Calculation of $K \to \pi\pi$ decay amplitudes with improved Wilson fermion in 2+1 flavor lattice QCD

at Lat2014 2014/06/23

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We present our results of $K \to \pi\pi$ decay amplitudes for both $\Delta I = 1/2$ and $3/2$.

- $N_f = 2+1$ improved Wilson fermion

  $a = 0.091\,\mathrm{fm}$, $La = 2.91\mathrm{fm}$

  $m_\pi = 280\mathrm{MeV}$ ($m_K \sim 2 \times m_\pi$)

  Decay process : $K(\mathbf{0}) \to \pi(\mathbf{0})\pi(\mathbf{0})$

1. Introduction

Previous direct calculations of $K \to \pi\pi$ decay amplitudes for $\Delta I = 1/2$:

RBC-UKQCD

$N_f = 2+1$ Domain wall fermion , $a = 0.114\,\mathrm{fm}$

- $m_\pi = 422\,\mathrm{MeV}$, $La = 1.8\,\mathrm{fm}$ PRD84(2011)114503
- $m_\pi = 330\,\mathrm{MeV}$, $La = 2.7\,\mathrm{fm}$ LAT2011 [arXiv:1110.2143]

For the Wilson fermion, operator renormalization for parity odd part of $\Delta S = 1$ op. :

$$Q_i^{\overline{\mathrm{MS}}}(\mu) = \sum_j Z_{ij}(\mu)\,\overline{Q}_j^{\mathrm{Lat}} \quad (i, j = 1, 2, \cdots 10)$$

$$\overline{Q}_j^{\mathrm{Lat}} = Q_j - \alpha_j P \ , \ \underline{P = \bar{s}\gamma_5 d}$$

P does not give finite contributions in the continuum.
This is not true for the Wilson fermion.

$Z_{ij}(\mu)$: same form as for the continuum (from *CPS* symmetry)

The direct calculation of the amplitudes is also possible with Wilson fermion, if we subtract the lower dimensional operator with a renormalization condition :

$$\alpha_j = \langle 0|Q_j|K\rangle / \langle 0|P|K\rangle$$

Calculation cost : Wilson fermion << Domain wall fermion

Statistical improvement is expected by using Wilson fermion.

2. Method

Parameter

$N_f = 2+1$ improved Wilson fermion + Iwasaki gauge action

$32^3 \times 64$, $a = 0.091\,\mathrm{fm}$, $La = 2.91\mathrm{fm}$

$m_\pi = 275.7(1.5)\,\mathrm{MeV}$, $m_K = 579.8(1.3)\,\mathrm{MeV}$

$$m_K \sim 2 \times m_\pi \qquad (m_K - 2 \cdot m_\pi = 28.3\,\mathrm{MeV})$$

Decay process : $K(\mathbf{0}) \to \pi(\mathbf{0})\pi(\mathbf{0})$

configurations :

PACS-CS : 2,000 traj

New : 10,000 traj

# of conf. in the present work = 480 (every 25 traj)

Extraction of the amplitudes

Time correlation function :

$$G^I(Q_i)(t) = \frac{1}{T}\sum_{\delta=1}^{T} \langle 0|\, W_K(t_K+\delta)\, \overline{Q}_i(t+\delta)\, W^I_{\pi\pi}(t_\pi+\delta)\, |0\rangle$$

(: periodic BC. in time)

$$\overline{Q}_i = Q_i - \alpha_i P \quad , \; P = \bar{s}\gamma_5 d \quad , \; \alpha_i = \langle 0|Q_i|K\rangle / \langle 0|P|K\rangle$$

$W_K(t)\,,\, W^I_{\pi\pi}(t)$: Wall source for K and $\pi\pi$ with the iso-spoin I

(: used with Coulomb gauge fixing at time slice of the wall sources)

Effective amplitude :

$$M^I(Q_i)(t) = G^I(Q_i)(t) \cdot F^I_{LL}/(N_K N^I_{\pi\pi}) \cdot \mathrm{e}^{m_K(t_K-t)+E^I_{\pi\pi}(t-t_\pi)} \times \underline{(-1)}$$

(for our convention of $K^0 = -\bar{s}\gamma_5 d$)

$$\longrightarrow \quad M^I(Q_i) = \langle K|\, \overline{Q}_i\, |\pi\pi; I\rangle \quad \text{for } t_K \gg t \gg t_\pi$$

$$(\; t_K = 24\,,\; t_\pi = 0\,,\; t : \text{run} \;)$$

F^I_{LL} : Lellouch-Lüscher factor

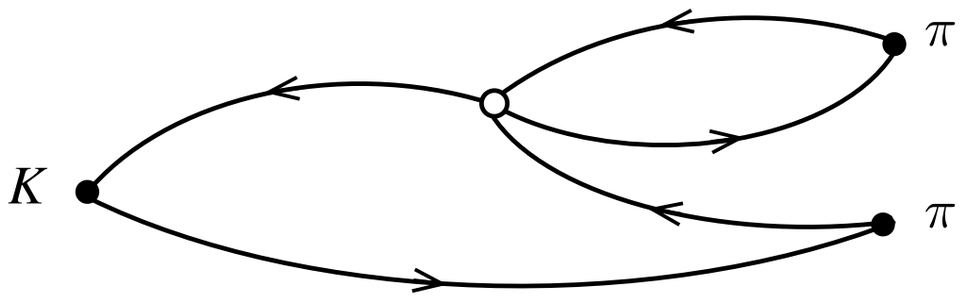
for *I=0* the factor for non-interacting case is used

$N_K = \langle 0|W_K|K\rangle \qquad E^I_{\pi\pi}$: energy of $|\pi\pi; I\rangle$

$N^I_{\pi\pi} = \langle 0|W^I_{\pi\pi}|\pi\pi; I\rangle$ (: extracted from K and $\pi\pi$ correlation function)

Calculation of quark loop at weak operators

Quark contractions :



