Resonances in πK scattering

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for the Hadron Spectrum Collaboration

based on work with J. J. Dudek, R. G. Edwards and C. E. Thomas, arXiv:1406.4158

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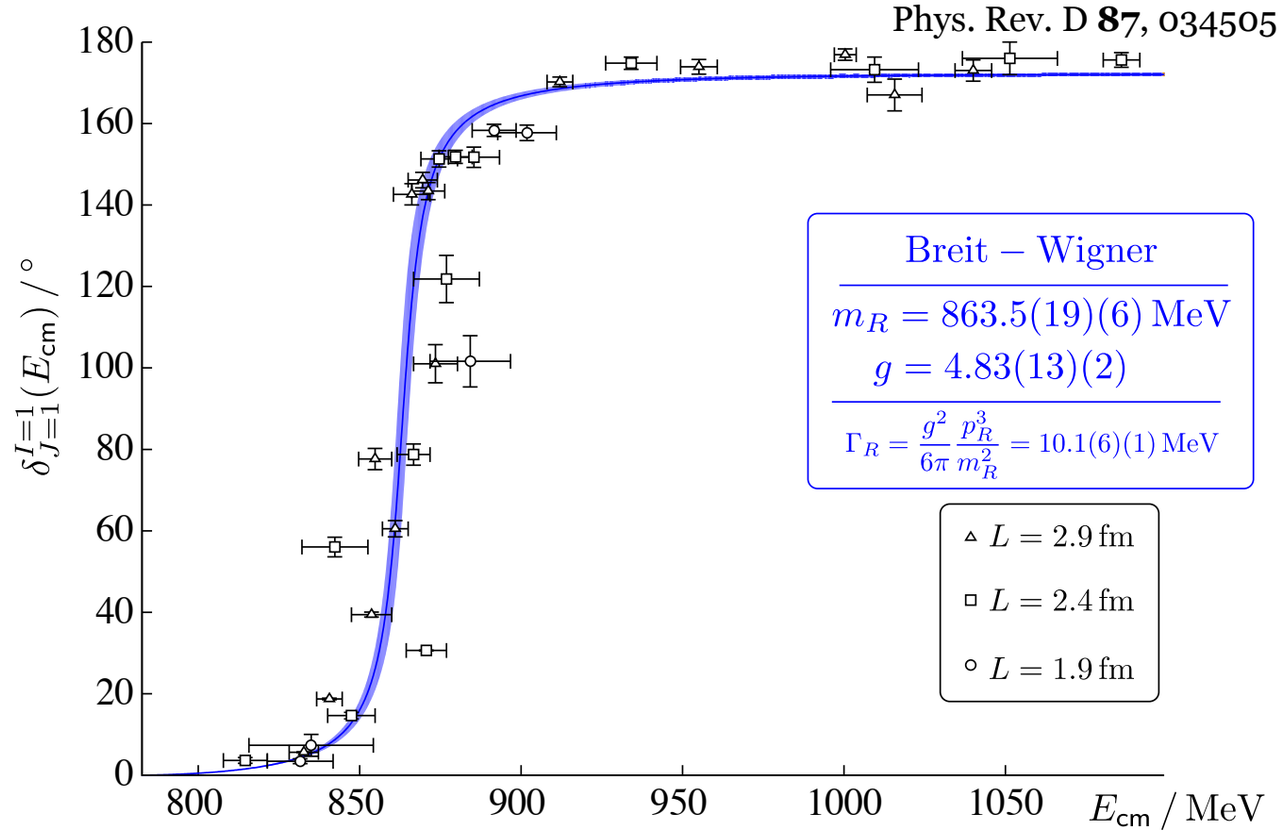
Resonances from QCD

- We want to understand the spectrum of hadrons directly from QCD.
- Most excited hadrons are resonant enhancements in the scattering of lighter stable particles: ρ resonance in I=1 $\pi\pi$ scattering.
- Many excited hadrons decay to the lightest pseudoscalar octet (π, K, η), which are long-lived.
- Excited states are often resonances in multiple channels: need to use the coupled-channel formalism.
- Several options, one choice is ***I*=1/2 *πK*-*ηK*** scattering.
- Physical amplitudes have resonant states in several partial waves:

J. J. Dudek, R. G. Edwards and C. E. Thomas
Phys. Rev. D **87**, 034505



$J^P=0^+$	κ, $K_0^*(1430)$, ...
$J^P=1^-$	$K^*(892)$, ...
$J^P=2^+$	$K_2^*(1430)$, ...

Calculation method

- Anisotropic lattices with 2+1 dynamical flavours.
  3.5× finer spacing in time: better energy resolution.
  Wilson clover action, Symanzik-improved gauge action.

- *Distillation* method for quark smearing.

- 3 volumes

L (fm)	$(L/a_s)^3 \times (T/a_t)$	N_{cfgs}	$N_{t_{\text{srcs}}}$	N_{vecs}
1.9	$16^3 \times 128$	479	4-8	64
2.4	$20^3 \times 128$	603	2-6	128
2.9	$24^3 \times 128$	553	2-6	162

$m_\pi = 391\text{MeV}, \quad m_K = 549\text{MeV}, \quad m_\eta = 589\text{MeV}$

- Large basis of operators including:

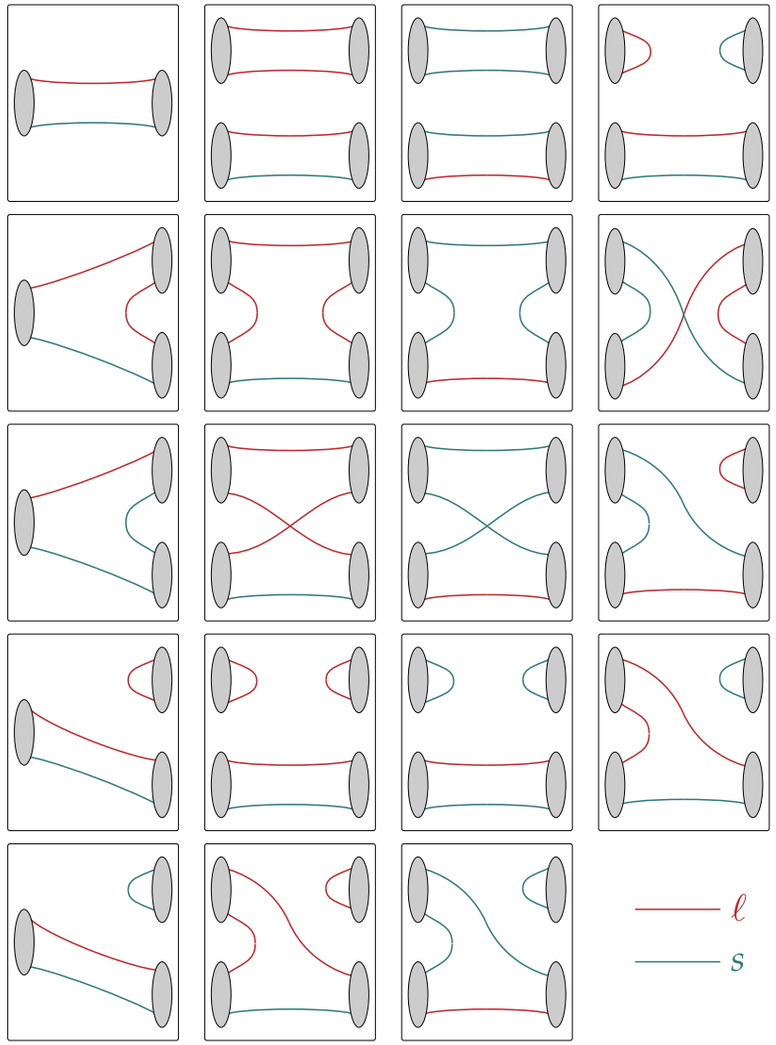
  "Single-meson" like operators: $\overline{\psi}\Gamma\psi\,,\ \overline{\psi}\Gamma\overleftrightarrow{D}\ldots\overleftrightarrow{D}\psi$

  "Meson-meson" like operators with definite momentum: $\pi(\vec{p}_1)K(\vec{p}_2), \quad \eta(\vec{p}_1)K(\vec{p}_2)$

- Include all Wick contractions, all disconnected contributions.

