# The effective Polyakov loop theory for finite temperature Yang-Mills theory and QCD

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# Effective Polyakov loop theory for QCD thermodynamics

$$\exp[-S_{\mathrm{eff}}] \equiv \int [dU_k]\ \exp\left[-S_g(\beta)\right] \prod_{f=1}^{N_f} \det\left[Q^f(\kappa)\right]$$

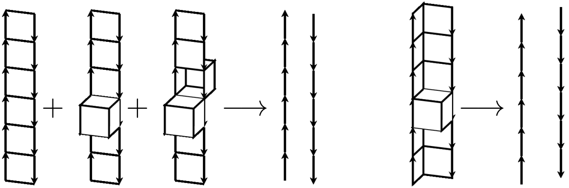
$$-S_g = \frac{\beta}{2N_c} \sum_p \left[\mathrm{tr}\, U_p + \mathrm{tr}\, U_p^\dagger\right]$$

- integration of spatial links $U_k$; three dimensional theory
  $U_\mu(x,t) \rightarrow U_0(x) \rightarrow$ Polyakov loops $L(x)$

$$Z = \int [dL] e^{-S_{\mathrm{eff}}[L]}$$

$\Rightarrow$ simulations of effective theory allow to circumvent/reduce the sign problem at finite chemical potential

# The improved strong coupling approach



$$S_{\text{eff}} = \lambda_1 S_{\text{nearest neighbors}} + \lambda_2 S_{\text{next to nearest neighbors}} + \dots$$

- ordering in expansion parameter $u = \frac{\beta}{18} + \dots < 1$
- simplest, most relevant contribution:

$$e^{-S_{\text{eff}}} \approx \prod_{<i,j>\ \text{nearest n.}} \left(1 + 2\lambda_1 \Re(L_i L_j^\dagger)\right)$$

- $\lambda_1$: resummed high orders in $u$

# Hopping parameter expansion

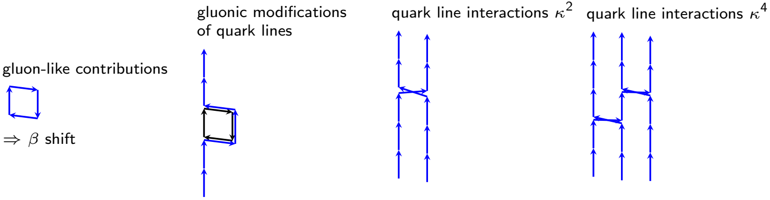
- Wilson-Dirac operator: $Q = 1 - \kappa H[U]$
- $H = T + S$ with temporal $T$ and spatial $S$ hopping
- static contribution with resummation of windings

$$\det(1 - \kappa T) = \prod_n (1 + cL_n + c^2 L_n^\dagger + c^3)^2 (1 + \bar{c} L_n^\dagger + \bar{c}^2 L_n + \bar{c}^3)^2$$

- chemical potential $\mu$ in fugacity factor

$$c = (2\kappa e^{a\mu})^{N_\tau} = \exp(\frac{\mu - m_{stat}}{T}); \quad \bar{c} = (2\kappa e^{-a\mu})^{N_\tau}$$

# Hopping parameter expansion: kinetic part



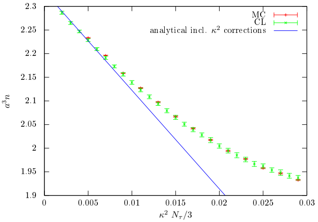

$$\det Q = \det(1-\kappa T)\,\det(1-(1-\kappa T)^{-1}S)$$

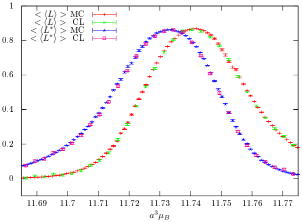
- kinetic term $\det(1-(1-\kappa T)^{-1}S)$: expansion in $\kappa$
- gluonic modifications by plaquette contributions $(u)$

⇒ interactions up to $\kappa^n + u^m$, $m+n=4$ included

⇒ $\kappa^n$ contributions can be automatized for higher $n$

# "Solution" of the sign problem: Numerical and analytic investigations of the effective theory



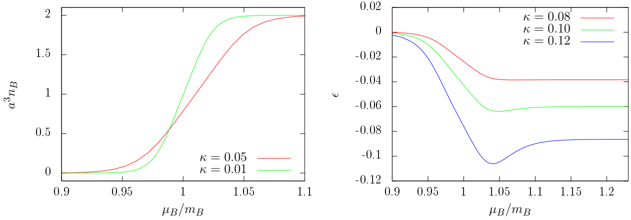

- effective theory inherits only mild version of sign problem
- solution 1: standard MC simulations and reweighting
- solution 2: complex Langevin algorithm
- correctness criteria checked, consistent results
- solution 3: perturbative expansion in effective couplings