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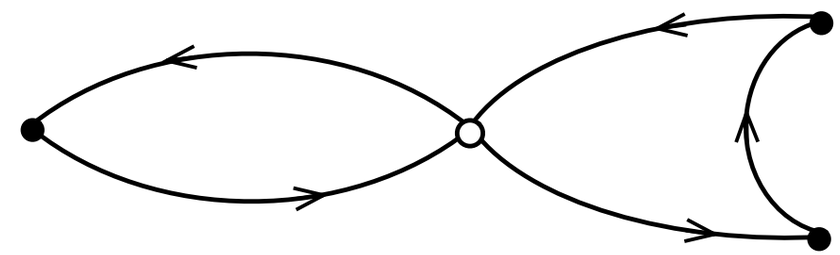

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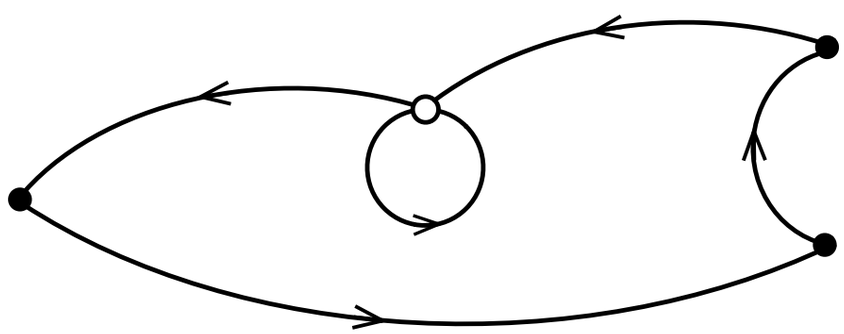

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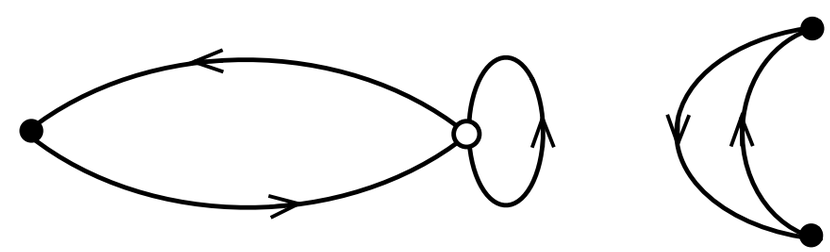


type4

Calculation of Quark loop :

Stochastic method

+ Hopping parameter expansion technique (HPE)

+ Truncated solver method (TSM)

(G.S.Bali et.al, Comp.Phys.Comm. 181(2010)1570.)

Hopping parameter expansion technique (HPE)

Wilson fermion :

$$S^W = \bar{\psi}\, W\, \psi = \bar{\psi}\,(M - D)\,\psi = \bar{\psi}\, M(1 - \overline{D})\,\psi \qquad \left(\overline{D} = M^{-1} D \right)$$

$$(M\psi)(x) = [\,1 - \kappa C_{SW}(\sigma \cdot F(x))/2\,]\,\psi(x)$$

$$(D\psi)(x) = \kappa \sum_{\mu} \left[\, (1-\gamma_\mu) U_\mu(x)\psi(x+\mu) \,+\, (1+\gamma_\mu) U^\dagger_\mu(x-\mu)\psi(x-\mu) \,\right]$$

Quark propagator :

$$Q = W^{-1} = (1-\overline{D})^{-1} M^{-1} = \sum_{n=0}^{\infty} \overline{D}^{\,n} M^{-1} = \sum_{n=0}^{k-1} \overline{D}^{\,n} M^{-1} + \overline{D}^{\,k} W^{-1} \qquad (\text{ for any } k\ )$$

$$= M^{-1} + \overline{D} M^{-1} + \overline{D}^{\,2} M^{-1} + \overline{D}^{\,3} M^{-1} + \overline{D}^{\,4} W^{-1} \qquad (\text{ for } k = 4\ )$$

Quark loop :

$$Q(x,x) = \left[\, M^{-1} + \cancel{\overline{D} M^{-1}} + \overline{D}^{\,2} M^{-1} + \cancel{\overline{D}^{\,3} M^{-1}} + \overline{D}^{\,4} W^{-1} \right] (x,x)$$

$$= \left[\, M^{-1} + \overline{D}^{\,2} M^{-1} + \overline{D}^{\,4} W^{-1} \right] (x,x)$$

Calculation of the quark loop by the stochastic method

$$Q(\mathbf{x},t;\mathbf{x},t) = \frac{1}{N_R} \sum_{i=1}^{N_R} \xi_i^*(\mathbf{x})\, S_i(\mathbf{x},t) \qquad \left(\delta^3(\mathbf{x}-\mathbf{y}) = \lim_{N_R \to \infty} \frac{1}{N_R} \sum_{i=1}^{N_R} \xi_i^*(\mathbf{x})\, \xi_i(\mathbf{y}) \right)$$

$$S_i(\mathbf{x},t) = \sum_{\mathbf{y}} \left[\, M^{-1} + \overline{D}^{\,2} M^{-1} + \overline{D}^{\,4} W^{-1} \right] (\mathbf{x},t;\mathbf{y},t)\ \xi_i(\mathbf{y})$$

Truncated solver method (TSM)

$$Q(\mathbf{x},t;\mathbf{x},t) = \frac{1}{N_T}\sum_{i=1}^{N_T} \xi_i^*(\mathbf{x})\, S_i^T(\mathbf{x},t) + \frac{1}{N_R}\sum_{i=N_T+1}^{N_T+N_R} \xi_i^*(\mathbf{x})\, \left[S_i(\mathbf{x},t) - S_i^T(\mathbf{x},t)\right]$$

$$S_i(\mathbf{x},t) = \sum_{\mathbf{y}} \left[\, M^{-1} + \overline{D}^{\,2} M^{-1} + \overline{D}^{\,4} W^{-1}\right] (\mathbf{x},t;\mathbf{y},t)\; \xi_i(\mathbf{y})$$

$S_i^T(\mathbf{x},t)$: with W^{-1} calculated with a loose stopping condition

$N_R = 1$ tor. $< 10^{-14}$ (tor. $= |WW^{-1} - \xi|/|\xi|$)

$N_T = 5$ tor. $< 1.2 \times 10^{-6}$

We find that 2nd term is negligible for all channels.

