Test of the Standard Model description of rare B decays using lattice QCD form factors

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1 Abstract

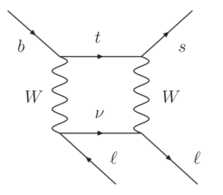
This poster reviews our recent calculation of $B \to K^*$, $B_s \to \phi$, and $B_s \to K^*$ form factors using nonrelativistic heavy quarks and improved staggered quarks on MILC lattices [1]. These unquenched calculations, performed in the low-recoil kinematic regime, provide a significant improvement over the use of extrapolated light cone sum rule results. We use the form factors along with Standard Model determinations of Wilson coefficients to give theoretical results for several observables [2]. Noting that the experimental measurements for the $B^0 \to K^{*0}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ branching fractions are smaller at low-recoil than the Standard Model predictions, we perform a fit of the relevant Wilson coefficients using experimental and lattice results. The favored values hint at deviations from the Standard Model that are consistent with fits done by other authors using complementary theoretical and experimental inputs.

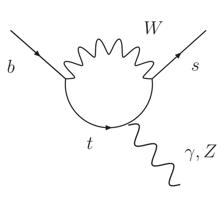
2 Theory

At energies well below the W mass, $b \to s$ decays are described by the effective Hamiltonian

$$\mathcal{H}_{\rm eff}^{b\to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i O_i \,.$$

The operators which dominate short-distance effects in the Standard Model are obtained as a consequence of box and penguin diagrams





$$O_7 = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_R b\, F_{\mu\nu}$$
$$O_9 = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_L b\, \bar{\ell}\gamma_\mu \ell$$
$$O_{10} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_L b\, \bar{\ell}\gamma_\mu\gamma^5 \ell$$

where $P_{L/R} = \frac{1}{2}(1 \mp \gamma^5)$ and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

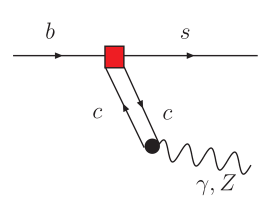
In the narrow-width approximation the $B \to K^*\ell^+\ell^-$ amplitude can be written as a sum of local ($\mathcal{A}_\mu$ & $\mathcal{B}_\mu$) and nonlocal ($\mathcal{T}_\mu$) terms.

$$\mathcal{M} = \frac{\alpha G_F V_{tb} V_{ts}^*}{\pi\sqrt{2}} \left[(\mathcal{A}_\mu + \mathcal{T}_\mu)\bar{u}_\ell \gamma^\mu v_\ell + \mathcal{B}_\mu \bar{u}_\ell \gamma^\mu\gamma^5 v_\ell\right]$$

with

$$\mathcal{A}_\mu = -\frac{2m_b}{q^2} q^\nu C_7 \langle K^*|\bar{s}\, i\sigma_{\mu\nu} P_R\, b|B\rangle + C_9 \langle K^*|\bar{s}\gamma_\mu P_L b\,|B\rangle$$
$$\mathcal{B}_\mu = C_{10}\langle K^*|\bar{s}\gamma_\mu P_L b\,|B\rangle$$
$$\mathcal{T}_\mu = -\frac{16 i\pi^2}{q^2} \sum_{i=1,\cdots,6,8} C_i \int d^4x\, e^{iq\cdot x} \langle K^*|\mathsf{T} O_i(0) j_\mu(x)|B\rangle$$

