Mesonic spectral functions and transport properties in the quenched QCD continuum

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H.T.Ding et. al., Phys.Rev.D83 (2011) 034504, arXiv:1312.5609

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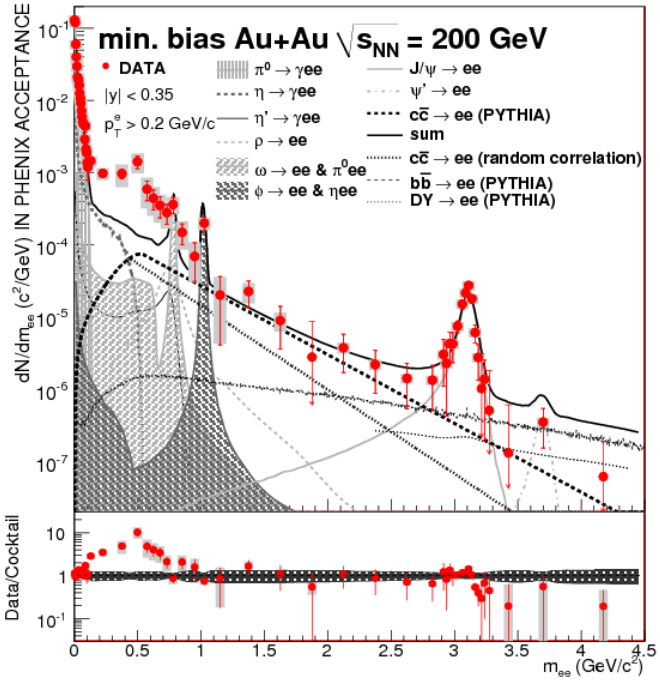
Outline

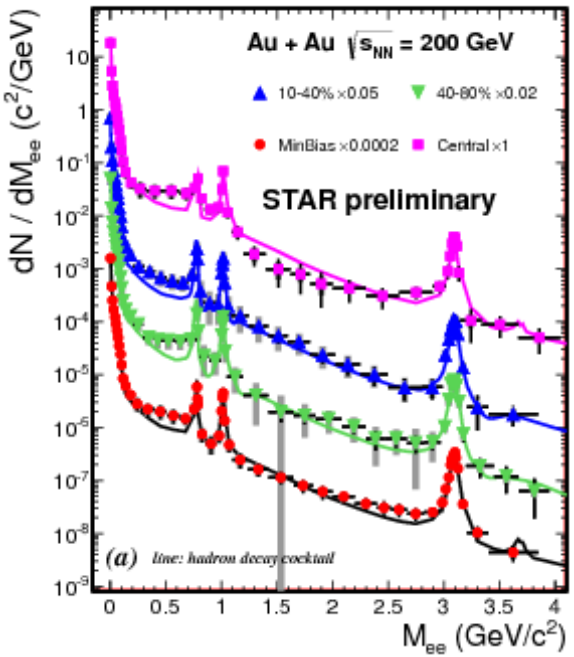
1. Introduction: spectral functions on the lattice
2. Produce data
3. Obtain the spectral function
4. Results

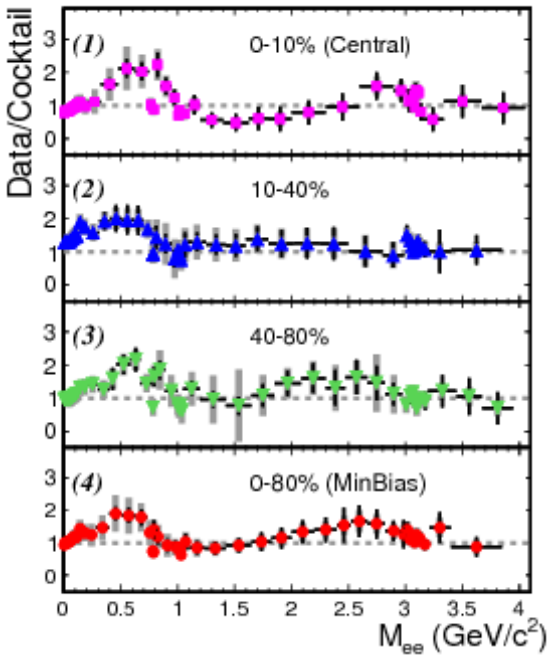
Introduction

Experiments

- Dilepton rates in *pp* collisions well described by hadron cocktail model
- Enhancement in low energy region of *AuAu* collisions
  [PHENIX PRC81, 034911 (2010)]
  $\Rightarrow$ thermal modifications?







