The $N_f = 3$-critical endpoint with smeared staggered quarks

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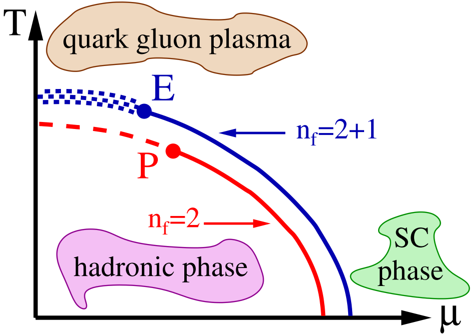
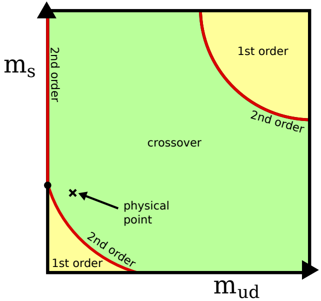
Introduction

- At low temperature color charges are confined.
- At higher temperature a deconfined phase is expected (quark-gluon-plasma).
- For physical quark masses it is known, that this transition is a analytic crossover.



Introduction

- What happens with different quark masses?



- Expected:
  - Crossover at intermediate quark masses.
  - At higher and lower quark masses: first order transition
  - Regions are separated by a line of a second order transition

[1] F. R. Brown, F. P. Butler, H. Chen, N. H. Christ, Z. -h. Dong, W. Schaffer, L. I. Unger and A. Vaccarino, Phys. Rev. Lett. **65** (1990) 2491.
[2] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, Nature **443** (2006) 675 [hep-lat/0611014].
[3] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29** (1984) 338.

What is known

$N_f = 3$-QCD with unimproved staggered action was studied at $N_t = 4$ lattices a long time ago

It was found, that the critical bare mass was at $ma = 0.033(1)$ and in physical mass it was at $m_{\pi,\xi_5} = 290\,\text{MeV}$. The transition was found to belong to the $Z(2)$ universality class as expected. [1]

By going over to an improved action it was possible to show, that the critical quark mass was considerable smaller. The same holds true when $N_t = 6$ lattices are considered. [2]

[1] Karsch, Laermann and Schmidt, Phys. Lett. B **520** (2001) 41
[2] de Forcrand, Kim and Philipsen, PoS LAT 2007 (2007) 178

What is known

The $N_f = 2+1$ theory was with Symmanzik improved gauge action and stout improved staggered fermion action was studied in [1]. They find $m_c < 0.12 m_{\text{phys}}$ on $N_t = 6$ lattices.

A study with Symmanzik improved gauge action and p4 staggered action has revealed $m_c a = 0.0007(4)$ on $N_t = 4$ lattices. [2]

The problem was also studied with Wilson fermions in [3]. They find a critical mass of approximately at $m_c \approx m_s$.

[1] G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, PoS LAT **2007** (2007) 182
[2] F. Karsch, C. R. Allton, S. Ejiri, S. J. Hands, O. Kaczmarek, E. Laermann and C. Schmidt, Nucl. Phys. Proc. Suppl. **129** (2004) 614
[3] Y. Nakamura, PoS LATTICE **2013**, 138 (2014).

What is known

To compare results at different actions one should look at the pion masses:

N_t	action	$m_{\pi,c}$	Ref.
4	stagg.,unimproved	260 MeV	[1]
6	stagg.,unimproved	150 MeV	[2]
4	stagg.,p4	70 MeV	[3]
6	stagg.,stout	$\leq$ 50 MeV	[4]
6	stagg.,HISQ	$\leq$ 45 MeV	[5]
6	Wilson-Clover	500 MeV	[6]

(Table from K. Szabo, PoS LAT **2014** 014, with extensions)

[1] Karsch, Laermann and Schmidt, Phys. Lett. B **520** (2001) 41
[2] de Forcrand, Kim and Philipsen, PoS LAT 2007 (2007) 178
[3] F. Karsch, C. R. Allton, S. Ejiri, S. J. Hands, O. Kaczmarek, E. Laermann and C. Schmidt, Nucl. Phys. Proc. Suppl. **129** (2004) 614
[4] G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, PoS LAT **2007** (2007) 182
[5] H. -T. Ding, A. Bazavov, P. Hegde, F. Karsch, S. Mukherjee and P. Petreczky, PoS LATTICE **2011** (2011) 191
[6] Y. Nakamura, PoS LATTICE **2013**, 138 (2014).

The idea

Starting with an unimproved action at coarse lattices, where we can locate the critical point. Then we continuously increase the improvement.

