = 7.7 \ldots 200$ has been scanned with heavy ion collisions.
Last inelastic scattering:
chemical freeze-out.
For each energy the chemical freeze-out is described as a grand canonical ensemble with one temperature and chemical potential.
Traditional method:
Hadron Resonance Gas (HRG)-based statistical fit of pion, kaon, proton, etc yields.
Fit result at $\sqrt{s_{\mathrm{NN}}} = 130\mathrm{GeV}$
$\mu_B = 38(12)$ MeV and
$T_{ch} = 165(5)$ MeV.

[Andronic et al nucl-th/0511071]

The idea

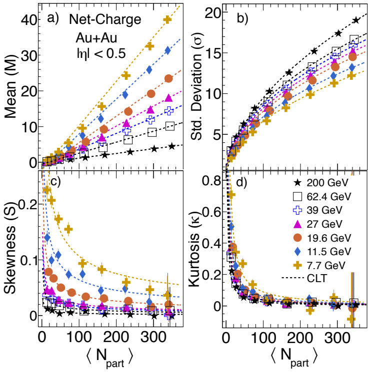
Let's not look at yields but things that exist on a lattice: conserved charges. Lattice calculates the grand canonical ensemble for a given charge (baryon number, electric charge or strangeness) and this is matched to the event-by-event statistics from the experiment

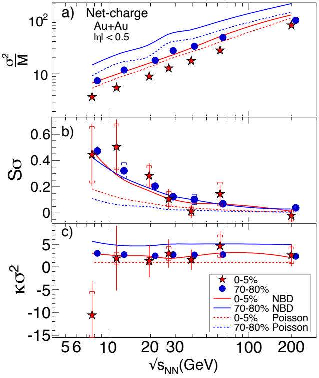
Net proton: number of protons - number of antiprotons
Net electric charge: number of positive - negative particles

Mean: $\langle N_X \rangle = -T \frac{\partial \log Z}{\partial \mu_X}$ Variance: $\langle \delta N_X^2 \rangle = -T^2 \frac{\partial^2 \log Z}{\partial {\mu_X}^2}$
On the lattice we have access to normalized quark number susceptibilities:

$$\chi_2^X = \frac{1}{VT^3} \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$$

At RHIC **STAR** has measured the mean, variance, skewness and kurtosis of the event-by-event **net charge** distribution at various energies and centralities.





[STAR: 1402.1558]

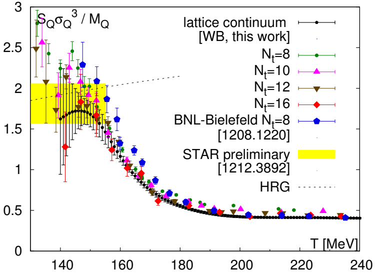
Thermometer from the skewness

A possible thermometer [BNL-Bielefeld 1208.1220] T_{ch} is found through

$$S\sigma^3/M\big|_{\text{experiment}}(\text{beam energy}) = S\sigma^3/M\big|_{\text{lattice}}(T_{ch})$$

Comparing Wuppertal-Budapest lattice results with STAR data:

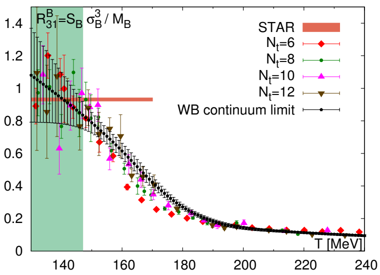
Net electric charge:



[Wuppertal-Budapest 1304.5161], [STAR 1402.1558].

Conclusion $T_{ch} \le 157\text{MeV}$

Net baryon number:

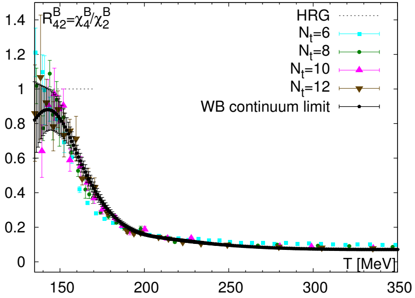


[Wuppertal-Budapest 1403.4576], [STAR 1309.5681] (protons).