Long-distance effects arise from multiple sources, one of the most important being the production of charmonium resonances via current-current operators



$$O_1 = \bar{s}^\alpha \gamma^\mu P_L c^\beta\, \bar{c}^\beta \gamma_\mu P_L b^\alpha$$
$$O_2 = \bar{s}^\alpha \gamma^\mu P_L c^\alpha\, \bar{c}^\beta \gamma_\mu P_L b^\beta$$

where α and β are color indices.
At high q^2 an OPE in $1/q^2$ has been developed to approximate the matrix elements of the nonlocal operator by a series of matrix elements of local operators [3]

$$\mathcal{T}_\mu = -\, T_7(q^2)\frac{2m_b}{q^2}\, q^\nu \langle \bar{K}^*|\,\bar{s}\, i\sigma_{\mu\nu} P_R b\,|\bar{B}\rangle + T_9(q^2)\langle \bar{K}^*|\bar{s}\gamma_\mu P_L b|\bar{B}\rangle + \mathcal{O}(\alpha_s \Lambda_{\rm QCD}/m_b,\, \Lambda_{\rm QCD}^2/m_b^2,\, m_c^4/q^4)$$

The functions $T_7(q^2)$ and $T_9(q^2)$ are computed perturbatively.

3 Form factors

The $B \to V$ hadronic matrix elements of $b \to q$ currents are parameterized by a set of 7 independent form factors. The traditional basis is definied through

$$\langle V(k,\varepsilon)|\bar{q}\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma$$
$$\begin{aligned}\langle V(k,\varepsilon)|\bar{q}\gamma^\mu\gamma^5 b|B(p)\rangle &= 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right) \\ &\quad - A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left[(p+k)^\mu - \frac{m_B^2-m_V^2}{q^2}q^\mu\right]\end{aligned}$$
$$q^\nu\langle V(k,\varepsilon)|\bar{q}\sigma_{\mu\nu} b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau k^\sigma$$
$$\begin{aligned}q^\nu\langle V(k,\varepsilon)|\bar{q}\sigma_{\mu\nu}\gamma^5 b|B(p)\rangle &= iT_2(q^2)[(\varepsilon^*\cdot q)(p+k)_\mu - \varepsilon^*_\mu(m_B^2-m_V^2)] \\ &\quad + iT_3(q^2)(\varepsilon^*\cdot q)\left[\frac{q^2}{m_B^2-m_V^2}(p+k)_\mu - q_\mu\right]\end{aligned}$$

We find it more convenient to eliminate A_2 and T_3 in favor of 2 form factors from the helicity basis, A_{12} and T_{23}.

$$A_{12}(q^2) = \frac{(m_B+m_V)^2(m_B^2-m_V^2-q^2)A_1(q^2) - \lambda A_2(q^2)}{16 m_B m_V^2 (m_B+m_V)}$$
$$T_{23}(q^2) = \frac{m_B+m_V}{8m_B m_V^2}\left[\left(m_B^2+3m_V^2-q^2\right)T_2(q^2) - \frac{\lambda T_3(q^2)}{m_B^2-m_V^2}\right]$$

4 Current matching

Matching between lattice and $\overline{\rm MS}$ schemes is done perturbatively at $\mathcal{O}(\alpha_s, \Lambda_{\rm QCD}/m_b)$. Writing $J_0^A = (\bar{\psi}_q\Gamma^A\Psi_b)|_{\rm latt}$ and $J_1^A = -\frac{1}{2m_b}(\bar{\psi}_q\Gamma^A\boldsymbol{\gamma}\cdot\nabla\Psi_b)|_{\rm latt}$, with the abbreviation $\Gamma^A \in [\gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}, \sigma^{\mu\nu}\gamma^5]$, the matched current is

$$\mathcal{J}^A = Z_{\Gamma^A}J_0^A + J_1^A - \alpha_s\zeta_{10}^{(A)}J_0^A$$

Truncation of $\mathcal{O}(\alpha_s^2)$ is the dominant systematic uncertainty, estimated to be 4%.

