Spectral Flow and Index Theorem for Staggered Fermions

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(Lattice 2014, Columbia University, NY, June 2014)

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Thanks: A. Hart for generating the configurations.

Outline

- Motivation
- Staggered fermions and topology.
- Index Theorem in the continuum.
- Index Theorem for Staggered Quarks with the spectral flow.
- Numerical results.
- Conclusions and outlook.

Motivation

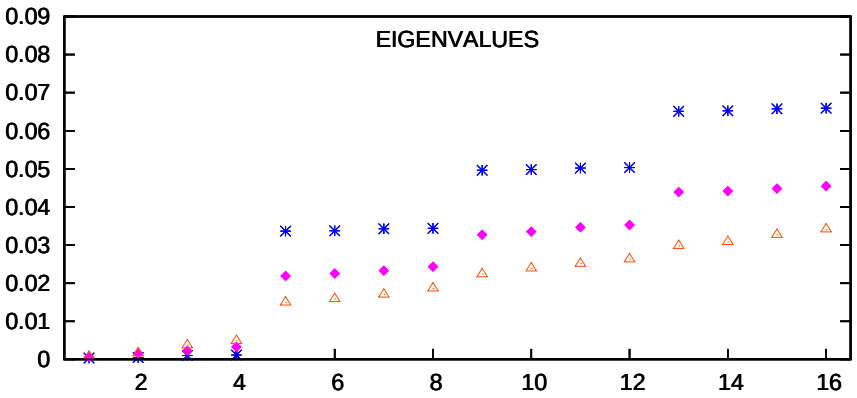
- Continuum QCD has interesting topological features:
  - Index theorem: quantized topological charge.
  - RMT description of the distribution of low-level eigenvalues for each sector of topological charge, in the ϵ-regime.
- Can we identify these properties in lattice QCD? (In particular, can we do so for staggered fermions?)
- For a long time, it was believed that staggered quarks are "topology-blind".
- No good (staggered) definition of the topological charge Q.
- No RMT description sensitive to Q.

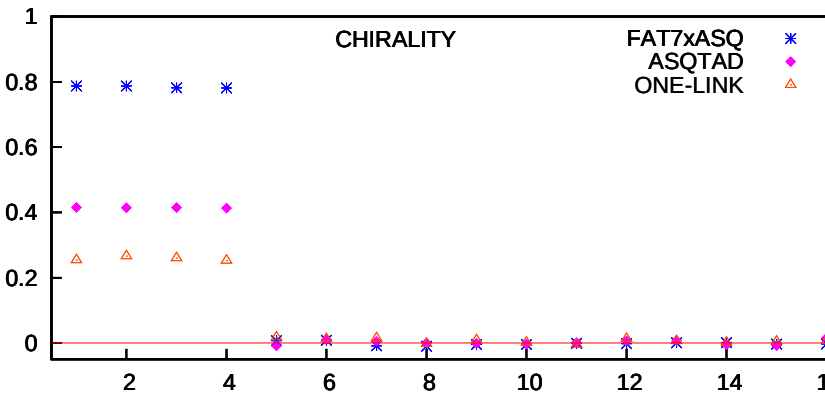
Staggered fermions and topology

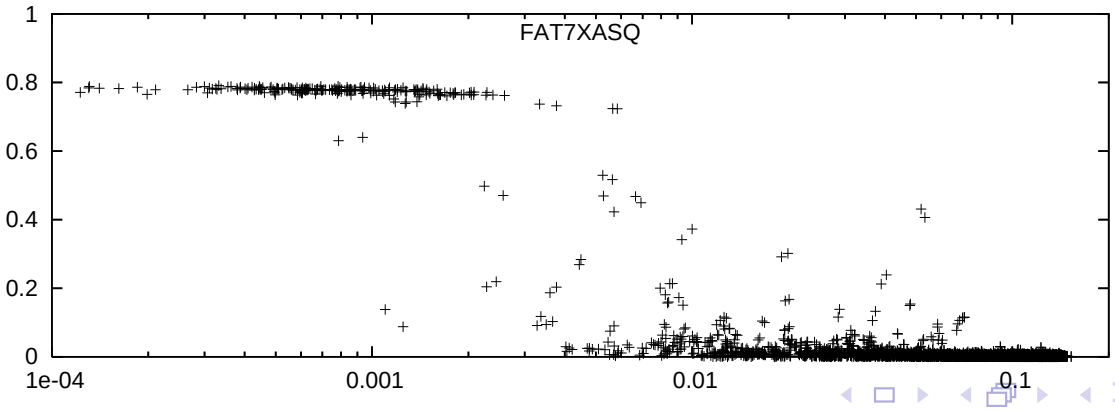
NOT SO!

- Staggered quarks display the expected behaviour, if we are close enough to the continuum $\Rightarrow$ improved actions (HISQ).

Improved Glue ($a \approx 0.077$ fm, $V = 20^4$)







Staggered fermions and topology

- We can define the index with the chirality of the low-lying eigenvalues.
- It works well in practice, and any discrepancy with other definitions is expected to vanish as $a \to 0$.
- However, it is not yet a topological definition in the same sense that the spectral flow of the Wilson operator is (or equivalently, the index of the overlap operator constructed with it.)
- D. H. Adams proposed a new definition for the index with staggered fermions, similar to the Wilson one (Phys.Rev.Lett.104:141602,2010 [arXiv:0912.2850])
- Preliminary results: E. Follana, V. Azcoiti, G. Di Carlo, A. Vaquero, PoS LATTICE2011 (2011) 100

Index Theorem in the continuum

Gluonic

- Smooth continuum gauge fields A_μ: integer Q

Fermionic

- $D = \gamma_\mu \left(\partial_\mu + A_\mu\right)$
  $D\Psi = 0$, $\gamma_5 \Psi = \pm \Psi$
- Index Theorem: $n_+ - n_- = Q$

Spectral flow

- $H(m) = \gamma_5 \left(D - m\right)$, hermitian.
- $H(m)^2 = D^\dagger D + m^2 \rightarrow$ semipositive definite.
- Zero modes of D of chirality ± 1: $\rightarrow$ eigenmodes of $H(m)$, whose eigenvalues $\lambda(m)$ cross the origin with slope ∓ 1.
- Spectral flow (net number of crossings, with sign depending on the slope) $= n_- - n_+$.