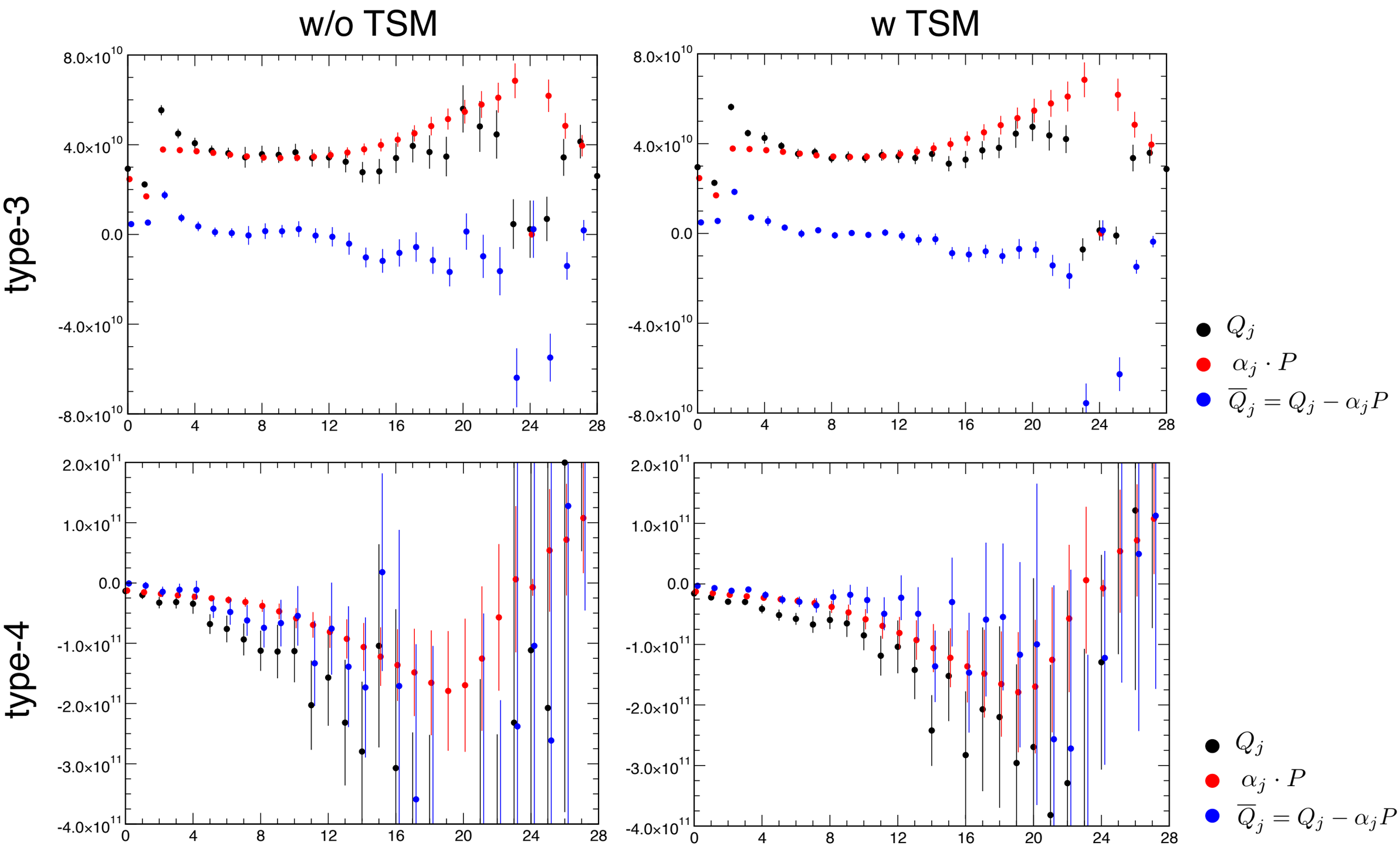
We omit 2nd term :

$$Q(\mathbf{x},t;\mathbf{x},t) = \frac{1}{N_T}\sum_{i=1}^{N_T} \xi_i^*(\mathbf{x})\, S_j^T(\mathbf{x},t)$$

