Hadron masses from fixed topological simulations: parity partners and SU(2) Yang-Mills results

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Outline

1. Introduction
   - BCNW-equation for the mass
   - Improvement of BCNW-equation
2. Considering parity mixing
   - Parity mixing : $\theta \neq 0$ and fixed topology
   - Consequences on mass extractions
3. Application to SU(2) Yang-Mills theory
   - Test method and parameters
   - Results

Outline

Mass relation

- Relation between mass in topological sector Q and mass of QCD $(\theta = 0)$ [1].

$$\boxed{M_Q = M(0) + \frac{M^{(2)}(0)}{2\chi_t V}\left(1 - \frac{Q^2}{\chi_t V}\right) + \mathcal{O}\left(\frac{1}{(\chi_t V)^2}\right)}$$

Conditions:

(C1) $1/\chi_t V \ll 1$ and $|Q|/\chi_t V \ll 1$: Taylor expansion and saddle point approximation.

(C2) $\left|M_H^{(2)}(0)t\right| / \chi_t V \ll 1$: Taylor expansion.

[1] R. Brower, S. Chandrasekharan, J.W. Negele, U.-J. Wiese, Phys. Lett. B560 (2003) 64

Extracting the mass

- Relation between mass in topological sector Q and mass of QCD ($\theta = 0$)

$$\boxed{M_Q = M(0) + \frac{M^{(2)}(0)}{2\chi_t V}\left(1 - \frac{Q^2}{\chi_t V}\right) + \mathcal{O}\left(\frac{1}{(\chi_t V)^2}\right)}$$

Method to extract the mass from fixed topological simulation.

- Compute $M_{Q,V}$ or equivalently $C_{Q,V}$ for different physical volumes and topological sectors ($C_{Q,V}$ two-point function at fixed topology and volume V)
- Fit the BCNW-Equation to those results (3 parameters $M(0)$, $M^{(2)}(0)$ and χ_t)
- Extracting $M(\theta = 0)$ and χ_t

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Motivations for improvements up to $1/(\chi_t V)^3$

Motivation to improve by $\mathcal{O}(1/(\chi_t V)^2)$ and $\mathcal{O}(1/(\chi_t V)^3)$

1. Complete $\mathcal{O}(1/(\chi_t V)^2)$ order in BCNW-equation.
2. Increase precision
3. Important when $\chi_t V$ is not to large e.g $\chi_t V \gtrsim 1$.
4. Helpful to estimate the error of $\mathcal{O}(1/(\chi_t V)^2)$ expansion.

Literature: General discussion of n-point functions at fixed topology including also higher orders in 1/V. [2]

Our contribution: Expansion of two-point correlation function up to $\mathcal{O}(1/(\chi_t V)^3)$

[2] S. Aoki, H. Fukaya, S. Hashimoto and T. Onogi, Phys. Rev. D 76, 054508 (2007)

Improvement

Improved equation [3]

$$C_{Q,V}(t) = \alpha(0)\exp\Bigg(- M_H(0)t - \frac{1}{\mathcal{E}_2 V}\frac{x_2}{2} - \frac{1}{(\mathcal{E}_2 V)^2}\left(\frac{x_4 - 2(\mathcal{E}_4/\mathcal{E}_2)x_2 - 2x_2^2}{8} - \frac{x_2}{2}Q^2\right)$$
$$- \frac{1}{(\mathcal{E}_2 V)^3}\Bigg(\frac{16(\mathcal{E}_4/\mathcal{E}_2)^2 x_2 + x_6 - 3(\mathcal{E}_6/\mathcal{E}_2)x_2 - 8(\mathcal{E}_4/\mathcal{E}_2)x_4 - 12x_2 x_4}{48}$$
$$+ \frac{18(\mathcal{E}_4/\mathcal{E}_2)x_2^2 + 8x_2^3}{48} - \frac{x_4 - 3(\mathcal{E}_4/\mathcal{E}_2)x_2 - 2x_2^2}{4}Q^2\Bigg)\Bigg)$$
$$+ \mathcal{O}\left(\frac{1}{(\mathcal{E}_2 V)^4}, \frac{1}{(\mathcal{E}_2 V)^4}Q^2, \frac{1}{(\mathcal{E}_2 V)^4}Q^4\right).$$