Spectral flow on the Lattice

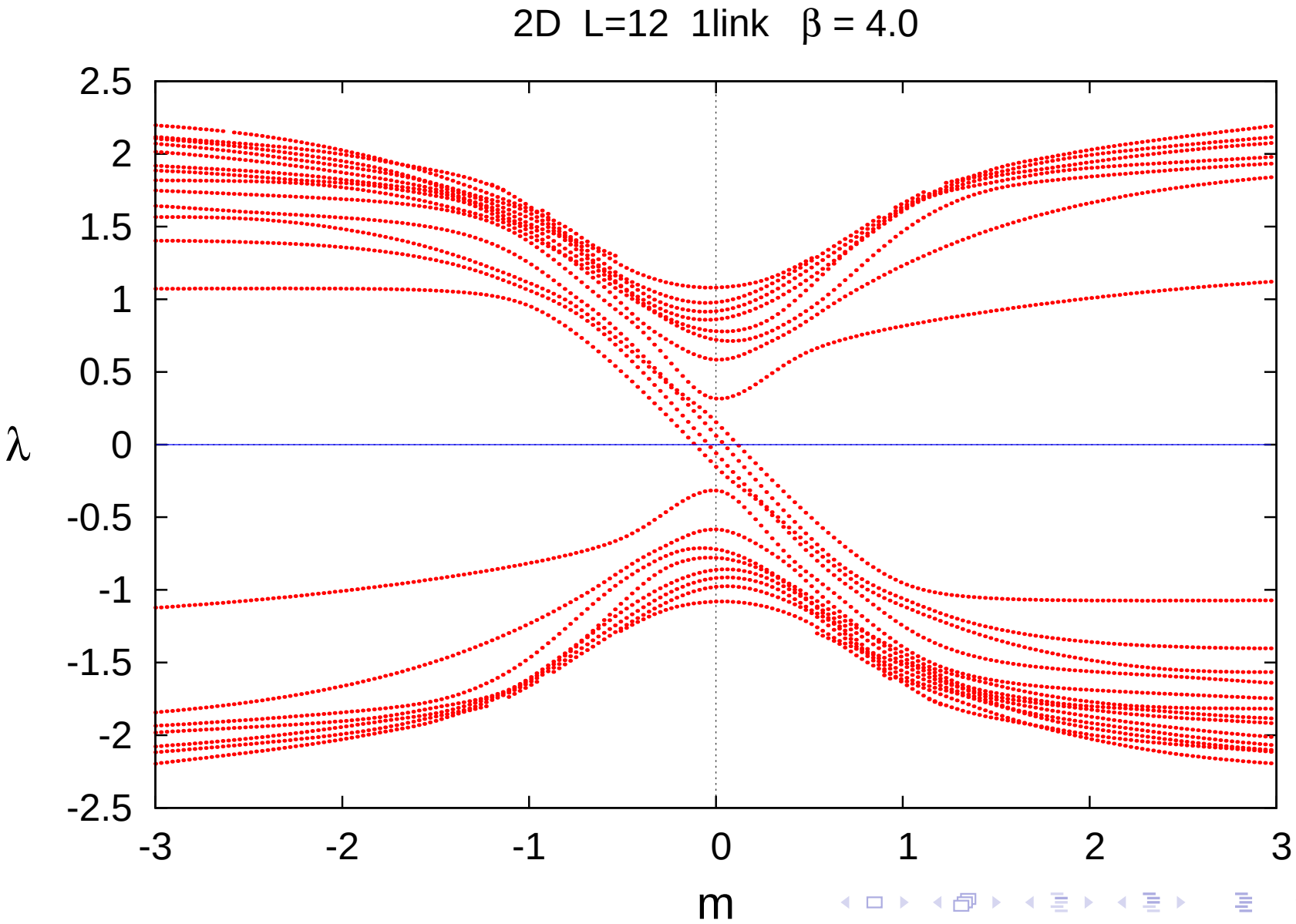
- For Wilson fermions, $H_W(m) = \gamma_5 (D_W - m)$
- This cannot be directly applied to staggered fermions ($\gamma_5 \to \Gamma_5$). But different operators could be used in the continuum, for example $H(m) = iD - m\gamma_5$
- This can be generalized directly to the staggered case, $H_{st}(m) = iD_{st} - m\Gamma_5$ (D. H. Adams).
- In a background which is not too rough, we should identify the would-be zero modes with the eigenmodes of $H(m)$ which cross zero at a low-lying value of m. The slope of the crossing is minus the chirality.
- This is a topological property, and thus stable under small deformations of the field.
- It provides an unambiguous definition of the index, as long as there is a good separation of low and high-lying crossings.

Numerical setup

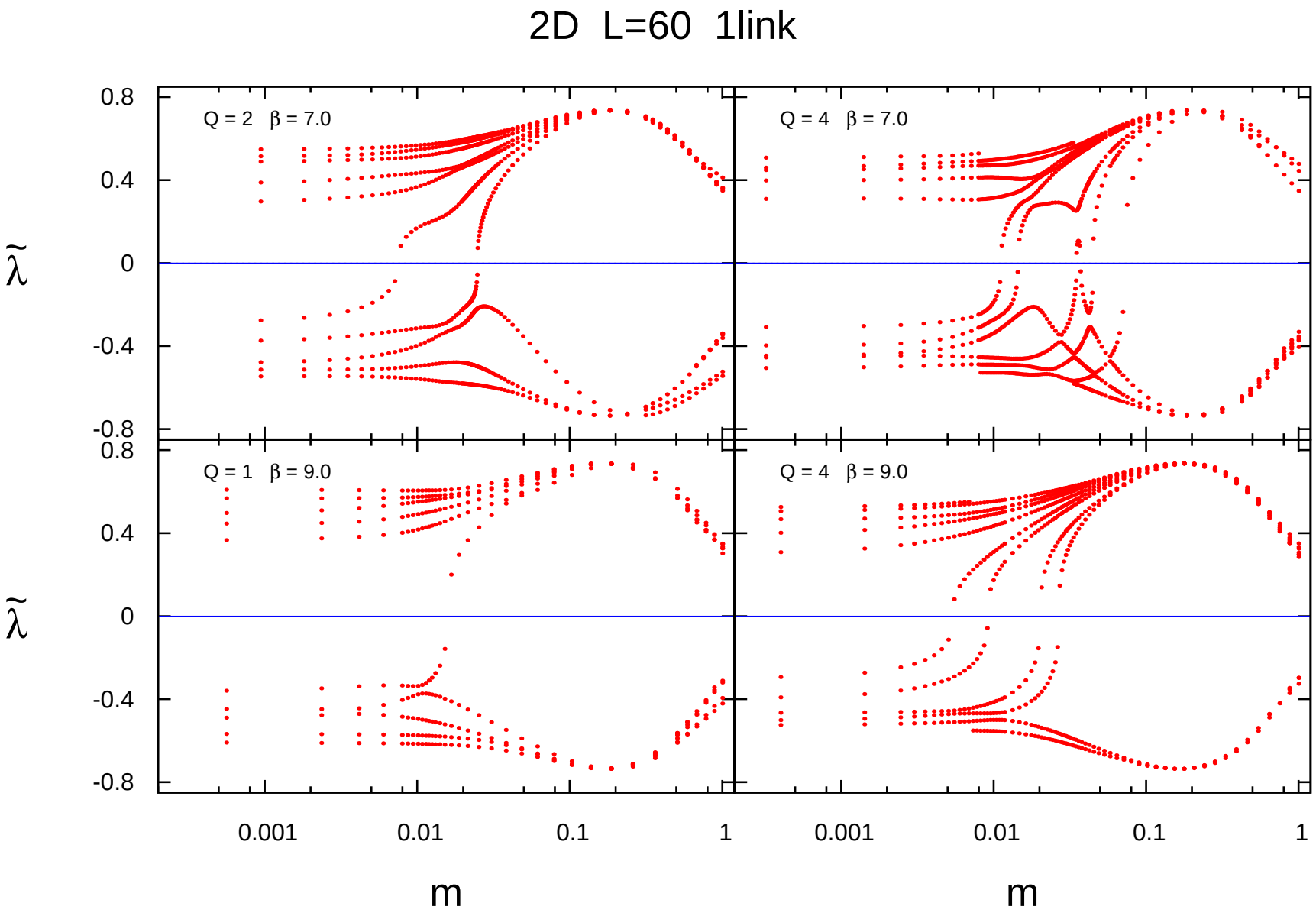
- Three ensembles of tree-level Symanzik and tadpole improved quenched QCD at different values of the lattice spacing (and roughly the same physical volume): .125fm (very coarse), .093 fm (coarse) and .0077 fm (fine).
- We study both the unimproved, 1link staggered Dirac operator and HISQ.
- The spectrum comes in pairs