5 Details of the lattice calculation

MILC lattices with 2+1 flavors of AsqTad improved staggered quarks. Meson propagators computed using the same AsqTad action for the light quarks and an $O(v^4)$ NRQCD action for the b quarks. Parameters of the calculation:

Ensemble	#conf	$N_x^3\times N_t$	sea m_ℓ/m_s	#src	valence m_ℓ/m_s	a^{-1}(GeV)
c007	2109	$20^3\times 64$	$0.007/0.05$	16872	$0.007/0.04$	$1.660(12)$
c02	2052	$20^3\times 64$	$0.02/0.05$	16416	$0.02/0.04$	$1.665(12)$
f0062	1910	$28^3\times 96$	$0.0062/0.031$	15280	$0.0062/0.031$	$2.330(17)$

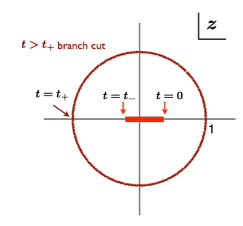
Results for meson masses:

Ensemble	m_B (GeV)	m_{B_s} (GeV)	m_π (MeV)	m_K (MeV)	m_{η_s} (MeV)	m_{K^*} (MeV)	m_ϕ (MeV)
c007	5.5439(32)	5.6233(7)	313.4(1)	563.1(1)	731.9(1)	1045(6)	1142(3)
c02	5.5903(44)	5.6344(15)	519.2(1)	633.4(1)	730.6(1)	1106(4)	1162(3)
f0062	5.5785(22)	5.6629(13)	344.3(1)	589.3(2)	762.0(1)	1035(4)	1134(2)
physical	5.279	5.366	140	495	686	892	1020

6 Form factor shape

We use a simplified series expansion to fit the numerical

Using $t = q^2$ and $t_\pm = (m_{B_{(s)}} \pm m_V)^2$, one constructs a dimensionless variable which is small,



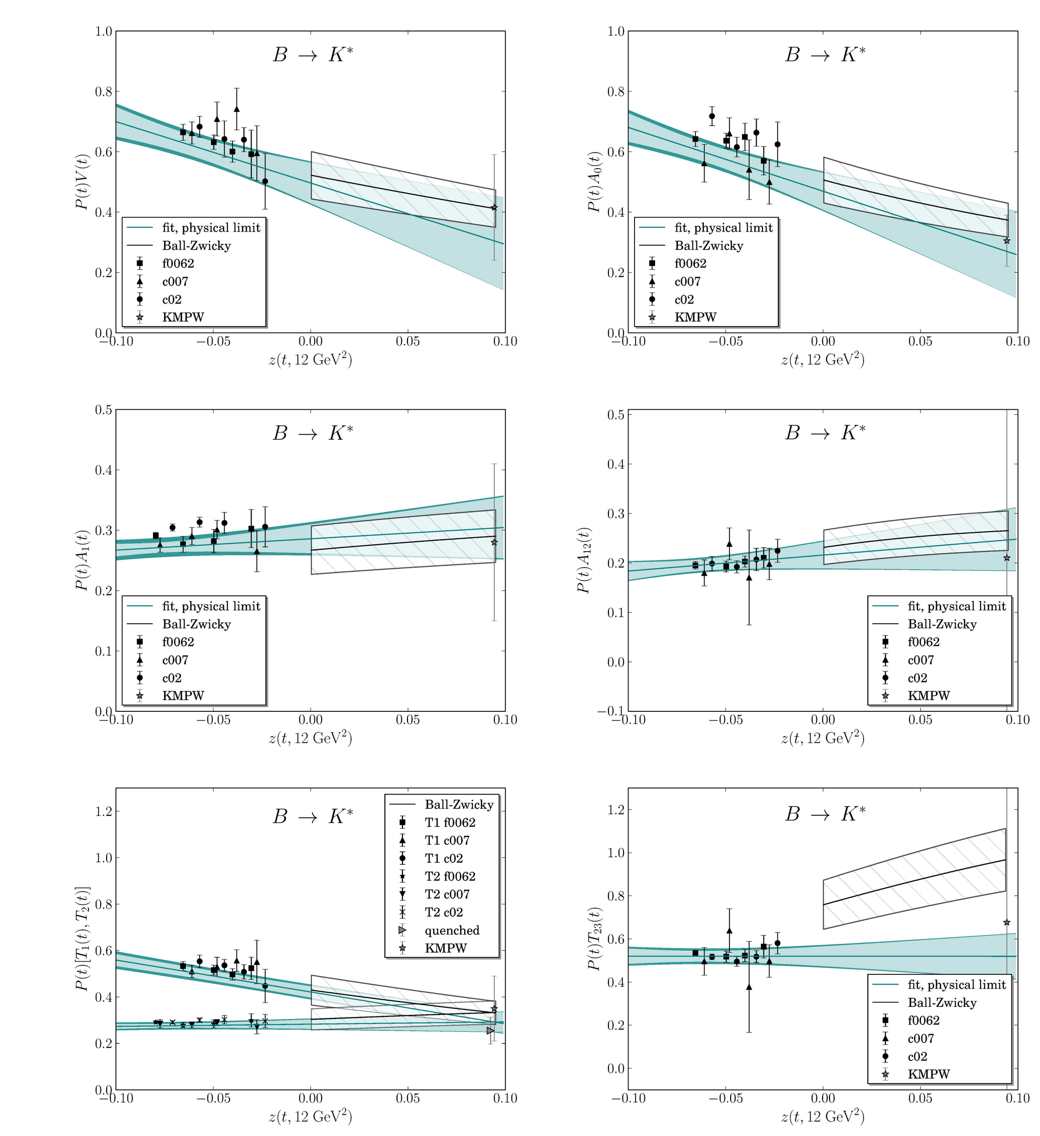
$$z(t,t_0) = \frac{\sqrt{t_+-t} - \sqrt{t_+-t_0}}{\sqrt{t_+-t} + \sqrt{t_+-t_0}}\,.$$