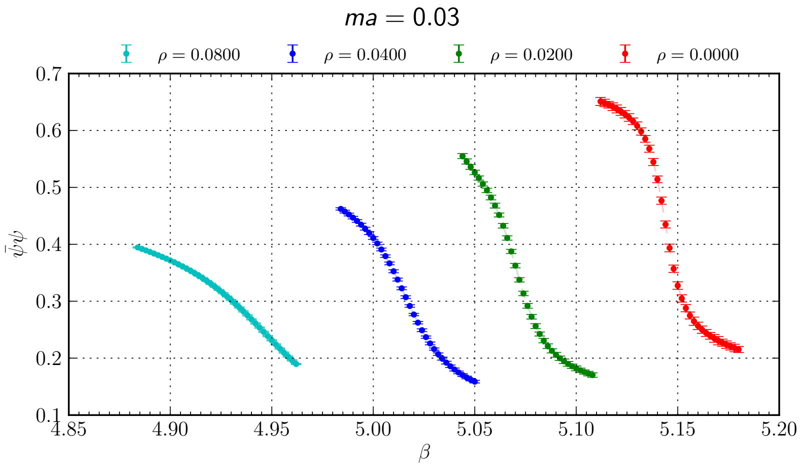
In this work: Using Wilson gauge action with $N_f = 3$ staggered fermions with two levels of stout-smearing. Smearing-parameter ρ can be varied.

Hope: Smearing should bring the theory closer to the continuum theory. We might get informations how to approach the continuum, and what action is best suited.

In this talk I will show **preliminary** results at $N_t = 4$ and $N_t = 6$.

The chiral condensate

The chiral condensate is the order parameter for $ma = 0$ and behaves approximately like an order parameter for $ma \neq 0$.



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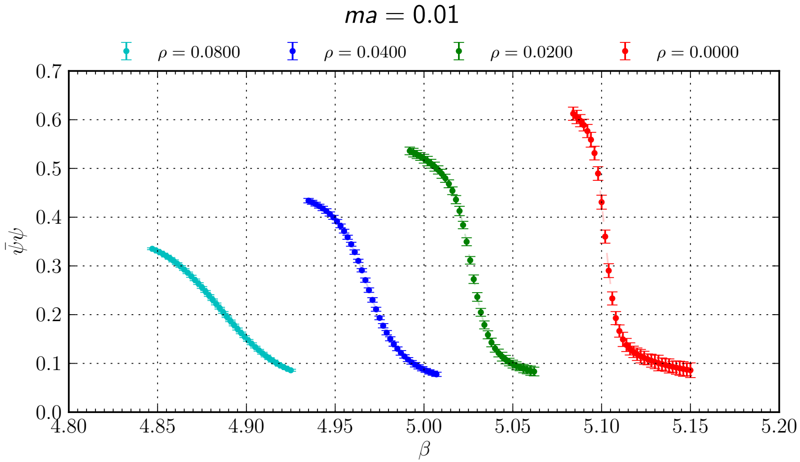

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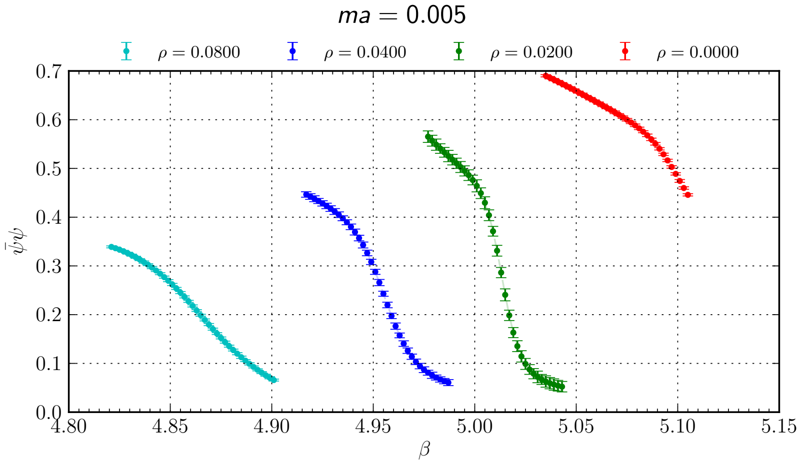
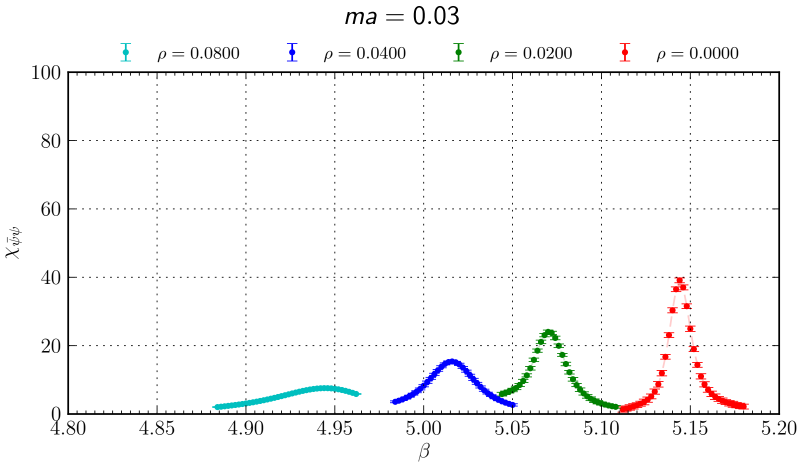


