# Chiral Properties of Pseudoscalar Meson in Lattice QCD with Domain-Wall Fermion

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# Introduction

Lattice QCD with exact chiral symmetry [1] provides an ideal framework to study the nonperturbative physics from the first principles of QCD. But it is rather nontrivial to perform HMC simulation to preserve high precision in chiral symmetry and also sample all topological sectors ergodically.

In this work, we study the chiral properties of the pseudoscalar meson in 2-flavors lattice QCD with optimal DWF [2]. We calculate the mass and the decay constant of the pseudoscalar meson, and compare our results with the NLO chiral perturbation theory (ChPT) [3].

[1] Kaplan ('92); Neuberger ('98); Narayanan & Neuberger ('95).

[2] Chiu ('03).

[3] Gasser & Leutwyler ('85).

# Introduction

Mathematically, optimal DWF [2] is a theoretical framework which preserves the chiral symmetry optimally with a set of analytical weights, $\{\omega_s\}$, $s = 1, \ldots, N_s$, one for each layer in the 5th dimension, such that the chiral symmetry breaking is reduced to minimum.

The 4D effective Dirac operator of massless optimal DWF is

$$D = m_0[1 + \gamma_5 S_{\rm opt}(H_w)], \quad S_{\rm opt}(H_w) = \frac{1 - \prod_{s=1}^{N_s} T_s}{1 + \prod_{s=1}^{N_s} T_s}, \quad T_s = \frac{1 - \omega_s H_w}{1 + \omega_s H_w}$$

which is exactly equal to Zolotarev optimal rational approximation of the overlap Dirac operator, i.e., $S_{\rm opt}(H_w) = H_w R_Z(H_w)$, where $R_Z(H_w)$ is the optimal rational approximation of $(H_w^2)^{-1/2}$.

We compute the 4D valence quark propagator via

$$\mathcal{D}(m_q)\,|Y\rangle = \mathcal{D}(2m_0)B^{-1}\,|\text{source vector}\rangle\,, \tag{1}$$

$$(D_c + m_q)^{-1}_{x,x'} = (2m_0 - m_q)^{-1}[(BY)_{x,1;x',1} - \delta_{x,x'}], \tag{2}$$

$$B^{-1}_{x,s';x',s'} = \delta_{x,x'}(P_-\delta_{s,s'} + P_+\delta_{s+1,s'}) \tag{3}$$

# Introduction

Simulation setups:

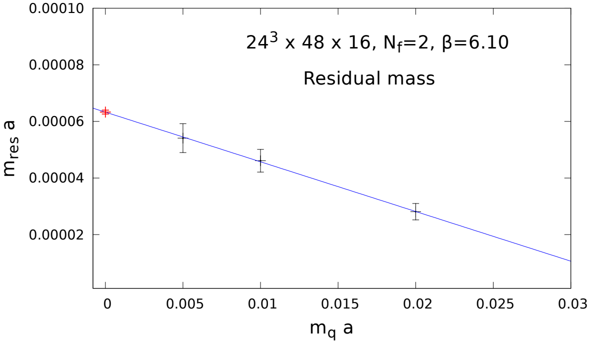
- We perform 2-flavors LQCD simulation with optimal DWF on a GPU cluster (see Ting-Wai Chiu's talk, on Mon 16:30, for more details).
  - Chiral symmetry is preserved to a good precision.
  - All topological sectors are sampled ergodically.
- Lattice size: $24^3 \times 48$ with $N_s = 16$.
- Gauge action: plaquette gauge action at $\beta = 6.1$.
- For each sea quark mass, we generate 3380 (for $m_q a = 0.005$) and $\geq 5000$ (for other $m_q a$) trajectories after thermalization.
- From the saturation of the binning errors of the plaquette, and the evolution of the top. charge, we estimate the autocorrelation time to be around 10 trajectories. Thus we sample one conf. every 10 trajatories.

| $m_q a$ | 0.005 | 0.01 | 0.02 |
|---|---|---|---|
| $N_{\mathrm{confs.}}$ | 338 | 506 | 501 |
| $a^{-1}$ (GeV) | 3.203(17) | 3.138(21) | 3.044(18) |
| $m_{\mathrm{res}}$ (MeV) | 0.173(16) | 0.145(13) | 0.086(9) |

# Introduction

To measure the chiral symmetry breaking due to finite $N_s$, we compute the residual mass with the formula [4] Chen & Chiu ('12)

$$m_{\rm res} = \frac{\left\langle {\rm tr}(D_c + m_q)^{-1}_{0,0} \right\rangle_{\{U\}}}{\left\langle {\rm tr}[(D_c^\dagger + m_q)(D_c + m_q)]^{-1}_{0,0} \right\rangle_{\{U\}}} - m_q \tag{4}$$




Using linear fit, we obtain the $m_{\rm res}$ in the chiral limit: $m_{\rm res}a = 6.329(68) \times 10^{-5}$, $m_{\rm res} = 0.204(3)$ MeV.