$$\lambda(m) \leftrightarrow -\lambda(-m)$$

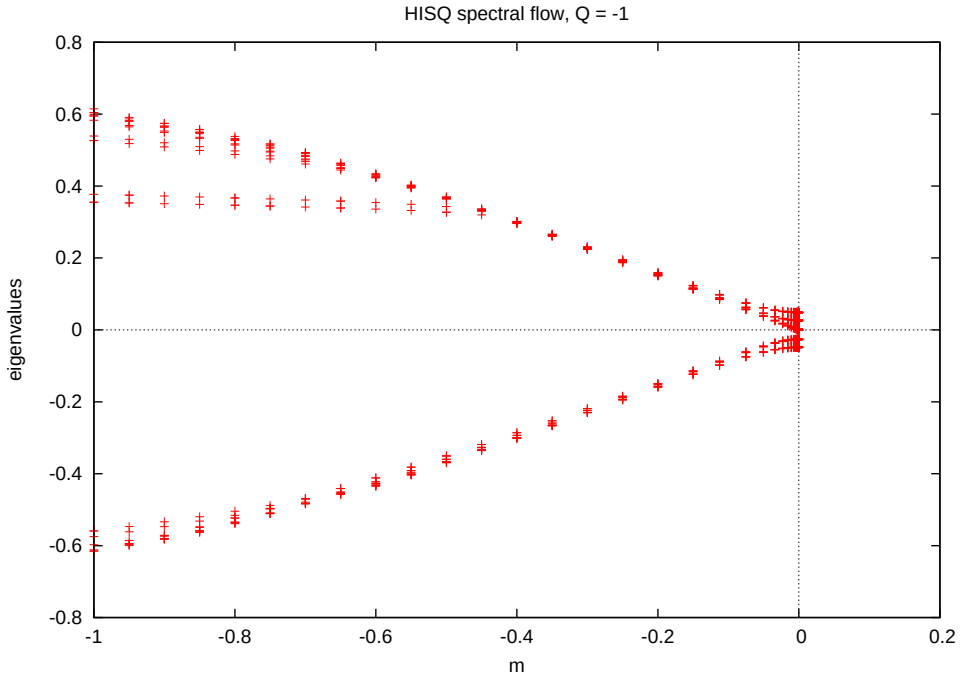
Numerical results: 2D

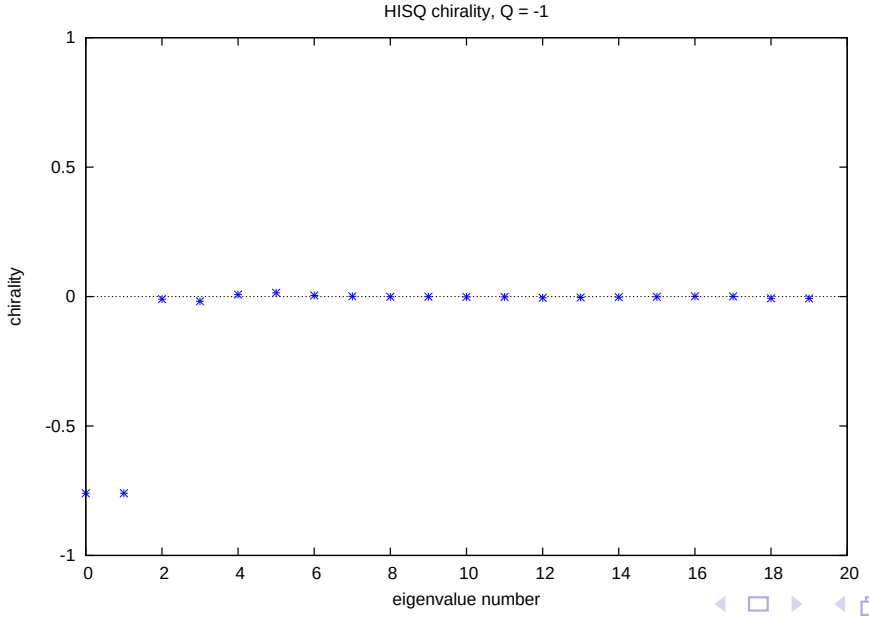


Numerical results: 2D



Numerical results





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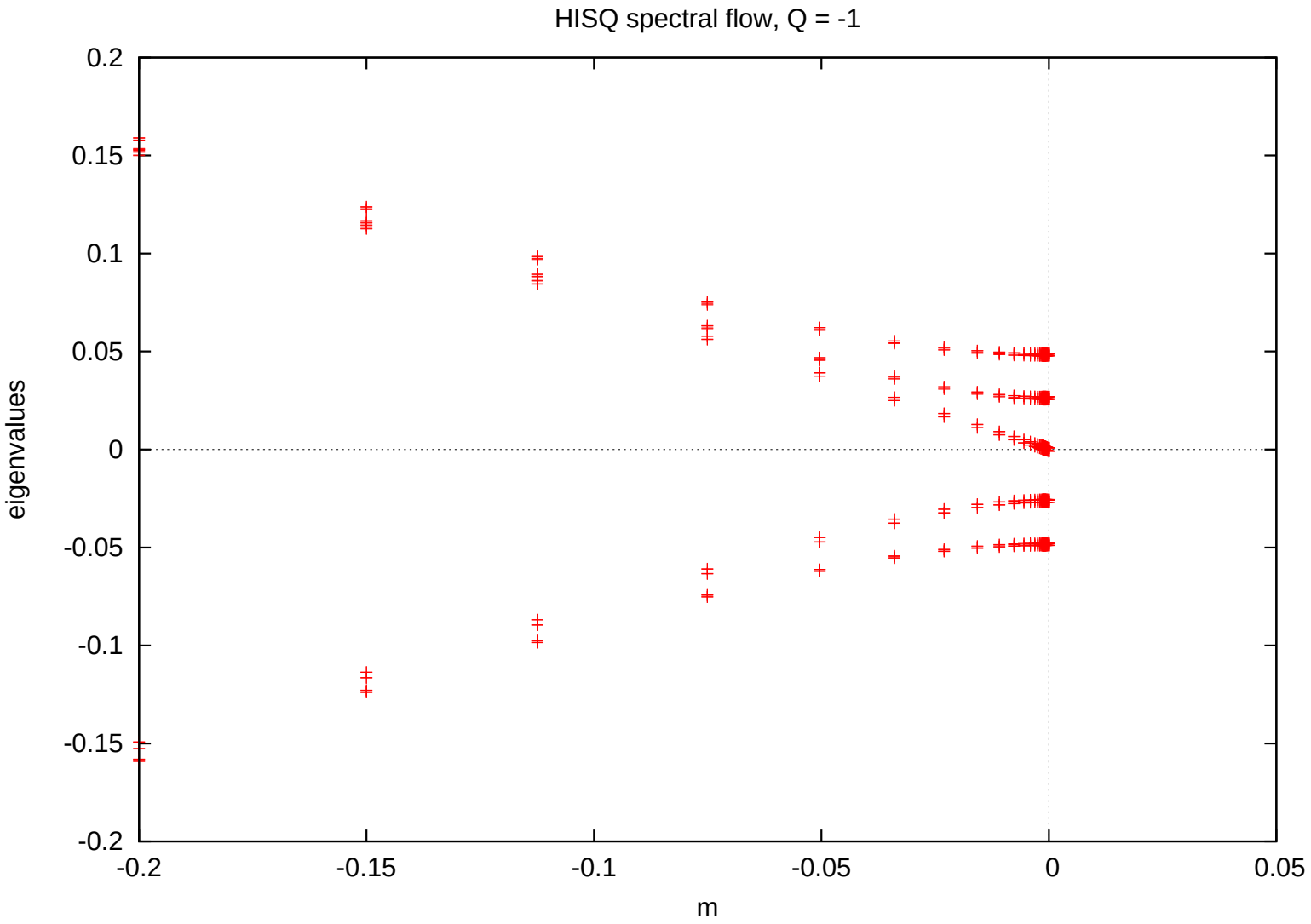

