Determining Sigma – Lambda mass mixing

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– QCDSF-UKQCD Collaboration –

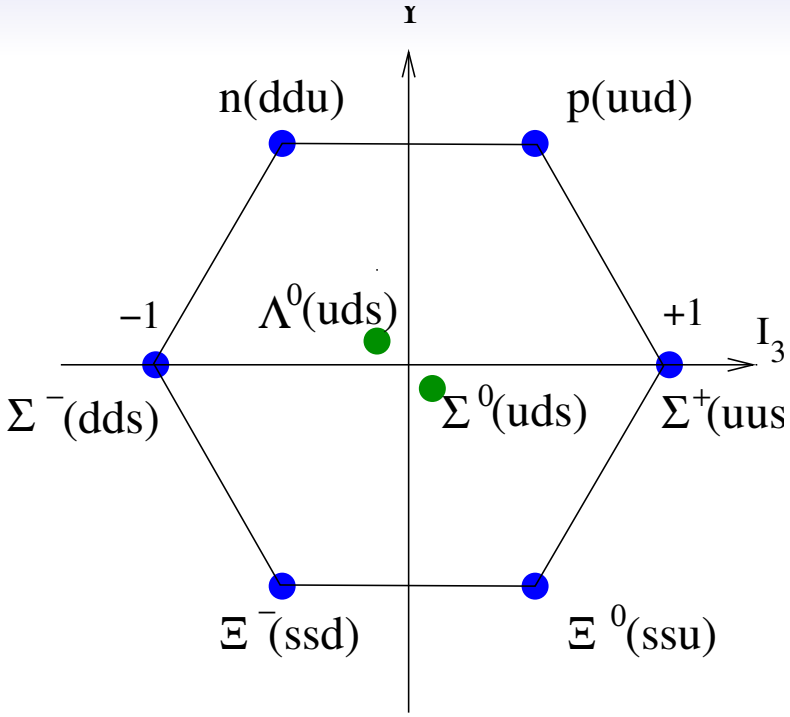
Edinburgh – Regensburg – RIKEN (Kobe) – Leipzig – FZ (Jülich) – Liverpool – DESY – Hamburg – Adelaide

[Lattice 2014, New York, USA]

QCDSF related talks with $2+1$ flavours:

- Holger Perlt — Parallels 7G (Wednesday 14:35)
  A Feynman-Hellmann approach to nonperturbative renormalization of lattice operators
- Gerrit Schierholz — Parallels 8B (Friday 14:15)
  Electromagnetic mass splittings from dynamical dynamical QCD + QED
- James Zanotti — Parallels 9D (Friday 16:30)
  A Feynman-Hellmann approach to the spin structure of hadrons

Introduction



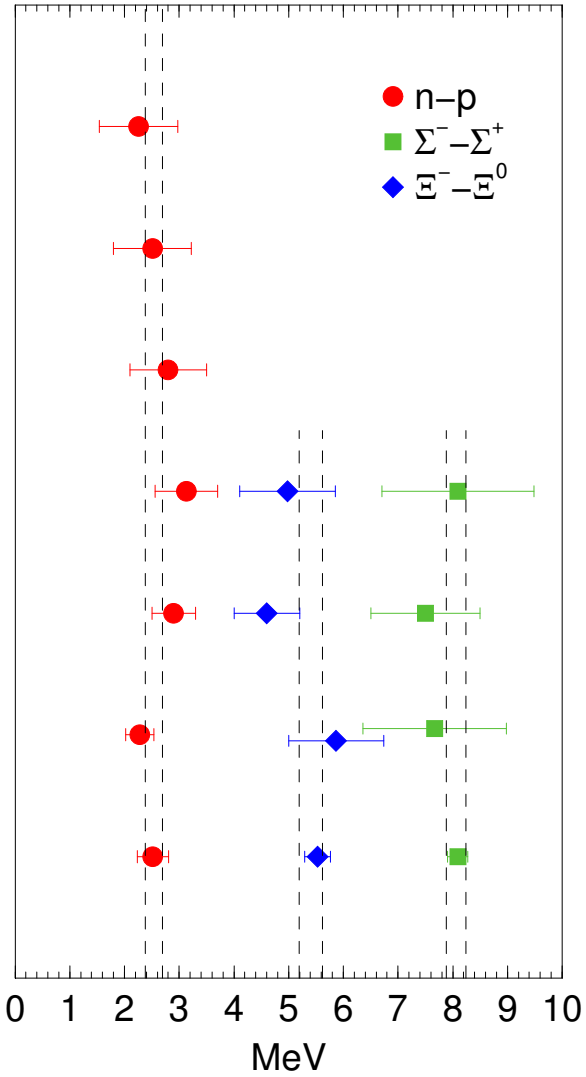
$$(M_n - M_p)^{\rm exp} = 1.293333(33)\,\text{MeV}$$
$$(M_{\Sigma^-} - M_{\Sigma^+})^{\rm exp} = 8.079(76)\,\text{MeV}$$
$$(M_{\Xi^-} - M_{\Xi^0})^{\rm exp} = 6.85(21)\,\text{MeV}\,.$$