The chiral susceptibility

One can trace out the peak of the disconnected chiral susceptibility for different smearing levels using reweighing:



One can see that the transition grows stronger with lower quark masses and weaker with higher smearing.

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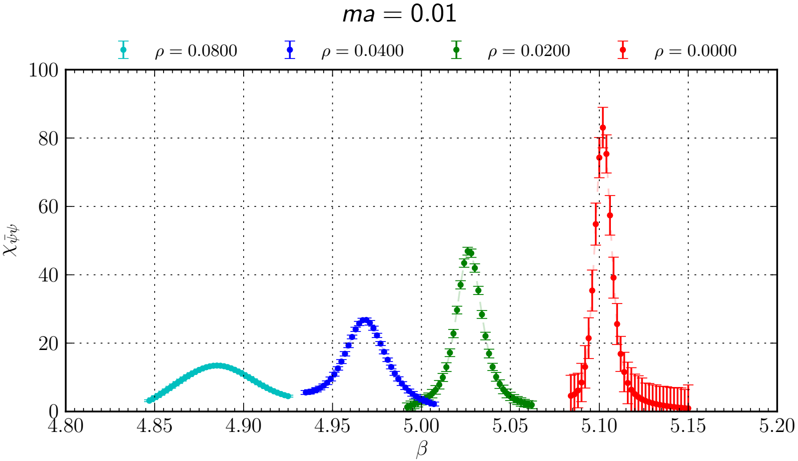

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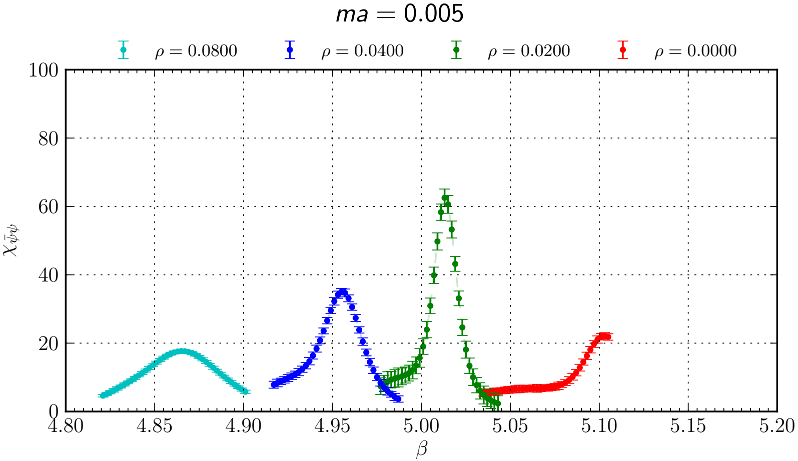

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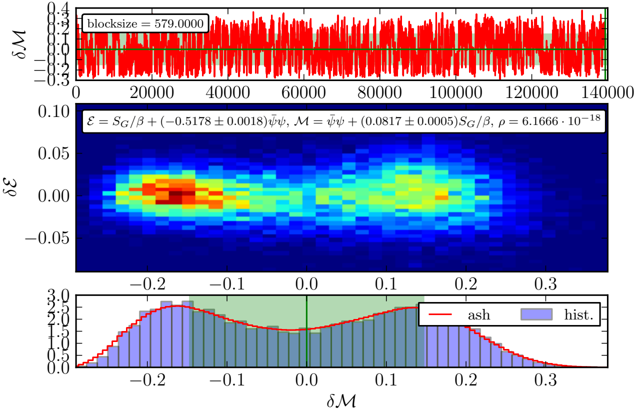
Distribution in the $\mathcal{E}/\mathcal{M}$-plane