Improvement cost

- Increasing the number of parameters (3 for BCNW-equations, 8 for $1/(\chi_t V)^2$, 11 for $1/(\chi_t V)^3$)

[3] A.D, M. Wagner: arXiv:1404.0247

Improvement (2)

Problem for limited statistical accuracy

Possibility to benefit of the improvement while keeping the number of parameters small.$\Rightarrow$ New parameters set to zero

$$C_{Q,V}(t) = \frac{\alpha(0)}{\sqrt{1+M_H^{(2)}(0)t/\chi_t V}} \exp\left(-M_H(0)t - \frac{1}{\chi_t V}\left(\frac{1}{1+M_H^{(2)}(0)t/\chi_t V} - 1\right)\frac{1}{2}Q^2\right)$$

- Evidence of the improvement in a toy-model [4] (quantum mechanics on a circle with well potential)

[4] A.D, M. Wagner: arXiv:1404.0247

Outline

Parity mixing due to the θ-term

$$S_E(\theta) = S_E - i\theta Q = S_E - i\theta \frac{1}{32\pi^2} \int d^4x F_{\mu\nu} \tilde{F}_{\mu\nu}$$

- The second term violates parity symmetry P
- Fixed topology superposition (Fourier transform) of theories with $\theta \neq 0 \Rightarrow$ not P invariant

Consequences

- Consider two states, which are parity partners:
  The heavier state has to be considered as an excitation, while the lighter is the ground state.
- A single correlator is generally not sufficient to determine the mass of an excited state precisely:
  $\Rightarrow$ use a correlation matrix.

Outline

Parity mixing at fixed Q

- Consider O_1 and O_2 with opposite parity.

$$C_Q = \begin{pmatrix} \langle O_1^\dagger(t) O_1(0)\rangle_Q & \langle O_1^\dagger(t) O_2(0)\rangle_Q \\ \langle O_2^\dagger(t) O_1(0)\rangle_Q & \langle O_2^\dagger(t) O_2(0)\rangle_Q \end{pmatrix}$$

If we neglect terms of order $\mathcal{O}\left(\frac{1}{(\chi_t V)^2}\right)$

$$\langle O_1^\dagger(t) O_1(0)\rangle_Q \approx a_{11} e^{-M_{H_1}(0)t}\left(1 - \frac{M_{H_1}^{(2)}(0)t}{2\chi_t V}\right) + \frac{b_{22}}{\chi_t V} e^{-M_{H_2}(0)t}$$

$$\langle O_1^\dagger(t) O_2(0)\rangle_Q \approx \frac{iQa_{12}}{\chi_t V} e^{-M_{H_1}(0)t} + \frac{iQb_{12}}{\chi_t V} e^{-M_{H_2}(0)t}$$

$$\langle O_2^\dagger(t) O_1(0)\rangle_Q \approx \frac{iQa_{21}}{\chi_t V} e^{-M_{H_1}(0)t} + \frac{iQb_{21}}{\chi_t V} e^{-M_{H_2}(0)t}$$

$$\langle O_2^\dagger(t) O_2(0)\rangle_Q \approx \frac{a_{22}}{\chi_t V} e^{-M_{H_1}(0)t} + b_{22} e^{-M_{H_2}(0)t}\left(1 - \frac{M_{H_2}^{(2)}(0)t}{2\chi_t V}\right)$$

Masses at fixed Q

Now let us assume that $M_{H_1}(\theta = 0) < M_{H_2}(\theta = 0)$:

The previous results yield:

- For H_1:
  $\langle O_1^\dagger(t) O_1(0) \rangle_Q =$
  $$a_{11} e^{\left(-M_{H_1}(0) t\right)} \left(1 - \frac{M_{H_1}^{(2)}(0) t}{2\chi_t V}\right) + \mathcal{O}\left(\frac{e^{-M_{H_2} t}}{\chi_t V}, \frac{1}{(\chi_t V)^2}\right)$$
  $\Rightarrow$ For large t, we can use what we have done in previous works