- Baryon octet 'Outer' ring
  - Small mass differences [expt precision way beyond what we achieve here]
  - Isospin breaking effects:
    - QED component [not considered here]
    - $m_d - m_u$ component — pure QCD
- Baryon octet 'centre'
  - $\Lambda^0(uds)$, $\Sigma^0(uds)$ have the same quark content (quantum numbers) but different wavefunctions

Present status

Baryon octet 'outer' ring

QCDSF-UKQCD 1206.3156



NPLQCD [arXiv:hep−lat/0605014]

Blum et al [arXiv:1006.1311]

RM123 [arXiv:1110.6294]

QCDSF−UKQCD [arXiv:1206.3156]

Shanahan et al [arXiv:1209.1892]

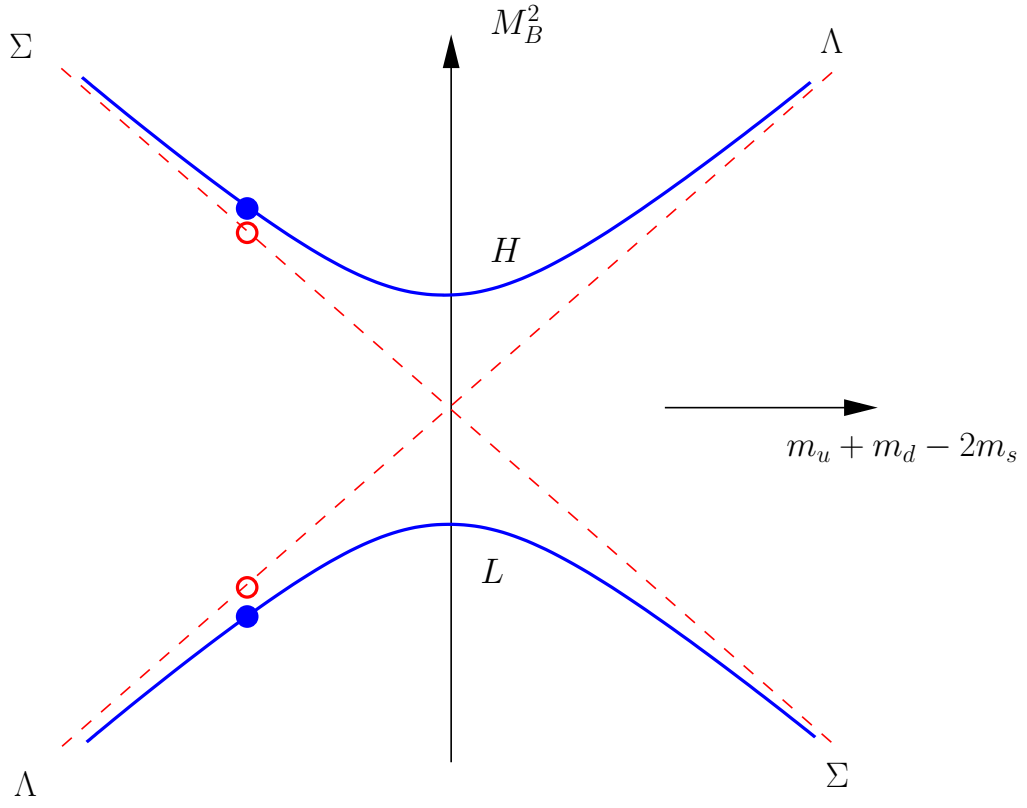
BMW [arXiv:1306.2287]

BMW [arXiv:1406.4088]

'pure' QCD part; weighted average

To complete the job: now consider octet 'centre'

Baryon octet 'centre'