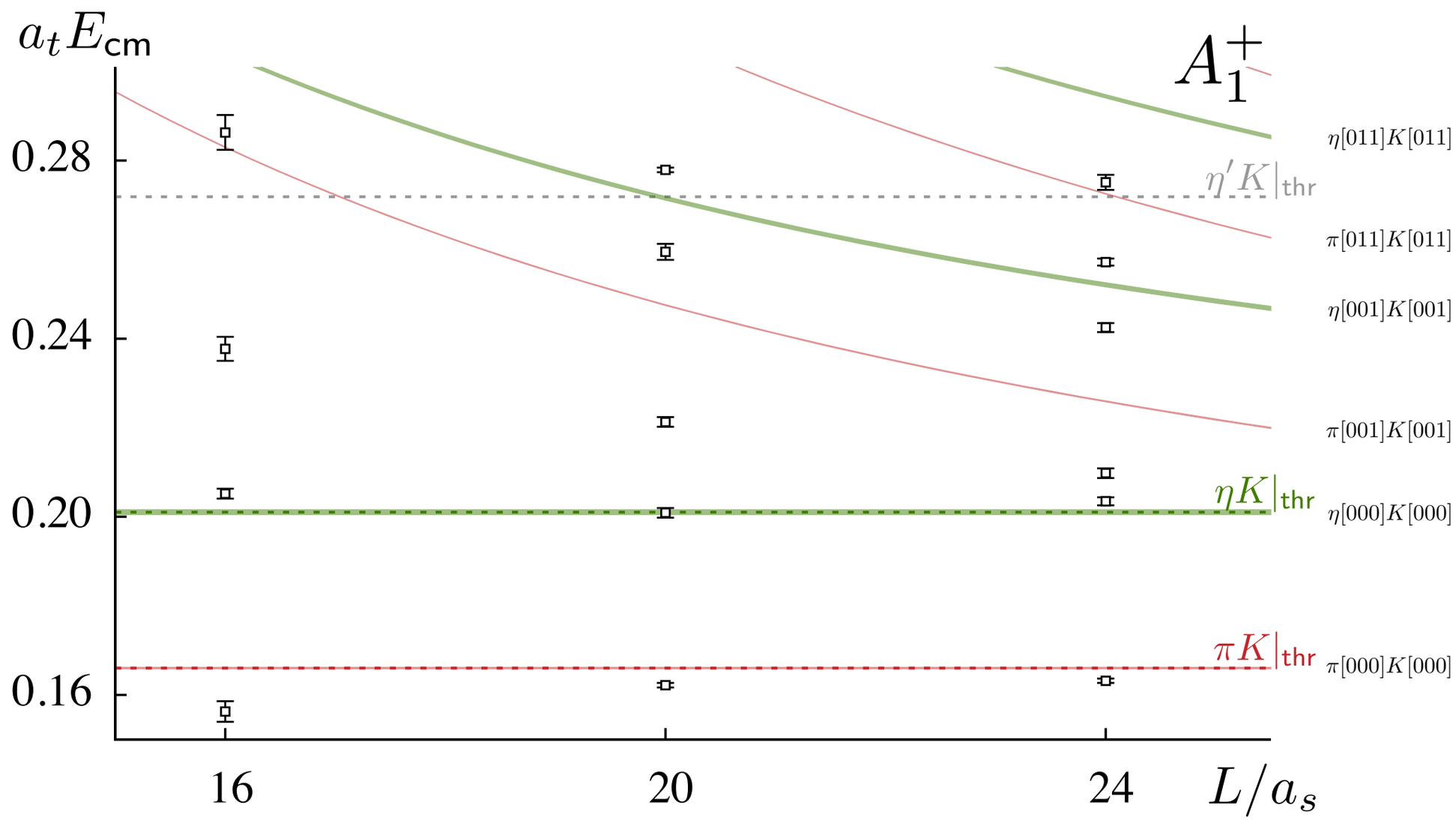
- All relevant irreps with boosts

  $p^2 = |\vec{p}_1 + \vec{p}_2|^2 \leq 4\left(\frac{2\pi}{L}\right)^2$



Spectrum extraction

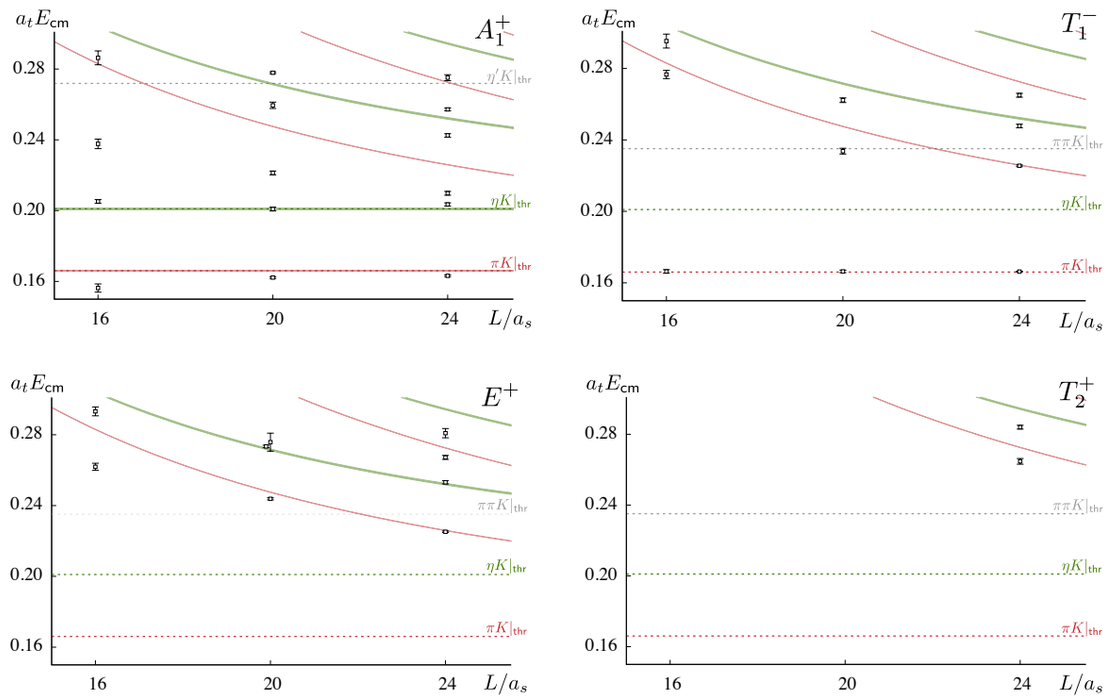
- Use variational method to obtain the spectrum in each irrep.
- A typical irrep: A_1^+. Contains πK-ηK with J^P=0$^+$, 4$^+$, ...
- Large shifts from non-interacting levels: strong interactions between hadrons. $E = \left(m_1^2 + |\vec{p}_1|^2\right)^{\frac{1}{2}} + \left(m_2^2 + |\vec{p}_2|^2\right)^{\frac{1}{2}}$



Many irreps

- >100 energy levels obtained
- 73 used for *S*+*P* wave analysis
- 24 used for *D*-wave analysis