We can write

$$\mathcal{E} = S_G/\beta + a\bar{\psi}\psi \quad \text{and} \quad \mathcal{M} = \bar{\psi}\psi + bS_G$$

and construct the order parameter by imposing the condition $\rho = \langle \delta\mathcal{E}\delta\mathcal{M}\rangle / \sqrt{\langle \delta\mathcal{M}^2\rangle\langle \delta\mathcal{E}^2\rangle} = 0$:



Finite-Size-Scaling-Fit

Close to a second order transition the free energy obeys

$$f(t, h, L) = b^{-1} f(tb^{y_t}, hb^{y_h}, Lb^{-1})$$

with t and h being mixtures of β and ma. From this the finite size scaling of the susceptibility can be derived:

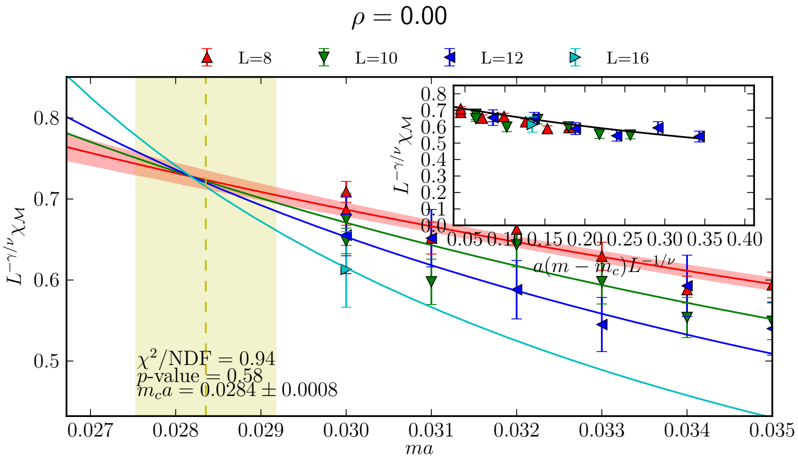
$$\chi_{\mathcal{M}} = L^{-\frac{\gamma}{\nu}} \phi^{\text{fss}}_{\chi_{\mathcal{M}}}(c(\beta - \beta_c) L^{\frac{1}{\nu}})$$

There are other exponents contributing to the scaling, but close to the transition this is the dominant contribution. (other exponents are smaller)

We make the ansatz $\phi^{\text{fss}}_{\chi_{\mathcal{M}}} = \frac{1}{c_1 + c_2 x}$.

The $\mathcal{M}$-susceptibility for $N_t = 4$

Do a scaling fit. ($\phi^{\text{fss}}_{\chi_\mathcal{M}} = \frac{1}{c_1 + c_2 x}$, $Z(2)$ critical exponents.)



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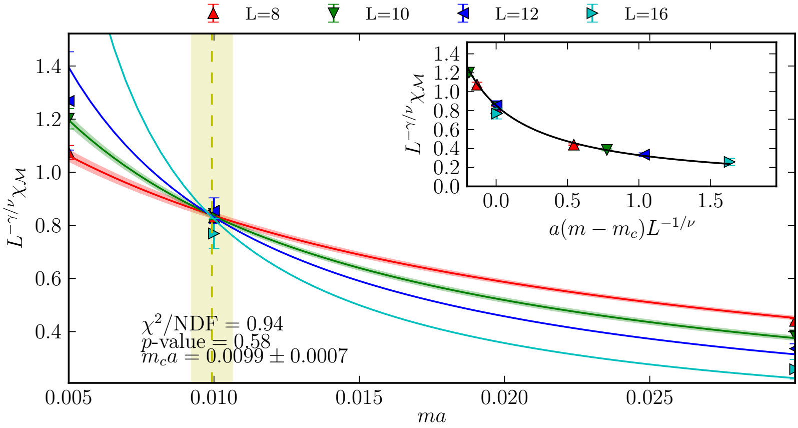
$\rho = 0.02$