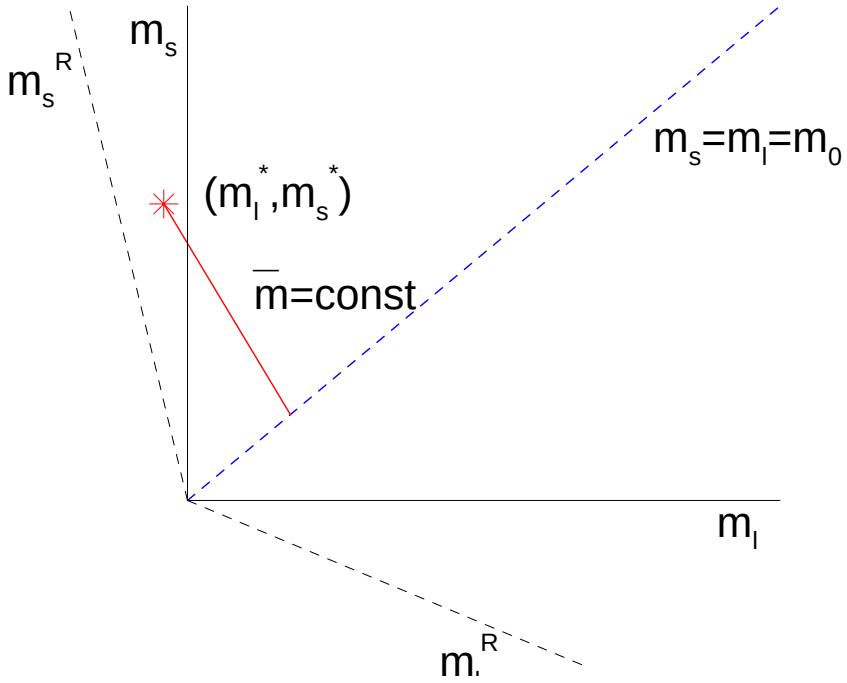


- $\Lambda^0(uds)$, $\Sigma^0(uds)$ have the same quark content (quantum numbers) but different wavefunctions
- Mass difference $(M_{\Sigma^0} - M_{\Lambda^0})^{\text{exp}} = 76.959(23)\,\text{MeV}$ as:
  - $m_s \neq m_l$, where $m_u = m_d = m_l$
  - mixing if $m_u \neq m_d$
    - avoided level crossing if isospin broken (Heavy, H, Light, L)

QCDSF strategy:

[arXiv:1102.5300]

$2+1$ simulations: many paths to approach the physical point [$m_u = m_d \equiv m_l$ case]



QCDSF: extrapolate from a point on the $SU(3)_F$ flavour symmetry line to the physical point

$$(m_0, m_0) \longrightarrow (m_l^*, m_s^*)$$

Choice here: keep the singlet quark mass $\overline{m}$ constant

$$\overline{m} = m_0 = \frac{1}{3}\,(2m_l + m_s)$$

QCDSF strategy

[arXiv:1102.5300]

- develop $SU(3)_F$ flavour symmetry breaking expansion for hadron masses
- expansion in:

$SU(3)$ flavour symmetric point $\delta m_q = 0$

$$\delta m_q = m_q - \overline{m}\,, \quad \overline{m} = \tfrac{1}{3}(m_u + m_d + m_s) = m_0$$

  - expansion coefficients are functions of $\overline{m}$
  - trivial constraint

$$\delta m_u + \delta m_d + \delta m_s = 0$$

- path called 'unitary line' as expand in both sea and valence quarks

Main observations:

- Provided $\overline{m}$ kept constant, then expansion coefficients $A(\overline{m}), \ldots$ remain unaltered whether
  - $1+1+1$
  - $2+1$
- Opens possibility of determining quantities that depend on $1+1+1$ from just $2+1$ simulations
- Furthermore can generalise to different valence quark masses, μ_q to sea quark masses m_q without increasing number of expansion coefficients

$$\delta\mu_q = \mu_q - \overline{m}$$

Mass matrix

Quark mass matrix

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Baryon mass matrix

$$M^2(\mathcal{M}) \quad = \quad \begin{pmatrix} M_n^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_p^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{\Sigma^-}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{\Sigma\Sigma}^2 & M_{\Sigma\Lambda}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{\Lambda\Sigma}^2 & M_{\Lambda\Lambda}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{\Sigma^+}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{\Xi^-}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{\Xi^0}^2 \end{pmatrix}$$

Demand that under all $SU(3)$ transformations

$$\mathcal{M} \to \mathcal{M}' = U\mathcal{M}U^\dagger \quad \leftrightarrow \quad M^2(\mathcal{M}') = UM^2(\mathcal{M})U^\dagger$$

Physically no change [to eigenvalues]

eg $m_d \leftrightarrow m_s$ equivalent to relabelling $M_n \leftrightarrow M_{\Xi^0}$, . . .