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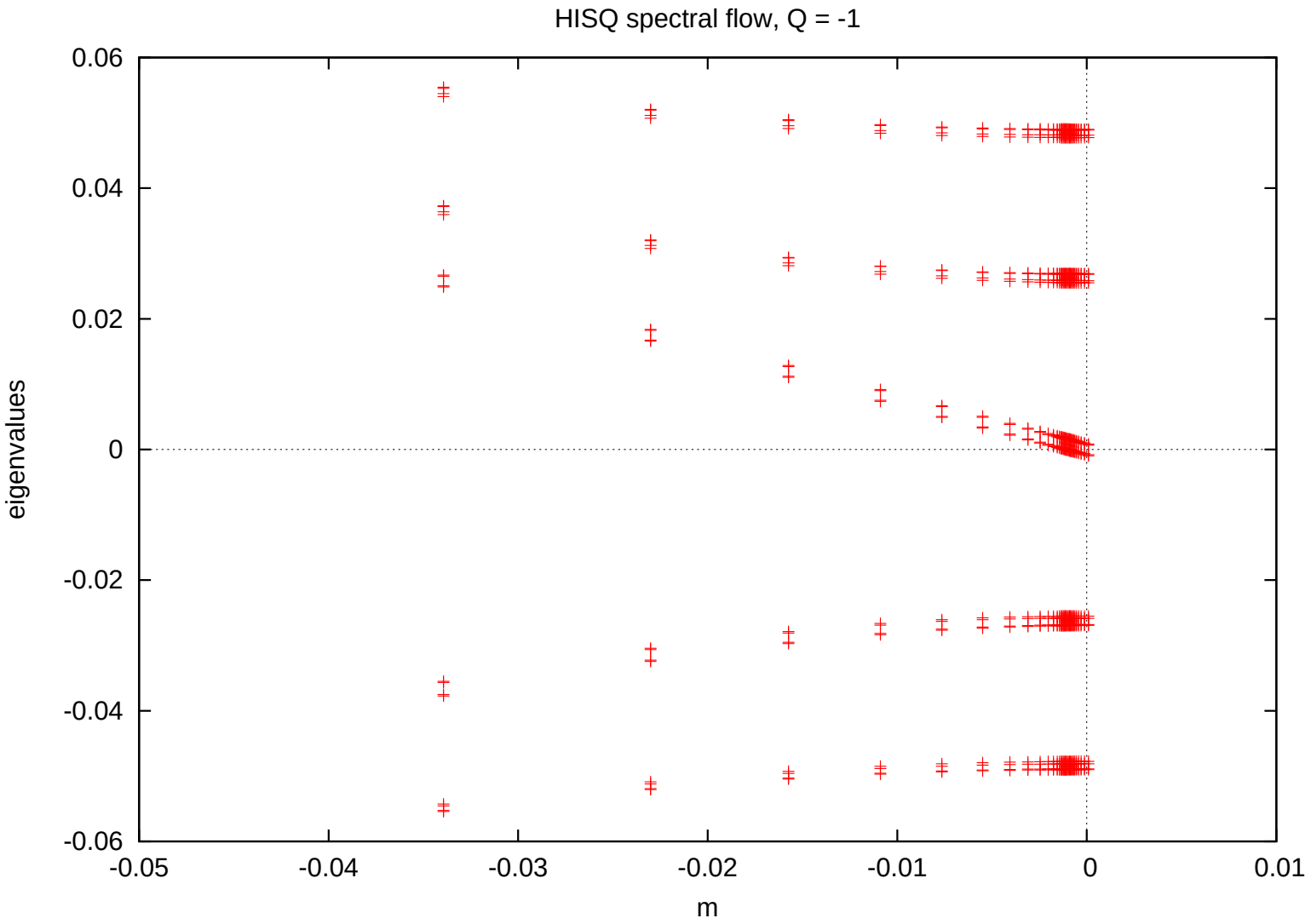

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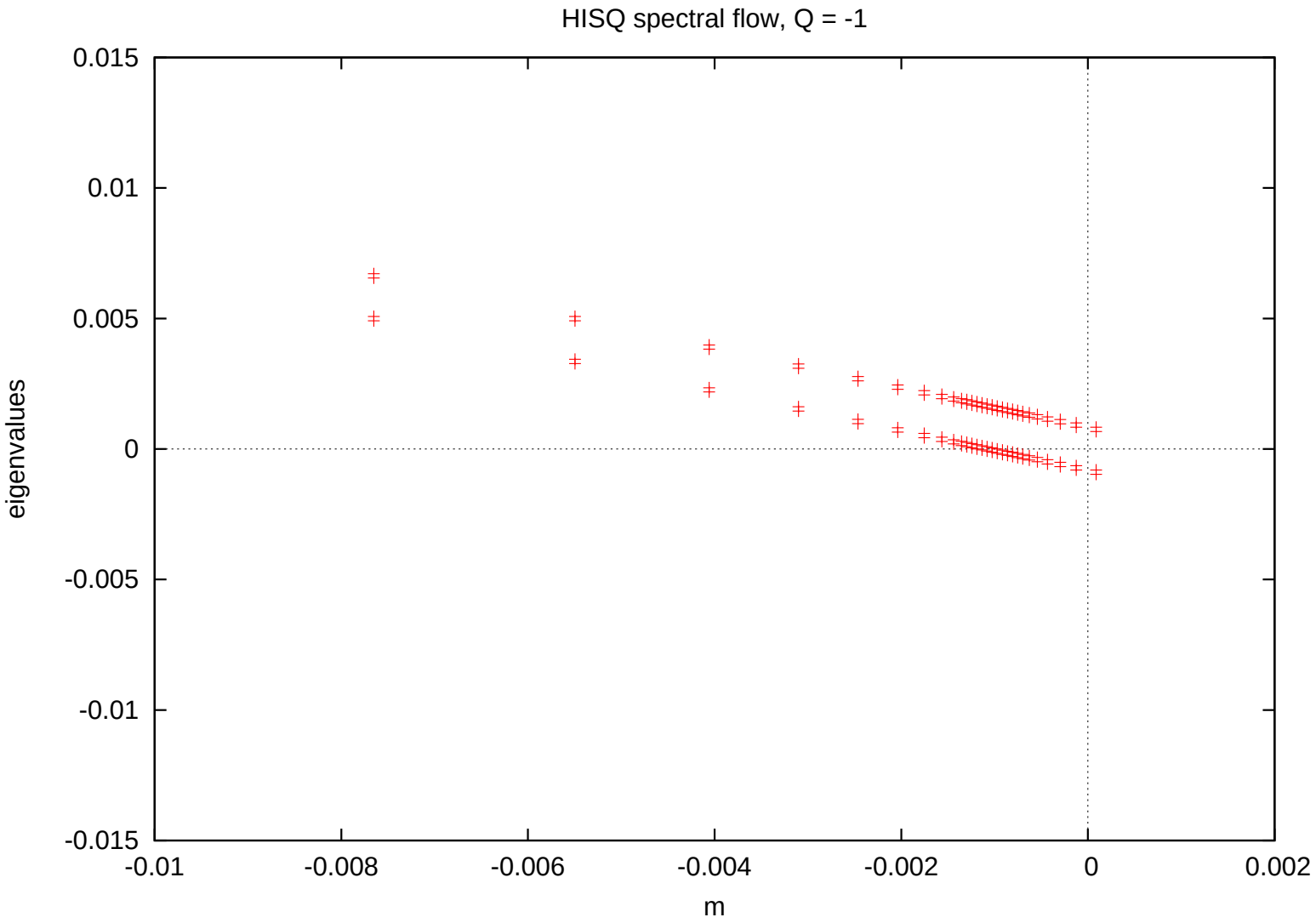

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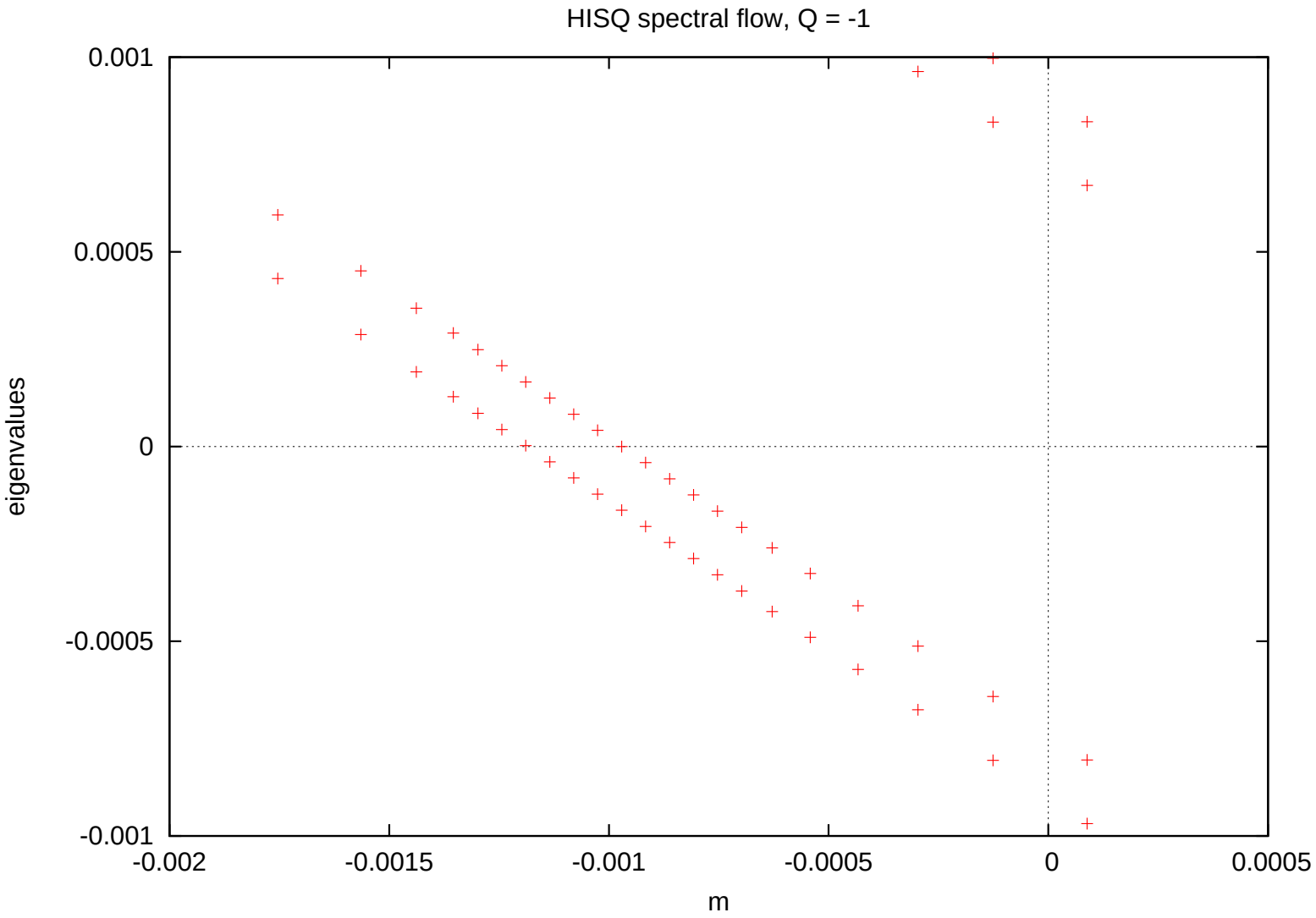