We fit the form factors $F = V, A_0, A_1, A_{12}, T_1, T_2, T_{23}$ to the following form:

$$F(t) = \frac{1}{1-t/m_{\rm pole}^2}[a_0(1 + c_{01}\Delta x + c_{01s}\Delta x_s) + a_1 z(t,t_0)]$$

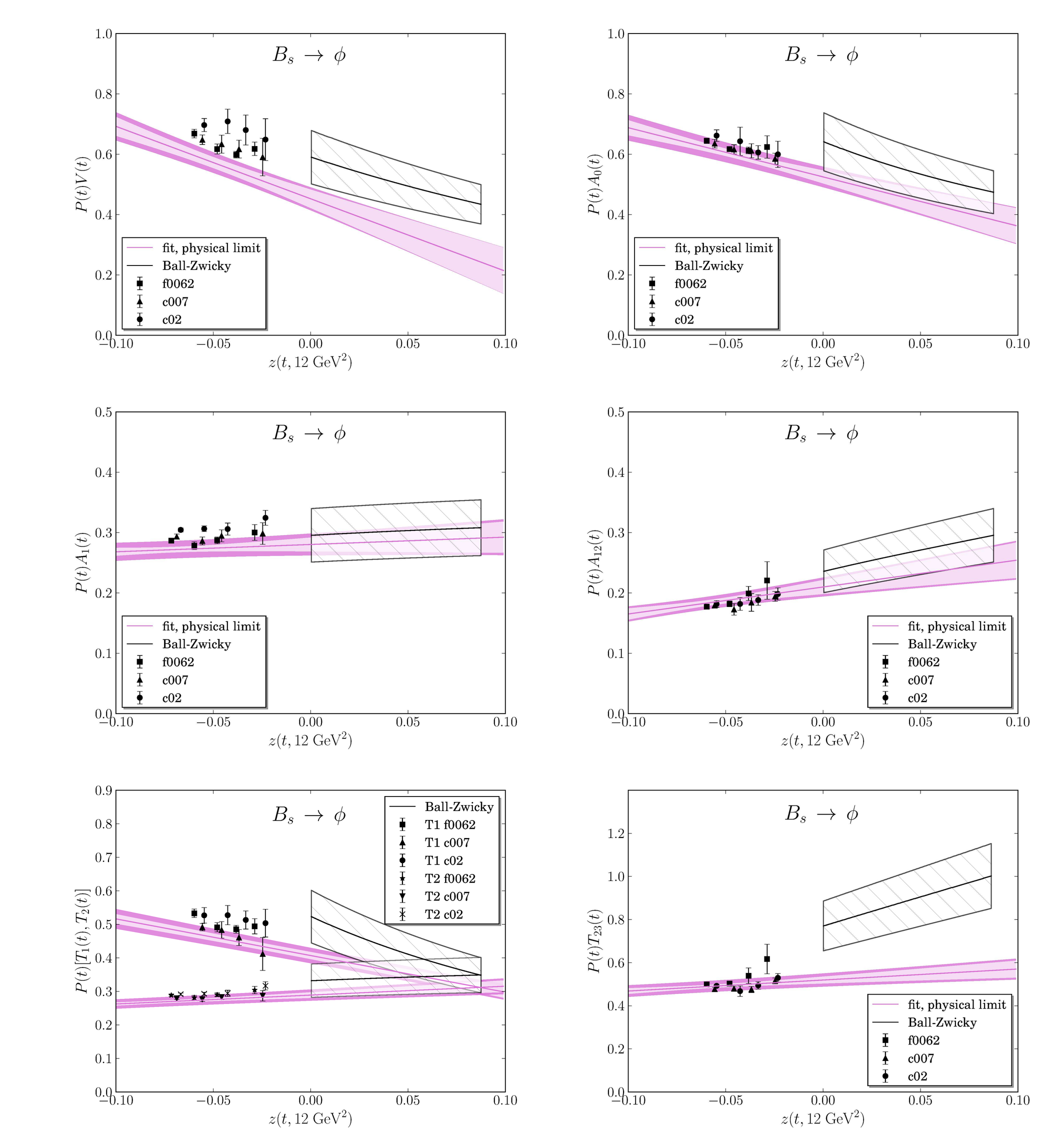
with $\Delta x = (m_\pi^2 - m_{\pi,{\rm phys}}^2)/(4\pi f_\pi)^2$ and $\Delta x_s = (m_{\eta_s}^2 - m_{\eta_s,{\rm phys}}^2)/(4\pi f_\pi)^2$.

7 $B \to K^*$ form factor results



Black points: Lattice results by ensemble. Solid curve: extrapolation to the physical quark mass limit with statistical (pale) and total (dark) error bands. Hatched band: LCSR results of [4] with a 15% uncertainty [5]. Gray star: LCSR [6]. Gray triangle: quenched lattice QCD [7].

8 $B_s \to \phi$ form factor results

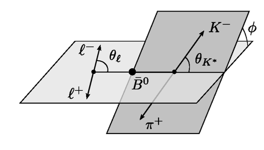


Points and curves as in Box 7.

9 Observables

Decay distribution for $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$:

$$\begin{aligned}\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi}\Big[& I_1^s \sin^2\theta_{K^*} + I_1^c\cos^2\theta_{K^*} \\ & + (I_2^s\sin^2\theta_{K^*} + I_2^c\cos^2\theta_{K^*})\cos 2\theta_\ell \\ & + I_3\sin^2\theta_{K^*}\sin^2\theta_\ell\cos 2\phi + I_4\sin 2\theta_{K^*}\sin 2\theta_\ell\cos\phi \\ & + I_5\sin 2\theta_{K^*}\sin\theta_\ell\cos\phi + (I_6^s\sin^2\theta_{K^*} + I_6^c\cos^2\theta_{K^*})\cos\theta_\ell \\ & + I_7\sin 2\theta_{K^*}\sin\theta_\ell\sin\phi + I_8\sin 2\theta_{K^*}\sin 2\theta_\ell\sin\phi \\ & + I_9\sin^2\theta_{K^*}\sin^2\theta_\ell\sin 2\phi\Big],\end{aligned}$$