- For H_2:
  $$\langle O_2^\dagger(t) O_2(0) \rangle_Q = \frac{a_{22}}{\chi_t V} e^{\left(-M_{H_1}(0) t\right)} + \mathcal{O}\left(e^{-M_{H_2} t}, \frac{1}{(\chi_t V)^2}\right)$$
  $\Rightarrow$ Extremely difficult to extract M_2, needs a lot of statistics
  $\Rightarrow$ Use the correlation matrix $\rightarrow$ fit all four expansions $\langle O_i^\dagger(t) O_j(0) \rangle$ at once.

Outline

Parameters and method

Parameters:

- Action: $S_E = \frac{1}{4}\int d^4x F_{\mu\nu}F_{\mu\nu}$, use standard plaquette action
- Observable: static potential $\mathcal{V}_{Q\bar{Q}}(R)$ for $R = 1$ to 6
- $\beta = 2.5$
- Volumes: $a^4 V = (aL)^4$ with $L \in \{14, 15, 16, 18\}$
- Number of configurations: 4000 per volume

Method to test the mass extraction from fixed topology:

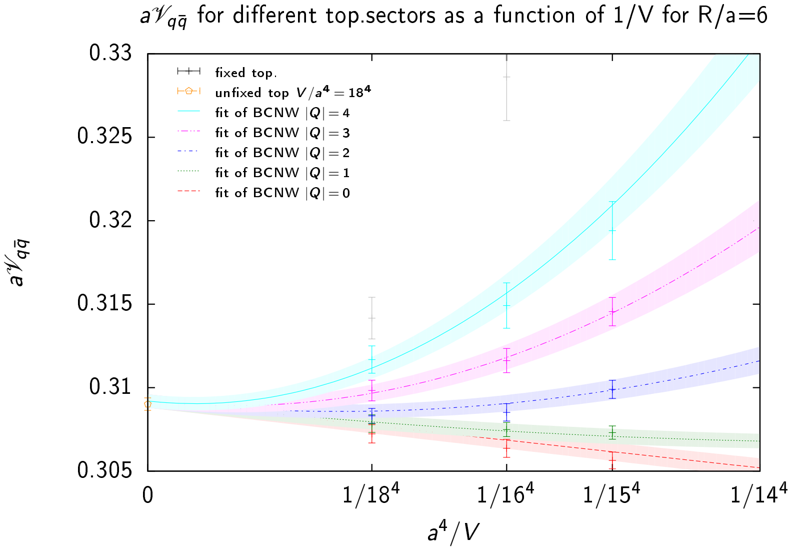
- Compute $C_{Q,V}(t)$ for different Q and V
- Fit the BCNW-equation or improvement
- Compare results to unfixed topology simulation

Systematic comparison:

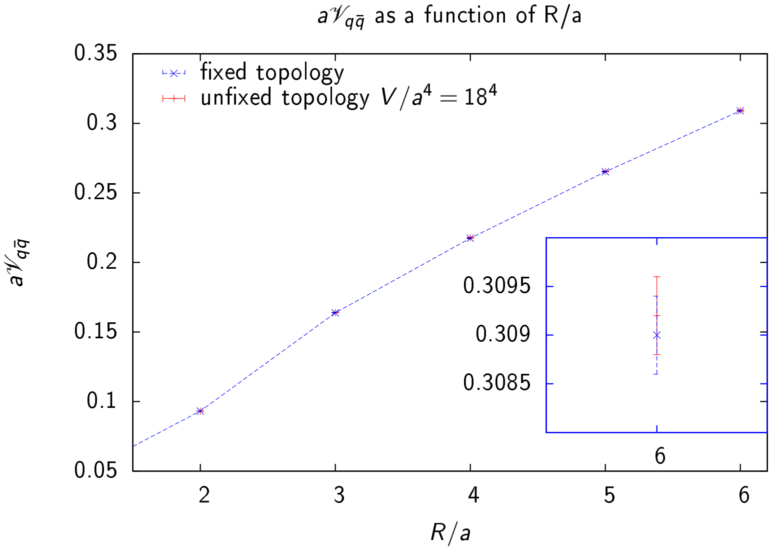
1. Using different number of volumes and different volumes
2. Using weaker criterion $\frac{|Q|}{\chi_t V} < 1$ or stronger one $\frac{|Q|}{\chi_t V} < 0.5$ to admit topological sectors and volumes in the fit.

