Critical behavior and continuum scaling of 3D Z(N) lattice gauge theories

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Introduction

$Z(N)$ lattice gauge theories (LGTs), at $T = 0$ and $T > 0$, are interesting on their own and can provide for useful insights into the universal properties of $SU(N)$ LGTs, being $Z(N)$ the center subgroup of $SU(N)$.

The most general action for the $Z(N)$ LGT is

$$S = \sum_x \sum_{n<m} \sum_{k=1}^{N-1} \beta_k \cos\left(\frac{2\pi k}{N}(s_n(x) + s_m(x+e_n) - s_n(x+e_m) - s_m(x))\right) ,$$

with fields defined on links, $s_n(x) = 0, 1, \cdots, N-1$.

Potts model: all β_k equal; Vector model: otherwise;
Conventional vector model: $\beta_1 \neq 0$, $\beta_2 = \cdots = \beta_{N-1} = 0$

- $T = 0$:
  Potts models: one phase transition (PT) from confinement to phase with zero string tension [1, 2, 3], 2nd order for $N = 2$, 1st order for $N \geq 3$
  Vector models: old numerical study for $N = 5, \ldots, 20$; one single PT, disappearing for $N \to \infty$ [4];
  $3D$ Ising class for $N = 2, 3, 4$ and $3D$ XY class for $N \geq 5$, but critical index α on one side of the PT compatible with $3D$ Ising (cusp?) [5]
- $T > 0$:
  Deconfinement PT for $N = 2, 3$: same class of $2D$ $Z(N)$ *spin*, 2nd order, in agreement with Svetitsky-Yaffe [6]
  Potts models: $N > 4$ expected 1st order, as $2D$ *spin* Potts
  Vector models: $N = 4$, 2nd order, as $2D$ $Z(4)$ *spin* [7]; $N > 4$, not much was known until recently
  - $3D$ vector $Z(N > 4)$ on anisotropic lattices with $\beta_s = 0$, $\beta_t \equiv \beta \neq 0$ [8]:
    reduce to a $2D$ generalized vector *spin* $Z(N)$ model, with the Polyakov loop as $Z(N)$ spin and exhibit two Berezinskii-Kosterlitz-Thouless (BKT) [9] PTs
    - $\beta < \beta_c^{(1)}$, low-temperature, confining phase; non-zero string tension σ; linear potential
    - $\beta_c^{(1)} < \beta < \beta_c^{(2)}$, intermediate phase; $Z(N)$ symmetry is enhanced to $U(1)$ symmetry string tension $\sigma = 0$; logarithmic (confining) potential
    - $\beta_c^{(2)} < \beta$, high-temperature, deconfining phase; spontaneous breaking of the $Z(N)$ symmetry
    - critical indices as in $2D$ vector spin $Z(N)$ models, $\eta(\beta_c^{(1)}) = 1/4$, $\nu^{(1)} = 1/2$, as in $2D$ XY
      $\eta(\beta_c^{(2)}) = 4/N^2$, $\nu^{(2)} = 1/2$
  - $3D$ vector $Z(N > 4)$ on isotropic lattices with $\beta_s = \beta_t \equiv \beta$ [10]:
    confirmed scenario as for $\beta_s = 0$ (small influence of spatial plaquettes on the Polyakov loop dynamics)

Aim of this study:
- to extend to other values of N and N_t our previous study on $3D$ vector $Z(N > 4)$ LGTs on isotropic lattices at $T > 0$
- to check the scaling near the continuum limit and establish the scaling with N of the critical couplings

Strategy

- BKT transition hard to study analytically, by, say, the RG of Ref. [11]
- Numerical simulations plagued by very slow, logarithmic convergence to the thermodynamic limit near the transition (large-scale simulations needed, together with FSS methods)
- Standard approach: use Binder cumulants and susceptibilities of the Polyakov loop to determine critical couplings and critical indices
- Our strategy: move to a dual formulation and use Binder cumulants and susceptibilities of *dual* $Z(N)$ *spins*
  - critical behavior of dual spins reversed with respect to that of Polyakov loops:
    - spontaneously-broken ordered phase mapped to symmetric phase and *vice versa*
    - critical indices η interchanged
    - index ν expected to be the same (=1/2) at both transitions
      (see Ref. [10] for further details)
  - cluster algorithms available