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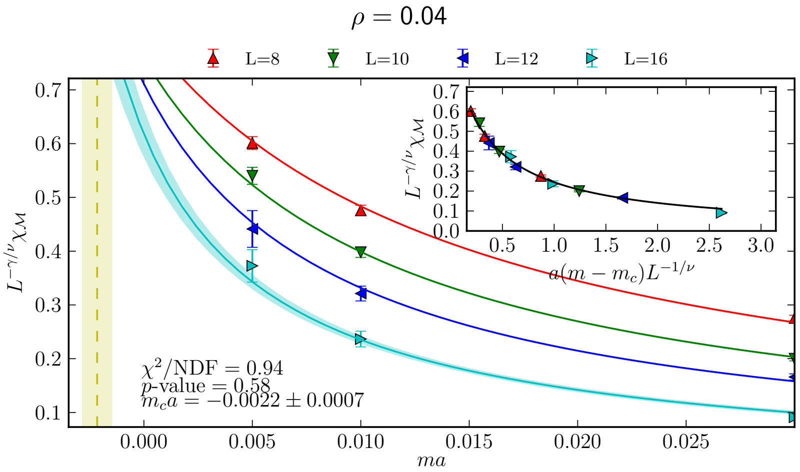

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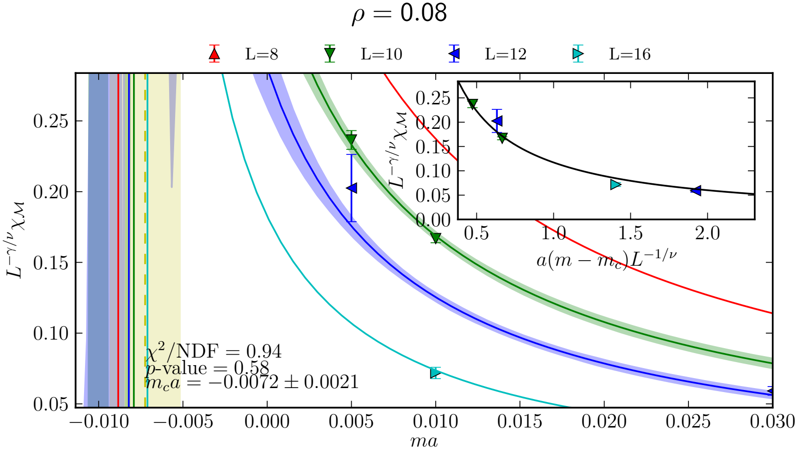
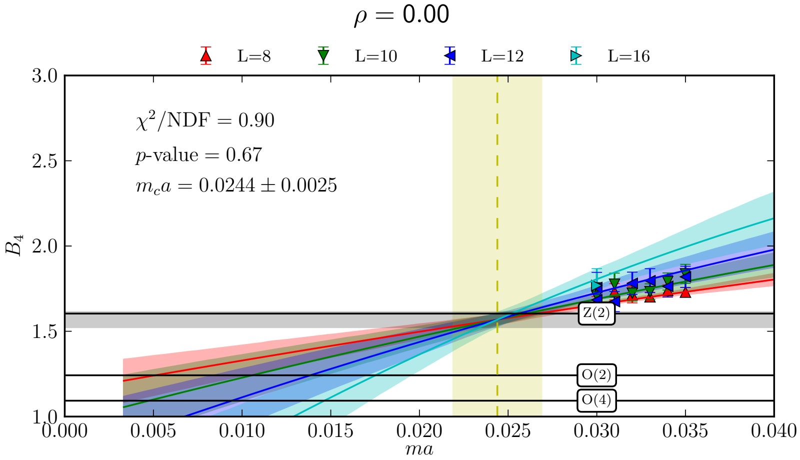


The binder cumulant for $N_t = 4$

Do a scaling fit for the binder cumulants. ($B_4(x) = a + bx + cx^2$ with free critical exponents.)



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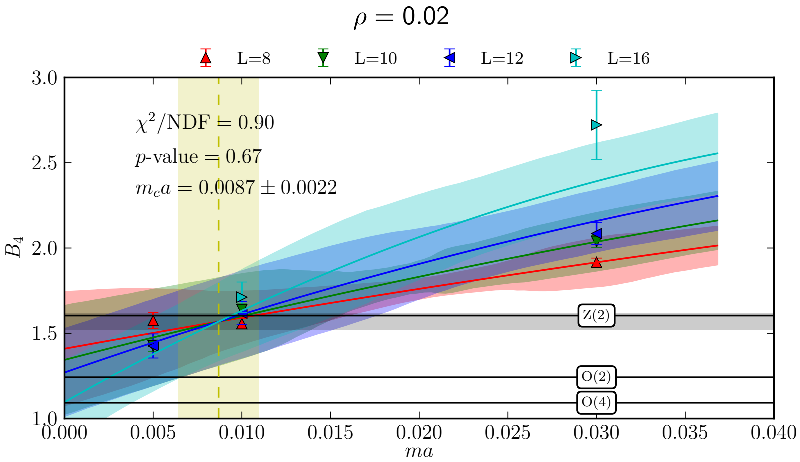

