Flux tubes in the SU(3) vacuum: London penetration depth and coherence length

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   - Confinement and the dual superconductor model
   - Chromoelectric field on the lattice

2 Flux tubes on the lattice
   - Details about simulations
   - The measuring process at a glance

3 Numerical data
   - Results from the fit and parameters vs smearing

4 Penetration depth and coherence length
   - From lattice to physical units
   - Scaling

5 Conclusions
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Confinement and the dual superconductor model

The color confinement problem

**Figure:** $q\bar{q}$ pair at distance $R$ in the QCD vacuum

<table>
<thead>
<tr>
<th>Deconfined phase</th>
<th>Confined phase</th>
</tr>
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<tbody>
<tr>
<td>$E_0(R) \overset{R \to \infty}{\to} 2m$</td>
<td>$E(R) \to \sigma R$, $\sqrt{\sigma} = 420$ MeV</td>
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At the scale of color confinement non perturbative methods are needed
Dual superconductivity

Dual superconductor picture of confinement in QCD proposed by Mandelstam and ’t Hooft.

[G. ’t Hooft, in High Energy Physics, EPS International Conference, (1975)]

QCD vacuum as a dual superconductor

- Color confinement due to the dual Meissner effect produced by the condensation of chromomagnetic monopoles
- Chromoelectric field connecting a $q\bar{q}$ static pair squeezed inside a tube structure: Abrikosov vortex

Relevance of nonperturbative study of chromoelectric flux tubes at $T \neq 0$ to clarify the formation of $c\bar{c}$ and $b\bar{b}$ bound states in heavy ion collisions.
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SUPERCONDUCTIVITY

$F_{\mu\nu}$

Electric charges condensate (Cooper pairs)

Magnetic Abrikosov flux tubes

DUAL SUPERCONDUCTIVITY

$\tilde{F}_{\mu\nu}$

Magnetic monopoles condensate

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Coherence length and London penetration depth

- $\lambda$ London penetration depth: characteristic length of the exponential decrease of $\vec{B}$ in a superconductor
- $\xi$ Coherence length: length scale on which the density of Cooper pairs can change appreciably

Here enters the dual superconductor model

- Ordinary superconductivity: magnetic field as function of the distance from a vortex line in the mixed state
- Two different expressions coming, by dual analogy, from the London model or, equivalently, the Ginzburg-Landau theory

1. Vortex as a line singularity

\[ E_l(x_t) = \frac{\phi}{2\pi} \mu^2 K_0(\mu x_t), \quad x_t > 0, \quad \lambda \gg \xi \leftrightarrow \kappa \gg 1 \]


2. Cylindrical vortex

\[ E_l(x_t) = \frac{\phi}{2\pi} \frac{1}{\lambda \xi_v} \frac{K_0(R/\lambda)}{K_1(\xi_v/\lambda)}, \]

[J. R. Clem, J. Low Temp. Phys. 18, 427 (1975)]
Confinement and the dual superconductor model

Fitting function in our work

\[ E_l(x_t) = \frac{\phi}{2\pi} \frac{\mu^2}{\alpha} \frac{K_0[(\mu^2 x_t^2 + \alpha^2)^{1/2}]}{K_1[\alpha]} \quad x_t \geq 0, \]

\[ R = \sqrt{x_t^2 + \xi_v^2}, \quad \mu = \frac{1}{\lambda}, \quad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}, \quad \kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha} \left[ 1 - K_0^2(\alpha)/K_1^2(\alpha) \right]^{1/2}. \]

1. \( \phi \) external flux
2. \( \mu = 1/\lambda \) London penetration depth inverse
3. \( 1/\alpha = \lambda/\xi_v \) with \( \xi_v \) variational core-radius parameter
4. \( \kappa = \lambda/\xi \) Ginzburg-Landau parameter
**Chromoelectric field on the lattice**

