The $N_f = 3$-critical endpoint with smeared staggered quarks

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Introduction

- At low temperature color charges are confined.
- At higher temperature a deconfined phase is expected (quark-gluon-plasma).
- For physical quark masses it is known, that this transition is a analytic crossover.

\[ n = 2 \]

\[ T \]

\[ \mu \]

\[ E \]

\[ n_f = 2 + 1 \]

\[ n_f = 2 \]

quark gluon plasma

SC phase

hadronic phase

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Introduction

- What happens with different quark masses?

![Diagram showing quark masses and transitions]

- Expected:
  - Crossover at intermediate quark masses.
  - At higher and lower quark masses: first order transition
  - Regions are separated by a line of a second order transition

$N_f = 3$-QCD with unimproved staggered action was studied at $N_t = 4$ lattices a long time ago.

It was found, that the critical bare mass was at $m_a = 0.033(1)$ and in physical mass it was at $m_{\pi, \xi_5} = 290$ MeV. The transition was found to belong to the $Z(2)$ universality class as expected. [1]

By going over to an improved action it was possible to show, that the critical quark mass was considerable smaller. The same holds true when $N_t = 6$ lattices are considered. [2]

What is known

The $N_f = 2 + 1$ theory was with Symanzik improved gauge action and stout improved staggered fermion action was studied in [1]. They find $m_c < 0.12m_{\text{phys}}$ on $N_t = 6$ lattices.

A study with Symanzik improved gauge action and p4 staggered action has revealed $m_ca = 0.0007(4)$ on $N_t = 4$ lattices. [2]

The problem was also studied with Wilson fermions in [3]. They find a critical mass of approximately at $m_c \approx m_s$.

What is known

To compare results at different actions one should look at the pion masses:

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>action</th>
<th>$m_{\pi,c}$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>stagg., unimproved</td>
<td>260 MeV</td>
<td>[1]</td>
</tr>
<tr>
<td>6</td>
<td>stagg., unimproved</td>
<td>150 MeV</td>
<td>[2]</td>
</tr>
<tr>
<td>4</td>
<td>stagg., p4</td>
<td>70 MeV</td>
<td>[3]</td>
</tr>
<tr>
<td>6</td>
<td>stagg., stout</td>
<td>$\leq 50$ MeV</td>
<td>[4]</td>
</tr>
<tr>
<td>6</td>
<td>stagg., HISQ</td>
<td>$\leq 45$ MeV</td>
<td>[5]</td>
</tr>
<tr>
<td>6</td>
<td>Wilson-Clover</td>
<td>500 MeV</td>
<td>[6]</td>
</tr>
</tbody>
</table>

(Table from K. Szabo, PoS LAT 2014 014, with extensions)

The idea

Starting with an unimproved action at coarse lattices, where we can locate the critical point. Then we continuously increase the improvement.

In this work: Using Wilson gauge action with $N_f = 3$ staggered fermions with two levels of stout-smearing. Smearing-parameter $\rho$ can be varied.

Hope: Smearing should bring the theory closer to the continuum theory. We might get informations how to approach the continuum, and what action is best suited.

In this talk I will show preliminary results at $N_t = 4$ and $N_t = 6$. 
The chiral condensate

The chiral condensate is the order parameter for $ma = 0$ and behaves approximately like an order parameter for $ma \neq 0$.

$$ma = 0.03$$

![Graph showing the behavior of the chiral condensate at $ma = 0.03$.](image)
The chiral condensate

The chiral condensate is the order parameter for $ma = 0$ and behaves approximately like an order parameter for $ma \neq 0$.

$m_0 = 0.01$
The chiral condensate

The chiral condensate is the order parameter for $ma = 0$ and behaves approximately like an order parameter for $ma \neq 0$.