Conclusion $T_{ch} \le 148\text{MeV}$

At LHC energies $\mu_B \approx 0$, we have to find two parameters only: temperature and volume, the latter cancels in ratios of cumulants. At the same time skewness and mean are both zero, the skewness thermometer hit a 0/0 limit.
What could be a good thermometer?

- Baryon fluctuations [shown here]: noisy, non-ideal T-dependence, in experiment protons, not baryons are measured.
  STAR at 200 GeV [1309.5681]: $\chi_4^B/\chi_2^B = 0.897(29)(20)$
- Electric charge fluctuations: large cut-off effects in the staggered formulation



[Wuppertal-Budapest 1304.5161]

Our goal: the electric charge kurtosis

We calculate the experimentally relevant χ_4^Q/χ_2^Q ratio as a function of tempearture with physical quark masses in the continuum.

2nd generation staggered program:

- `4stout` staggered action (taste breaking similar to HISQ), tree-level Symanzik gauge action, smeared one-link fermions
- 2+1+1 dynamical flavors, also used e.g. for charmed equation of state [S. Krieg, Thu]
- Bare masses tuned to M_π/f_π, M_K/f_π, scale setting: f_π
- Charm mass set to $m_c/m_s = 11.85$ [HPQCD: 0910.3102]

Electric charge is mostly carried by pions: taste breaking must be brought under control.

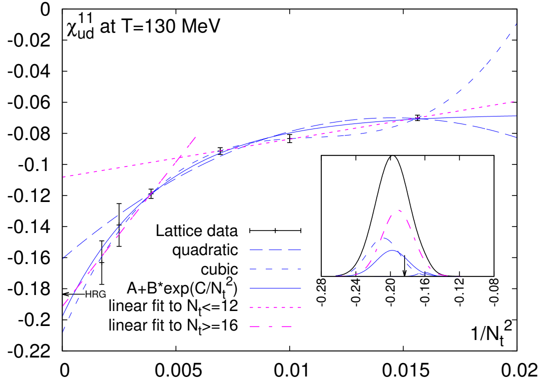
Finite temperature ensembles

T [MeV]	$24^3 \times 6$	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$	$64^3 \times 16$	$80^3 \times 20$	$64^3 \times 24$
125	-	10515	10080	10008	5027	2060	1024
130	-	5766	5326	10253	5099	600	617
135	-	14762	10590	10060	10189	2000	1108
140	6477	14863	5381	15043	4959	5097	1015
145	6292	5784	5020	10014	5019	700	-
150	3514	5464	5067	11043	5064	1000	1135
155	2668	5613	5001	4000	5015	999	-
157	4775	5526	5409	10018	5160	1065	-
160	5270	5247	5017	4973	5073	1082	1311
165	5429	8169	10086	10496	5000	1000	-
170	7313	6005	6113	5600	5111	600	1195
175	26197	12018	5375	5058	5104	972	-
180	6024	5007	5089	5034	5013	1000	1079
190	10156	4900	5031	5121	5045	992	-
200	9666	5989	5002	6722	1012	1000	1069
220	12036	5514	5000	7231	1003	1000	347

Number of analyzed configurations, separated by ten RHMC trajectories.
We used 4×128 random sources for the kurtosis analysis.

Extrapolating up-down correlator

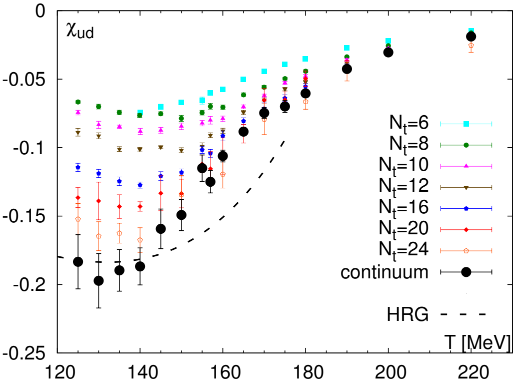
This correlator is driven by pions in the confined phase and is extremely sensitive to taste breaking.