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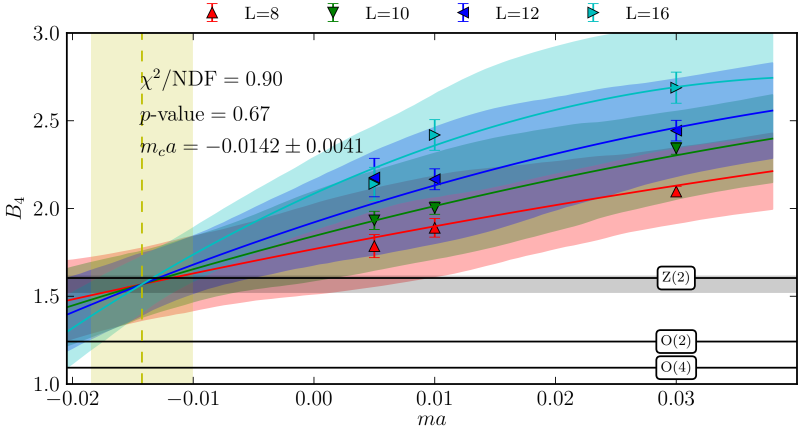
$\rho = 0.04$



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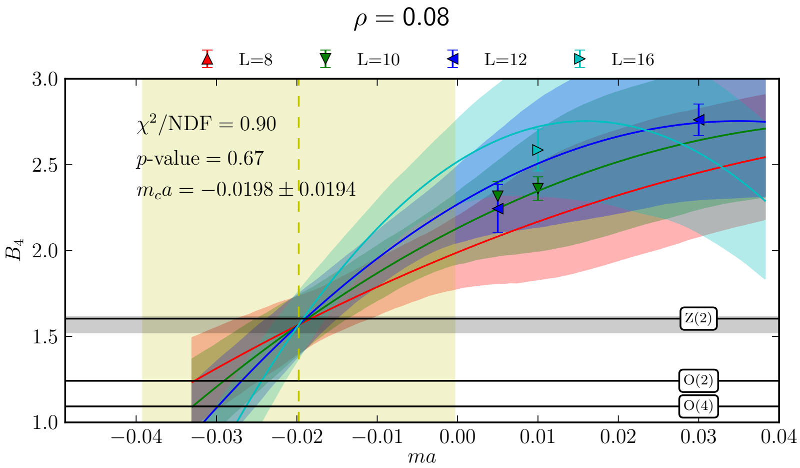

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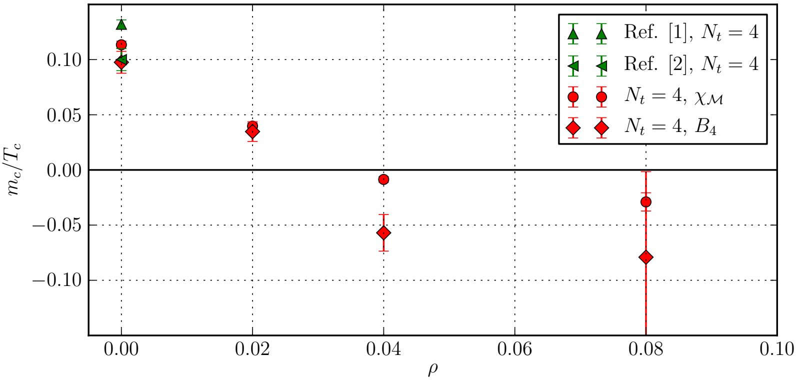
The critical mass for $N_t = 4$



Binder cumulant works well for interpolation, but not good for extrapolations. Scaling fir for the $\mathcal{M}$ susceptibility seems to be more reliable.

Unexpected: critical quark mass becomes formally (slightly) negative.