$ma = 0.005$

\[ \bar{\psi} \psi (\beta, \rho) \]
The chiral susceptibility

One can trace out the peak of the disconnected chiral susceptibility for different smearing levels using reweighing:

\[ ma = 0.03 \]

One can see that the transition grows stronger with lower quark masses and weaker with higher smearing.
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Distribution in the $\mathcal{E}/\mathcal{M}$-plane

We can write

$$\mathcal{E} = S_G/\beta + a\bar{\psi}\psi \quad \text{and} \quad \mathcal{M} = \bar{\psi}\psi + bS_G$$

and construct the order parameter by imposing the condition

$$\rho = \langle \delta \mathcal{E} \delta \mathcal{M} \rangle / \sqrt{\langle \delta \mathcal{M}^2 \rangle \langle \delta \mathcal{E}^2 \rangle} = 0:
Close to a second order transition the free energy obeys

\[ f(t, h, L) = b^{-1} f(tb^{y_t}, hb^{y_h}, Lb^{-1}) \]

with \( t \) and \( h \) being mixtures of \( \beta \) and \( ma \). From this the finite size scaling of the susceptibility can be derived:

\[ \chi_M = L^{-\frac{\gamma}{\nu}} \phi_{\chi_M}^{\text{fss}} (c(\beta - \beta_c)L^{\frac{1}{\nu}}) \]

There are other exponents contributing to the scaling, but close to the transition this is the dominant contribution. (other exponents are smaller)

We make the ansatz \( \phi_{\chi_M}^{\text{fss}} = \frac{1}{c_1 + c_2 x} \).
The $\mathcal{M}$-susceptibility for $N_t = 4$

Do a scaling fit. ($\phi_{\chi \mathcal{M}}^{fss} = \frac{1}{c_1+c_2x}$, $Z(2)$ critical exponents.)

$$\rho = 0.00$$

$L = 8$  $L = 10$  $L = 12$  $L = 16$

$m_\sigma = 0.0284 \pm 0.0008$

$\chi^2/\text{NDF} = 0.94$

$p$-value $= 0.58$
The $\mathcal{M}$-susceptibility for $N_t = 4$

Do a scaling fit. ($\phi_{\chi \mathcal{M}}^{\text{fss}} = \frac{1}{c_1 + c_2 x}$, $Z(2)$ critical exponents.)

$$\rho = 0.02$$

\begin{align*}
\chi^2/\text{NDF} &= 0.94 \\
p\text{-value} &= 0.58 \\
m_c a &= 0.0099 \pm 0.0007
\end{align*}
The $\mathcal{M}$-susceptibility for $N_t = 4$

Do a scaling fit. ($\phi_{\chi_M}^{\text{fss}} = \frac{1}{c_1 + c_2 x}$, $Z(2)$ critical exponents.)

$$\rho = 0.04$$

$L = 8$  
$L = 10$  
$L = 12$  
$L = 16$

\[\frac{\chi^2}{\text{NDF}} = 0.94\]

$p$-value = 0.58

$m_c a = -0.0022 \pm 0.0007$
The $\mathcal{M}$-susceptibility for $N_t = 4$

Do a scaling fit. ($\phi^{fss}_{\chi_{\mathcal{M}}} = \frac{1}{c_1 + c_2 x}$, $Z(2)$ critical exponents.)

\[\rho = 0.08\]

$\chi^2 / NDF = 0.94$

$p$-value = 0.58

$m_c a = -0.0072 \pm 0.0021$
The binder cumulant for $N_t = 4$

Do a scaling fit for the binder cumulants. ($B_4(x) = a + bx + cx^2$ with free critical exponents.)

$$\rho = 0.00$$

$\chi^2/\text{NDF} = 0.90$

$p$-value = 0.67

$m_c a = 0.0244 \pm 0.0025$

$a = 1.604$ is expected for $Z(2)$ universality class.
The binder cumulant for $N_t = 4$

Do a scaling fit for the binder cumulants. ($B_4(x) = a + bx + cx^2$ with free critical exponents.)

$$\rho = 0.02$$

$\chi^2/\text{NDF} = 0.90$
$p$-value = 0.67
$m_c a = 0.0087 \pm 0.0022$

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$$\rho = 0.08$$

$\chi^2/NDF = 0.90$

$p$-value $= 0.67$

$m_c a = -0.0198 \pm 0.0194$

$a = 1.604$ is expected for $Z(2)$ universality class.
The critical mass for $N_t = 4$

Binder cumulant works well for interpolation, but not good for extrapolations. Scaling fir for the $\mathcal{M}$ susceptibility seems to be more reliable.