$N_T = 6$ tor. $< 1.2 \times 10^{-6}$

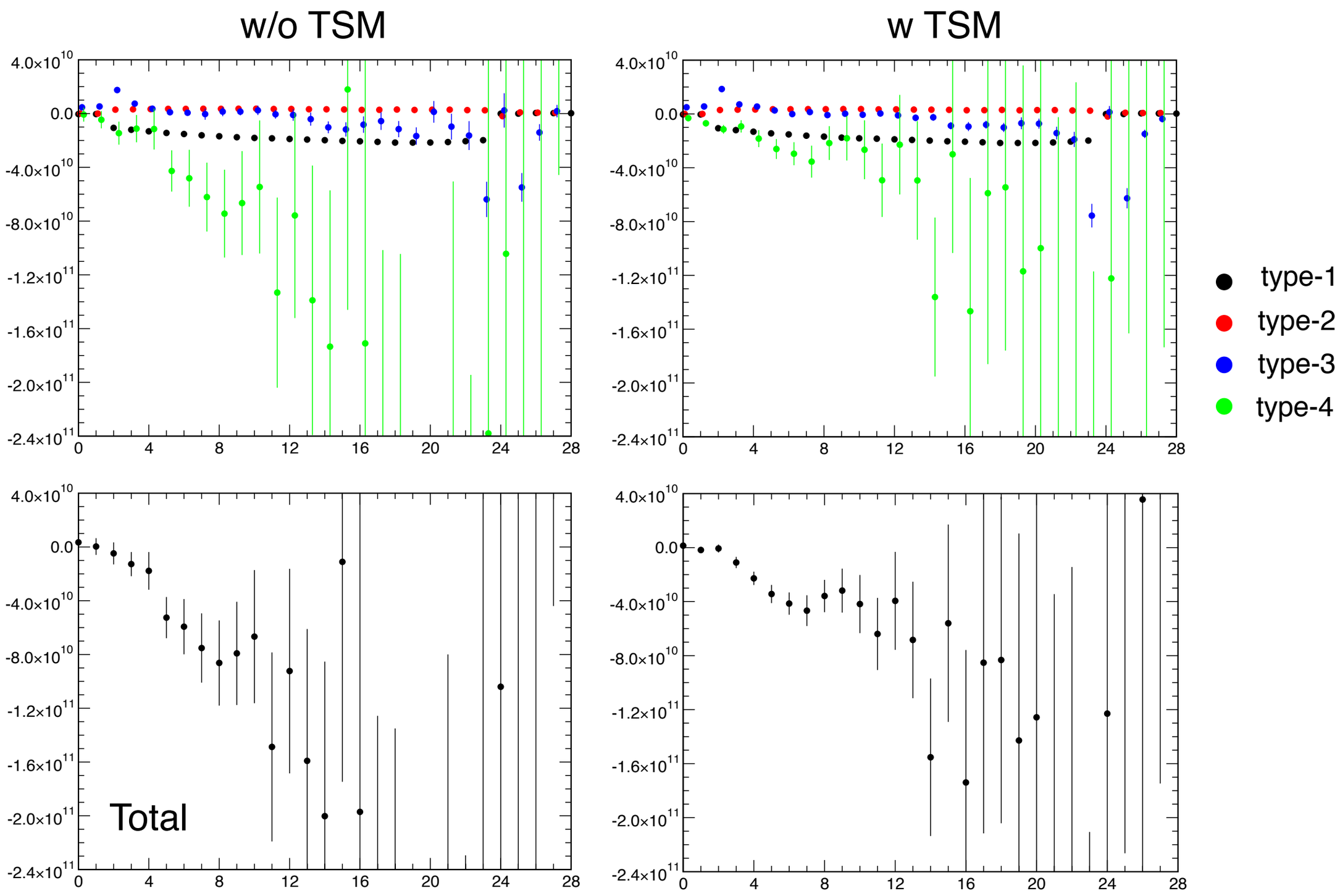
2. Results

$G^{I=0}(Q_2)$ from type-3 and type-4 $(\ t_K = 24\,,\ t_\pi = 0\,,\ Q(t) : \mathrm{run}\ )$



TSM reduces the statistical error

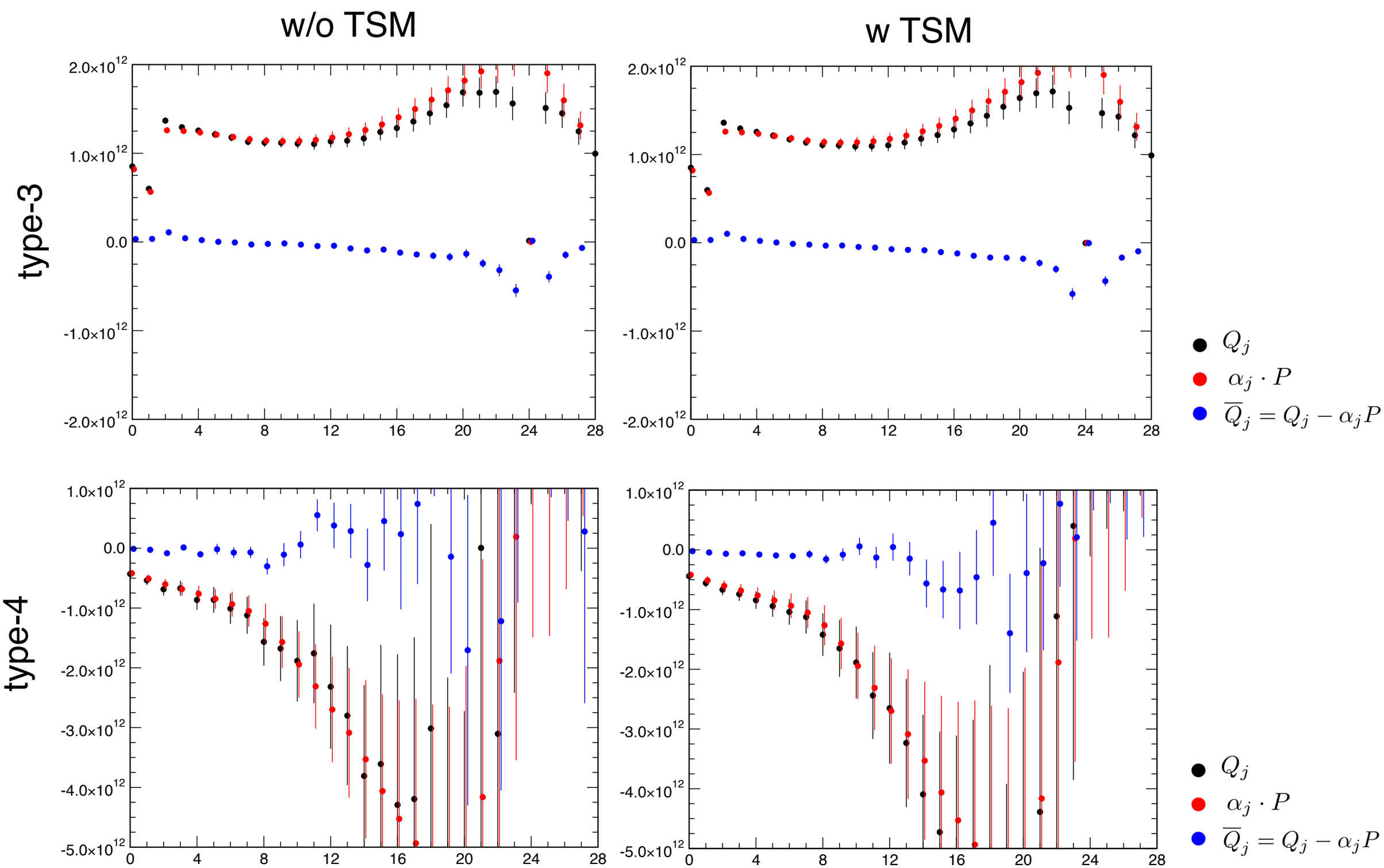
$G^{I=0}(Q_2)$ $(t_K = 24 , t_\pi = 0 , Q(t) : \mathrm{run})$