N matrices

$$M^2 = \sum_{i=1}^{10} K_i(m_q, \mu_q) N_i$$

N_i classified under S_3, $SU(3)$ symmetry; $K(m_q, \mu_q)$ coefficients

								S_3	$SU(3)$
1	1	1	1	1	1	1	1	A_1	1
−1	−1	0	0	0	0	1	1	E^+	8_a
−1	1	−2	0	0	2	−1	1	E^-	8_a
1	1	−2	−2	2	−2	1	1	E^+	8_b
−1	1	0	mix		0	1	−1	E^-	8_b
1	1	1	−3	−3	1	1	1	A_1	27
1	1	−2	3	−3	−2	1	1	E^+	27
−1	1	0	mix		0	1	−1	E^-	27
1	−1	−1	0	0	1	1	−1	A_2	$10,\overline{10}$
0	0	0	mix		0	0	0	A_2	$10,\overline{10}$

- N_i mostly diagonal, eg $N_1 = \mathrm{diag}(1,1,1,1,1,1,1,1)$, except N_5, N_8, N_{10}
- S_3 symmetry group (equilateral triangle C_{3v}); 3 irreducible representations:
  - two singlets A_1, A_2
  - one doublet E, elements $E^{\pm}$

$$N_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad N_2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$N_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad N_4 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$N_5 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2/\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2/\sqrt{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad N_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$N_7 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad N_8 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$N_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad N_{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Demand that under all $SU(3)$ transformations

$$\mathcal{M} \to U\mathcal{M}U^\dagger \quad \leftrightarrow \quad M^2(\mathcal{M}') = UM^2(\mathcal{M})U^\dagger$$

giving [for B not at octet centre ie $\neq \Sigma, \Lambda$]

$$M_B^2 = P_{A_1} + P_{E^+}$$

and

$$\begin{pmatrix} M^2_{\Sigma\Sigma} & M^2_{\Sigma\Lambda} \\ M^2_{\Lambda\Sigma} & M^2_{\Lambda\Lambda} \end{pmatrix}$$
$$= P_{A_1}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + P_{E^+}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + P_{E^-}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + P_{A_2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

P_G are functions of the quark masses with the symmetry G under the S_3 permutation group

$$
\begin{aligned}
P_{A_1} &= M_0^2 + 3A_1\delta\overline{\mu} \\
&+\tfrac{1}{6}B_0(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + B_1(\delta\mu_u^2 + \delta\mu_d^2 + \delta\mu_s^2) \\
&+\tfrac{1}{4}(B_3 + B_4)\left[(\delta\mu_s - \delta\mu_u)^2 + (\delta\mu_s - \delta\mu_d)^2 + (\delta\mu_u - \delta\mu_d)^2\right] + O(3)
\end{aligned}
$$

$$
\begin{aligned}
P_{E^+} &= \tfrac{3}{2}A_2(\delta\mu_s - \delta\overline{\mu}) \\
&+\tfrac{1}{2}B_2(2\delta\mu_s^2 - \delta\mu_u^2 - \delta\mu_d^2) \\
&+\tfrac{1}{4}(B_3 - B_4)\left[(\delta\mu_s - \delta\mu_u)^2 + (\delta\mu_s - \delta\mu_d)^2 - 2(\delta\mu_u - \delta\mu_d)^2\right] + O(3)
\end{aligned}
$$

$$
\begin{aligned}
P_{E^-} &= \tfrac{\sqrt{3}}{2}A_2(\delta\mu_d - \delta\mu_u) \\
&+\tfrac{\sqrt{3}}{2}B_2(\delta\mu_d^2 - \delta\mu_u^2) + \tfrac{\sqrt{3}}{4}(B_3 - B_4)\left[(\delta\mu_s - \delta\mu_d)^2 - (\delta\mu_s - \delta\mu_u)^2\right] + O(3)
\end{aligned}
$$

$$
P_{A_2} = 0 + O(3)
$$

[$O(3)$ terms have also been determined]

Diagonalisation

$$
\begin{aligned}
M_H^2 &= P_{A_1} + \sqrt{P_{E^+}^2 + P_{E^-}^2 + P_{A_2}^2} \\
M_L^2 &= P_{A_1} - \sqrt{P_{E^+}^2 + P_{E^-}^2 + P_{A_2}^2}
\end{aligned}
$$

write eigenvectors as

$$
e_H = \begin{pmatrix} \cos\theta \\ e^{-i\phi}\sin\theta \end{pmatrix}, \qquad e_L = \begin{pmatrix} -e^{i\phi}\sin\theta \\ \cos\theta \end{pmatrix}
$$

giving for the mixing angle, θ, and phase, ϕ

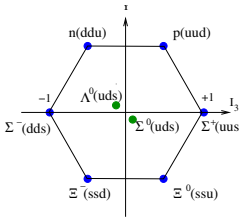
$$
\tan 2\theta = \frac{\sqrt{P_{E^-}^2 + P_{A_2}^2}}{P_{E^+}}, \qquad \tan\phi = \frac{P_{A_2}}{P_{E^-}}
$$

$$
M_{\Sigma^0} = M_H^*, \qquad M_{\Lambda^0} = M_L^*, \qquad \theta^*
$$

Defining the scale – using singlet quantities

- octet baryons (centre of mass):

$$
\begin{aligned}
X_N^2 &= \frac{1}{6}(M_p^2 + M_n^2 + M_{\Sigma^+}^2 + M_{\Sigma^-}^2 + M_{\Xi^0}^2 + M_{\Xi^-}^2) = (1.160\ \mathrm{GeV})^2 \\
&= M_0^2 + \frac{1}{6}(B_0 + B_1 + B_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_0^2 + O(\delta m_q^2)
\end{aligned}
$$



stable under strong ints.

