Mesonic spectral functions and transport properties in the quenched QCD continuum

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Lattice 2014 Columbia University, New York
1. Introduction: spectral functions on the lattice
2. Produce data
3. Obtain the spectral function
4. Results
Introduction

Experiments

- Dilepton rates in $pp$ collisions well described by hadron cocktail model
- Enhancement in low energy region of $AuAu$ collisions
  [PHENIX PRC81, 034911 (2010)]
  ⇒ thermal modifications?
The spectral function (SPF)

QGP probes

- Goal is to compute vector channel SPF because
  1. Photons and dileptons are produced in the QGP
  2. Leave it almost undisturbed

Experimental observables

- Dilepton rate linked to the vector SPF:
  \[ \frac{dW}{d\omega d^3p} \sim \frac{\rho_V(\omega, \vec{p}, T)}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)} \]

- Photon rate linked to the vector SPF:
  \[ \omega \frac{dR_\gamma}{d^3p} \sim \frac{\rho_V^T(\omega = |\vec{p}|, T)}{e^{\omega/T} - 1} \]

Transport properties

- SPF relates to transport properties via Kubo formulae
- Here: electrical conductivity from spatial part:
  \[ \frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \to 0} \frac{\rho_{ii}}{\omega} \]
Lattice observables

Observables

- (Renormalized) vector current \( J_H = Z_V \bar{\psi}(x)\gamma_H \psi(x) \)
- Euclidean correlator \( G_H(\tau, \vec{x}) = \langle J_H(\tau, \vec{x})J_H^\dagger(0, \vec{0}) \rangle \)
- In momentum space \( G_H(\tau, \vec{p}) = \sum_{\vec{x}} G_H(\tau, \vec{x})e^{i\vec{p}\cdot\vec{x}} \)
- Relation to SPF \( \rho_H \):

\[
G_H(\tau, \vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega, \vec{p}, T) K(\omega, \tau, T)
\]

with kernel \( K(\omega, \tau, T) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})} \)

- different parts of the vector SPF: sum over spatial \( H = ii \), temporal \( H = 00 \) and full \( H = V \)
Determining the SPF

An ill posed problem

- Inversion problematic: $\sim \mathcal{O}(10)$ data points $G(\tau)$, far finer resolution in $\omega$ required for $\rho(\omega)$

Two possible solutions

1. Maximum Entropy method
2. Here: use phenomenologically motivated ansatz and fit to correlator data
Choosing the Ansatz

Free case is known

- \( \rho_{ii}(\omega, T) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi} \omega^2 \tanh\left(\frac{\omega}{4T}\right) \)
- \( \rho_{00}(\omega, T) = 2\pi T^2 \omega \delta(\omega) \)

With interactions

- net quark number conservation: \( \rho_{00} \to 2\pi \chi_q \omega \delta(\omega) \)
- \( \rho_{ii} \) is modified: delta peak gets smeared out
- Describe as Breit-Wigner peak + free part for large \( \omega \)

\( \Rightarrow \) Phenomenologically inspired ansatz

\[
\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega \Gamma}{\omega^2 + \left(\frac{\Gamma}{2}\right)^2} + \frac{2}{3\pi} (1 + k) \omega^2 \tanh\left(\frac{\omega}{4\pi}\right)
\]

with parameters \( c_{BW}, \Gamma, k \) (\( = \frac{\alpha_s}{\pi} \) at leading order)
- Nonperturbatively improved Wilson-Clover action
- No dynamical Sea Quarks
- Finite volume effects under control
- All quark masses $m_{\overline{MS}}(\mu = 2\text{GeV}) \sim O(10\text{MeV})$
- Nonperturbative renormalization constants
- Fixed aspect ratio $\frac{N_\sigma}{N_\tau} = 3$ for $T = 1.1T_c$ and $1.2T_c$
- Continuum extrapolation of the vector correlator

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Continuum extrapolation

Observables

- Form ratio \( \frac{T^2}{\chi_q} \frac{G_{ii}(\tau T)}{G_{V}^{\text{free, lat}}(\tau T)} \) with \( \chi_q = -\frac{G_{00}}{T} \)
  \( \Rightarrow \) Renormalization constant drops out
- Ratio to free lattice correlator reduces cutoff effects