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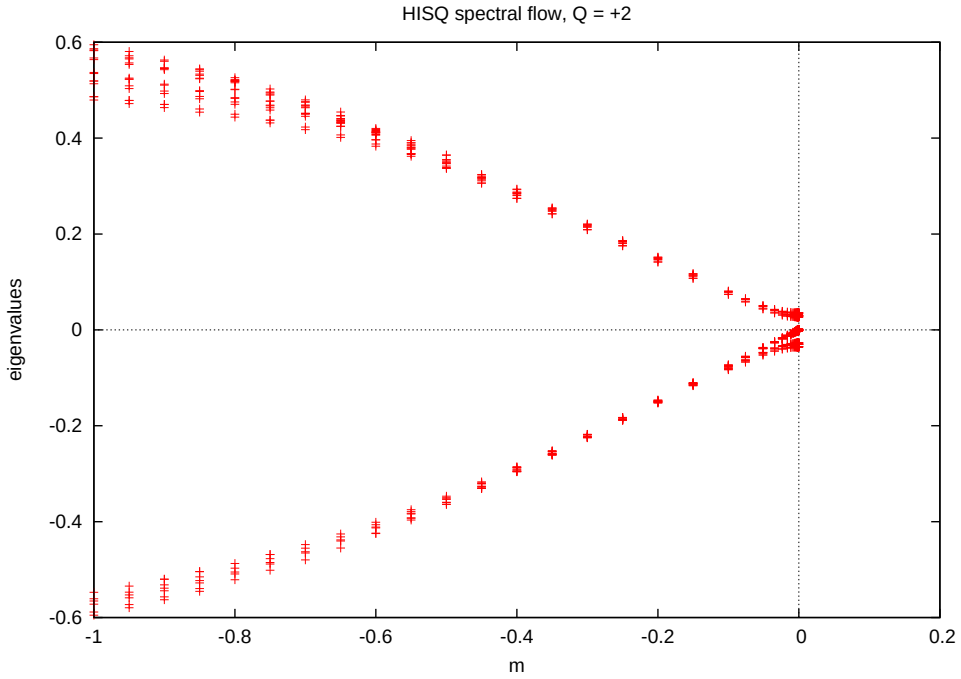

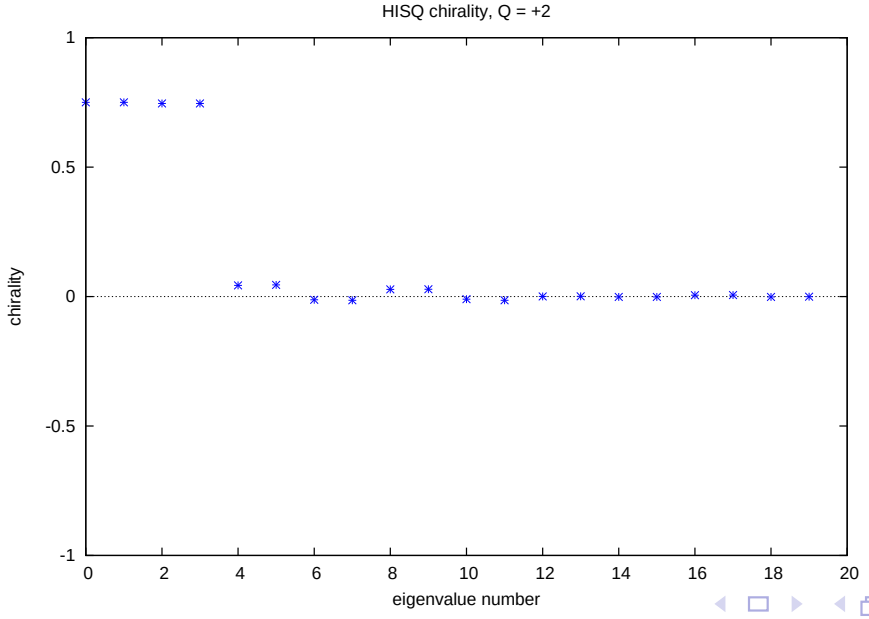

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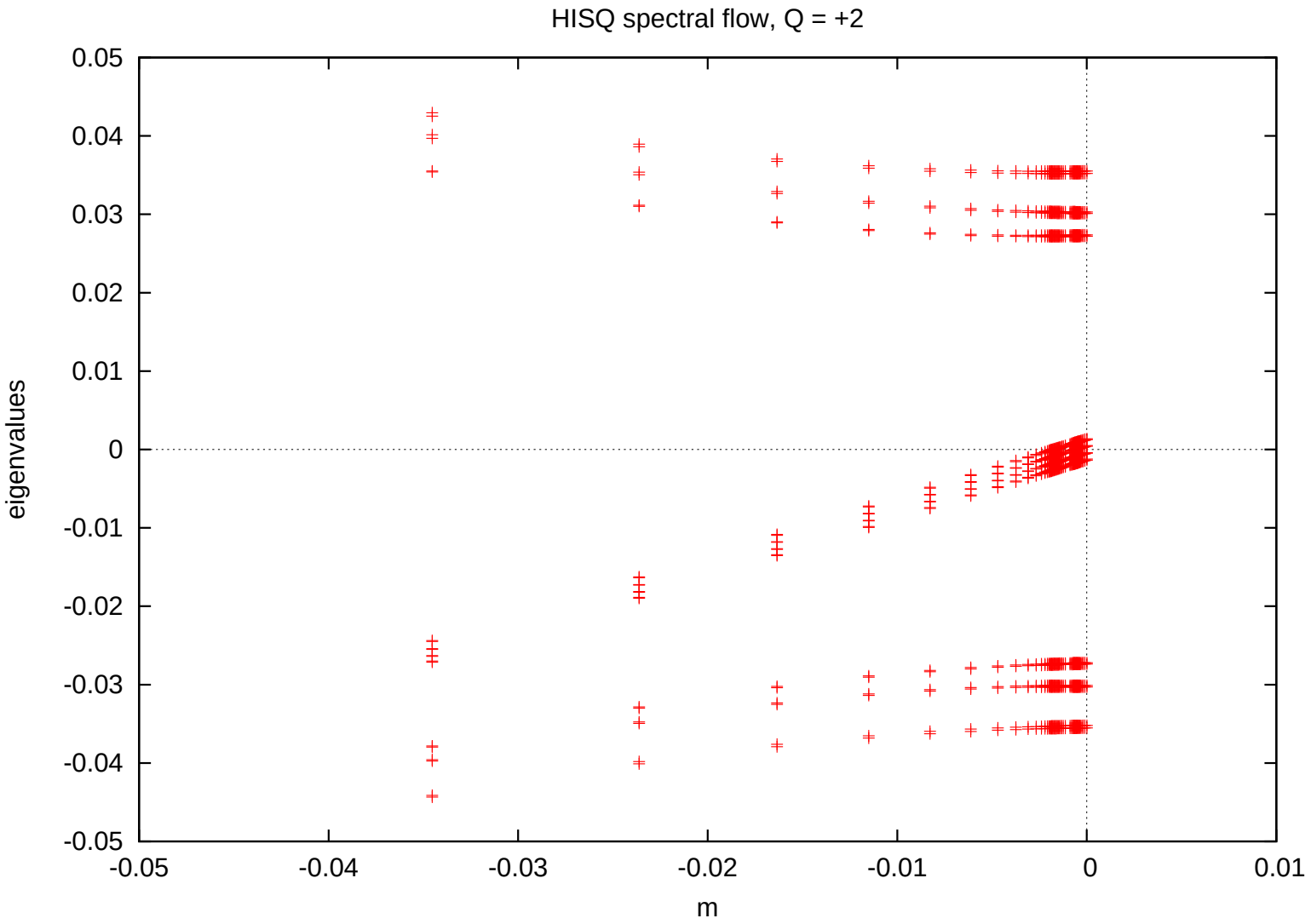