- all singlet quantities

$$X_S^2 = \# + \#(\delta m_q^2)$$

(almost) constant

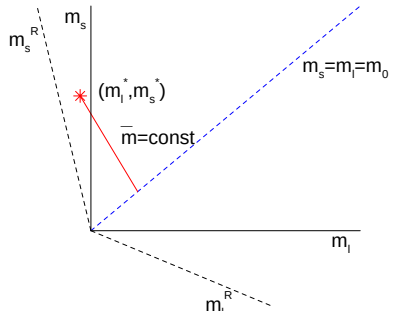
[$\Longrightarrow$ scale determination]

- form dimensionless ratios (within a multiplet):

$$\tilde{M}^2 \equiv \frac{M^2}{X_S^2}\,, \quad S = \pi, N, \ldots, \qquad \tilde{A}_i \equiv \frac{A_i}{M_0^2}\,, \ldots \quad \text{in expansions}$$

Lattice

- $O(a)$ NP improved clover action
  - tree level Symanzik glue
  - mildly stout smeared $2+1$ clover fermion
  - $\beta = 5.50$, $32^3 \times 64$
- κ_0 is start point on $SU(3)_F$ symmetric line κ_{0c} is chiral limit along $SU(3)_F$ symmetric line



$$m_q = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{0c}}\right)$$

giving

$$m_0 = \frac{1}{2}\left(\frac{1}{\kappa_0} - \frac{1}{\kappa_{0c}}\right) = \overline{m} = \frac{1}{3}(2m_l + m_s) = \frac{1}{2}\left(\frac{2}{\kappa_l} + \frac{1}{\kappa_s} - \frac{1}{\kappa_{0c}}\right)$$

So $1/\kappa_{0c}$ cancels: given κ_0 and κ_l gives κ_s

$$\delta m_q = m_q - m_0 = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0}\right)$$

Method

- Use PQ data to determine expansion coefficients
  - $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ – baryon octet
- Determine physical quark masses
  - Use PQ data to determine expansion coefficients
    $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ – pseudoscalar octet [arXiv:1206.3156]
  - $$\delta m_u^*\,, \quad \delta m_d^*\,, \quad \delta m_s^*$$

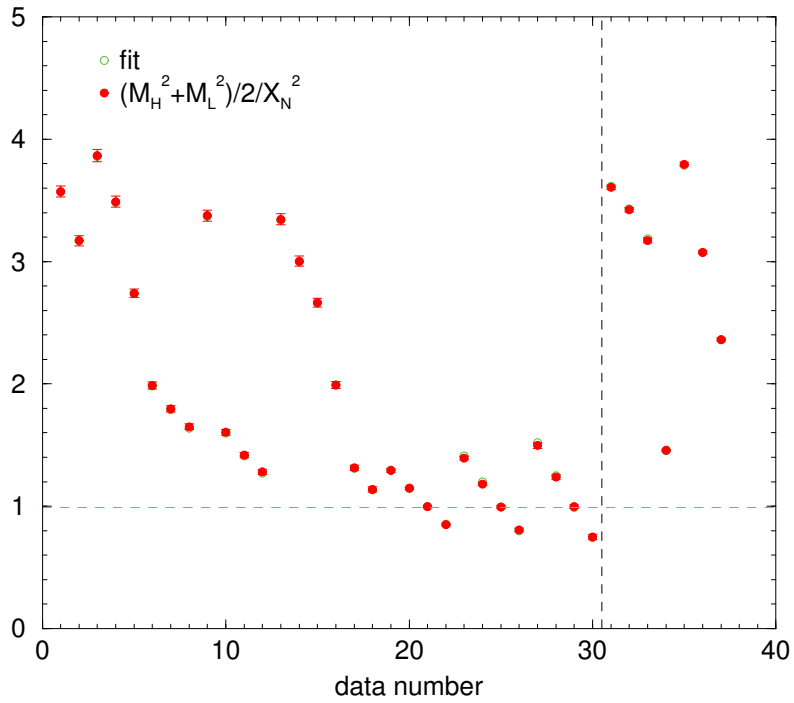
    by fitting/using [Dashen's thm]