The spectral function (SPF)

QGP probes

- Goal is to compute vector channel SPF because
  1. Photons and dileptons are produced in the QGP
  2. Leave it almost undisturbed

Experimental observables

- Dilepton rate linked to the vector SPF: $\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3 p} \sim \frac{\rho_V(\omega, \vec{p}, T)}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)}$
- Photon rate linked to the vector SPF: $\omega \frac{\mathrm{d}R_\gamma}{\mathrm{d}^3 p} \sim \frac{\rho_V^T(\omega = |\vec{p}|, T)}{e^{\omega/T} - 1}$

Transport properties

- SPF relates to transport properties via **Kubo formulae**
- Here: electrical conductivity from spatial part: $\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}}{\omega}$

Lattice observables

Observables

- (Renormalized) vector current $J_H = Z_V \bar{\psi}(x)\gamma_H \psi(x)$
- Euclidean correlator $G_H(\tau, \vec{x}) = \langle J_H(\tau, \vec{x}) J_H^\dagger(0, \vec{0}) \rangle$
- In momentum space $G_H(\tau, \vec{p}) = \sum_{\vec{x}} G_H(\tau, \vec{x}) e^{i\vec{p}\vec{x}}$
- Relation to SPF ρ_H:

$$G_H(\tau, \vec{p}) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho_H(\omega, \vec{p}, T) K(\omega, \tau, T)$$

with kernel $K(\omega, \tau, T) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$

- different parts of the vector SPF: sum over spatial ($H = ii$), temporal ($H = 00$) and full ($H = V$)

Determining the SPF

An ill posed problem

- Inversion problematic: $\sim \mathcal{O}(10)$ data points $G(\tau)$, far finer resolution in ω required for $\rho(\omega)$

Two possible solutions

1. Maximum Entropy method
2. **Here:** use phenomenologically motivated ansatz and fit to correlator data

Choosing the Ansatz

Free case is known

- $\rho_{ii}(\omega, T) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\omega^2 \tanh(\frac{\omega}{4T})$
- $\rho_{00}(\omega, T) = 2\pi T^2 \omega \delta(\omega)$

With interactions

- net quark number conservation: $\rho_{00} \to 2\pi \chi_q \omega \delta(\omega)$
- ρ_{ii} is modified: delta peak gets smeared out
  G.Aarts, J.M.Martinez Resco, JHEP 0204 (2002) 053
  J.Hong, D.Teaney, Phys.Rev.C82 (2010) 044908
- Describe as Breit-Wigner peak + free part for large ω

$\Rightarrow$ Phenomenologically inspired ansatz

$$\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega\Gamma}{\omega^2 + (\Gamma/2)^2} + \frac{2}{3\pi}(1+k)\omega^2 \tanh\left(\frac{\omega}{4\pi}\right)$$

with parameters c_{BW}, Γ, k ($= \frac{\alpha_s}{\pi}$ at leading oder)

Lattice Data

- Nonperturbatively improved Wilson-Clover action
- No dynamical Sea Quarks
- Finite volume effects under control
- All quark masses $m_{\overline{MS}}(\mu = 2GeV) \sim \mathcal{O}(10MeV)$
- Nonperturbative renormalization constants
- Fixed aspect ratio $\frac{N_\sigma}{N_\tau} = 3$ for $T = 1.1\,T_c$ and $1.2\,T_c$
- Continuum extrapolation of the vector correlator

N_σ	N_τ	β	κ	1/a[GeV]	#
$T = 1.1T_c$					
32	96	7.192	0.13440	9.65	314
48	144	7.544	0.13383	13.21	358
64	192	7.793	0.13345	19.30	242
$T = 1.2T_c$					
28	96	7.192	0.13440	9.65	232
42	144	7.544	0.13383	13.21	417
56	192	7.793	0.13345	19.30	273
$T = 1.45T_c$					
16	128	6.827	0.13495	6.43	191
24	128	7.192	0.13440	9.65	340
32	128	7.457	0.13390	12.86	255
48	128	7.793	0.13340	19.30	456