Systematic errors are calculated from the spread of various fit models:
Histogram method *[BMW Science **322** 1224]*
Weight: using the Akaike Information Criterion (AIC)

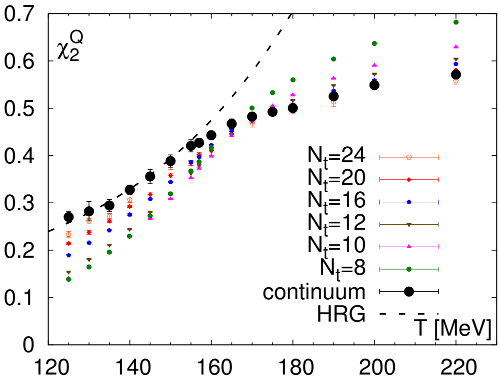
The up-down correlator

This correlator is driven by pions in the confined phase and is extremely sensitive to taste breaking. Below T_c: agreement with HRG



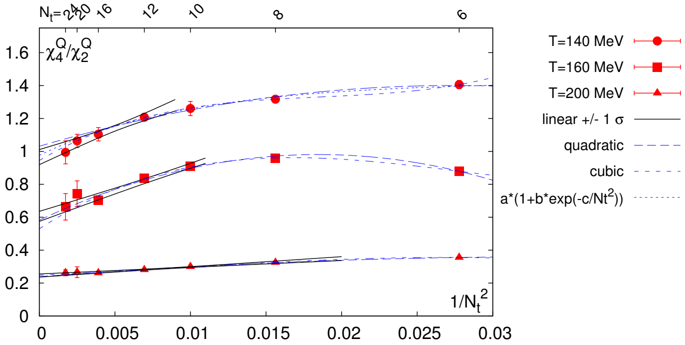
The errors are combined (statistical+systematic) using the histogram method with AIC weights.

We update the continuum limit of the charge susceptibility:
agreement with HRG shown to high precision



For earlier calculations see [Wuppertal-Budapest: 1112.4416, HotQCD: 1203.0784]

Continuum extrapolation of the kurtosis



Linear extrapolation on fine lattices or non-linear fit to all lattices give consistent results.

Systematic errors from the histogram metod [BMW Science (2008) **322**, 1224]

$$\mathrm{Data}(T; N_t) = X(T) + Y(T)/N_t^2$$

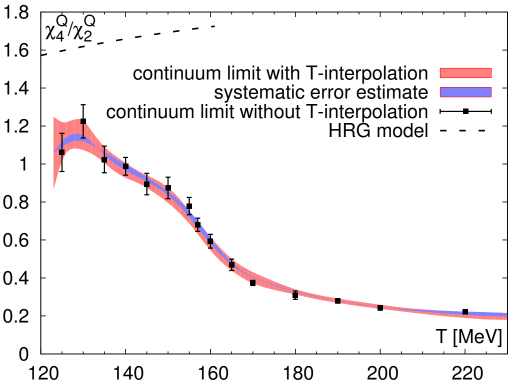
Assuming the smoothness of $X(T)$ *and* $Y(T)$ can significantly improve the errors, and correlation is introduced. The result will depend on the set of node points. [G. Endrődi 1010.2952]

Method to include the systematics:

- Random node point set: place a node point with 0.5 probability between each subsequent pair of data points, its location is evenly distributed in that interval.
- Use the AIC weight as likelihood $\exp[-(\chi^2 + 2N_{\mathrm{parameter}})/2]$ with $N_{\mathrm{parameter}} = 2N_{\mathrm{nodepoint}}$
- Do a Markov-chain with an AIC-based accept/reject step to automatically find several optimal number and position of node points.
- Use the fluctuation of these splines in the final error

Continuum curve for χ_4^Q/χ_2^Q

The continuum limit is away from the HRG result.