$p = [000]$



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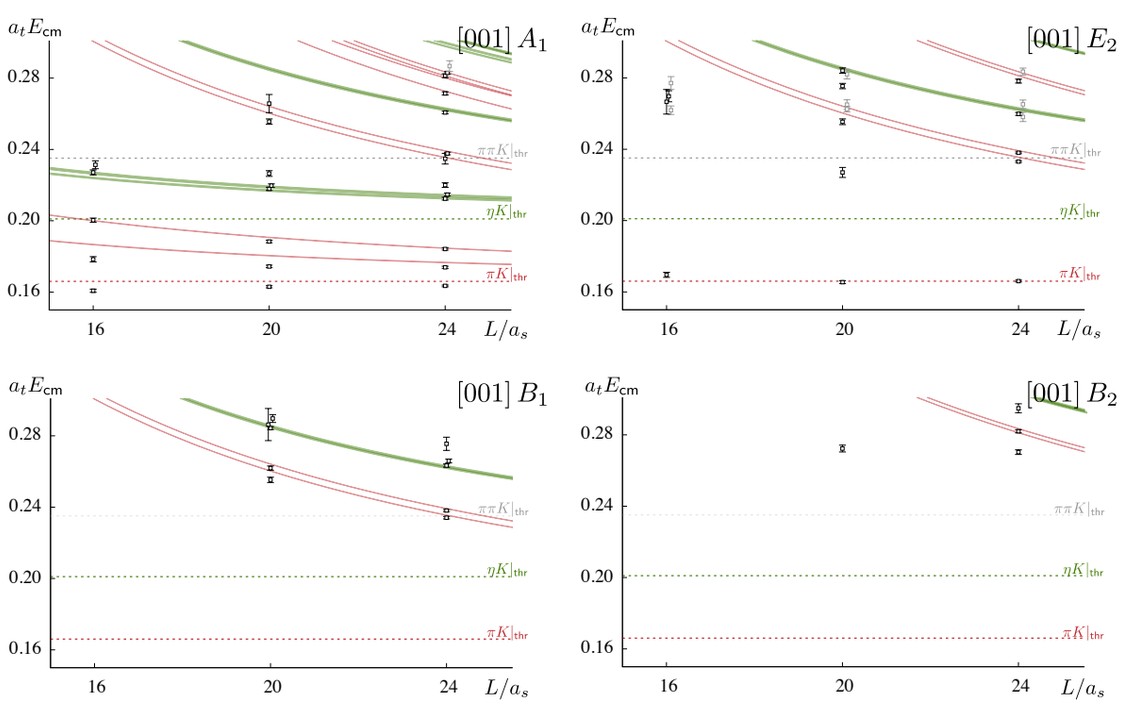
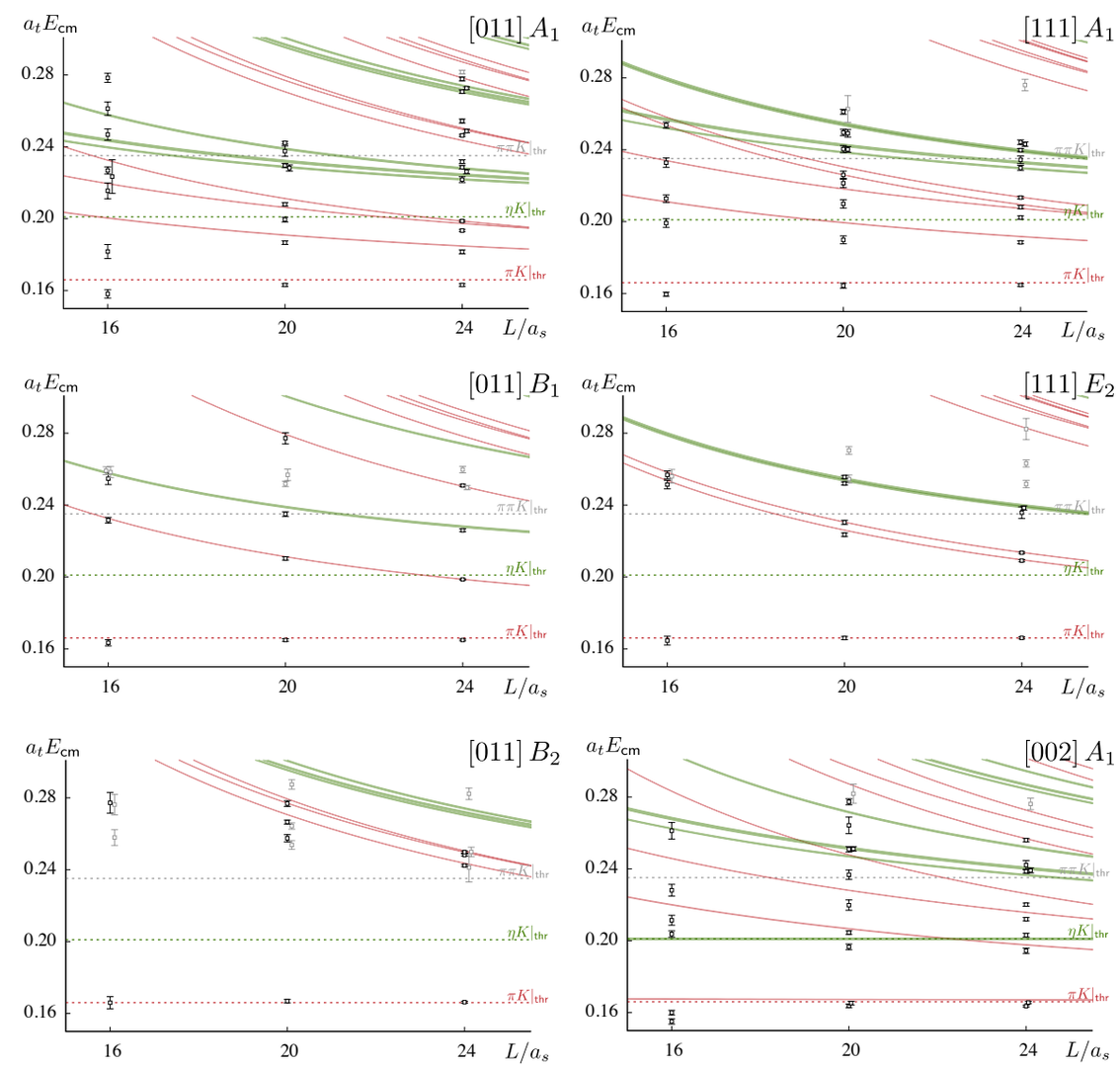
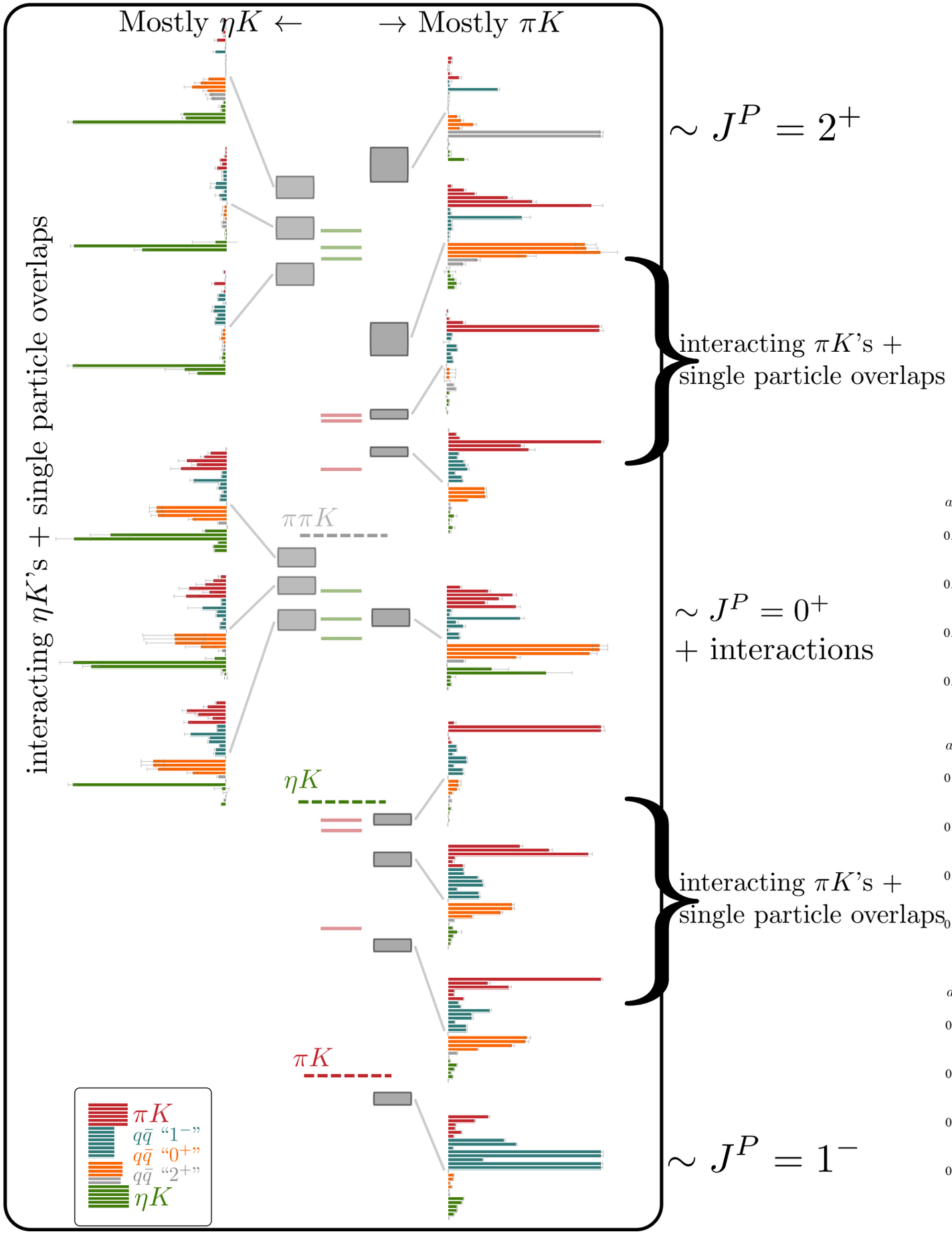
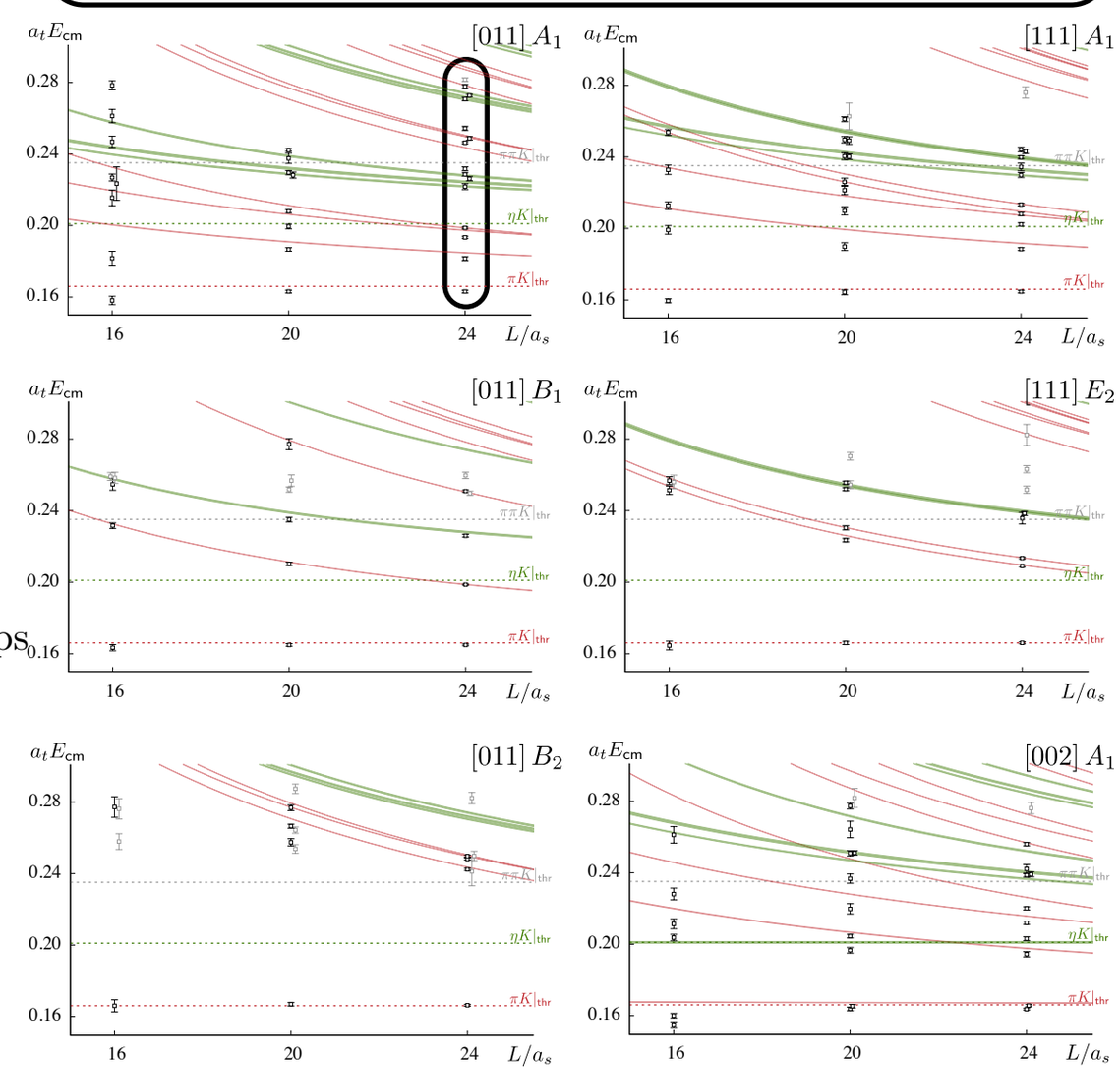


$p = [011], [111], [002]$





- Overlaps ~ guide to resonant content $Z_i^{\mathfrak{n}} = \langle \mathfrak{n} | \mathcal{O}_i^\dagger | 0 \rangle$
- Shifted πK-like and ηK-like states
- J^P=1⁻ state near to πK threshold, J^P=2⁺ state, extra J^P=0⁺.
- Considerable partial-wave mixing. [011] A_1 contains J^P=0⁺, 1, 2, ...



Coupled-channel scattering

$$\rho_i = \frac{2p_i}{E_{\text{cm}}}$$
$$q_i^2 = p_i^2 (L/2\pi)^2$$

- Use coupled-channel extension of Lüscher formalism:

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{nn'} + i\rho_i\, t_{ij}^{(\ell)}\left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{d},\Lambda}_{\ell n;\ell' n'}(q_i^2)\right)\right] = 0$$

Couples channels $i\,j$, diagonal in ℓ

Couples partial waves ℓ

- t-matrix in ∞-volume ⟶ finite volume energies.
- Actual problem: Finite volume energy levels ⟶ t-matrix.
- Many unknowns for each energy level: channel-coupling, partial wave mixing.

A solution:

➔ Parameterise t_{ij} using a few free parameters.

➔ Given many energy levels, the problem is constrained.