Continuum extrapolation

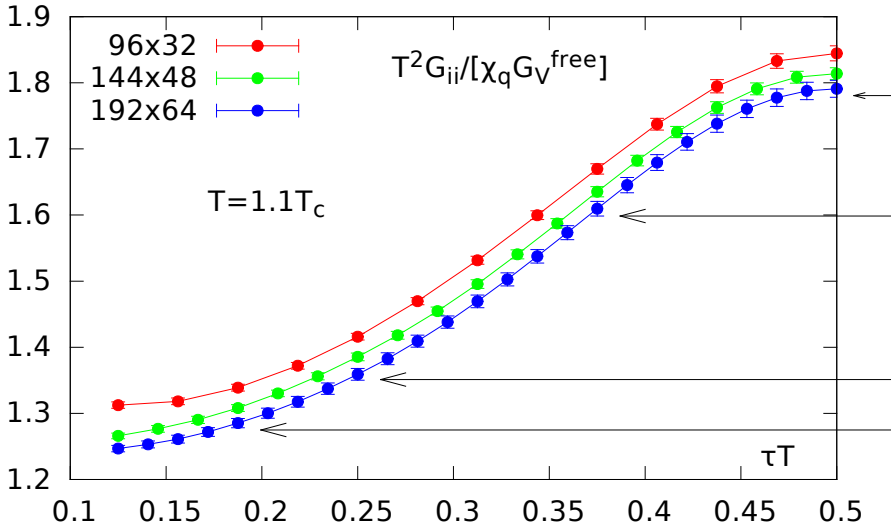
Observables

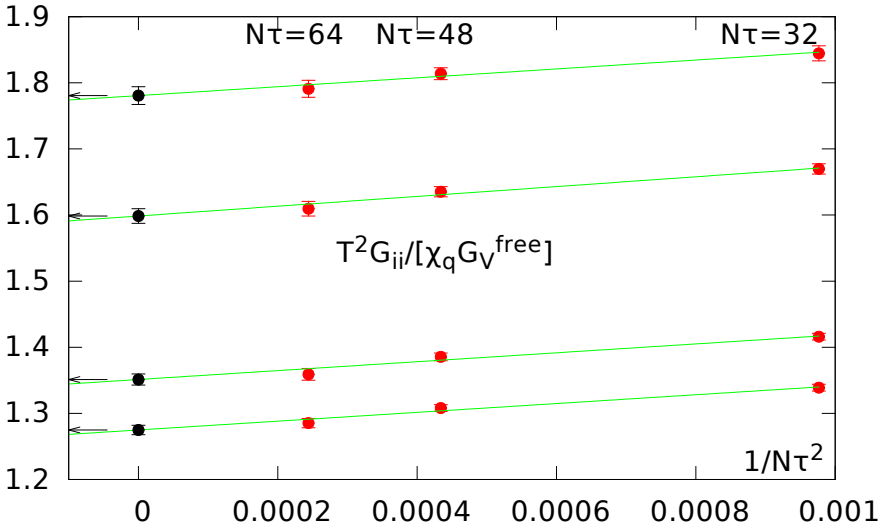
- Form ratio $\frac{T^2}{\chi_q} \frac{G_{ii}(\tau T)}{G_V^{free,lat}(\tau T)}$ with $\chi_q = -G_{00}/T$
  $\Rightarrow$ Renormalization constant drops out
- Ratio to free *lattice* correlator reduces cutoff effects

Continuum limit

- Use natural cubic splines $\Rightarrow N_\tau^{max}$ data points per T
- Linear extrapolation in a^2 due to Clover term
- Error estimation via bootstrap