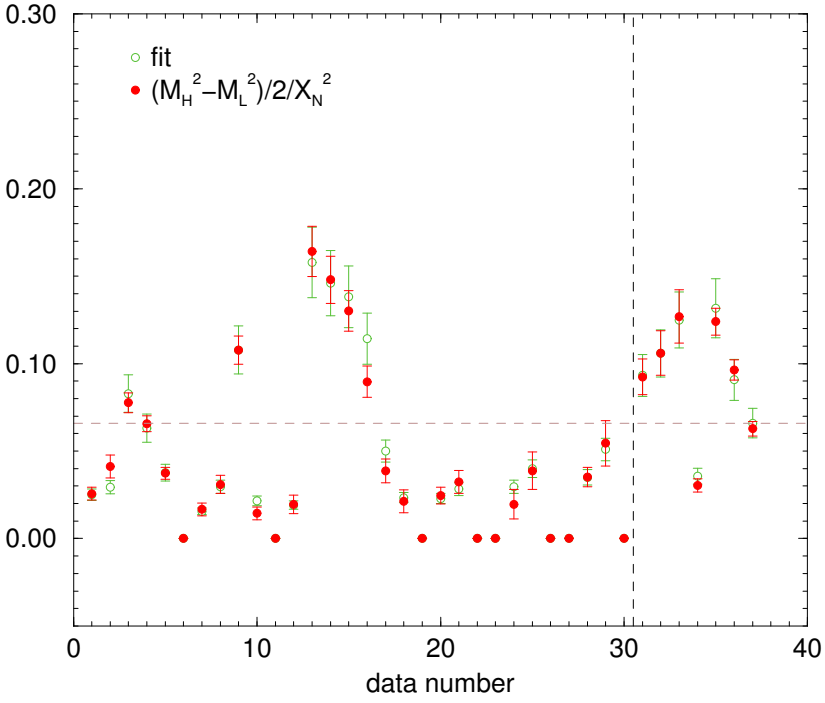
    $$M_{\pi^+}^{*\,2} = M_{\pi^0}^{\text{exp}\ 2}\,, \quad M_{K^+}^{*\,2} = M_{K^+}^{\text{exp}\ 2} - (M_{\pi^+}^{\text{exp}\ 2} - M_{\pi^0}^{\text{exp}\ 2})\,, \quad M_{K^0}^{*\,2} = M_{K^0}^{\text{exp}\ 2}$$

    [together with κ_0, so 3 inputs needed]

Data: $M_H(aab)$, $M_L(aa'b)$, $M_H(abc)$, $M_L(abc) \lesssim 2.0\,\mathrm{GeV}$

[wf $\Sigma(abc)$, $\Lambda(abc)$]





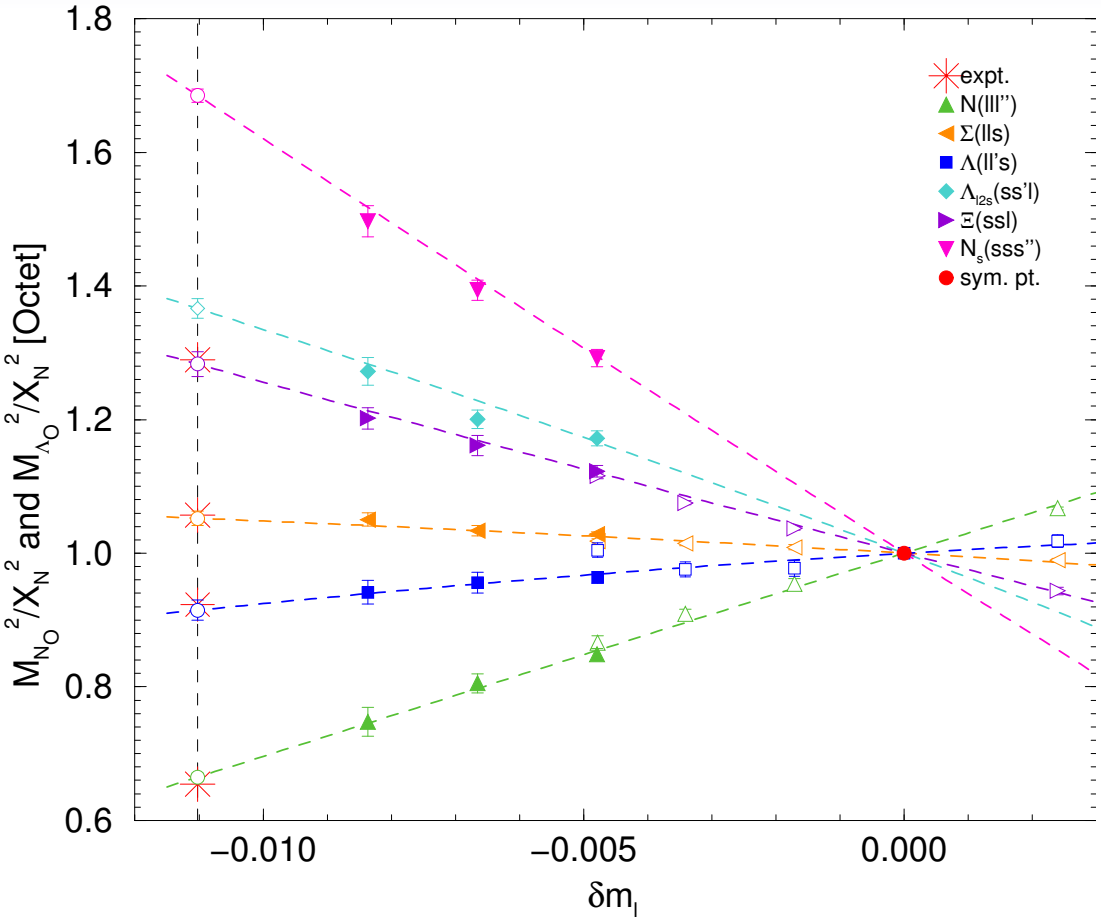
- LH plot side: $M_H(aab)$, $M_L(aa'b)$ [already diagonal]
- RH plot side: $M_H(abc)$, $M_L(abc)$