# Time Correlation Function

We compute the time-correlation function of the pseudoscalar interpolator and its effective mass:

$$C(t) = \sum_{\vec{x}} \mathrm{tr}[\gamma_5 (D_c + m_q)^{-1}_{0,x} \gamma_5 (D_c + m_q)^{-1}_{x,0}] \tag{5}$$

$$m_{\rm eff}(t) = \ln[A(t)/A(t+1)], \quad A(t) = C(t) + \sqrt{(C(t))^2 - (C(T/2))^2} \tag{6}$$

We fit $\langle C(t) \rangle$ to the formula

$$Z[e^{-M_\pi t} + e^{-M_\pi (T-t)}]/(2M_\pi) \tag{7}$$

to extract the pion mass $M_\pi$ and decay constant $F_\pi = m_q \sqrt{2Z}/M_\pi^2$.

# Preliminary Results

We make the correction for the finite volume effect using the estimate within ChPT calculated up to $\mathcal{O}(M_\pi^4/(4\pi F_\pi)^4)$ [5] ($M_\pi L \simeq 2.15$ for our lightest pion). The errors are estimated by jackknife method with the bin size of 20 confs. of which the statistical error saturate.

| $m_q a$ | $[t_1, t_2]$ | $\chi^2/\mathrm{dof}$ | $M_\pi$ [GeV] | $F_\pi$ [GeV] | $1 + R_{M_\pi}$ | $1 + R_{F_\pi}$ |
|---|---|---|---|---|---|---|
| 0.005 | [17-21] | 0.806 | 0.255(21) | 0.0956(71) | 1.123 | 0.709 |
| 0.010 | [18-21] | 1.036 | 0.347(13) | 0.1003(25) | 1.048 | 0.894 |
| 0.020 | [18-23] | 1.591 | 0.479(9) | 0.1056(28) | 1.014 | 0.971 |

[5] Colangelo, Durr, Haefeli ('05)

# Preliminary Results

With the finite volume correction for $M_\pi^2/m_q$ and $F_\pi$, we fit our data to the formulas of NLO ChPT [6]

$$\frac{M_\pi^2}{m_q} = \frac{2\Sigma}{F^2}\left[1+\left(\frac{\Sigma m_q}{16\pi^2 F^4}\right)\ln\left(\frac{2\Sigma m_q}{F^2\Lambda_3^2}\right)\right] \quad (8)$$

$$F_\pi = F\left[1-\left(\frac{\Sigma m_q}{8\pi^2 F^4}\right)\ln\left(\frac{2\Sigma m_q}{F^2\Lambda_4^2}\right)\right] \quad (9)$$

where

$$\bar{l}_3 = \ln\left(\frac{\Lambda_3^2}{m_{\pi^\pm}^2}\right), \quad \bar{l}_4 = \ln\left(\frac{\Lambda_4^2}{m_{\pi^\pm}^2}\right), \quad m_{\pi^\pm} = 0.140 \text{ GeV} \quad (10)$$

[6] Gasser & Leutwyler ('85)

# Preliminary Results

The strategy of our data fitting is to search for the values of the parameters $\Sigma$, $F$, $\Lambda_3$, and $\Lambda_4$ such that they minimize

$$\chi^2 = \sum_i V_i^T C_i^{-1} V_i \,, \tag{11}$$

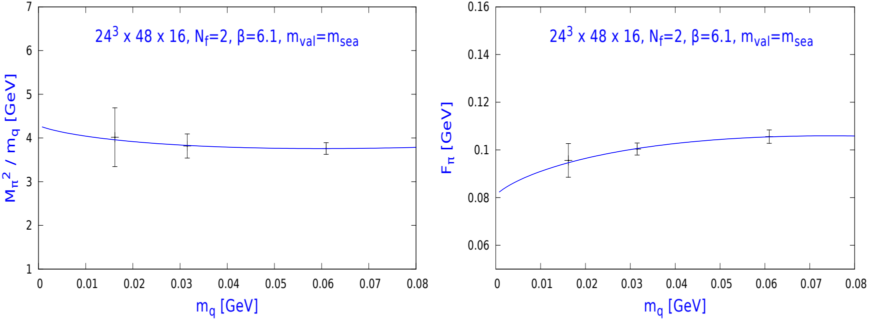
where $i$ is the index of sea-quark masses, and $C_i$ is the $2 \times 2$ covariance matrix for $M_\pi^2/m_q$ and $F_\pi$,