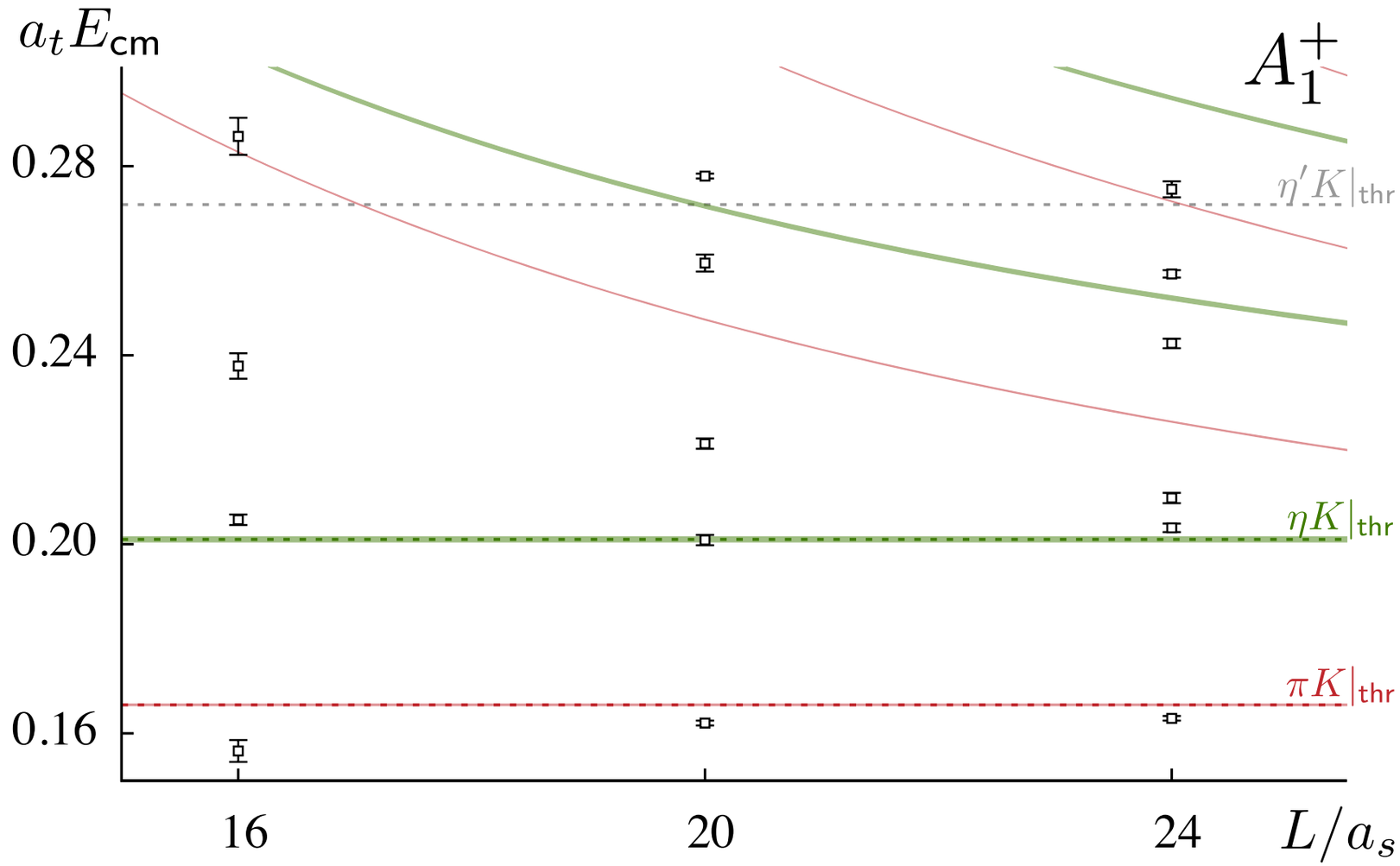
S-wave amplitudes from A_1^+

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{nn'} + i\rho_i\, t_{ij}^{(\ell)}\left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{d},\Lambda}_{\ell n;\ell' n'}(q_i^2)\right)\right] = 0$$

- Expect J^P=0$^+$ dominant. Neglect J^P=4$^+$ and higher.
- Use *K*-matrix, respects unitarity and has the flexibility needed for resonances.

$$t_{ij}^{-1}(s) = K_{ij}^{-1}(s) + I_{ij}(s) \qquad K = \frac{1}{m^2 - s}\begin{bmatrix} g_{\pi K}^2 & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g_{\eta K}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

- Minimise a χ^2 by varying m, g's and γ's to obtain best possible description of the lattice energies.



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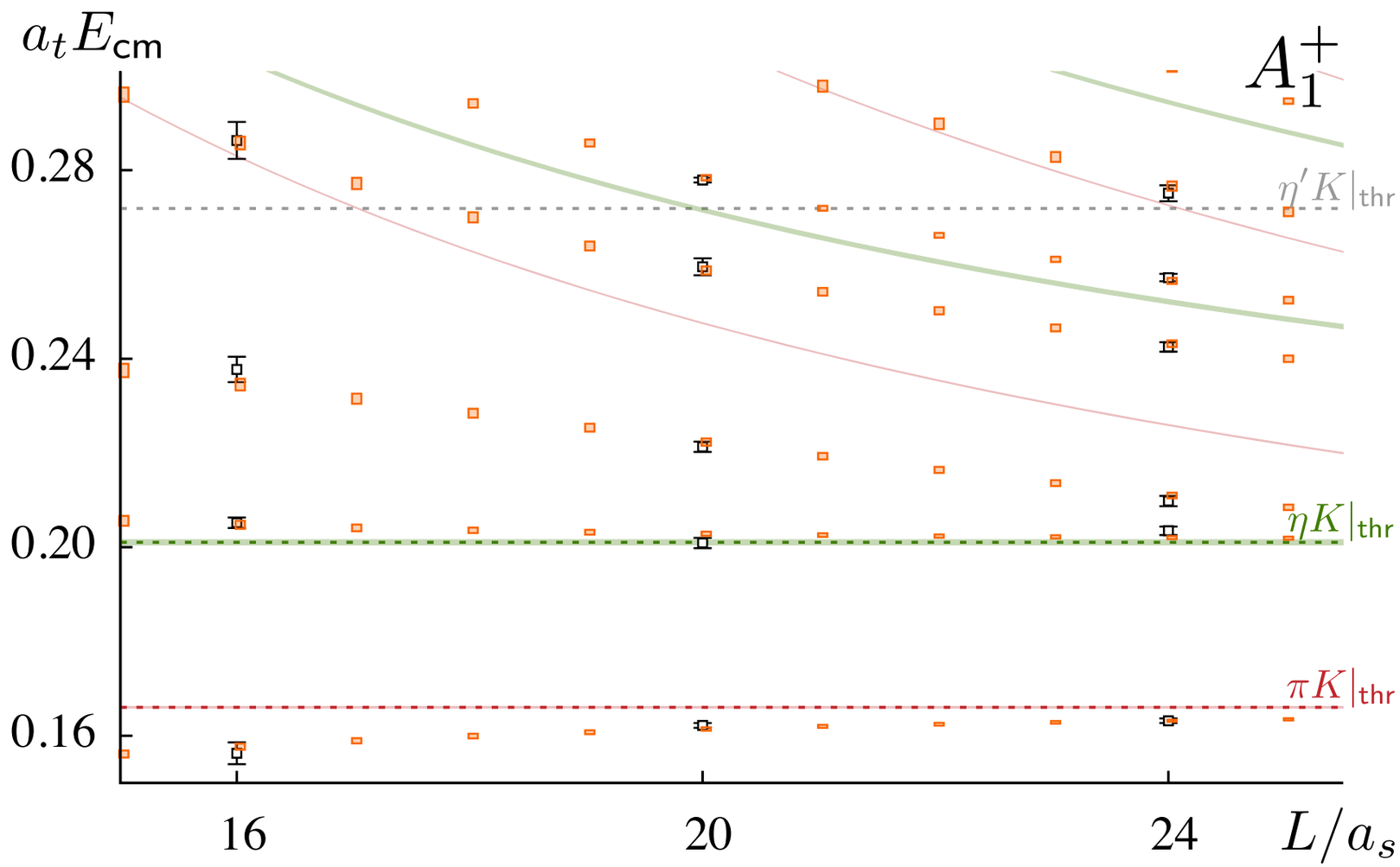
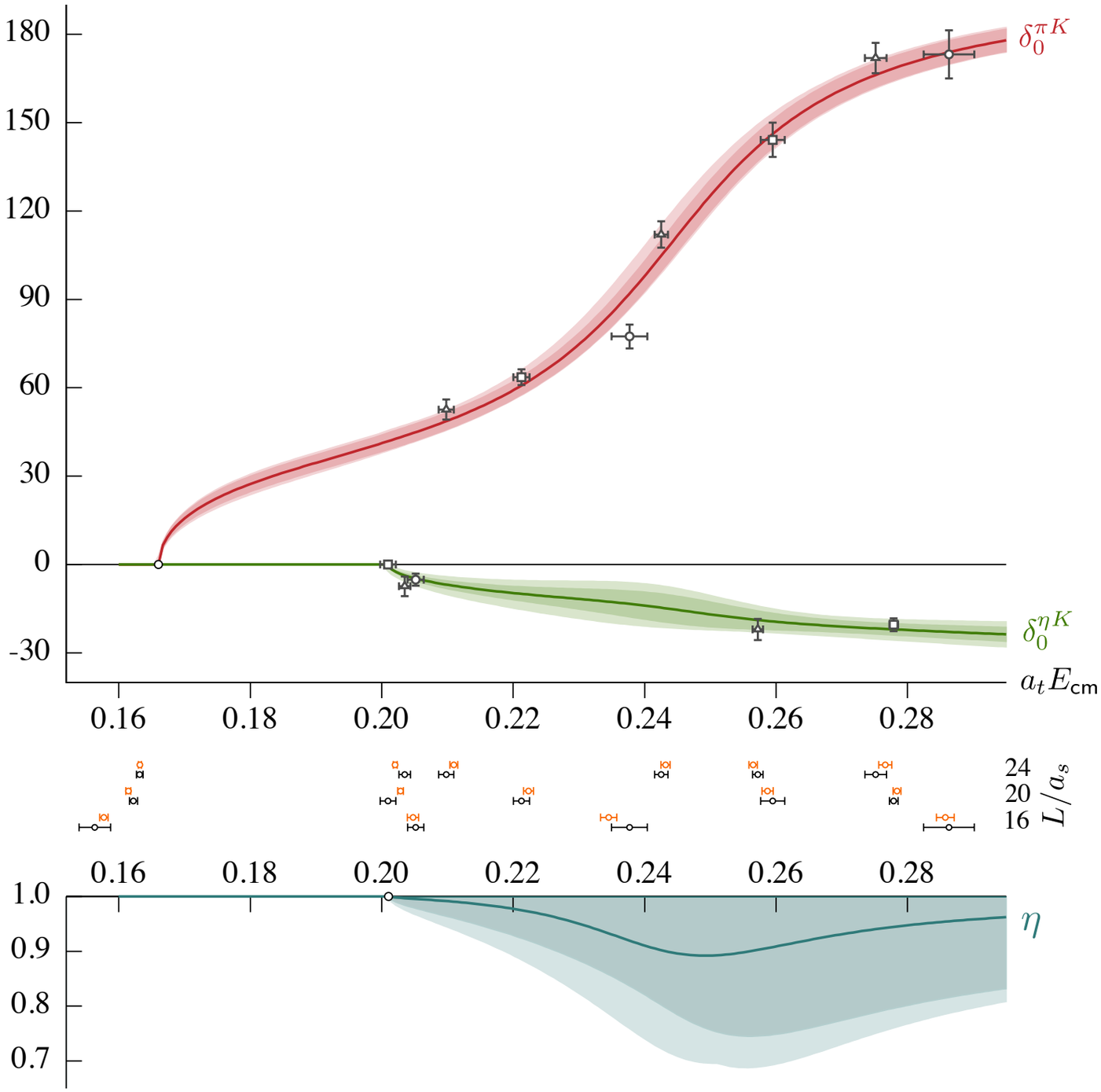


$$\chi^2/N_{\text{dof}} = \frac{9.56}{15-6} = 1.06$$

S-wave amplitudes



$$\chi^2/N_{\mathrm{dof}} = \frac{9.56}{15-6} = 1.06$$

- t_{ij} represented as phase and inelasticity

$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i = j) \\ \dfrac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

- Discrete phase shift points obtained in decoupling ($\eta \to 1$) limit.
- Broad resonance in *S*-wave πK.
- ηK coupling is small.
- 3 subthreshold points, naturally included in an energy-level fit.