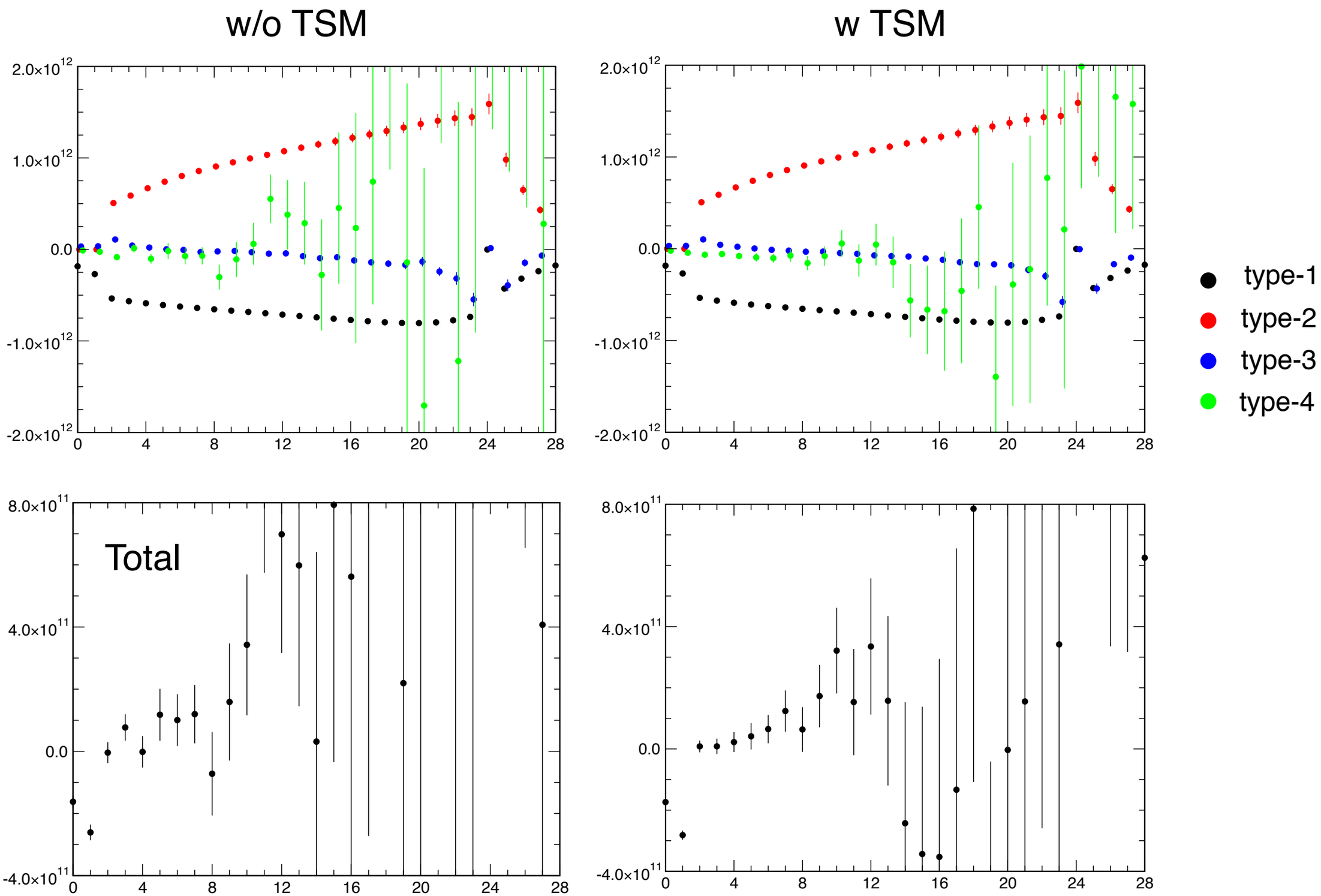
type-4 is large !!

type-4 ~ type-1

$G^{I=0}(Q_6)$ from type-3 and type-4 $(\ t_K = 24\,,\ t_\pi = 0\,,\ Q(t) : \mathrm{run}\ )$