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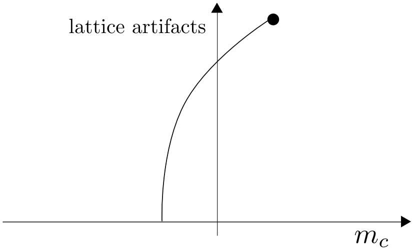
[2] de Forcrand, Kim and Philipsen, PoS LAT 2007 (2007) 178

Interpretation

In the continuum the critical mass should not depend on the details of the smearing. Several things could happen:

- All points move downward: No critical mass in the continuum:



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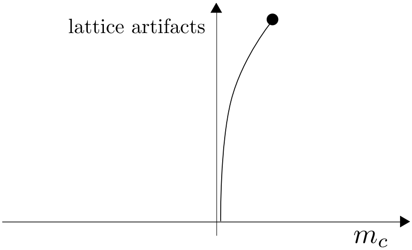

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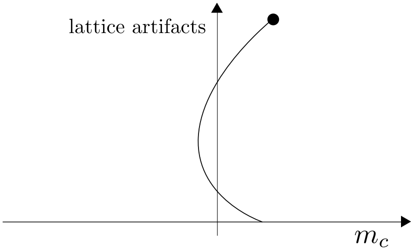

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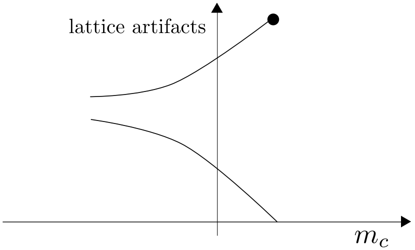
- The points at lower smearing move down and the points with higher smearing move up: There is a finite critical mass in the continuum and we can give lower and upper limit.
- The critical mass becomes formally negative, but becomes positive again before reaching continuum.
- There is a transition in the continuum, but it is unrelated to the one observed at $N_t = 4$.



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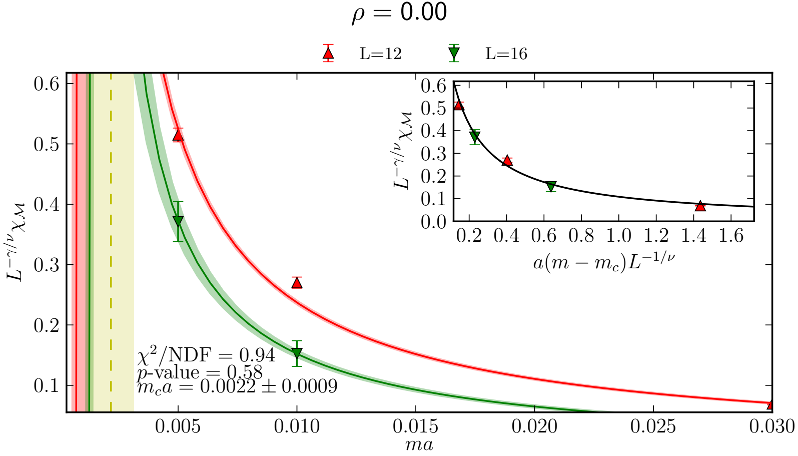

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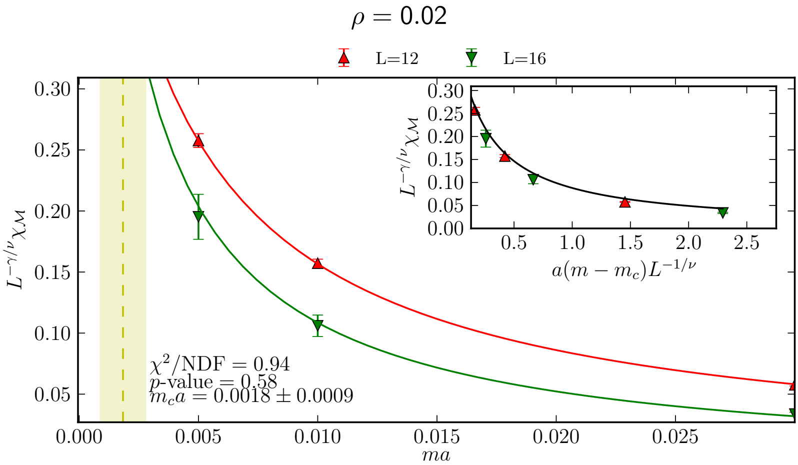

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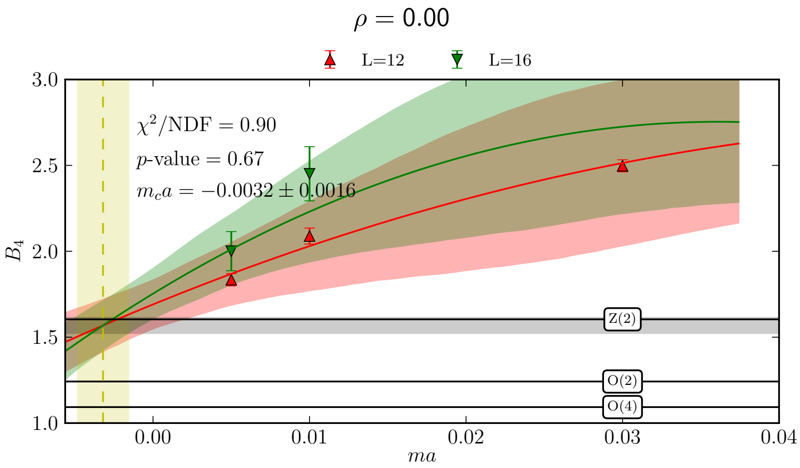