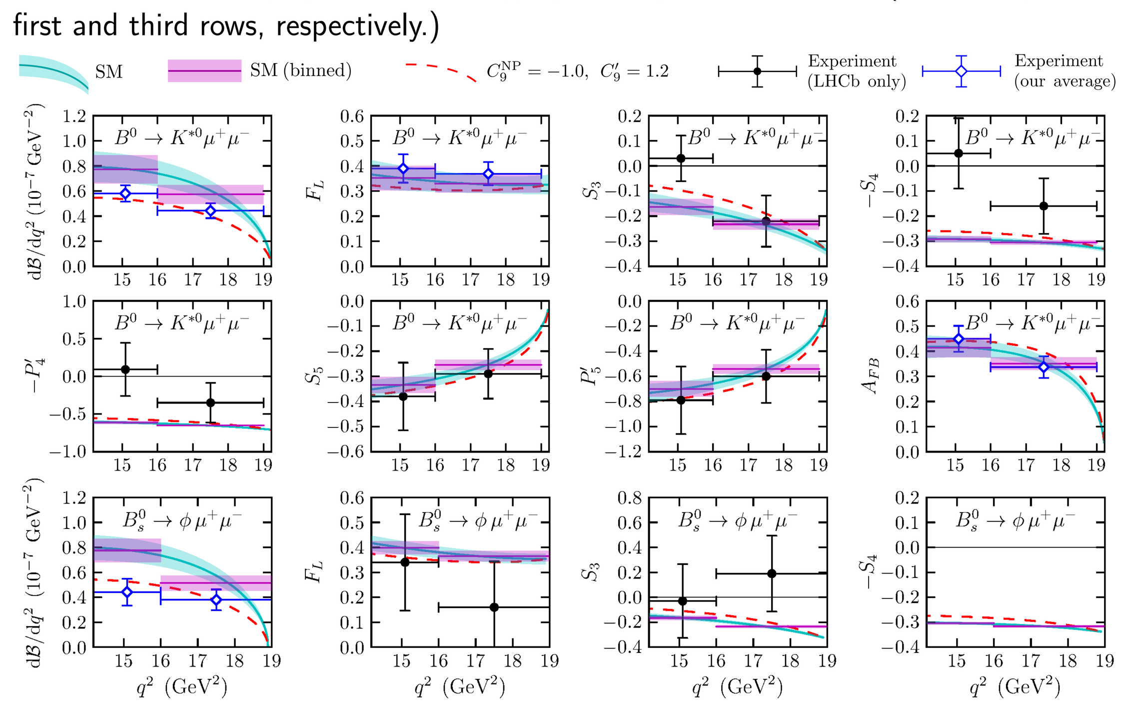
Similar for CP-conjugated mode, with $I_{1,2,3,4,7}^{(a)} \mapsto \bar{I}_{1,2,3,4,7}^{(a)}$, $I_{5,6,8,9}^{(a)} \mapsto -\bar{I}_{5,6,8,9}^{(a)}$.
Differential decay rate: $d\Gamma/dq^2 = \frac{3}{4}(2I_1^s + I_1^c) - \frac{1}{4}(2I_2^s + I_2^c)$. CP averages and CP aysymmetries:

$$S_i^{(a)} = \frac{I_i^{(a)} + \bar{I}_i^{(a)}}{{\rm d}(\Gamma+\bar{\Gamma})/{\rm d}q^2} \qquad A_i^{(a)} = \frac{I_i^{(a)} - \bar{I}_i^{(a)}}{{\rm d}(\Gamma+\bar{\Gamma})/{\rm d}q^2} \qquad \langle P'_{4,5,6,8}\rangle = \frac{\langle S_{4,5,7,8}\rangle}{2\sqrt{-\langle S_2^c\rangle\langle S_2^s\rangle}}$$

where $\langle\cdot\rangle$ indicates binning over a range of q^2.