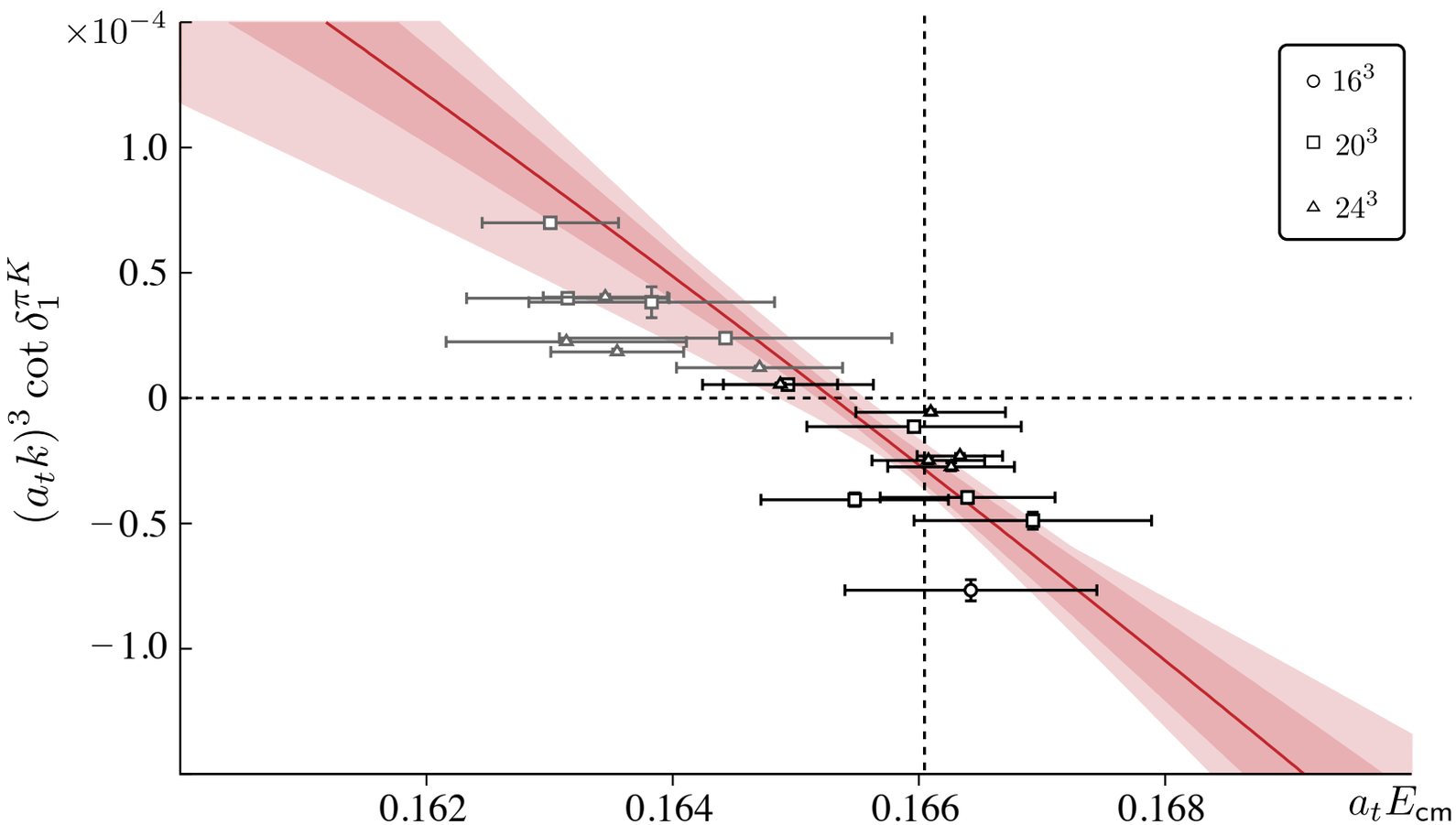
P-wave near-threshold state

In irreps with *P*-wave overlap:

T_1^-, [001] A_1, [001] E_2, [011] A_1, [011] $B_{1,2}$, [111] A_1, [111] E_2, [002] A_1

"extra" level near πK threshold.

Fitting the energy levels using an elastic Breit-Wigner in πK: $k^3 \cot \delta_1 = (m_R^2 - s)\dfrac{6\pi s^{\frac{1}{2}}}{g_R^2}$



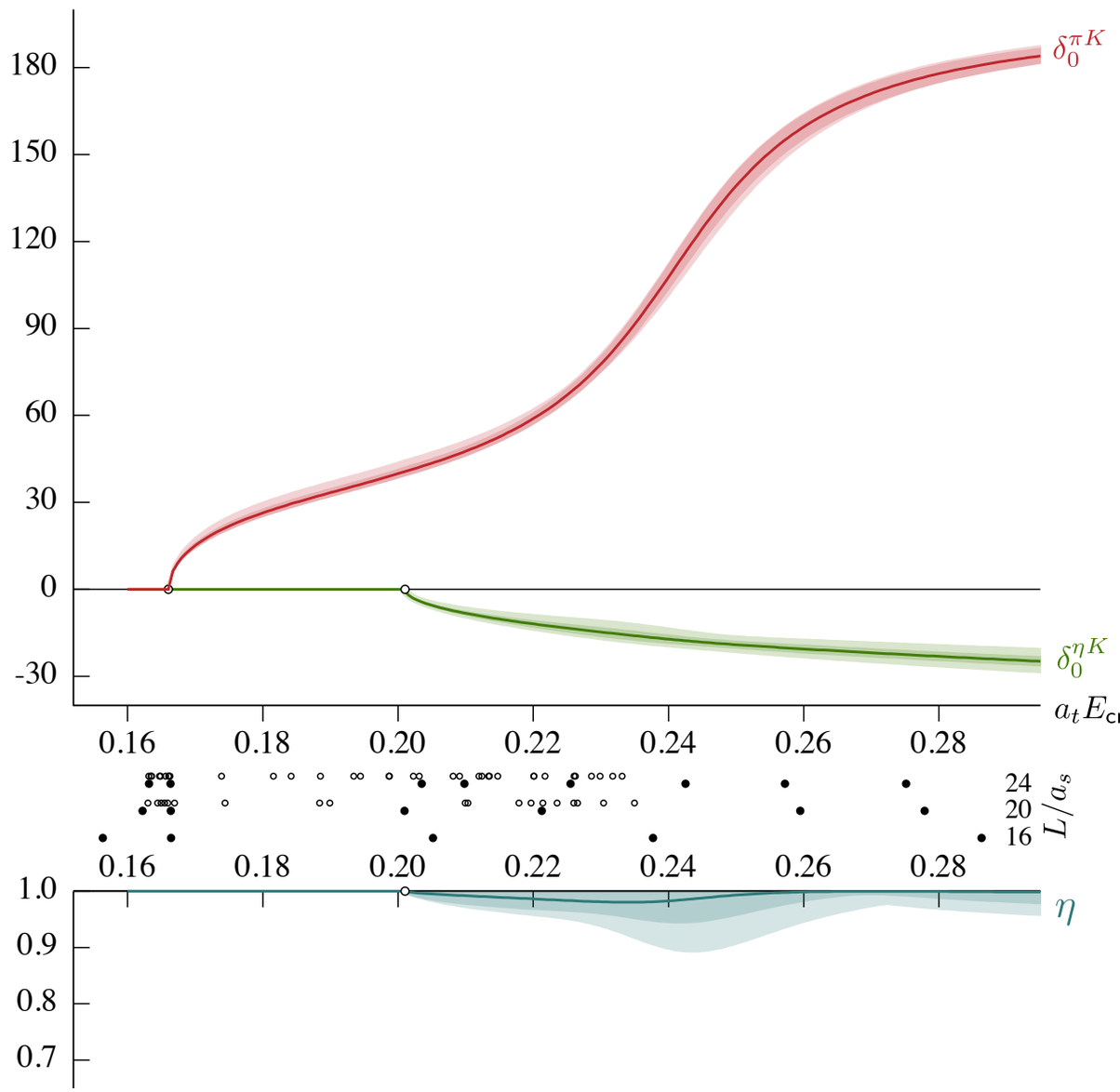
In *t* there is a pole on the real axis just below πK threshold:

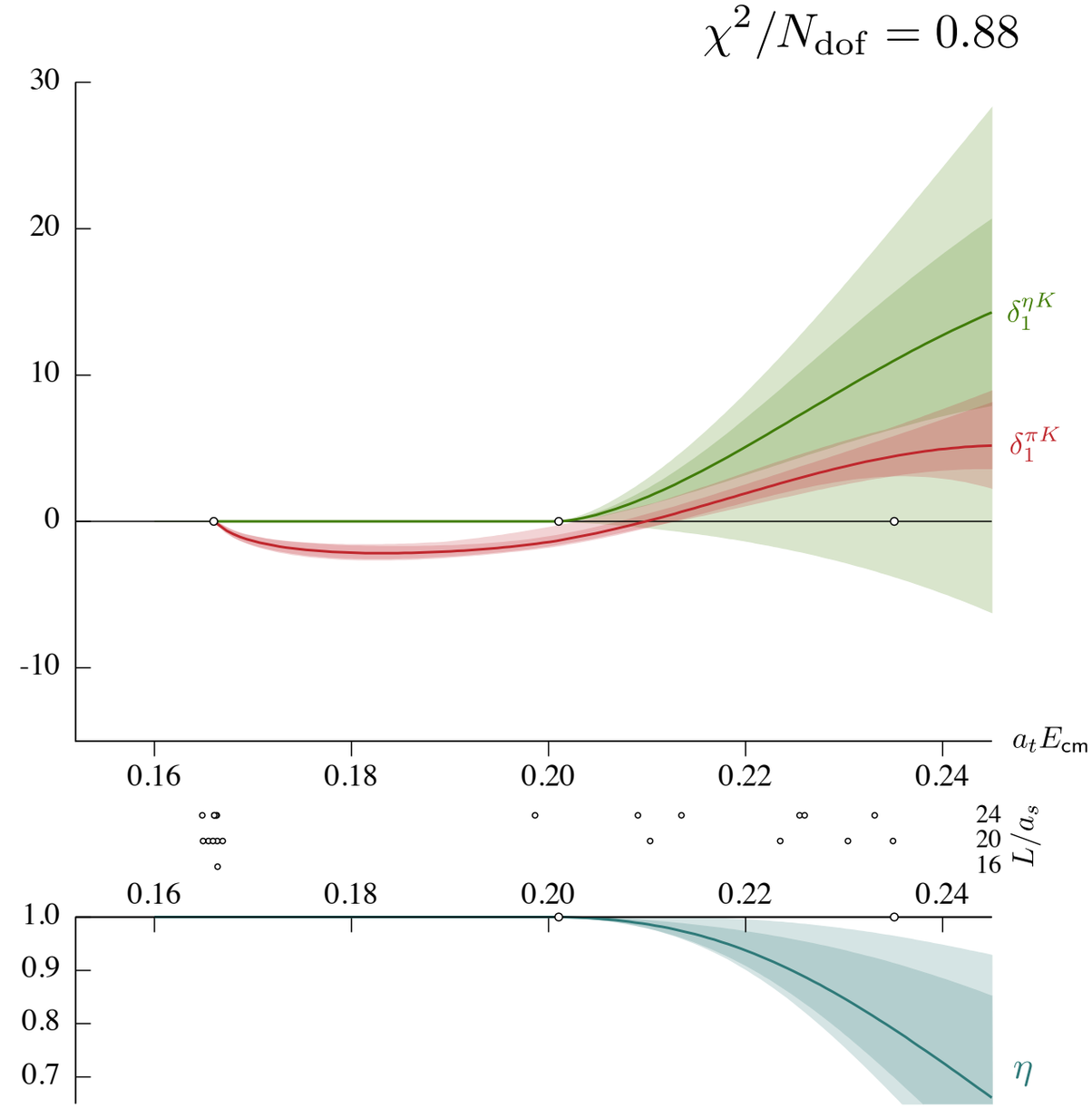
Bound state in J^P=1$^-$

$$m_R = (0.16530 \pm 0.00011 \pm 0.00013) \cdot a_t^{-1} \qquad \begin{bmatrix} 1 & -0.13 \\ & 1 \end{bmatrix}$$
$$g_R = 5.22 \pm 0.41 \pm 0.09$$

$$\chi^2/N_{\text{dof}} = \tfrac{9.24}{19-2} = 0.54\ .$$

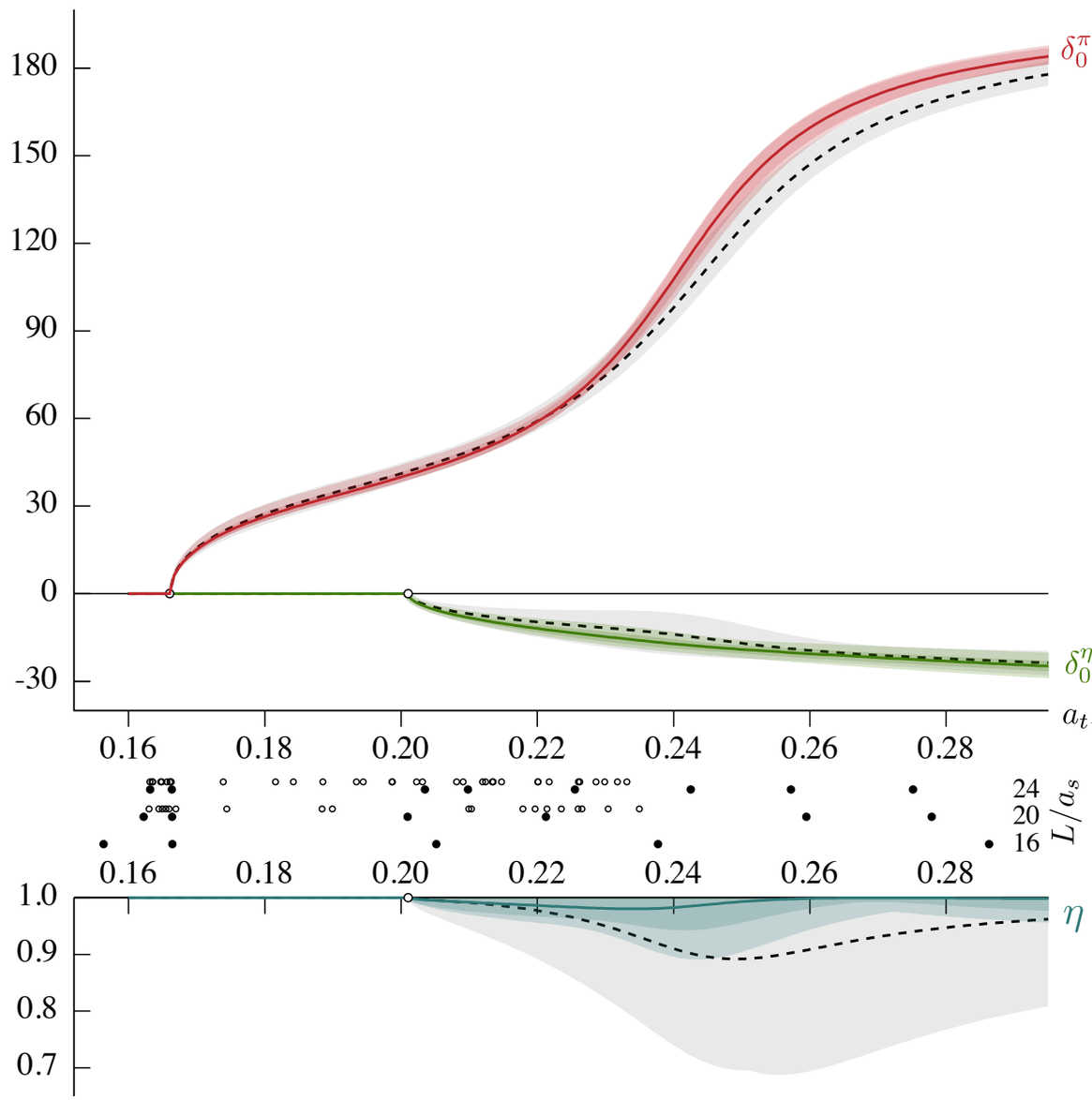
S+*P*-waves from 73 energy levels

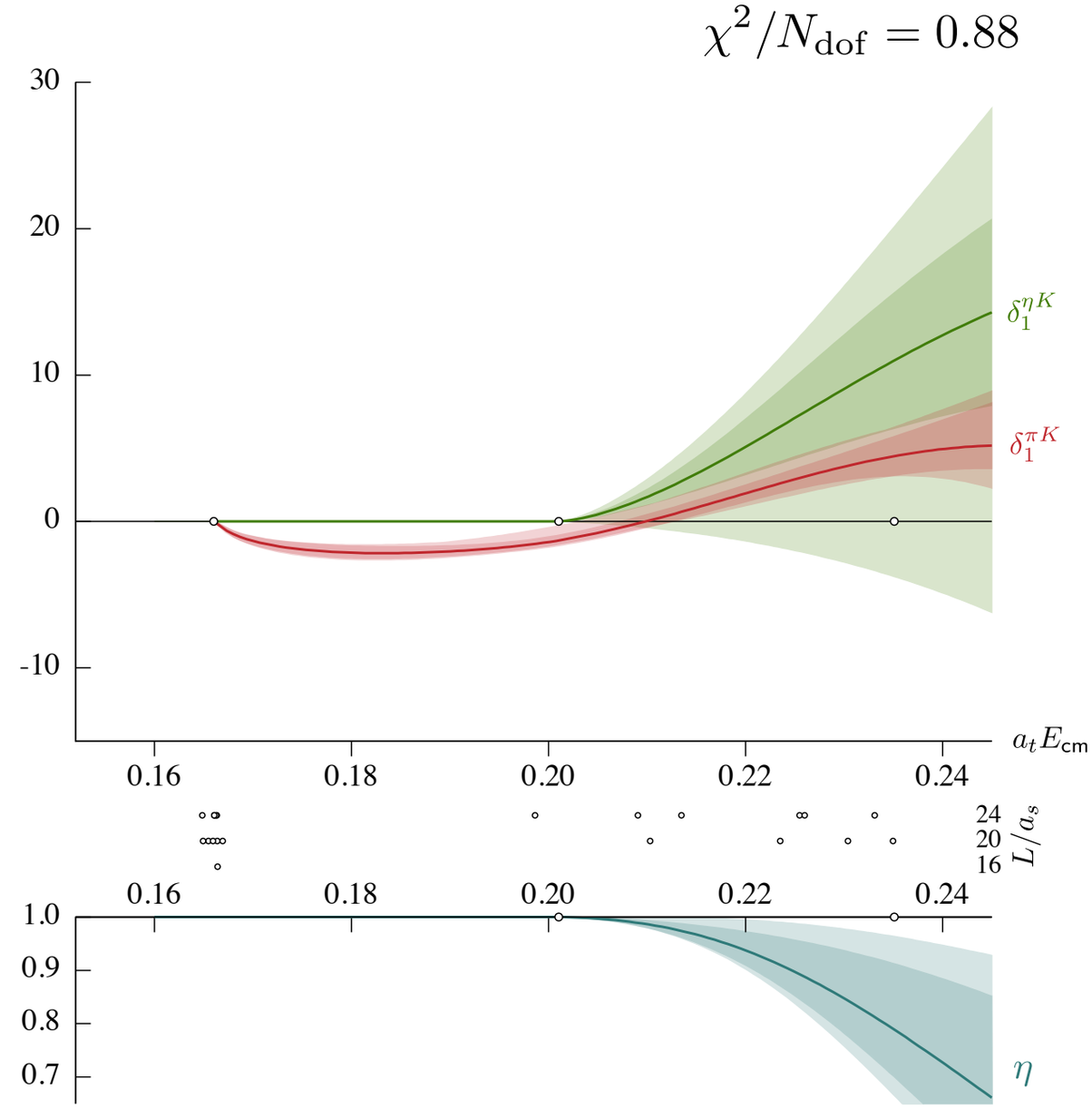




- Makes use of in-flight A_1 energies to constrain *S*-wave.
- Partial wave mixing ➔ obtain both *S* and *P* simultaneously
- Use in-flight and *P*-wave energy levels below $\pi\pi K$ threshold ($a_t E_{cm}$=0.235).
- *D*-wave and higher are negligible in this region.