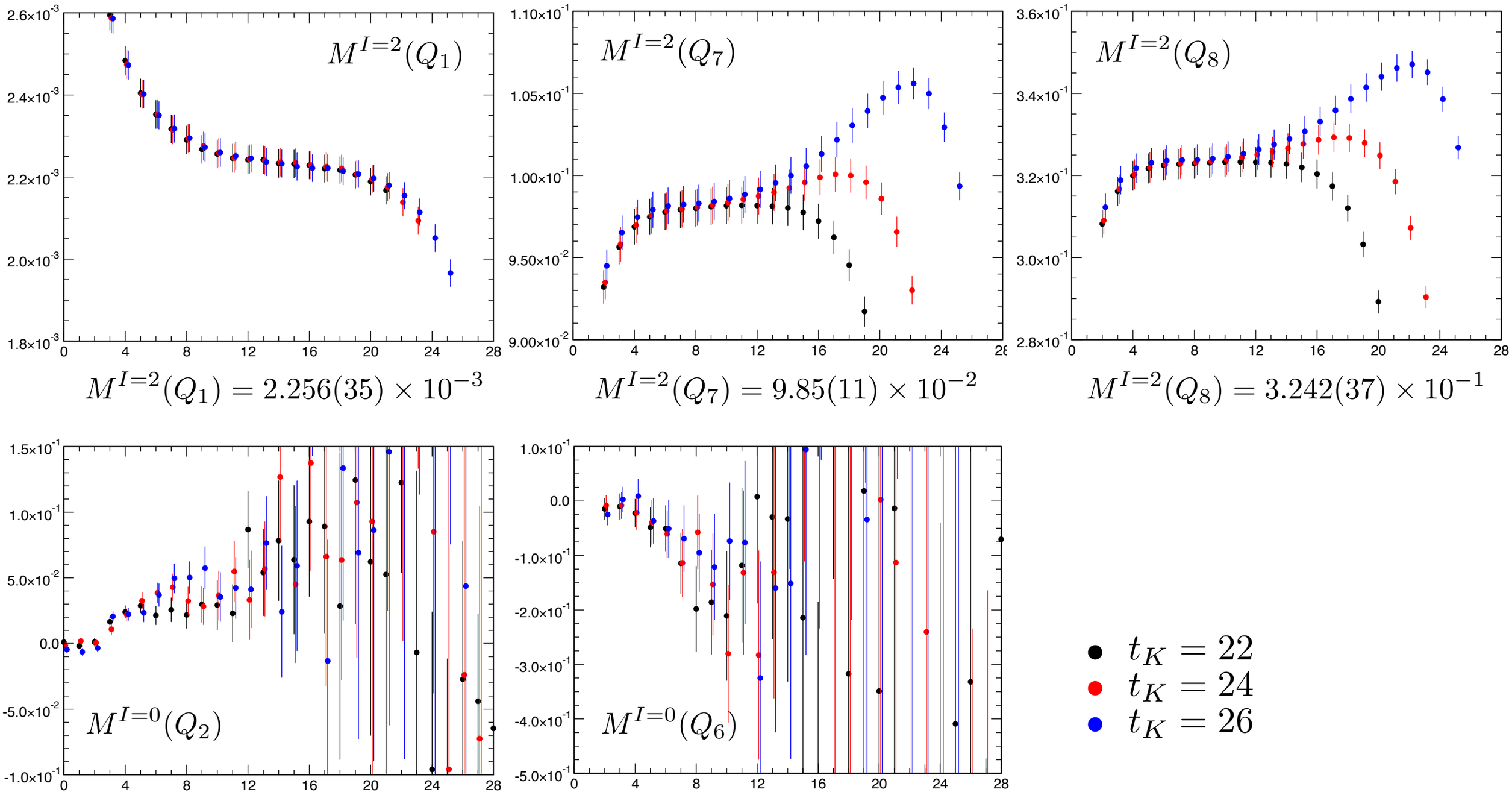


$G^{I=0}(Q_6)$ ($t_K = 24$, $t_\pi = 0$, $Q(t)$: run)



Large cancelation between type-1 and type-2.

Effective amplitudes $(t_K = 22, 24, 26 \, , \, t_\pi = 0 \, , \, Q(t) : \mathrm{run} \,)$



$M^{I=2}(Q_1) = 2.256(35) \times 10^{-3}$ $\quad$ $M^{I=2}(Q_7) = 9.85(11) \times 10^{-2}$ $\quad$ $M^{I=2}(Q_8) = 3.242(37) \times 10^{-1}$

$M^{I=0}(Q_2) = 3.55(1.43) \times 10^{-2}$ $\quad$ $M^{I=0}(Q_6) = -1.96(1.06) \times 10^{-1}$

The around-the-world effect for two pion state can be avoided for the time range t=[9,12].

Physical amplitudes

From the lattice to the continuum :

$$Q_i^{\overline{\rm MS}}(\mu) = \sum_j Z_{ij}(\mu)\, \overline{Q}_j^{\rm Lat} \qquad (\ i,j = 1,2,\cdots 10\ )$$

with perturbative renormalization factor (1 loop).

Y. Taniguchi, JHEP04(2012)143.

matching point : $\mu = 1/a$

(also $\mu = \pi/a$ to estimate higher order effect)

Coefficient function : G. Bychalla, A.J.Buras, M.E. Lautenbacher, RMP 68(1996)125.

$$H = \sum_i C_i(\mu) Q_i^{\overline{\rm MS}}(\mu) = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_i \left(z_i(\mu) + \tau y_i(\mu)\right) Q_i^{\overline{\rm MS}}(\mu)$$

Physical decay amplitudes :

$$A_I = \langle K|H|\pi\pi; I\rangle = \sum_{ij} C_i(\mu) Z_{ij}(\mu) M^I(Q_j)$$

$M^I(Q_i)$: amplitudes on the lattice

Physical decay amplitudes

	$\mu = 1/a$	$\mu = \pi/a$	RBC-UKQCD		Exp
$a\,(\mathrm{fm})$	0.091		0.114	0.114	
$m_\pi\,(\mathrm{MeV})$	280		330	422	140
$\mathrm{ReA}_2\,(\times 10^{-8}\,\mathrm{GeV})$	2.426(38)	2.460(38)	2.668(14)	4.911(31)	1.479(4)
$\mathrm{ReA}_0\,(\times 10^{-8}\,\mathrm{GeV})$	60(36)	56(32)	31.1(4.5)	38.0(8.2)	33.2(2)
$\mathrm{Re}A_0/\mathrm{Re}A_2$	25(15)	23(13)	12.0(1.7)	7.7(1.7)	22.45(6)
$\mathrm{ImA}_2\,(\times 10^{-12}\,\mathrm{GeV})$	−1.14(13)	−0.746(8)	−0.651(3)	−0.550(4)	
$\mathrm{ImA}_0\,(\times 10^{-12}\,\mathrm{GeV})$	−67(56)	−52(48)	−33(15)	−25(22)	
$\mathrm{Re}(\epsilon'/\epsilon)\,(\times 10^{-3})$	0.8(2.5)	0.9(2.5)	2.0(1.7)	1.11(91)	1.66(23)

(used $|\epsilon^{\mathrm{EXP}}| = 2.22 \times 10^{-3}$)

- $\mathrm{Re}A_I > \mathrm{Exp}$ $\qquad \mathrm{Re}A_I \propto (m_K^2 - m_\pi^2)$
- Matching point dependence is very large for ImA_2 .
- Enhancement of $\Delta I = 1/2$ process is seen.
- Further improvement of statistics is necessary for ϵ'/ϵ .