**Connected correlator from previous studies**

\[
\rho_{\mathcal{W}}^{\text{conn}} = \frac{\langle \text{tr} (\mathcal{W} L U_P L^\dagger) \rangle}{\langle \text{tr}(\mathcal{W}) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(U_P) \text{tr}(\mathcal{W}) \rangle}{\langle \text{tr}(\mathcal{W}) \rangle}
\]


- **Continuum limit**

  \[
  \rho_{\mathcal{W}}^{\text{conn}} \xrightarrow{a \to 0} a^2 g \left[ \langle F_{\mu\nu} \rangle_{q\bar{q}} - \langle F_{\mu\nu} \rangle_0 \right]
  \]

- **Color field strength tensor**

  \[
  F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}} \rho_{\mathcal{W}}^{\text{conn}}(x)
  \]

- \(W\) Wilson loop
- \(L\) Schwinger line
- \(U_p\) Plaquette

- \(E_i(x), B_i(x)\) by changing \(U_P = U_{\mu\nu}(x)\) orientation.

- \(E_i(x_t)\) component dominates at \(T=0\).
Chromoelectric field on the lattice

Connected correlator with Polyakov loops

\[
\rho_{P}^{\text{conn}} = \frac{\left\langle \text{tr} \left( P(x) L U P L^\dagger \right) \text{tr} P(y) \right\rangle}{\left\langle \text{tr} \left( P(x) \right) \text{tr} \left( P(y) \right) \right\rangle} - \frac{1}{3} \frac{\left\langle \text{tr} \left( P(x) \right) \text{tr} \left( P(y) \right) \text{tr} \left( U_P \right) \right\rangle}{\left\langle \text{tr} \left( P(x) \right) \text{tr} \left( P(y) \right) \right\rangle}
\]

- Color field strength tensor

\[
F_{\mu\nu}(x) = \sqrt{\frac{\beta}{6}} \rho_{P}^{\text{conn}}(x)
\]

- \(\rho_{P}^{\text{conn}}\) suited for the \(T \neq 0\) case


- \(P(x), P(y)\) Polyakov lines separated by a distance \(\Delta\)
- \(L\) Schwinger line
- \(U_P\) Plaquette
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## Technicalities

### Lattice and correlator features

- Size $20^4$ and periodic boundary conditions
- Distance between Polyakov loops $\Delta = 4a, 6a, 8a$

### LGT and action

- $SU(3)$ pure gauge LGT
- Wilson action $S = \beta \sum_{x, \mu > \nu} \left[ 1 - \frac{1}{3} \text{Re} \text{Tr} U_{\mu \nu}(x) \right]$, with $5.9 < \beta < 6.1$

### Algorithms

- Cabibbo-Marinari algorithm combined with overrelaxation
- APE smearing procedure to increase signal-to-noise ratio
Smearing procedure: motivations and method

- Replacement of the previously used cooling mechanism
- Possibility to check previous results in many different cases
  - Wilson correlator and Smearing
  - Polyakov correlator and Cooling
  - Polyakov correlator and Smearing

APE smearing procedure


\[
C_{\mu\nu}(x) = U_\nu(x)U_\mu(x + \hat{\nu})U_\nu^\dagger(x + \hat{\mu})
+ U_\nu^\dagger(x - \hat{\nu})U_\mu(x - \hat{\nu})U_\nu(x - \hat{\nu} + \hat{\mu})
\]

\[
\tilde{U}_\mu(x) = \mathcal{P}_{SU(3)}[(1 - \alpha)U_\mu(x) + \frac{\alpha}{6} \sum_{\mu \neq \nu} C_{\mu\nu}(x)],
\]

\[\alpha = 0.5, \quad 16 < n_{ape} < 50\]
The measuring process at a glance

Our investigation in few steps

For different values of $\beta$

- Smearing over a thermalized field configuration
- Measurement of $E_i(x_t)$ through $\rho_P^{\text{conn}}$ by varying plaquette position
- Fit of the shape of $E_i(x_t)$ to extract the parameters $\phi$, $\mu$, $\lambda/\xi_v$, $\kappa$
- Analysis of the behavior of $\phi$, $\mu$, $\lambda/\xi_v$, $\kappa$ with smearing, looking for a plateau
- Estimate of $\lambda$ and $\xi$ from a scaling analysis
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Results from the fit and parameters vs smearing