Duality transformation

General $3D$ $Z(N)$ gauge theory on an anisotropic lattice:

$$Z(\beta_t, \beta_s;) = \prod_{l \in \Lambda} \left(\frac{1}{N} \sum_{s(l)=0}^{N-1}\right) \prod_{p_s} Q(s(p_s)) \prod_{p_t} Q(s(p_t)) ,$$

$$Q(s) = \exp\left[\sum_{k=1}^{N-1} \beta_p(k) \cos\frac{2\pi k}{N} s\right] ,$$

$\Lambda = L^2 \times N_t$ is the $3D$ lattice; $s(p)$ the plaquette angle ($p_{t,s}$ stands for temporal/spatial),

$$s(p) = s_n(x) + s_m(x+e_n) - s_n(x+e_m) - s_m(x) .$$

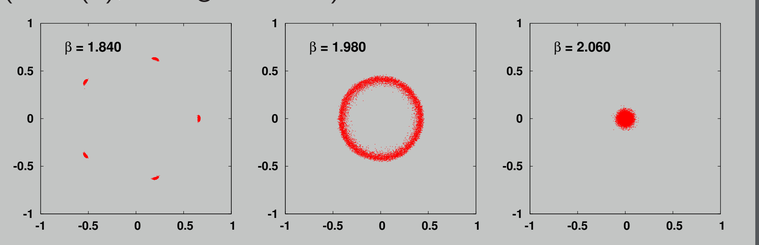
Wilson action: $\beta_p(1) = \beta$, $\beta_p(k > 1) = 0$;
isotropic lattice, $\beta_s = \beta_t = \beta$

Duality: generalized $3D$ spin $Z(N)$ model,

$$S = \sum_x \sum_{n=1}^{3} \sum_{k=1}^{N-1} \beta_k \cos\left(\frac{2\pi k}{N}(s(x) - s(x+e_n))\right)$$

$$\beta_k = \frac{1}{N} \sum_{p=0}^{N-1} \ln\left[\frac{Q_d(p)}{Q_d(0)}\right] \cos\left(\frac{2\pi pk}{N}\right) .$$

- strongly ordered β_k, near criticality; $3D$ vector spin model with only $\beta_1 \neq 0$ is a good approximation;
- weak and strong coupling regimes are interchanged (for $Z(5)$, see figure below)



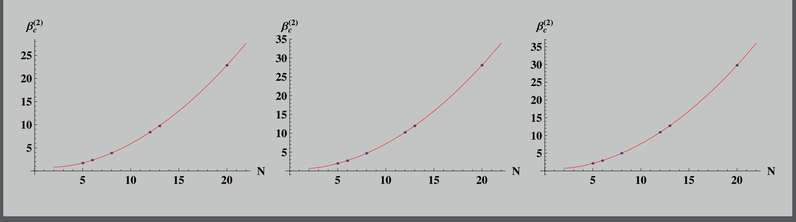
Critical couplings

$\beta_c^{(2)}$ determined through Binder cumulants of *standard magnetization* $M_L = |M_L| e^{i\psi}$, with $L = 128 \div 768$;
$\beta_c^{(1)}$ through Binder cumulants of the *rotated magnetization*, $M_R = |M_L| \cos(N\psi)$, with $L = 128 \div 2048$

N	N_t	$\beta_c^{(1)}$	$\beta_c^{(2)}$
5	2	1.617(2)	1.6972(14)
5	4	1.943(2)	1.9885(15)
5	6	2.05(1)	2.08(1)
5	8	2.085(2)	2.1207(9)
5	12	2.14(1)	2.16(1)
6	2		2.3410(15)
6	4		2.725(12)
6	8		2.899(4)
8	2		3.8640(10)
8	4	2.544(8)	4.6864(15)
8	8	3.422(9)	4.9808(5)