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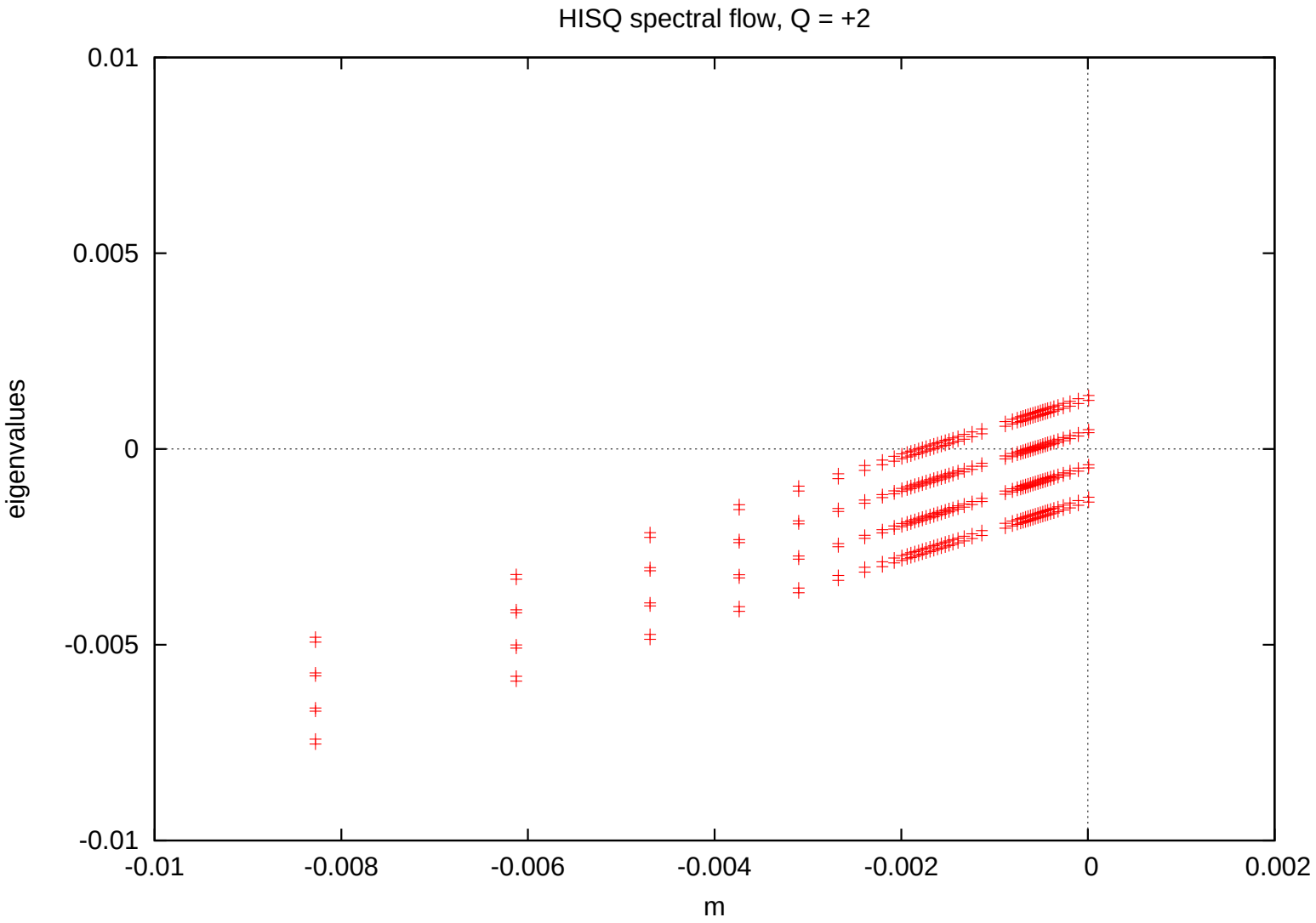