10 Theory vs. experimental results

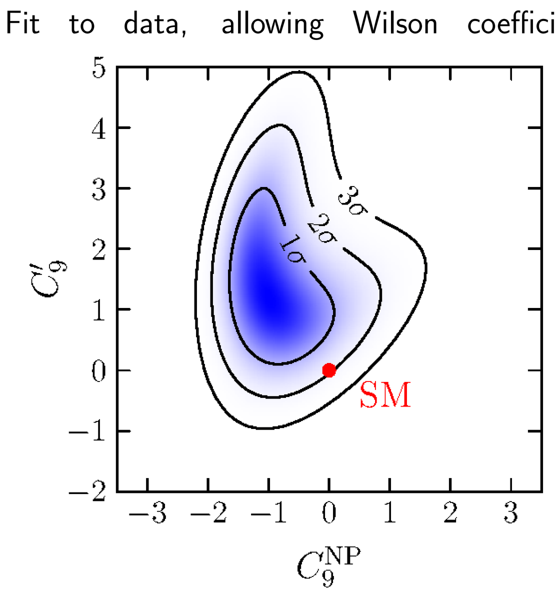
Standard Model (SM) predictions using these lattice QCD form factors are compared to experimental measurements. Note that the differential decay rate $d\Gamma/dq^2$ is lower for both $B^0 \to K^{*0}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ compared to the SM. (Leftmost plots in first and third rows, respectively.)



In Box 11 we show that a better fit to the data is achieved by allowing the Wilson coefficients C_9 and C_9' to vary from their SM values. The red dashed curve above gives the theory predictions for our preferred values.

11 Beyond the Standard Model

Fit to data, allowing Wilson coefficients to differ from their SM values

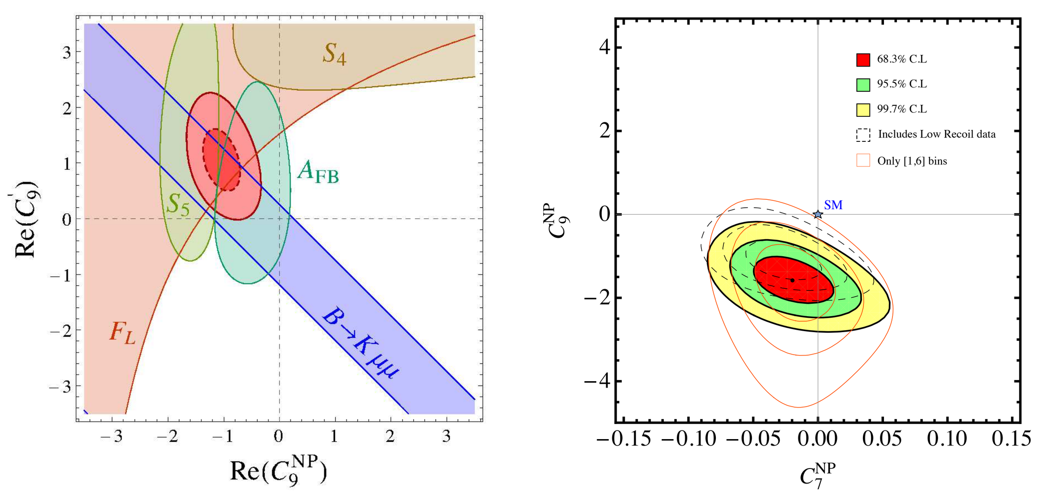


$C_0^{\rm NP}$ is the deviation from the SM C_9. C_9' multiplies the matrix element of the chirality-flipped operator

$$O_9' = \frac{e^2}{16\pi^2}\bar{s}\gamma^\mu P_R b\,\bar{\ell}\gamma_\mu\ell$$

No deviation from SM was seen for $O_7(')$ or $O_{10}(')$.

12 Comparison to other analyses



Figures from [8] (left) and [9] (right). A Bayesian analysis of $b \to s$ data treating theoretical uncertianties as nuisance parameters finds values for Wilson coefficients which are consistent with the Standard Model and with our best fit values [10].

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