N	N_t	$\beta_c^{(1)}$	$\beta_c^{(2)}$
12	2		8.3745(5)
12	4		10.240(7)
12	8		10.898(5)
13	2	1.795(4)	9.735(4)
13	4	2.74(5)	11.959(6)
13	8	3.358(7)	12.730(2)
20	2		22.87(4)
20	4	2.57(1)	28.089(3)
20	8	3.42(5)	29.758(6)

At fixed N_t, $\beta_c^{(2)}(N)$ well fitted by $A/(1 - \cos 2\pi/N) + B(1 - \cos 2\pi/N)$ (the plots below refer to $N_t = 2, 4, 8$, respectively, for the latter function)



Continuum limit of the two transitions

At fixed N, $\beta_c^{(1,2)}(N_t)$ fitted by the scaling function $\beta_c^{(1,2)}(N_t) = \beta_c^{(1,2)} - (N_t a T_c)^{-1/\nu}$ ($\beta_c^{(1,2)}$ should match β_c of $T = 0$; also ν is the $T = 0$ index)

N	aT_c	$\beta_c^{(1)}$	ν	χ_r^2
5	0.790(5)	2.198(9)	0.84(3)	1.21
	0.764(14)	2.144(9)	0.670	23.1
	0.758(16)	2.135(11)	0.640	33.6
	0.786(7)	2.17961	0.788(10)	2.66
	0.722(16)	2.17961	0.670	105
	0.709(19)	2.17961	0.640	171

N	aT_c	$\beta_c^{(2)}$	ν	χ_r^2
5	0.810(13)	2.17961	0.670	81.8
	0.776(31)*	2.17961	0.670	74.2*
	0.731(18)*	2.17961	0.640	31.4*
6	0.6769(76)	2.977(10)	0.674	5.02
	0.6740(85)	2.969(12)	0.642	6.90
	0.6832(46)	3.00683	0.768(15)	1.14
	0.572(13)*	3.00683	0.674	1.44*
	0.542(21)*	3.00683	0.642	4.48*
8	0.42378(12)	5.14299(25)	0.672	0.19
	0.4294(12)	5.12829	0.648(6)	33.0
	0.4216(10)*	5.12829	0.637	2.21*
12	0.24559(13)	11.2640(23)	0.670	0.22
	0.2602(32)	11.1962	0.630(11)	14.2
	0.25742(10)	11.1962	0.640	12.7
13	0.21872(53)	13.1656(56)	0.671	5.88
	0.22851(40)	13.1199(42)	0.642	3.40
	0.22928(67)	13.1077	0.642	16.0
20	0.1357(26)	30.6729	0.642(19)	58.2
	0.13199(13)*	30.6729	0.673	1.57*
	0.13519(49)*	30.6729	0.647	23.9*

No error bar means "fixed parameter"; the mark * means only $N_t = 4, 8$ included in the fit

Critical indices at the two transitions

The scaling laws at the critical couplings,

$$M_{R,L} = AL^{-\beta/\nu} , \qquad \chi_L^{(M_{R,L})} = AL^{-\gamma/\nu}$$

are used to extract $\beta^{(1,2)}/\nu$ and $\gamma^{(1,2)}/\nu \equiv 2 - \eta^{(1,2)}$ at the two transitions.

The reference value for $\eta^{(1)}$ is $4/N^2$.