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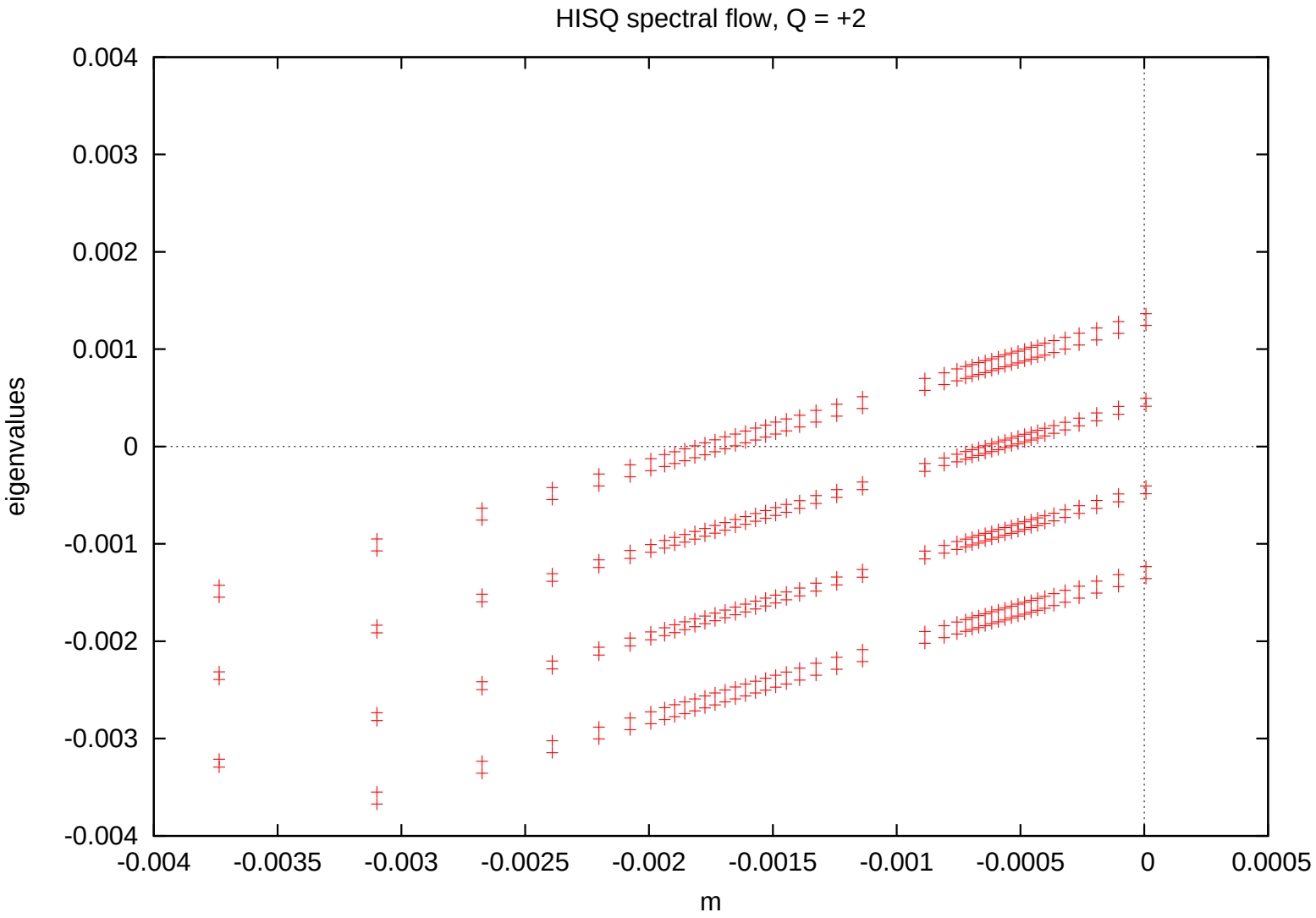
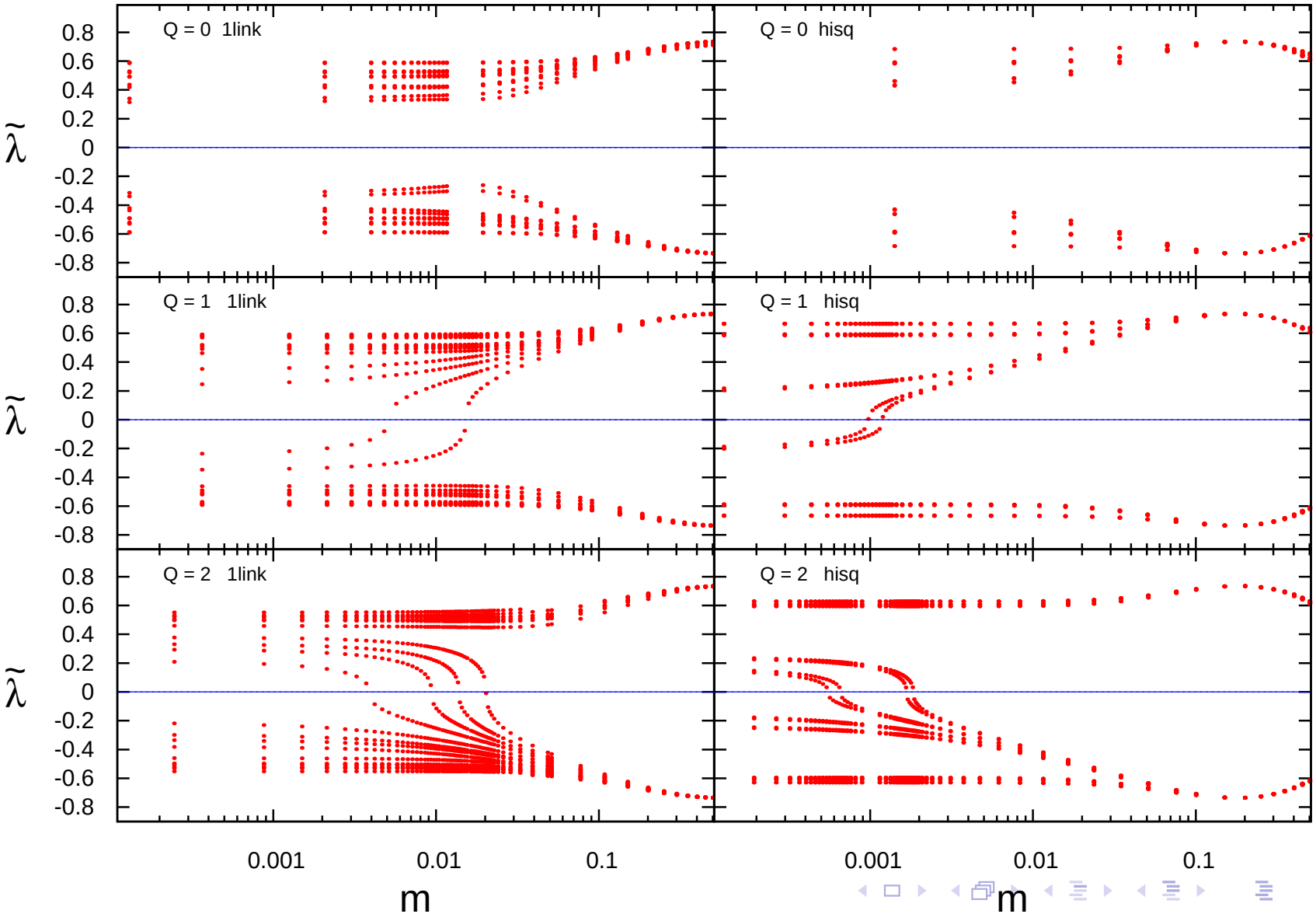