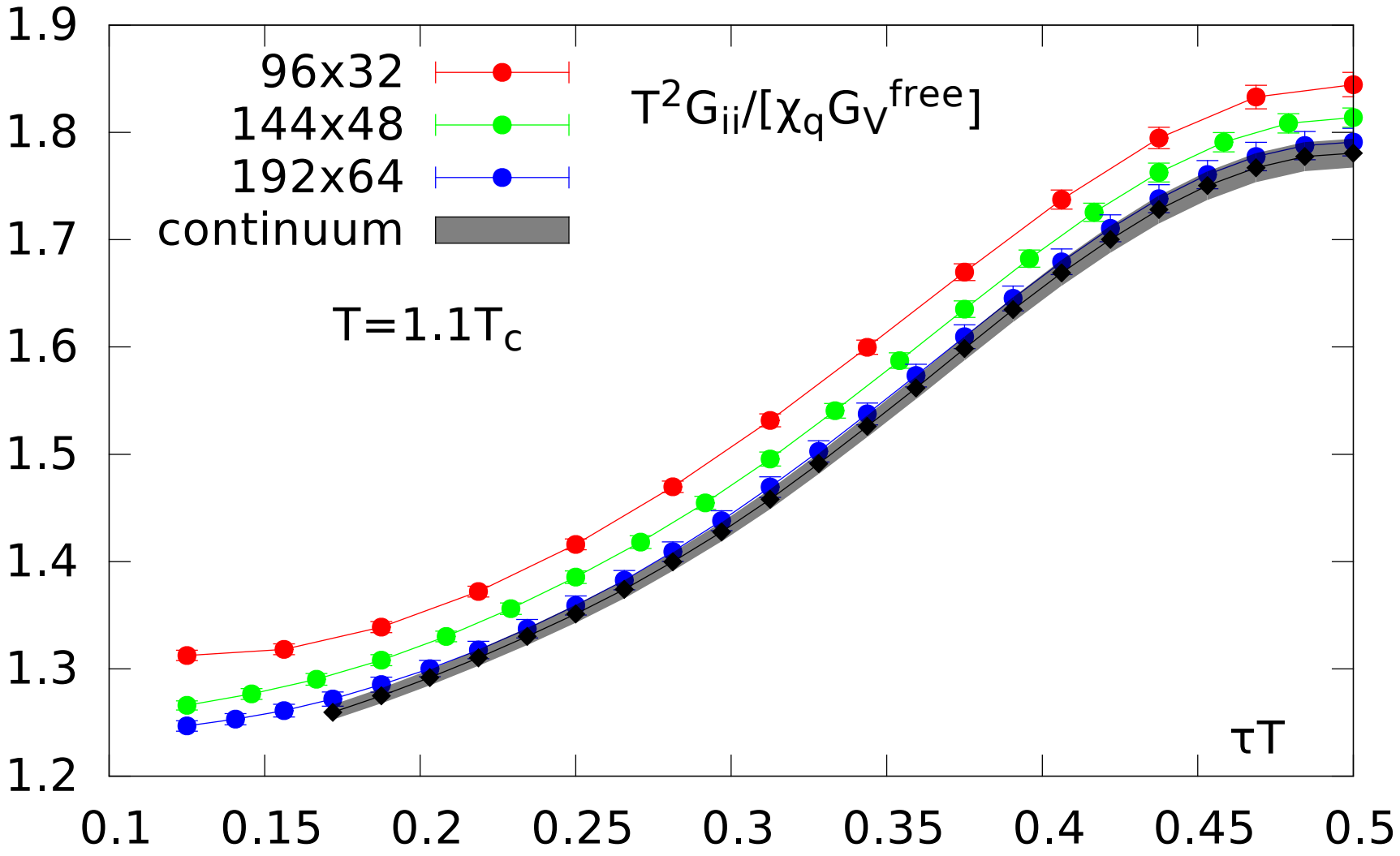
Continuum extrapolation





- Continuum extrapolation for each τT in $1/N_\tau^2$
- Well behaved extrapolation at all τT