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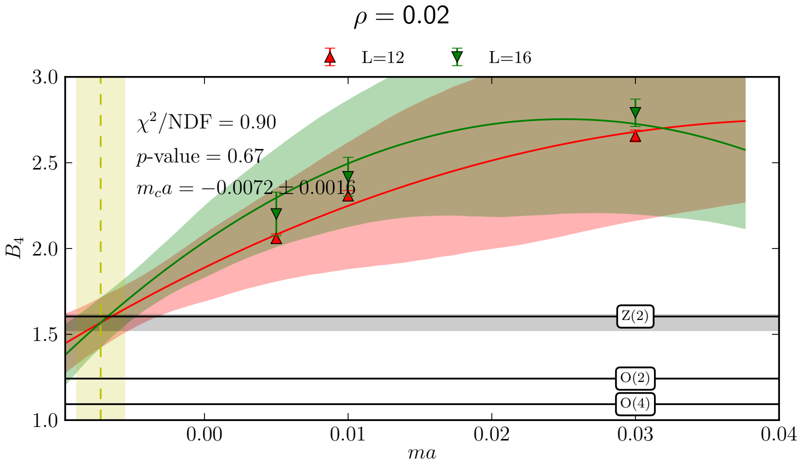

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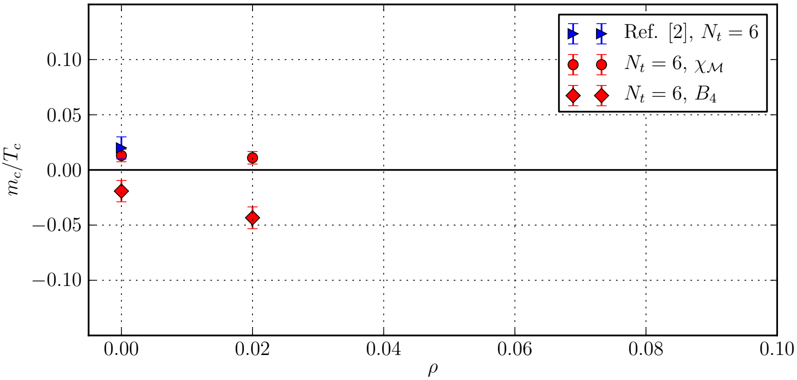

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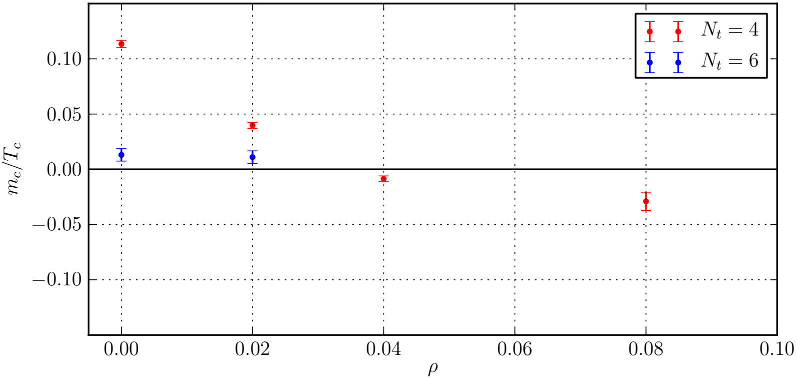
The critical mass



Critical mass similar for $\rho \approx 0.03$.
Possibility: For $\rho \lesssim 0.03$ continuum value can be approach from above, for $\rho \gtrsim 0.03$ continuum value can be approached from below.

Conclusions

- The chiral phase transition has been studied at $N_t = 4$ and $N_t = 6$ lattices.
- The dependence of m_c at fixed N_t on the smearing parameter has been studied.
- The dependence on the smearing parameter is very large for coarse lattices and the transition eventually vanishes completely.
- On $N_t = 6$ this dependence is reduced drastically $\rightarrow$ Results are more reliable

Thank you for the attention!