$$C_{ij}(t) = \frac{1}{V_s}\,\mathrm{Tr}_D\Gamma_{\mathrm{unpol}}\left\langle \sum_{\vec{y}} \mathcal{B}_i(\vec{y},t) \sum_{\vec{x}} \overline{\mathcal{B}}_j(\vec{x},0)\right\rangle$$

$$\propto A_i A_j e^{-M_L t} + B_i B_j e^{-M_H t} \qquad i,j = H,L$$

Diagonalise, C_{ij} yielding M_H and M_L

'Fan' plot for the $2+1$: $\tilde{M}_N(aab)^2$, $\tilde{M}_\Lambda(aa'b)^2$



- $N(lll'')[=\Lambda_{3l}(ll'l'')]$, $\Sigma(lls)$, $\Xi(ssl)$, $N_s(sss'')[=\Lambda_{3s}(ss's'']$, $\Lambda(ll's)$, $\Lambda_{l2s}(ss'l)$
- As diagonal: $\tilde{M}_N^2 = P_{A_1} + P_{E^+}$, $\tilde{M}_\Lambda^2 = P_{A_1} - P_{E^+}$
- $\delta m_l^* = (\delta m_u^* + \delta m_d^*)/2$, $M_N^{*\,2}(lll'') = (M_n^{\exp\,2}(ddu) + M_p^{\exp\,2}(uud))/2$, $M_\Lambda^{*\,2}(ll's) = M_{\Lambda^0}^{\exp\,2}(uds)$,

$$M_\Sigma^{*\,2}(lls) = (M_{\Sigma^-}^{\exp\,2}(dds) + M_{\Sigma^+}^{\exp\,2}(uus))/2,\ M_\Xi^{*\,2}(ssl) = (M_{\Xi^-}^{\exp\,2}(ssd) + M_{\Xi^0}^{\exp\,2}(ssu))/2$$

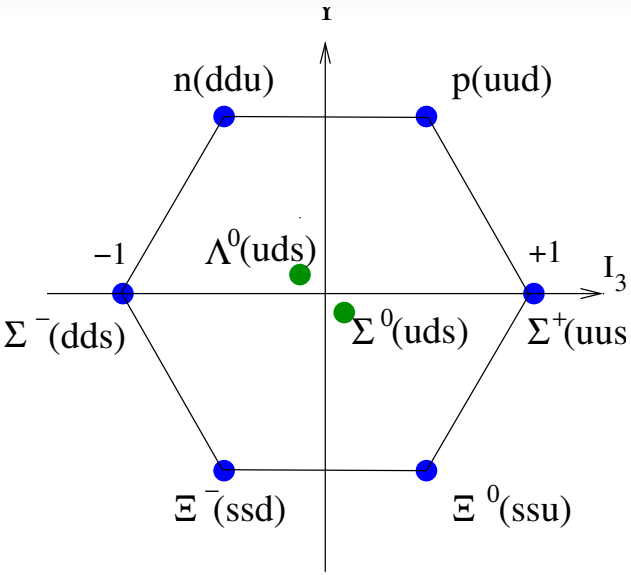
$\Sigma^0 - \Lambda^0$ mixing at NLO – unitary limit