Continuum extrapolation



Fitting the SPF

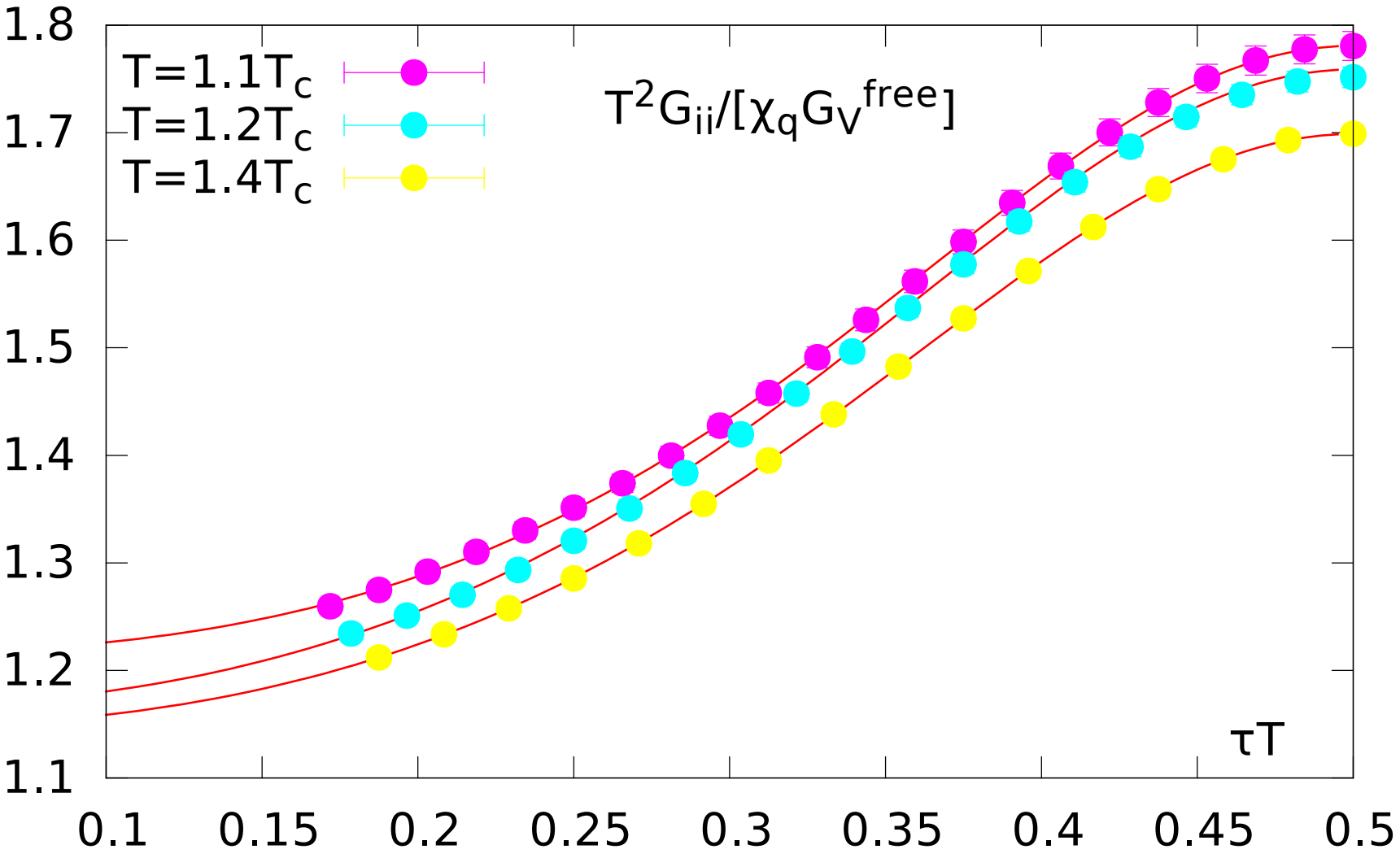
Fit

- Ansatz $\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega\Gamma}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\omega^2 \tanh\left(\frac{\omega}{4T}\right)$
- fit parameters c_{BW}, Γ, k
- k insensitive to low ω region
- Electrical conductivity $\frac{\sigma}{T} = \frac{2}{3} C_{em} \chi_q \frac{c_{BW}}{\Gamma}$
- Fit ansatz to continuum extrapolated ratio $\frac{T^2}{\chi_q} \frac{G_{ii}(\tau T)}{G_V^{free}(\tau T)}$
- Constrain fit further by curvature at midpoint

Numerics

- Standard gaussian quadrature for Breit-Wigner part
- Fit done using Levenberg-Marquardt minimization algorithm