S+*P*-waves from 73 energy levels





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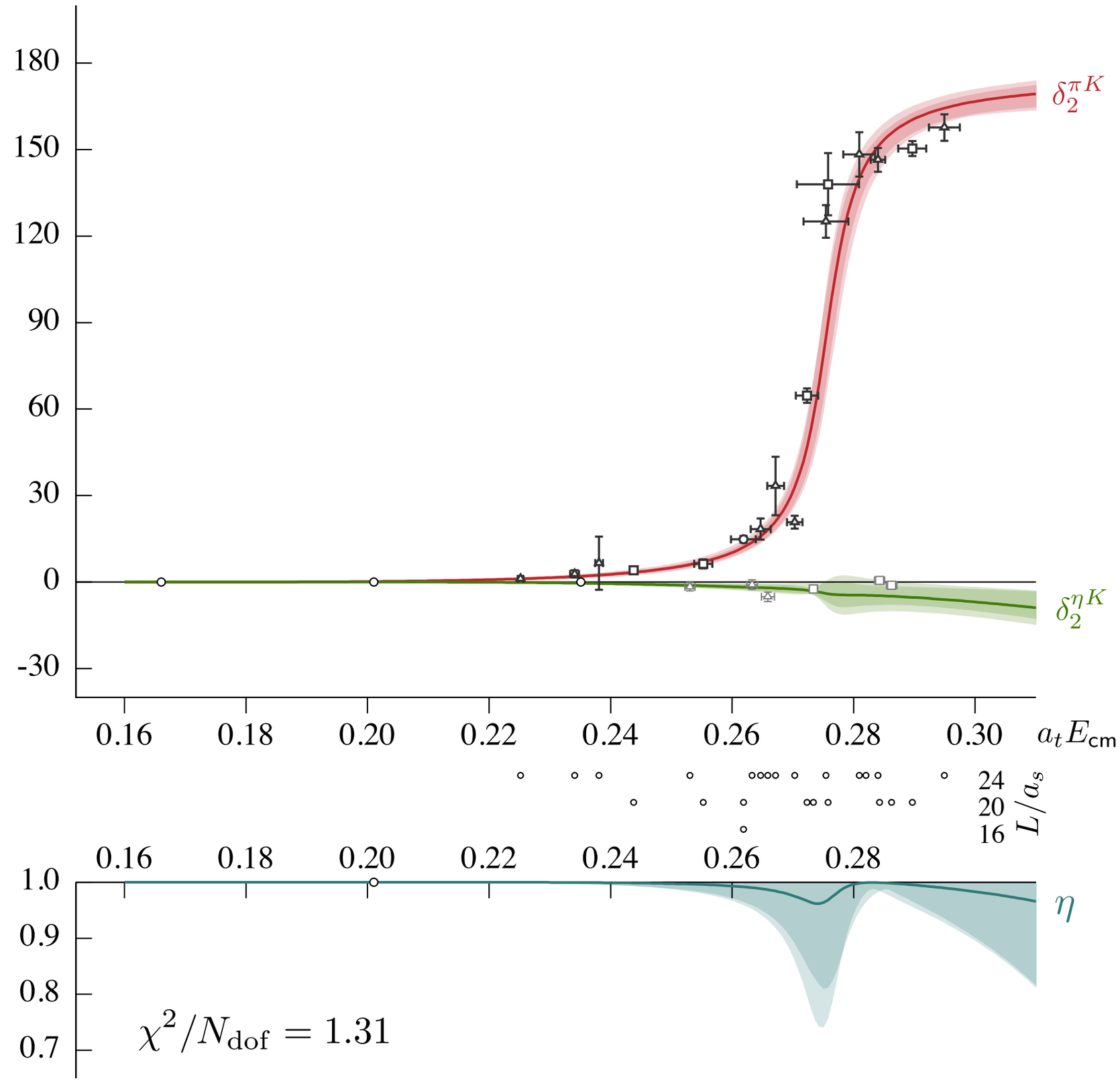
Narrow *D*-wave resonance

- No time for full discussion.
- Many other energy levels containing scattering amplitude information.
- Using only irreps with J=2 and higher (E^+, T_2^+, [100]$B_{1,2}$) we find a narrow resonance:
- Fit to energies. Discrete points obtained in η→1 limit.
- In J≥1 scattering $\pi\pi K$ can contribute (for $a_t E_{cm}$ > 0.235).
- Ideally requires 3-body formalism. Although not strictly rigorous, we can apply the 2→2 formalism anyway.

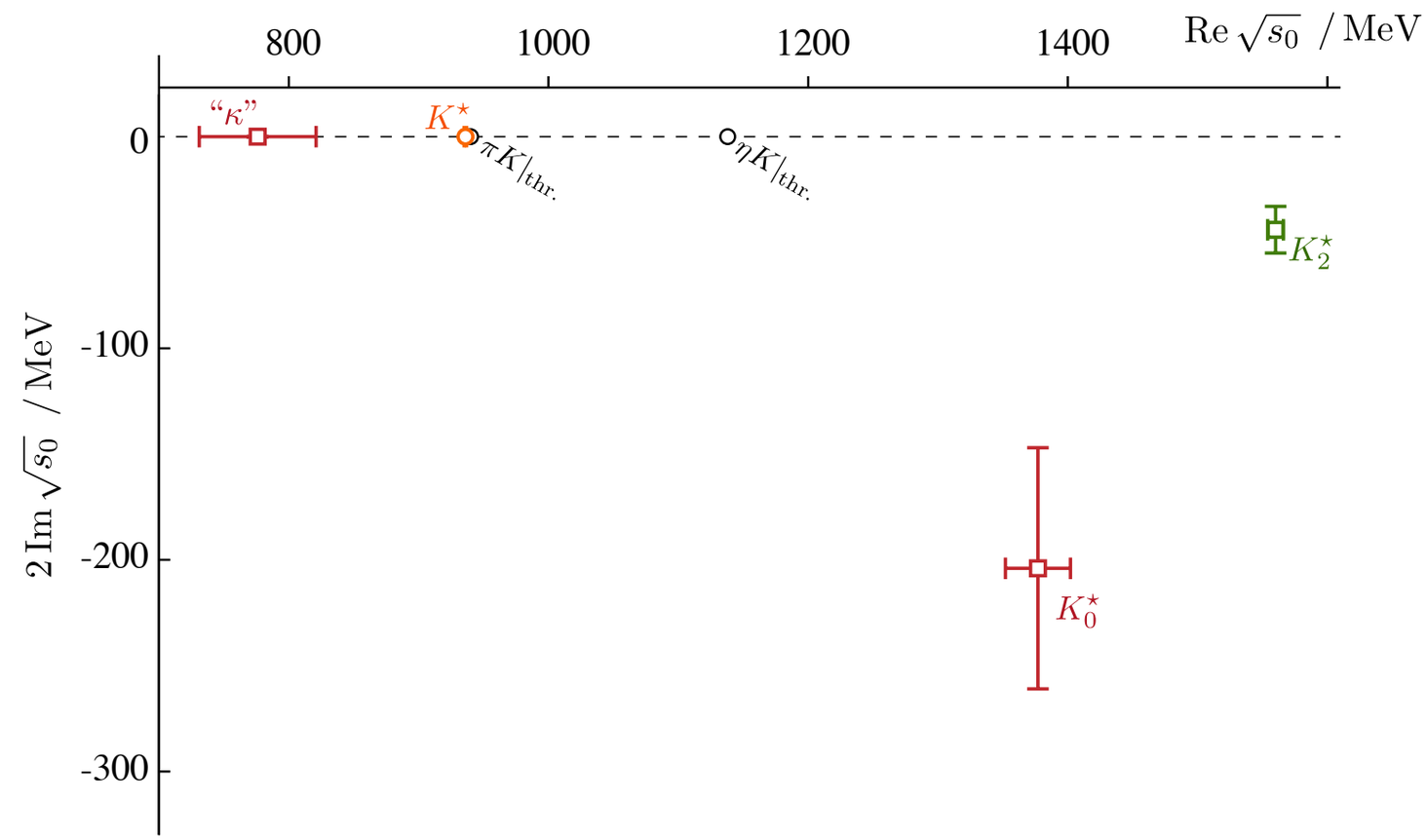


Scattering amplitude poles

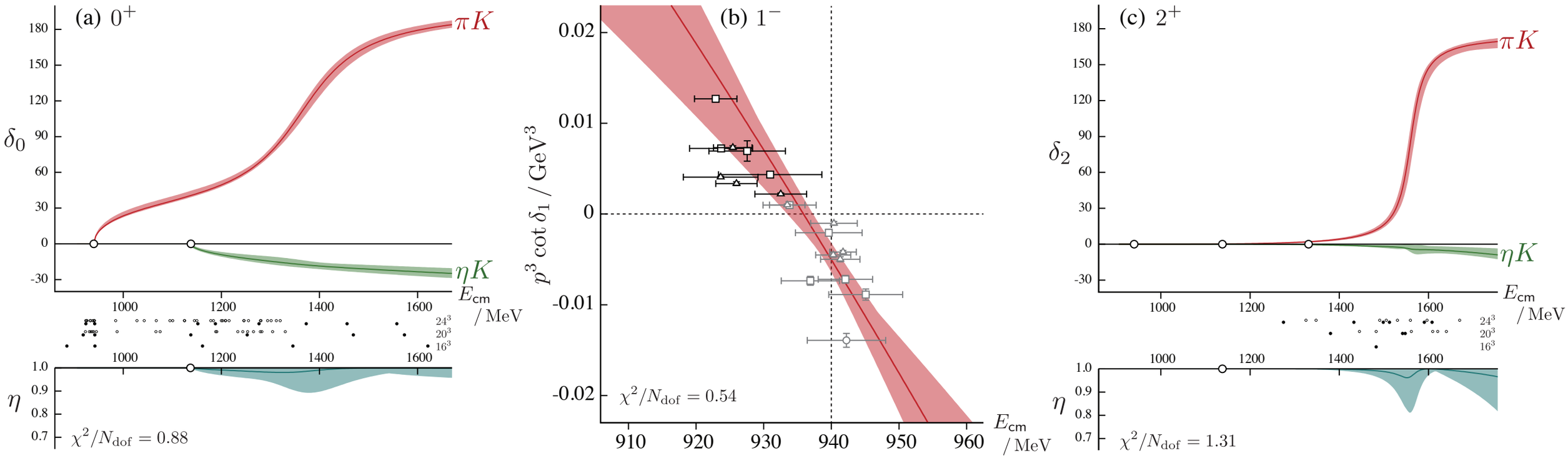
- t-matrix singularities have similar pattern to experiment.
- Unitarised χPT: $\kappa(800)$ ➔ virtual bound state (for $m_\pi \gtrsim 2.5\, m_\pi^{(\mathrm{phys})}$)
  Pole found below threshold on the unphysical sheet.
- A pole on the physical sheet below threshold found in $J^P=1^-$.
  Similar to $K^\star(892)$, but just bound at $m_\pi = 391$ MeV.

Poles are found on unphysical sheets:

- Poles in S-wave, large width.
  Dominant coupling to πK.
  Similar to the $K_0^\star(1430)$.
- Narrow poles found in D-wave.
  Dominant coupling to πK.
  Similar to the $K_2^\star(1430)$.



Summary



- Parameterise energy dependence
  ➔ determine coupled-channel πK-ηK scattering amplitudes.
- Thorough analysis of ~100 energy levels
  ➔ obtain tight constraints on the amplitudes
- Methods appear to work quite generally for a wide range of interactions.
- Very different scattering features found:
  (a) Broad resonance (in coupled channels)
  (b) Bound state
  (c) Narrow resonance
- Approximate decoupling between πK-ηK
- Applicable to scattering in many other coupled-channels.