Measurements at integer and noninteger distances

- Nonintegers distances included to check for rotational invariance restoration
- Restriction only to points at integer distances:
  - Smaller $\chi_r^2$
  - CPU time saved

Consistent values for parameters in both cases

**Figure:** Longitudinal chromoelectric field $E_l$ versus $x_t$, in lattice units for $\Delta = 4a$ and after 10 smearing steps
Results from the fit and parameters vs smearing

Parameters vs smearing: looking for a plateau

Figure: $\phi$ vs smearing ($\Delta = 6a$)

Figure: $\mu$ vs smearing ($\Delta = 6a$)
Parameters vs smearing: looking for a plateau

Figure: $\lambda/\xi_v$ vs smearing ($\Delta = 6a$)

Figure: $\kappa$ vs smearing ($\Delta = 6a$)
Plateau values vs $\beta$ comparing all the sizes

$\Delta$ variation to study contamination effects due to the proximity of the static color sources.

Figure: Plateau values for $\mu$ vs $\beta$ ($\Delta = 4a, 6a, 8a$)

Figure: Plateau values for $\lambda/\xi_v$ vs $\beta$ ($\Delta = 4a, 6a, 8a$)

$\Delta = 6a$ good compromise between contaminations and signal-to-noise.
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Scaling of the plateau values of $a\mu$ with the string tension through the parametrization.

$$\sqrt{\sigma}(g) = f_{SU(3)}(g^2)[1 + 0.2731 \hat{a}^2(g) - 0.01545 \hat{a}^4(g) + 0.01975 \hat{a}^6(g)]/0.01364$$

$$\hat{a}(g) = \frac{f_{SU(3)}(g^2)}{f_{SU(3)}(g^2(\beta = 6))}, \quad \beta = \frac{6}{g^2}, \quad 5.6 \leq \beta \leq 6.5$$

$$f_{SU(3)}(g^2) = \left(b_0 g^2\right)^{-\frac{b_1}{2b_0^2}} \exp\left(\frac{-1}{2b_0 g^2}\right), \quad b_0 = \frac{11}{(4\pi)^2}, \quad b_1 = \frac{102}{(4\pi)^4}$$

Figure: Longitudinal chromoelectric field $E_l$ versus $x_t$, in lattice units and in physical units, for $\Delta = 6a$ and after 30 smearing steps.
Parameters scaling behavior: sizes compared

\[ \frac{\mu}{\sigma} \]

\[ \kappa \]
Parameters scaling behavior: $\Delta = 6a$

$\mu/\sqrt{\sigma} = 2.684(97)$  \hspace{1cm} $\mu/\sqrt{\sigma}_{old} = 2.799(38)$

$\kappa = 0.178(21)$  \hspace{1cm} $\kappa_{old} = 0.243(88)$

$\lambda = 1/\mu = 0.1750(63)$ fm  \hspace{1cm} $\xi = 0.983(121)$ fm

Here ‘old‘ means Wilson connected correlator and cooling as in [P. Cea, L. Cosmai, and A. Papa, Phys. Rev. D 86, (2012)]
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Summary and outlook

- SU(3) vacuum as a type-I dual superconductor in agreement with [A. Shibata, K.-I. Kondo, S. Kato, and T. Shinohara, Phys. Rev. D 87, (2013)]

- Finite temperature
- Introduction of dynamical quarks d.o.f. (implementation of $\rho_P^{\text{conn}}$ within the MILC code)
- Check of the validity of the model (goodness of the fit): $R$ and $x_t$ ranges
THANK YOU