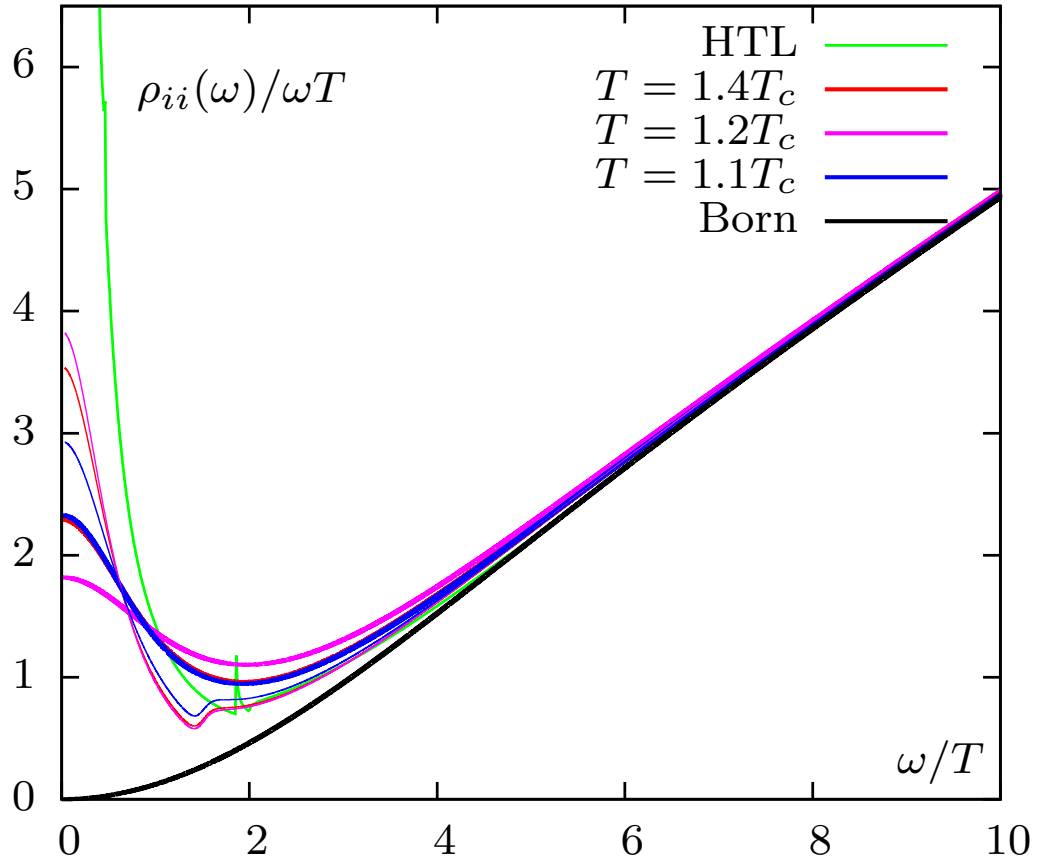
Correlators reconstructed from the fits



Analysis of systematics

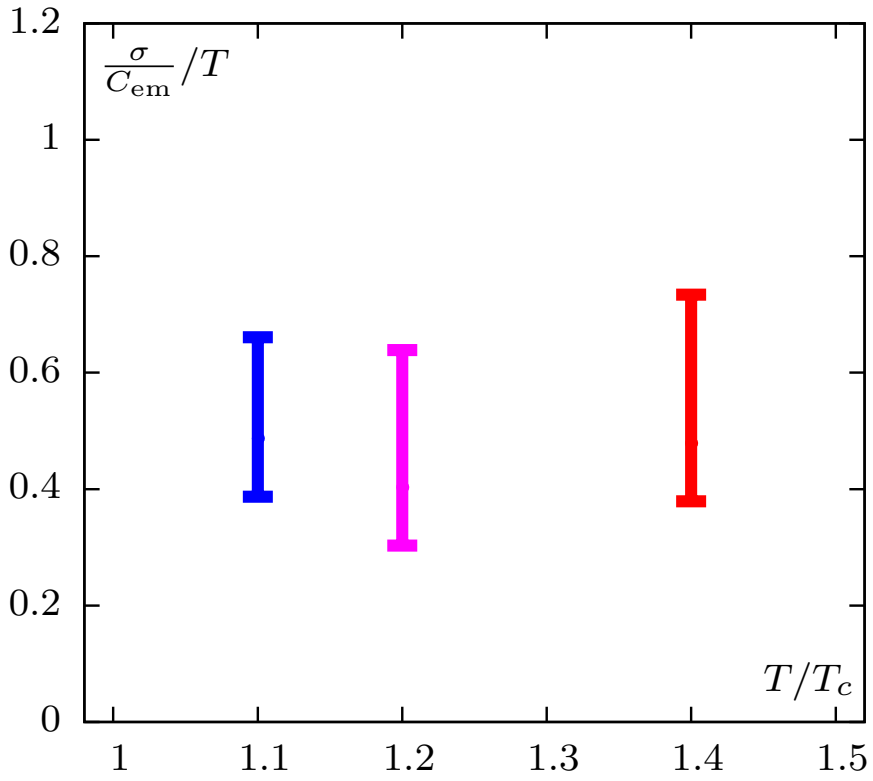
Modifying the ansatz

- Multiply the high ω part with smoothed out Θ-function
  $\Rightarrow$ control where the high ω behaviour sets in