5. Summary

We calculate $K \to \pi\pi$ decay amplitudes
for the process $K(\mathbf{0}) \to \pi(\mathbf{0})\pi(\mathbf{0})$ at $m_\pi = 280\mathrm{MeV}$ ($m_K \sim 2 \times m_\pi$)

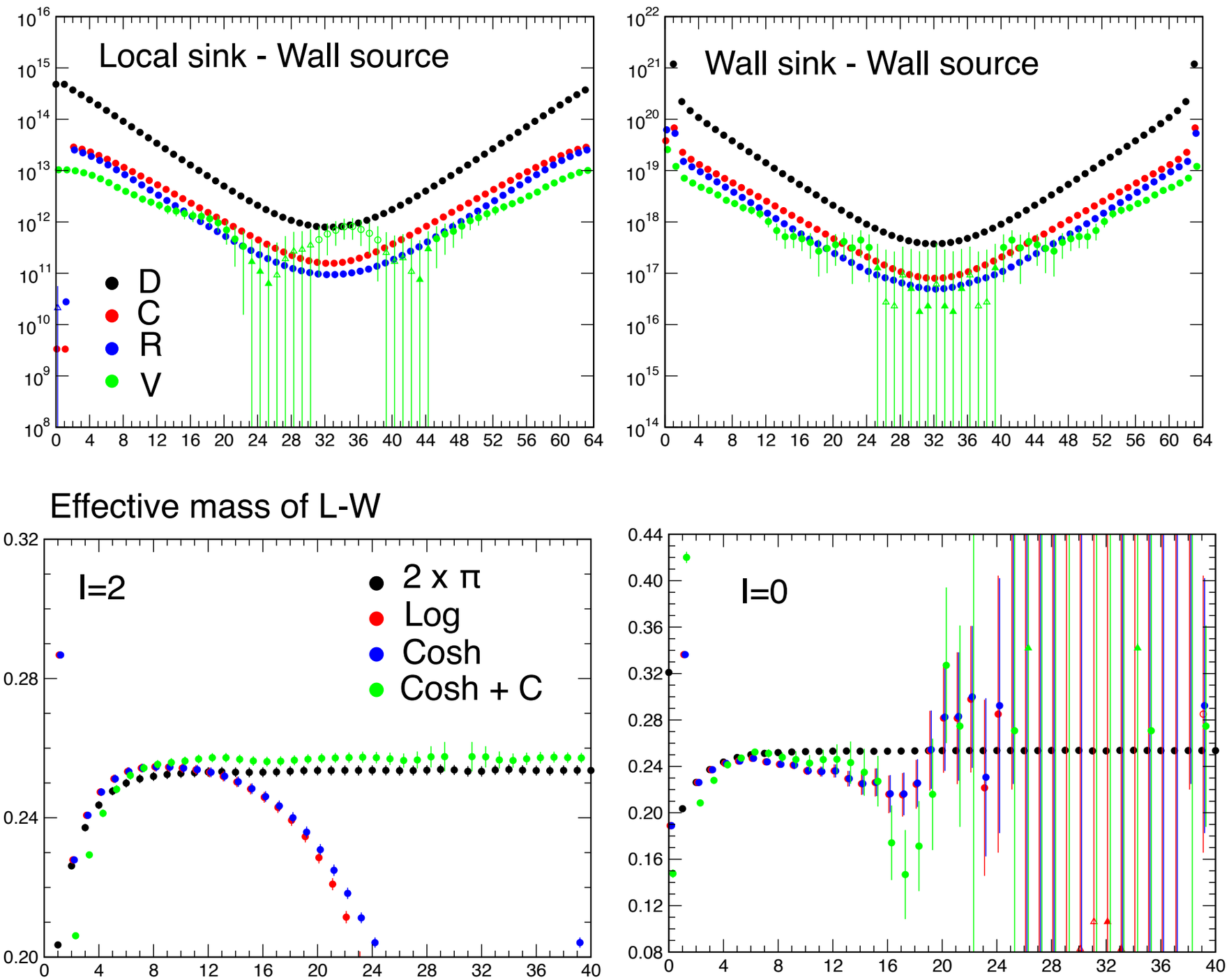
- $N_f = 2+1$ improved Wilson fermion with
  Non-perturbative subtraction of the lower dimensional operator .
- Calculation of quark loop by
  Stochastic method with HPE and TSM

We found :

- TSM is an efficient method.
- For Q_2 , the contribution of type-4 (OZI-suppression diag.) is large.
  type-4 ~ type-1 (: Wilson fermion ?)
- Matching point dependence is very large for ImA_2 .
  Non-perturbative renormalization factor is needed.
- Enhancement of $\Delta I = 1/2$ process is seen.
  $\mathrm{Re}A_0/\mathrm{Re}A_2 = 25 \pm 15$
- Further improvement of statistics is necessary for ϵ'/ϵ .
  Improvement of the *K* and *ππ* operator.