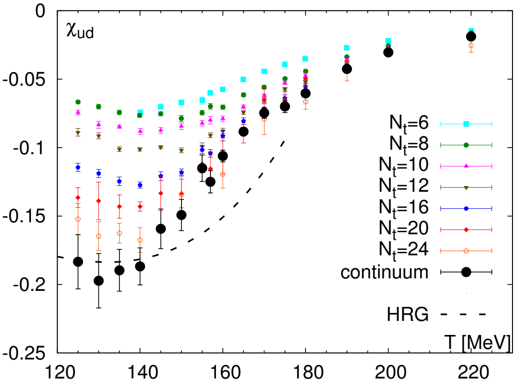
Here three methods for the continuum extrapolation are compared:

a) Temperature-by-temperature linear fitting though the finer lattices (points)

b) Spline extrapolation $X(T) + Y(T)/N_t^2$ using $N_t \geq 12$ (red band)

c) Spline using $N_t \geq 10$ (blue band shows the deviation from b))

This correlator is driven by pions in the confined phase and is extremely sensitive to taste breaking. Below T_c: agreement with HRG



The errors are combined (statistical+systematic) using the histogram method with AIC weights.

If we know that the hadron resonance gas model gives a proper discription of $\chi_{ud}(T)$ for $T < T_c$, then we can quantify lattice artefacts as an effective pion mass:

At fixed T^* (e.g. 125 MeV):

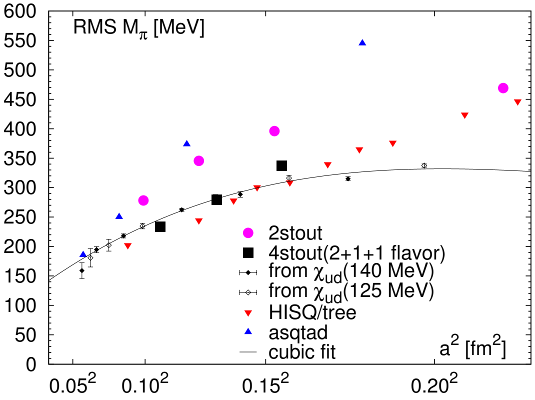
Which pion mass shall we put into HRG so that we get the same χ_{ud} as the lattice finds for a given lattice spacing

$$\chi_{ud}^{\mathrm{LATTICE}}(T^*)\big|_{N_t} \stackrel{!}{=} \chi_{ud}^{\mathrm{HRG}}(T^*)\big|_{M_{\mathrm{eff}}}$$

M_{eff} then corresponds to $a = 1/N_t T^*$

Useful is this definition if $M_{\mathrm{eff}}(a)$ is not (strongly) T^* dependent.
This is an a finite temperature estimate for the pion splitting.

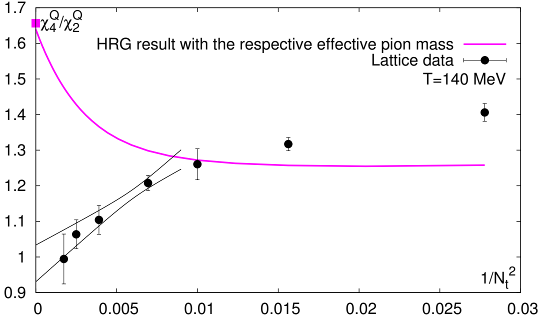
The finite temperature pion splitting estimate is in the ballpark of the RMS pion mass (Root mean square of all 16 pion levels).



The mass estimates with $T^* = 125$ or 140 MeV are fairly consistent.

Are the lattices fine enough for χ_4^Q/χ_2^Q?

What does HRG give for χ_4^Q/χ_2^Q using the effective pion masses?