Unexpected: critical quark mass becomes formally (slightly) negative.

Interpretation

In the continuum the critical mass should not depend on the details of the smearing. Several things could happen:

- All points move downward: No critical mass in the continuum:
- The points at lower smearing move down and the points with higher smearing move up: There is a finite critical mass in the continuum and we can give lower and upper limit.
- The critical mass becomes formally negative, but becomes positive again before reaching continuum.
- There is a transition in the continuum, but it is unrelated to the one observed at $N_t = 4$. 

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\[ m_c \]

[Graph showing lattice artifacts and critical mass]
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\[ \text{lattice artifacts} \]

\[ m_c \]
The $\mathcal{M}$-susceptibility for $N_t = 6$

Do a scaling fit. ($\phi_{\chi \mathcal{M}}^{fss} = \frac{1}{c_1 + c_2 x}$, $Z(2)$ critical exponents.)

$$\rho = 0.00$$

\[\chi^2/\text{NDF} = 0.94\]
\[p\text{-value} = 0.58\]
\[m_c a = 0.0022 \pm 0.0009\]
The $\mathcal{M}$-susceptibility for $N_t = 6$

Do a scaling fit. ($\phi^\text{fss}_{\chi_M} = \frac{1}{c_1 + c_2 x}$, $Z(2)$ critical exponents.)

$$\rho = 0.02$$

\[ L \gamma/\nu \chi_M \]

\[ \chi^2/\text{NDF} = 0.94 \]

\[ p\text{-value} = 0.58 \]

\[ m_c a = 0.0018 \pm 0.0009 \]
The binder cumulant for $N_t = 6$

Do a scaling fit for the binder cumulants. ($B_4(x) = a + bx + cx^2$ with free critical exponents.)

\[ \rho = 0.00 \]

\[ \chi^2 / \text{NDF} = 0.90 \]
\[ p\text{-value} = 0.67 \]
\[ m_c a = -0.0032 \pm 0.0016 \]

\[ a = 1.604 \text{ is expected for } Z(2) \text{ universality class.} \]
The binder cumulant for $N_t = 6$

Do a scaling fit for the binder cumulants. ($B_4(x) = a + bx + cx^2$ with free critical exponents.)

$$\rho = 0.02$$

![Graph showing $B_4$ vs $ma$ with data points and error bars, indicating $\chi^2/NDF = 0.90$, $p$-value = 0.67, and $mca = -0.0072 \pm 0.0016$.]

$a = 1.604$ is expected for $Z(2)$ universality class.
The critical mass for $N_t = 6$

Binder cumulant works well for interpolation, but not good for extrapolations. Scaling for the $\mathcal{M}$ susceptibility seems to be more reliable.

Unexpected: critical quark mass becomes formally (slightly) negative.

The critical mass

Critical mass similar for $\rho \approx 0.03$.
Possibility: For $\rho \lesssim 0.03$ continuum value can be approach from above, for $\rho \gtrsim 0.03$ continuum value can be approached from below.
Conclusions

- The chiral phase transition has been studied at $N_t = 4$ and $N_t = 6$ lattices.
- The dependence of $m_c$ at fixed $N_t$ on the smearing parameter has been studied.
- The dependence on the smearing parameter is very large for coarse lattices and the transition eventually vanishes completely.
- On $N_t = 6$ this dependence is reduced drastically → Results are more reliable

Thank you for the attention!