Backup Slides

Energy level fitting

$$\det\left[\delta_{ij}\delta_{\ell\ell'}\delta_{nn'} + i\rho_i\, t_{ij}^{(\ell)}\left(\delta_{\ell\ell'}\delta_{nn'} + i\mathcal{M}^{\vec{d},\Lambda}_{\ell n;\ell' n'}(q_i^2)\right)\right] = 0$$

A convenient unitarity-preserving coupled-channel parameterisation is the *K*-matrix:

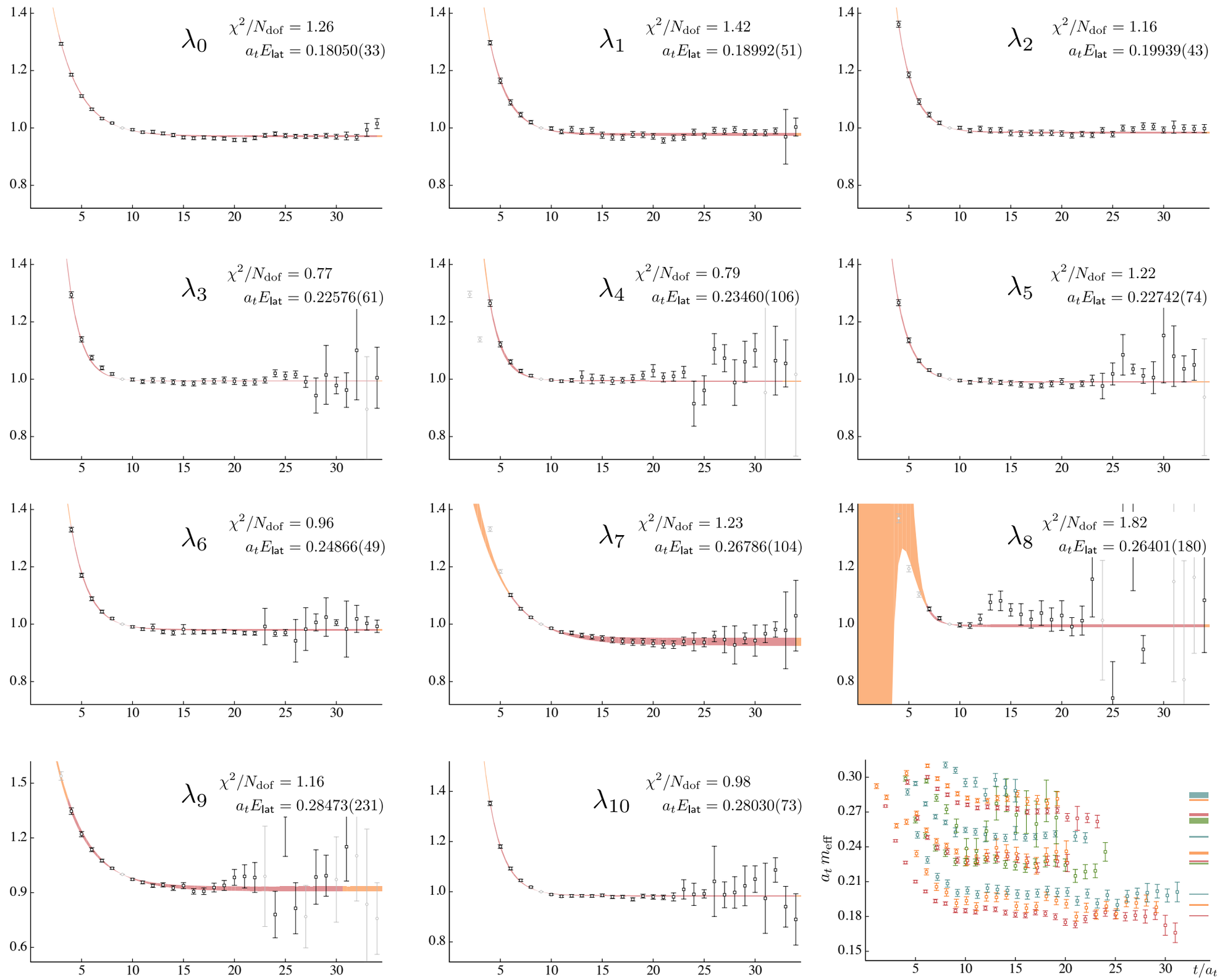
$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s).$$

$$K_{ij}(s) = \sum_p \frac{g_i^{(p)} g_j^{(p)}}{m_p^2 - s} + \sum_n \gamma_{ij}^{(n)} s^n$$

- Unitarity of the *S*-matrix is preserved when the parameters m, g and γ are real, and

$$\mathrm{Im}\, I_{ij}(s) = -\rho_i \delta_{ij}$$

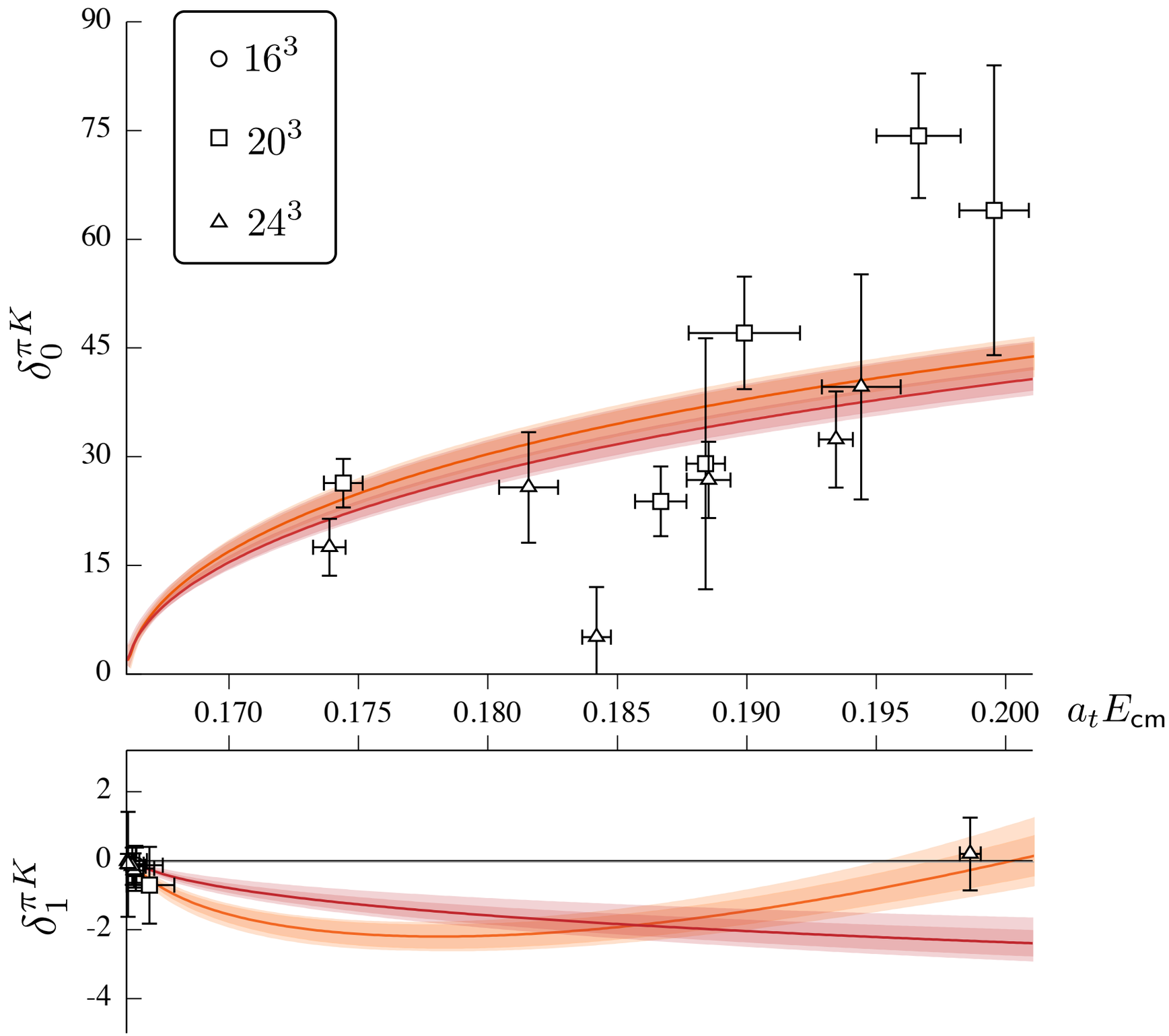
Principal Correlators - [100] A_1



Elastic Region

Scattering length and *K*-matrix in *S*-wave.

Breit-Wigner and *K*-matrix in *P*-wave.



Partial wave contributions in each irrep

$\vec{P}$	LG($\vec{P}$)	Λ	$J^P(\vec{P}=0)$ $\lvert\lambda\rvert^{(\tilde{\eta})}$ $(\vec{P}\neq 0)$	$(K\pi)\ \ell^N$
$[0,0,0]$	O_h^D	A_1^+	0^+, 4^+	0^1, 4^1
		T_1^-	1^-, 3^-, (4^-)	1^1, 3^1
		T_1^+	(1^+), (3^+), 4^+	4^1
		T_2^-	(2^-), 3^-, (4^-)	3^1
		T_2^+	2^+, (3^+), 4^+	2^1, 4^1
		E^+	2^+, 4^+	2^1, 4^1
		A_2^-	3^-	3^1
$[0,0,n]$	Dic_4	A_1	0^+, 4	0^1, 1^1, 2^1, 3^1, 4^2
		A_2	(0^-), 4	4^1
		E_2	1, 3	1^1, 2^1, 3^2, 4^2
		B_1	2	2^1, 3^1, 4^1
		B_2	2	2^1, 3^1, 4^1
$[0,n,n]$	Dic_2	A_1	0^+, 2, 4	0^1, 1^1, 2^2, 3^2, 4^3
		A_2	(0^-), 2, 4	2^1, 3^1, 4^2
		B_1	1, 3	1^1, 2^1, 3^2, 4^2
		B_2	1, 3	1^1, 2^1, 3^2, 4^2
$[n,n,n]$	Dic_3	A_1	0^+, 3	0^1, 1^1, 2^1, 3^2, 4^2
		A_2	(0^-), 3	3^1, 4^1
		E_2	1, 2, 4	1^1, 2^2, 3^2, 4^3

Table 2: The separation of $K\pi$ spins, helicities and partial waves in each lattice irrep, with $J \leq 4$. The first three columns are taken from Refs. [13, 8, 7]. The final column is derived by considering how the helicity components are projected on each ℓ. The brackets around (J^P) donote a J^P at rest that does not contribute to elastic $K\pi$ scattering (e.g. 0^-, 1^+, 2^-, ...) and also $|\lambda|^{\tilde{\eta}} = 0^-$.