- Breit-Wigner part compensates
  $\rightarrow$ conductivity rises
- At some point the fit becomes poor
  $\rightarrow$ error estimate

Electrical conductivity

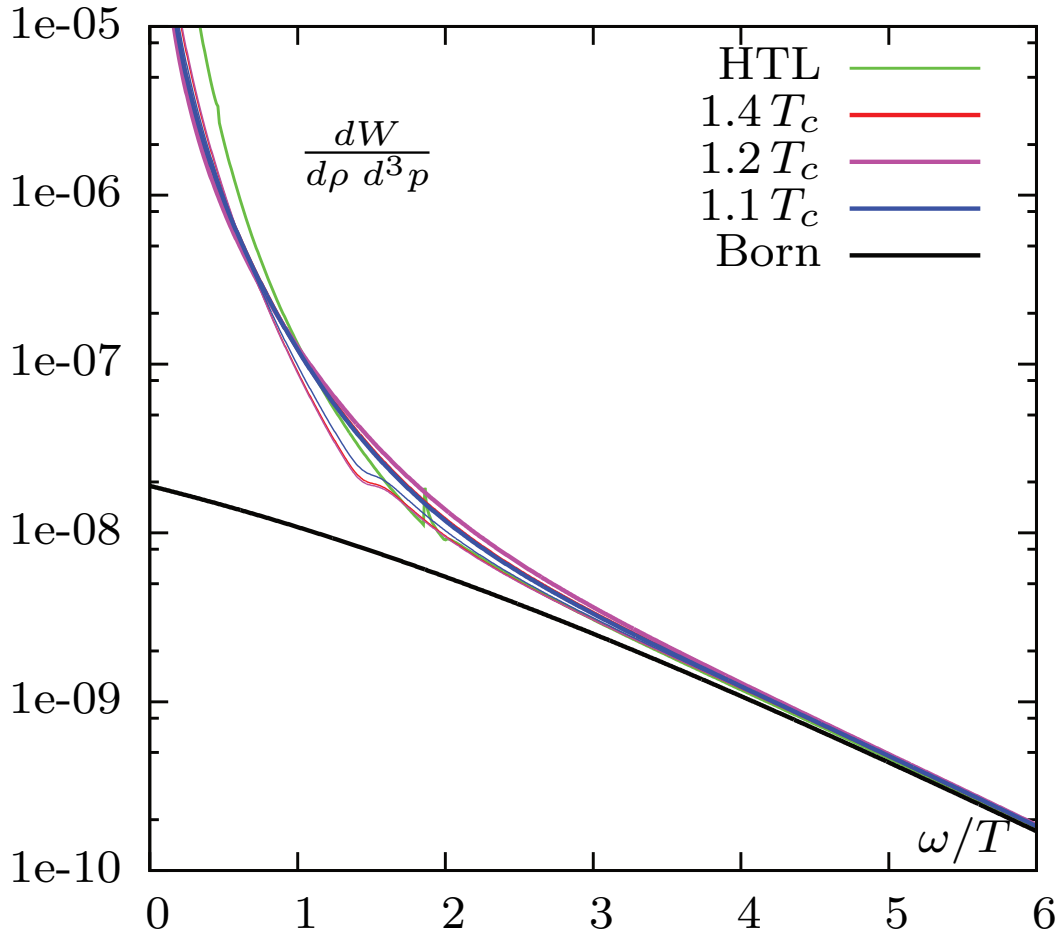


Similar studies: also Wilson fermions, but w/o continuum limit

- A.Amato et al., arXiv:1307.6763
- B.B.Brandt et al., JHEP 1303 (2013) 100

Dilepton rate

- $\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3 p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T}-1)} \rho_V(\omega, T)$



Conclusion / Outlook

Conclusion

- Continuum extrapolation of correlator-ratios at three temperatures above T_c
- Ratios reconstructed from the SPF
- Results for electrical conductivity and dilepton rate

Outlook

- Also consider finite momenta $\rightarrow$ results for e.g. the photon rate
- Use dynamical fermions
- Reduce systematic errors $\rightarrow$ study temperature dependence of transport properties