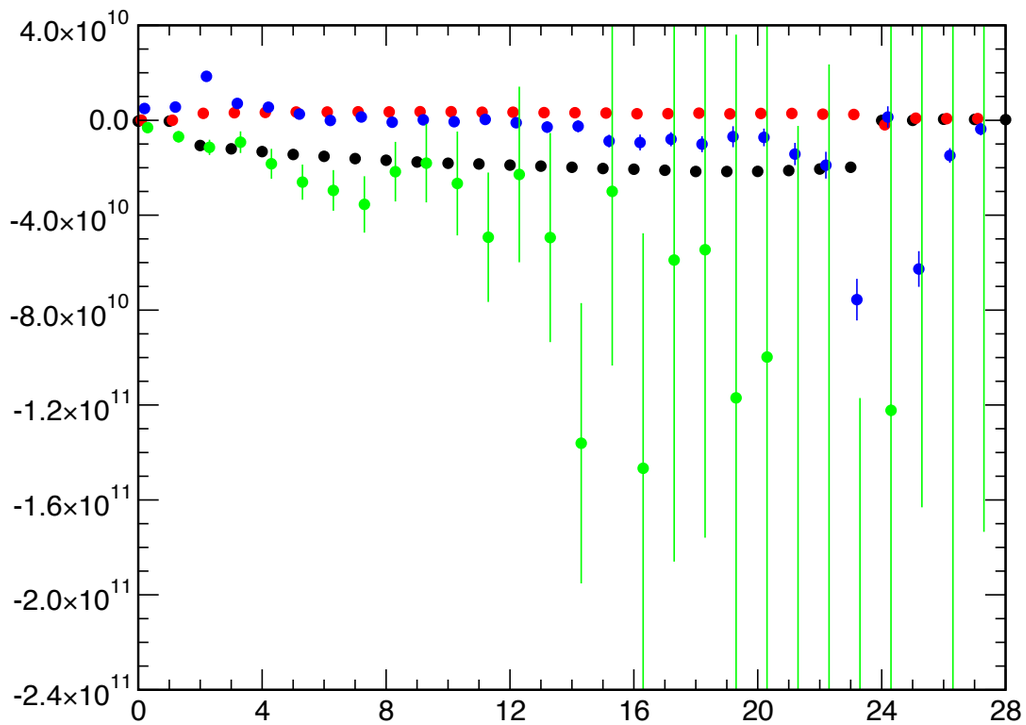
Back up

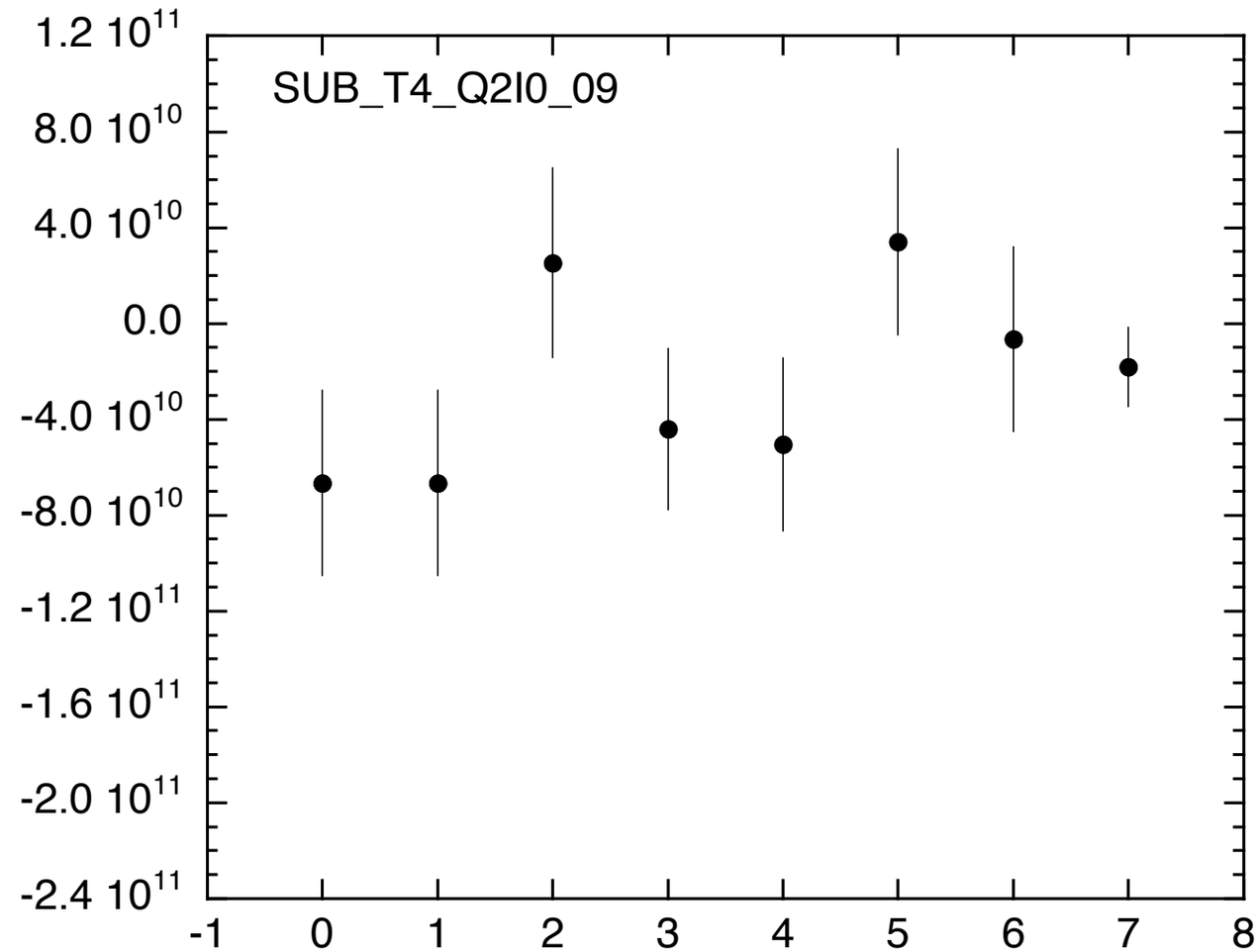
$\pi\ \pi$


Local sink - Wall source
Wall sink - Wall source
D
C
R
V
Effective mass of L-W
I=2
I=0
2 x π
Log
Cosh
Cosh + C

Back up

dependence of the dimension of the random number





Back up

contribution of the amplitudes on the lattice :

	Re(A2)[GeV]		Im(A2)[GeV]		Re(A0)[GeV]		Im(A0)[GeV]	
1	-1.8871D-08	2.94D-10	--		-4.0191D-08	1.11D-07	--	
2	4.3299D-08	6.75D-10	--		6.8169D-07	2.75D-07	--	
3	--		--		-1.2535D-08	6.47D-09	-2.5521D-11	1.32D-11
4	--		--		5.3176D-08	2.01D-08	6.6713D-11	2.52D-11
5	--		--		1.4731D-09	5.85D-09	1.7021D-12	6.76D-12
6	--		--		-8.4411D-08	4.58D-08	-1.0293D-10	5.58D-11
7	1.0534D-10	1.20D-12	2.7716D-13	3.17D-15	2.5879D-10	1.89D-11	6.8090D-13	4.98D-14
8	-2.7219D-10	3.13D-12	-1.6698D-12	1.92D-14	-6.2597D-10	4.49D-11	-3.8403D-12	2.76D-13
9	-1.1396D-12	1.78D-14	3.7616D-13	5.87D-15	1.0249D-11	4.81D-12	-3.3828D-12	1.59D-12
10	3.7718D-12	5.88D-14	-1.7564D-13	2.74D-15	1.9542D-13	1.39D-11	-9.1000D-15	6.45D-13
Tot.	2.4264D-08	3.79D-10	-1.1922D-12	1.34D-14	5.9885D-07	3.60D-07	-6.6592D-11	5.63D-11