$$C_i = \begin{pmatrix} (\delta_1)_i & (\delta_3)_i \\ (\delta_3)_i & (\delta_2)_i \end{pmatrix} , \quad V_i = \begin{pmatrix} \langle M_\pi^2/m_q \rangle_i - (M_\pi^2/m_q)_i^{\mathrm{ChPT}} \\ \langle F_\pi \rangle_i - (F_\pi)_i^{\mathrm{ChPT}} \end{pmatrix}$$

$$\begin{aligned}
(\delta_1)_i &= \left\langle \left( (M_\pi^2/m_q)_i - \langle M_\pi^2/m_q \rangle_i \right)^2 \right\rangle \\
(\delta_2)_i &= \left\langle \left( (F_\pi)_i - \langle F_\pi \rangle_i \right)^2 \right\rangle \\
(\delta_3)_i &= \left\langle \left( (M_\pi^2/m_q)_i - \langle M_\pi^2/m_q \rangle_i \right) \left( (F_\pi)_i - \langle F_\pi \rangle_i \right) \right\rangle
\end{aligned}$$

# Preliminary Results

Physical results of 2 flavor QCD with optimal DWF: (a) $M_\pi^2/m_q$, and (b) $F_\pi$. The solid lines are the simultaneous fits to the NLO ChPT for three sea quark masses ($m_q a = 0.005 - 0.02$).




(a) (b)

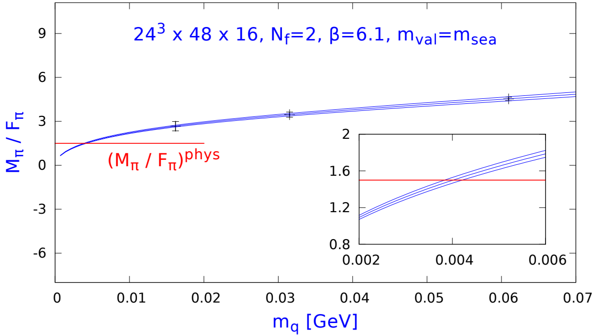
# Preliminary Results

For 3 sea-quark masses (corresponding to $255 \leq M_\pi \leq 479$ MeV), our fit gives

$$\Sigma = [0.2412(19)(32)\ \mathrm{GeV}]^3\,, \qquad \bar{l}_3 = 3.57(15)(51)\,,$$
$$F = 0.0809(10)(46)\ \mathrm{GeV}\,, \qquad \bar{l}_4 = 3.79(8)(20) \qquad (12)$$

To obtain the physical bare quark mass, we use the physical ratio $(M_\pi/F_\pi)^{\mathrm{phys}} = 0.135/0.093 = 1.45$ as the input, we have

$$m_q^{\mathrm{phys}} = 3.95(20)(23)\ \mathrm{MeV} \qquad (13)$$



# Summary

- We perform a 2-flavors lattice QCD simulation on a $24^3 \times 48 \times 16$ lattice with optimal DWF, for sea-quark masses $m_q a = 0.005, 0.01, 0.02$.
- We measure the time-correlation function of pseudoscalar meson from the quark propagators with $m_{\rm val} = m_{\rm sea}$, from which we extract the mass and the decay constant of the pseudoscalar meson.
- Our results of mass and the decay constant of the pseudoscalar meson versus the sea-quark mass are in good agreement with the prediction of the NLO ChPT.
- Our results of ChPT fit are

$$\Sigma = [0.2412(19)(32)\ \mathrm{GeV}]^3 , \qquad \bar{l}_3 = 3.57(15)(51) ,$$
$$F = 0.0809(10)(46)\ \mathrm{GeV} , \qquad \bar{l}_4 = 3.79(8)(20)$$

- Using the input $(M_\pi / F_\pi)^{\rm phys}$ and Eq.(8), (9), we obtain the physical bare quark mass, pion decay constant, and the pion mass:

$$m_q^{\rm phys} = 3.95(20)(23)\ \mathrm{MeV} , \ F_\pi^{\rm phys} = 86.3(1)(4)\ \mathrm{MeV} , \ M_\pi^{\rm phys} = 131(4)(3)\ \mathrm{MeV}$$