Outline

Fitting


$a\mathscr{V}_{q\bar{q}}$ for different top.sectors as a function of 1/V for R/a=6
fixed top.
unfixed top $V/a^4 = 18^4$
fit of BCNW $|Q| = 4$
fit of BCNW $|Q| = 3$
fit of BCNW $|Q| = 2$
fit of BCNW $|Q| = 1$
fit of BCNW $|Q| = 0$
$a\mathscr{V}_{q\bar{q}}$
0.33
0.325
0.32
0.315
0.31
0.305
0
$1/18^4$
$1/16^4$
$1/15^4$
$1/14^4$
a^4/V

String Tension



Topological susceptibility

Examples:

For $a^4 V = 15^4$ and 16^4

eq.	$\mathcal{V}_{Q\bar{Q}}$ (a)	$\mathcal{V}_{Q\bar{Q}}$ (2a)	$\mathcal{V}_{Q\bar{Q}}$ (3a)	$\mathcal{V}_{Q\bar{Q}}$ (4a)	$\mathcal{V}_{Q\bar{Q}}$ (5a)	$\mathcal{V}_{Q\bar{Q}}$ (6a)
	from P. de Forcrand, M. Garcia Perez, I. Stamatescu in Nucl.Phys. B499 (1997) 409-449					
	$\chi_t = 7.0(0.9)$					
	fixed topology, a single combined fit for all separations r/a					
BCNW	6.7(3.3)					
improved	7.0(3.5)					
	fixed topology, a separate fit for each separation r/a					
BCNW	8.2(5.8)	5.9(5.0)	7.6(4.7)	7.5(5.0)	7.8(4.9)	7.6(5.0)
improved	8.2(5.7)	6.6(5.1)	7.5(4.7)	7.7(4.8)	8.3(5.0)	8.2(4.9)

For $a^4 V = 15^4, 16^4$ and 18^4

eq.	$\mathcal{V}_{Q\bar{Q}}$ (a)	$\mathcal{V}_{Q\bar{Q}}$ (2a)	$\mathcal{V}_{Q\bar{Q}}$ (3a)	$\mathcal{V}_{Q\bar{Q}}$ (4a)	$\mathcal{V}_{Q\bar{Q}}$ (5a)	$\mathcal{V}_{Q\bar{Q}}$ (6a)
	from P. de Forcrand, M. Garcia Perez, I. Stamatescu in Nucl.Phys. B499 (1997) 409-449					
	$\chi_t = 7.0(0.9)$					
	fixed topology, a single combined fit for all separations r/a					
BCNW	10.4(5.2)					
improved	7.2(1.5)					

- Large statistical errors

Summary of results for SU(2)

- **Clear Discrepancy** between topological sectors observed $\rightarrow$ motivates our work (need to have a method to extract masses from fixed topological sectors)
- **Rather precise to determine masses**: $\left(\frac{|Q|}{\chi_t V} < 1\right)$
    - Improvements when using more volumes (still works reasonably well with two volumes)
    - Reducing statistical errors using larger number of points for the fit (more topological sectors, more volumes)
- Topological susceptibility: **large statistical error!**
    - Improved by increasing the number of topological sectors or number of volumes
    - Not possible to determine χ_t with high precision.
- Improved equation is slightly better or as good as BCNW-equation

Outlook

Outlook

1. Apply to QCD
2. Apply it for the heaviest parity partner

Thank you for your attention!