# Low temperature limit in the heavy dense regime

- low temperature: $N_\tau$ large; heavy: $\kappa \ll 1$; dense: $c \approx 1$; $\bar{c} \approx 0$

$\Rightarrow$ dominated by short range quark line interactions

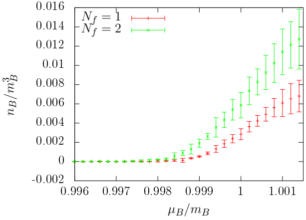
- physics of the nuclear liquid-gas transition

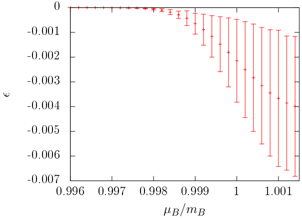



Features:

- $n_B$: silver blaze and saturation by Pauli exclusion principle
- nuclear binding energy $\epsilon = \frac{e - n_B m_B}{n_B m_B}$: negative due to attractive quark interaction

# Continuum limit in the heavy dense low temperature regime




- continuum limit $a \to 0$ at fixed $\frac{m_B}{T}$ and $T = \frac{1}{aN_\tau}$ requires larger value of $\kappa$
- truncation error: difference of $\kappa^2$ and $\kappa^4$ results
- main features persist in the continuum limit

$\Rightarrow$ higher orders in hopping expansion of particular importance for the continuum limit

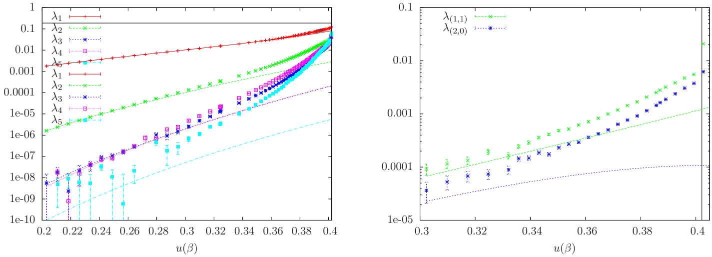
# Numerical determination of effective couplings

- strong coupling motivated form of effective action

$$e^{-S_{\mathrm{eff}}} = \prod_{x,i=1,\ldots 3} \Big(1 + \lambda_1 (L(x)L(x+\hat{i})^\dagger + L(x)^\dagger L(x+\hat{i}))\Big)$$
$$\prod_{[x,y]} \Big(1 + \lambda_2 (L(x)L(y)^\dagger + L(x)^\dagger L(y))\Big) \ldots$$

- include in the same way: long range interactions, different representations of Polyakov loops, n-point interactions
- assumptions: effective couplings $<< 1$, long range interactions and higher representations suppressed
- coupling constants of the effective theory can be approximately related to correlators of Polyakov loops
- allows to estimate the couplings and check the strong coupling results

# Numerical results for the effective couplings

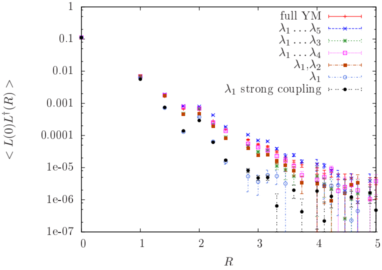



- next neighbor interaction: strong coupling result good approximation in wide parameter range
- long range interaction: strong coupling result less precise
- interactions of higher representations, higher n-point terms same size as long range interactions of fundamental Polyakov loop

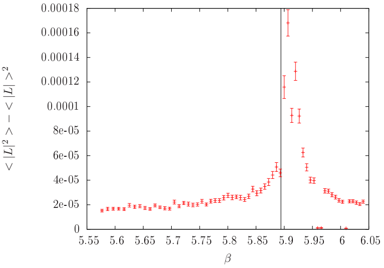
# Relevance of the difference to strong coupling approach

Depends on observable:

- correlators of fundamental Polyakov loops: significant influence of long range two point fund. interactions
- phase transition: dominated by short range interactions
- thermodynamics: higher rep. as important as long range two point fund. interactions




$\beta = 5.4$; $N_\tau = 4$




$N_\tau = 6$; two coupling model

# Conclusions

- effective Polyakov loop theory allows to circumvent the sign problem by analytic and numerical methods
- derivation of effective theory: improved strong coupling and hopping parameter expansion
- advantage of this approach: functional dependence of effective couplings on the bare parameters
- interesting application: heavy quark nuclear liquid-gas transition
- approach can be improved/checked using the effective couplings obtained in numerical simulations of the full theory
- implications of the differences to strong coupling approach depend on the observables
- If the small corrections of long range interactions of fund. loops are considered important for thermodynamics other, more involved, terms should be considered as well.