N	N_t	$\beta_c^{(2)}$	$\beta^{(1)}/\nu$	χ_r^2	$\gamma^{(1)}/\nu$	χ_r^2	d	$\eta^{(1)}$	$4/N^2$
5	2	1.617(2)	0.097(6)	0.101	1.847(5)	0.561	2.04(2)	0.153(5)	
5	4	1.943(2)	0.11(1)	1.25	1.841(1)	0.70	2.07(3)	0.159(1)	0.16
5	8	2.085(2)	0.09(2)	0.77	1.844(1)	0.78	2.01(4)	0.156(1)	
8	4	2.544(8)	−0.26(2)	1.79	1.930(3)	1.58	1.41(5)	0.070(3)	0.0625
8	8	3.422(9)	−0.52(5)	0.21	1.959(1)	0.21	0.9(1)	0.040(1)	
13	2	1.795(4)	0.07(5)	1.28	1.968(9)	0.97	2.1(1)	0.032(9)	
13	4	2.74(5)	−0.26(2)	1.81	1.976(3)	1.80	1.5(1)	0.024(3)	0.0236..
13	8	3.358(7)	−0.9(1)	1.17	1.973(4)	1.25	0.3(2)	0.027(4)	
20	4	2.57(1)	−0.25(2)	0.37	1.991(3)	1.91	1.49(5)	0.009(3)	0.01
20	8	3.42(5)	−0.72(6)	0.41	1.9790(16)	0.33	0.55(13)	0.0210(16)	

The reference value for $\eta^{(2)}$ is $1/4$.

N	N_t	$\beta_c^{(2)}$	$\beta^{(2)}/\nu$	χ_r^2	$\gamma^{(2)}/\nu$	χ^2	d	$\eta^{(2)}$
5	2	1.6972(14)	0.1259(2)	1.22	1.750(3)	0.50	2.001(4)	0.250(3)
5	4	1.9885(15)	0.1061(3)	2.67	1.758(9)	2.45	1.971(9)	0.242(9)
5	8	2.1207(9)	0.1376(6)	2.04	1.747(15)	1.62	2.022(16)	0.253(15)
6	2	2.3410(15)	0.26(3)	1.8	1.6(6)	1.21	2.1(6)	0.4(6)
6	4	2.725(12)	0.1056(13)	1.84	1.76(7)	2.05	1.97(8)	0.24(7)
6	8	2.899(4)	0.0949(4)	1.67	1.731(8)	0.71	1.920(9)	0.269(8)
8	2	3.8640(10)	0.1336(4)	0.36	1.743(15)	0.73	2.010(16)	0.257(15)
8	4	4.6864(15)	0.1278(4)	3.85	1.753(6)	1.34	2.009(7)	0.247(6)
8	8	4.9808(5)	0.1379(5)	0.77	1.745(18)	1.82	2.020(19)	0.255(18)
12	2	8.3745(5)	0.1283(16)	1.22	1.73(4)	0.78	1.99(4)	0.27(4)
12	4	10.240(7)	0.1303(4)	1.52	1.746(9)	0.87	2.007(10)	0.254(9)
12	8	10.898(5)	0.149(3)	0.64	1.78(16)	1.19	2.07(17)	0.22(16)
13	2	9.735(4)	0.1251(2)	0.22	1.744(5)	0.09	1.995(5)	0.256(5)
13	4	11.959(6)	0.1265(2)	1.43	1.746(3)	0.48	1.999(4)	0.254(3)
13	8	12.730(2)	0.1357(18)	3.55	1.75(2)	0.82	2.02(2)	0.25(2)
20	2	22.87(4)	0.1322(14)	1.06	1.78(3)	0.68	2.04(4)	0.22(3)
20	4	28.089(3)	0.1384(4)	0.17	1.748(14)	0.17	2.025(15)	0.252(14)
20	8	29.758(6)	0.1278(7)	1.60	1.713(17)	1.15	1.968(18)	0.287(17)

Conclusions

- All $3D$ vector $Z(N > 4)$ LGTs at $T > 0$ considered in Ref. [10] and in the present study feature two BKT-like phase transitions.
- Critical indices suggest that these models belong to the universality class of $2D$ $Z(N)$ vector spin models, in agreement with the Svetitsky-Yaffe conjecture.
- We proposed and checked a formula for the scaling with N of the critical coupling of the second phase transition.
- Using the value of the index ν obtained by us at $T = 0$, we checked the continuum scaling and predicted an approximate value for aT_c in the continuum limit.

Acknowledgments

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