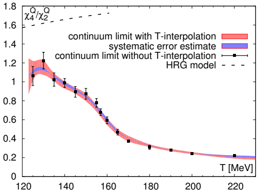


The continuum trend is the opposite of what we might expect from HRG.

Summary

We calculated the electric charge fluctuations on very fine staggered lattices: $N_t = 6, 8, 10, 12, 16, 20$ and 24.

- Strong enough T-dependence, good thermometer for heavy ion applications at LHC
- Around the expected freeze-out temperature kurtosis data are inconsistent with the HRG estimate



We demonstrated that the taste breaking can be kept under control on the example of the up-down correlator.

We characterized the staggered lattice artefacts by an effective mass based on mathing HRG to finte-temperature results.

What is the physical reason for the failure of the hadron resonance gas model here?

Backup slides

Translating from quark numbers to B,Q and S

In terms of physical derivatives

$$
\begin{aligned}
\frac{d}{d\mu_B} &= \frac{1}{3}\partial_u + \frac{1}{3}\partial_d + \frac{1}{3}\partial_s\,, \\
\frac{d}{d\mu_Q} &= \frac{2}{3}\partial_u - \frac{1}{3}\partial_d - \frac{1}{3}\partial_s\,, \\
\frac{d}{d\mu_S} &= -\partial_s
\end{aligned}
$$

we have

$$
\begin{aligned}
\chi_2^B &= \frac{1}{VT}\frac{1}{9}\left[2\partial_u^2 + \partial_s^2 + 4\partial_u\partial_s + 2\partial_u\partial_d\right]\log Z\,, \\
\chi_2^Q &= \frac{1}{VT}\frac{1}{9}\left[5\partial_u^2 + \partial_s^2 - 2\partial_u\partial_s - 4\partial_u\partial_d\right]\log Z\,, \\
\chi_2^I &= \frac{1}{VT}\frac{1}{2}\left[\partial_u^2 - \partial_u\partial_d\right]\log Z\,.
\end{aligned}
$$

Translating from quark numbers to B,Q and S

$$
\begin{aligned}
\chi_4^B &= \frac{T}{V}\frac{1}{81} && [2\partial_u^4 + \partial_s^4 + 6\partial_u^2\partial_d^2 + 12\partial_u^2\partial_s^2 \\
& && +8\partial_s^3\partial_u + 8\partial_u^3\partial_s + 8\partial_u^3\partial_d \\
& && +24\partial_u^2\partial_d\partial_s + 12\partial_s^2\partial_u\partial_d] \log Z\,, \\
\chi_4^Q &= \frac{T}{V}\frac{1}{81} && [17\partial_u^4 + \partial_s^4 + 24\partial_u^2\partial_d^2 + 30\partial_u^2\partial_s^2 \\
& && -4\partial_s^3\partial_u - 28\partial_u^3\partial_s - 40\partial_u^3\partial_d \\
& && +24\partial_u^2\partial_d\partial_s - 24\partial_s^2\partial_u\partial_d] \log Z\,, \\
\chi_4^S &= \frac{T}{V} && \partial_s^4 \log Z\,.
\end{aligned}
$$

Similar expressions can be derived for the mixed derivatives.

Caveats

- *Effects due to volume variation because of finite centrality bin width*
  Experimentally corrected by centrality-bin-width correction method
- *Finite reconstruction efficiency*
  Experimentally corrected based on binomial distribution
  [A. Bzdak, V. Koch, PRC (2012)]
- *Spallation protons*
  Experimentally removed with proper cuts in p_T
- *Canonical vs Gran Canonical ensemble*
  Experimental cuts in the kinematics and acceptance
  [V. Koch, S. Jeon, PRL (2000)]
- *Proton multiplicity distributions vs baryon number fluctuations*
  Numerically very similar once protons are properly treated
  [M. Asakawa and M. Kitazawa], [PRC (2012), M. Nahrgang et al., 1402.1238]
- *Final-state interactions in the hadronic phase* [J.Steinheimer et al., PRL (2013)]
  Consistency between different charges = fundamental test