Continuum limit

- Use natural cubic splines \( \Rightarrow N_T^{\text{max}} \) data points per \( T \)
- Linear extrapolation in \( a^2 \) due to Clover term
- Error estimation via bootstrap
Continuum extrapolation for each $\tau T$ in $1/N_T^2$

Well behaved extrapolation at all $\tau T$
Continuum extrapolation

\[ T^2 G_{ii}/[\chi q G_V^{\text{free}}] \]

\[ T = 1.1 T_c \]
Fitting the SPF

Fit

- Ansatz $\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega \Gamma}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} (1 + k) \omega^2 \tanh \left( \frac{\omega}{4T} \right)$
- fit parameters $c_{BW}, \Gamma, k$
- $k$ insensitive to low $\omega$ region
- Electrical conductivity $\sigma_T = \frac{2}{3} C_{em} \chi_q \frac{c_{BW}}{\Gamma}$
- Fit ansatz to continuum extrapolated ratio $\frac{T^2 \chi_q G_{ii}(\tau T)}{G_{\text{free}}^V(\tau T)}$
- Constrain fit further by curvature at midpoint

Numerics

- Standard gaussian quadrature for Breit-Wigner part
- Fit done using Levenberg-Marquardt minimization algorithm
Correlators reconstructed from the fits

\[ T^2 G_{ii} / [\chi_q G_V^{\text{free}}] \]
Analysis of systematics

Modifying the ansatz

- Multiply the high $\omega$ part with smoothed out $\Theta$-function
  $\Rightarrow$ control where the high $\omega$ behaviour sets in

![Graph showing spectral functions and dilepton rates](attachment:image.png)

- Breit-Wigner part compensates
  $\Rightarrow$ conductivity rises
- At some point the fit becomes poor
  $\Rightarrow$ error estimate
Electrical conductivity

Figure 5.19: Electrical conductivity for temperatures 1.1, 1.2, 1.3, 1.4, 1.5 or $T/T_c$, with systematic error estimates as laid out in section 5.6.2, with a maximal $\omega_0 = 1.5$ and $\Delta \omega = 0.1$, see also tables 5.5 and 5.6. To compare the temperature dependence, results are given in units of temperature $T$ (left) and in units of the critical temperature $T_c$ (right).

The ratios are smooth so they allow for a cubic spline interpolation on the coarser lattices. The continuum extrapolation is well behaved and removes lattice cutoff effects down to distances $\tau_T = 0.125$ at 1.1 $T_c$ and $\tau_T = 0.142$ at 1.2 $T_c$. Spectral functions have been extracted successfully from the continuum extrapolated correlators at all three temperatures by using a phenomenologically motivated ansatz. This rather simple ansatz, consisting of a Breit-Wigner peak and a continuum contribution, see eq. (5.7), is found to provide a good description of the data set at all three temperatures. A systematic error analysis was performed via a parametrized modification of the ansatz, by truncating the continuum contributions, see eq. (5.19). It is found that an increasing continuum cutoff can not be fully compensated by an enhanced Breit-Wigner peak, thus the truncated ansatz yields an inferior description of the data set. This allows to find an upper limit for the Breit-Wigner contribution. The spectral function is linked to the dilepton rate, which thus can be calculated for all three temperatures. A summarizing plot of the spectral functions and associated dilepton rates is provided in fig. 5.13.

In the low frequency limit, the spectral function also gives access to the electrical conductivity as an important transport coefficient. With the systematic error estimates in place, lower and upper bounds for the electrical conductivity have been calculated. Within these limits, the conductivity shows no clear temperature dependence, see fig. 5.19. The systematic error analysis – as currently employed – implies a lower limit for the electrical conductivity at each temperature. The upper limit is influenced by

Similar studies: also Wilson fermions, but w/o continuum limit

- A.Amato et al., arXiv:1307.6763
- B.B.Brandt et al., JHEP 1303 (2013) 100
\[ \frac{dW}{d\omega d^3p} = \frac{5\alpha^2}{54\pi^3} \frac{1}{\omega^2(e^{\omega/T} - 1)} \rho_V(\omega, T) \]
Conclusion

- Continuum extrapolation of correlator-ratios at three temperatures above $T_c$
- Ratios reconstructed from the SPF
- Results for electrical conductivity and dilepton rate

Outlook

- Also consider finite momenta → results for e.g. the photon rate
- Use dynamical fermions
- Reduce systematic errors → study temperature dependence of transport properties