$$\tilde{M}_{\Sigma^0} - \tilde{M}_{\Lambda^0} = \sqrt{\frac{3}{2}}\tilde{A}_2\sqrt{\delta m_u^2 + \delta m_d^2 + \delta m_s^2} \times \left[1 + \frac{3}{2}\left(\frac{2\tilde{B}_2 + 3\tilde{B}_3 - 3\tilde{B}_4}{\tilde{A}_2}\right)\frac{\delta m_u \delta m_d \delta m_s}{\delta m_u^2 + \delta m_d^2 + \delta m_s^2}\right]$$

$$\tan 2\theta = \frac{(\delta m_d - \delta m_u)}{\sqrt{3}\delta m_s} \times \left[1 - \frac{1}{3}\left(\frac{2\tilde{B}_2 + 3\tilde{B}_3 - 3\tilde{B}_4}{\tilde{A}_2}\right)\frac{(\delta m_s - \delta m_u)(\delta m_s - \delta m_d)}{\delta m_s}\right]$$

- mixing only when $\delta m_d \neq \delta m_u$, so mixing contribution to $\tilde{M}_{\Sigma^0} - \tilde{M}_{\Lambda^0}$ very small
- small electromagnetic effects

cf Baryon 'Outer' ring



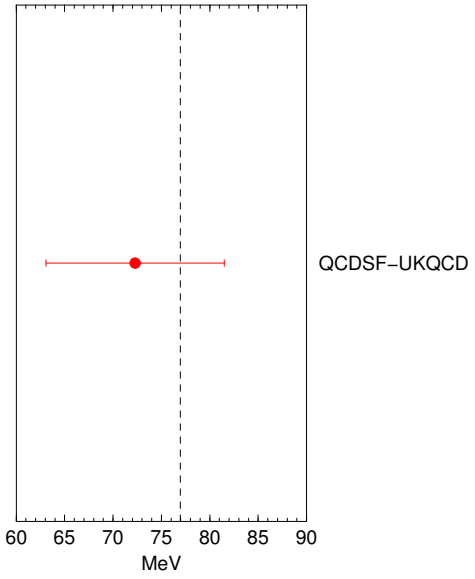
$$\tilde{M}_n - \tilde{M}_p = (\delta m_d - \delta m_u)\left[\tilde{A}'_1 - 2\tilde{A}'_2 + (\tilde{B}'_1 - 2\tilde{B}'_2)(\delta m_d + \delta m_u)\right]$$

$$\tilde{M}_{\Sigma^-} - \tilde{M}_{\Sigma^+} = (\delta m_d - \delta m_u)\left[2\tilde{A}'_1 - \tilde{A}'_2 + (2\tilde{B}'_1 - \tilde{B}'_2 + 3\tilde{B}'_3)(\delta m_d + \delta m_u)\right]$$

$$\tilde{M}_{\Xi^-} - \tilde{M}_{\Xi^0} = (\delta m_d - \delta m_u)\left[\tilde{A}'_1 + \tilde{A}'_2 + (\tilde{B}'_1 + \tilde{B}'_2 + 3\tilde{B}'_3)(\delta m_d + \delta m_u)\right]$$

- A', B' simply related to A, B
- electromagnetic effects arising from difference $\delta m_d - \delta m_u$
- results at beginning

$M_{\Sigma^0} - M_{\Lambda^0}$



- Mixing angle [as anticipated very small $\theta \lesssim 1^o$]

$$\tan 2\theta = 0.0130(29)$$

- Mass difference [mixing contribution to mass difference $\lesssim$ 1 MeV]

$$M_{\Sigma^0} - M_{\Lambda^0} = 72.29(9.22)\,\text{MeV}$$

$$[(M_{\Sigma^0} - M_{\Lambda^0})^{\text{exp}} = 76.959(23)\,\text{MeV}]$$

Conclusions

- Programme: Developed a $SU(3)$ flavour symmetry breaking expansion keeping the average quark mass $\overline{m}$ constant advantages:
  - can use $2+1$ simulations, ie $m_u = m_d = m_l$
  - can use cheap PQ results
- Generalised previous method to now also include mixing

$$\Sigma^0 - \Lambda^0$$

mass splitting as well as

$$n - p\,, \quad \Sigma^- - \Sigma^+\,, \quad \Xi^- - \Xi^0$$

mass splittings due to difference in $u - d$ quark masses ('pure' QCD isospin effects)
Now can include $B(abc)$ as well as $B(aa'b)$ PQ quarks

- Encouraging first results
- Repeat for $\eta - \eta'$ mixing (but then disconnected pieces to compute)