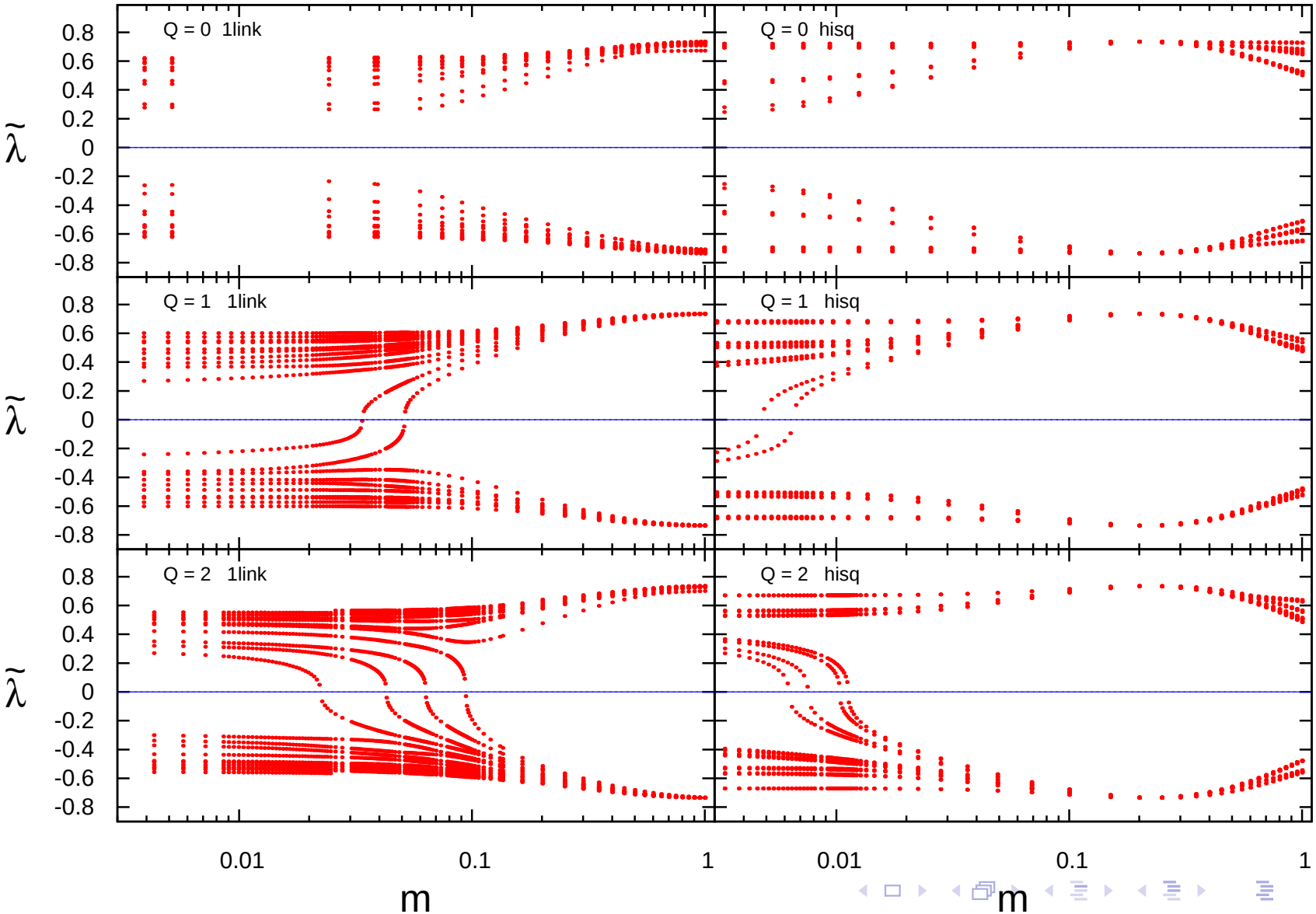
Numerical results

$$\beta = 5.0$$



Numerical results

$$\beta = 4.6$$



Numerical results

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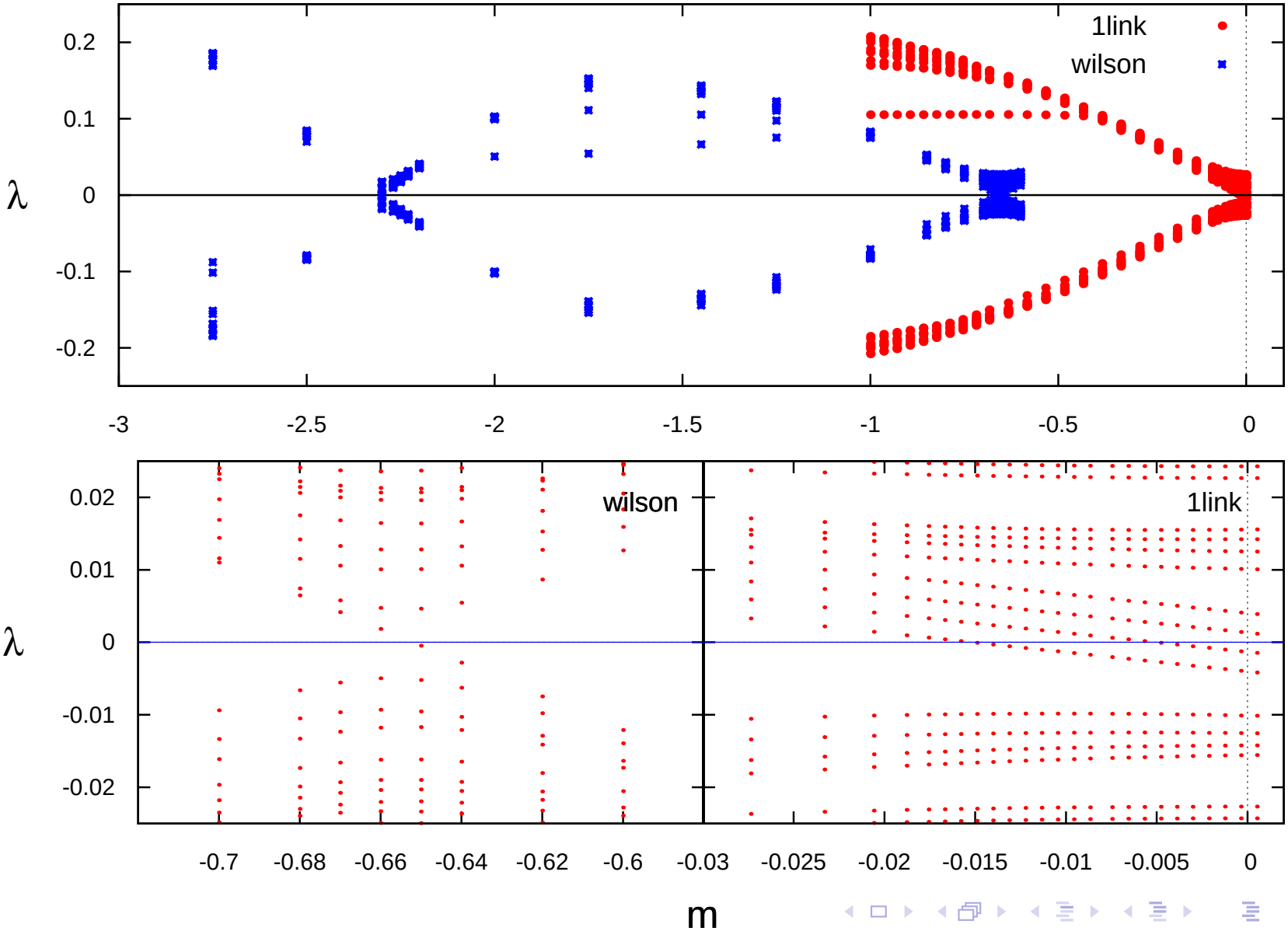

Numerical results

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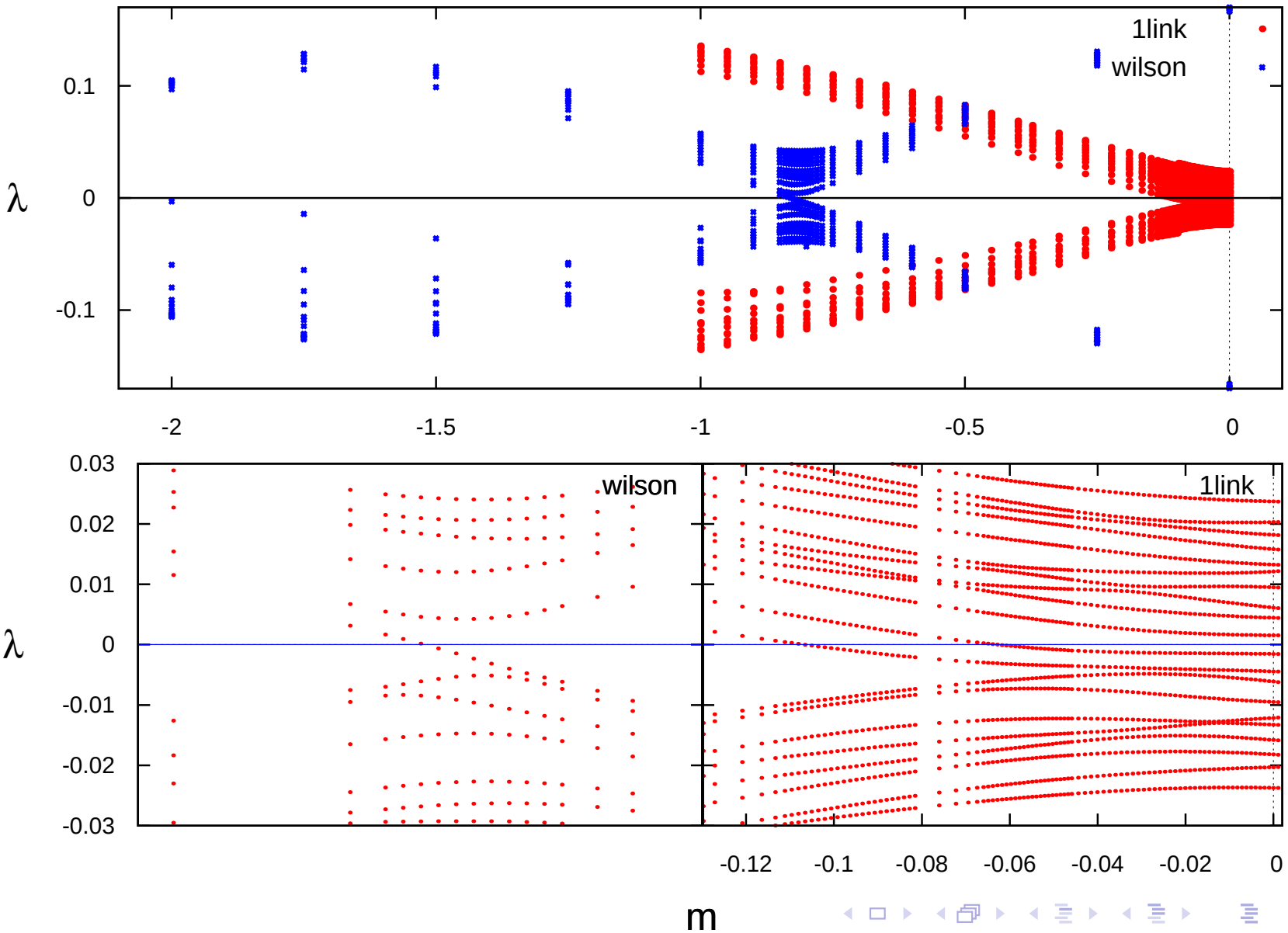

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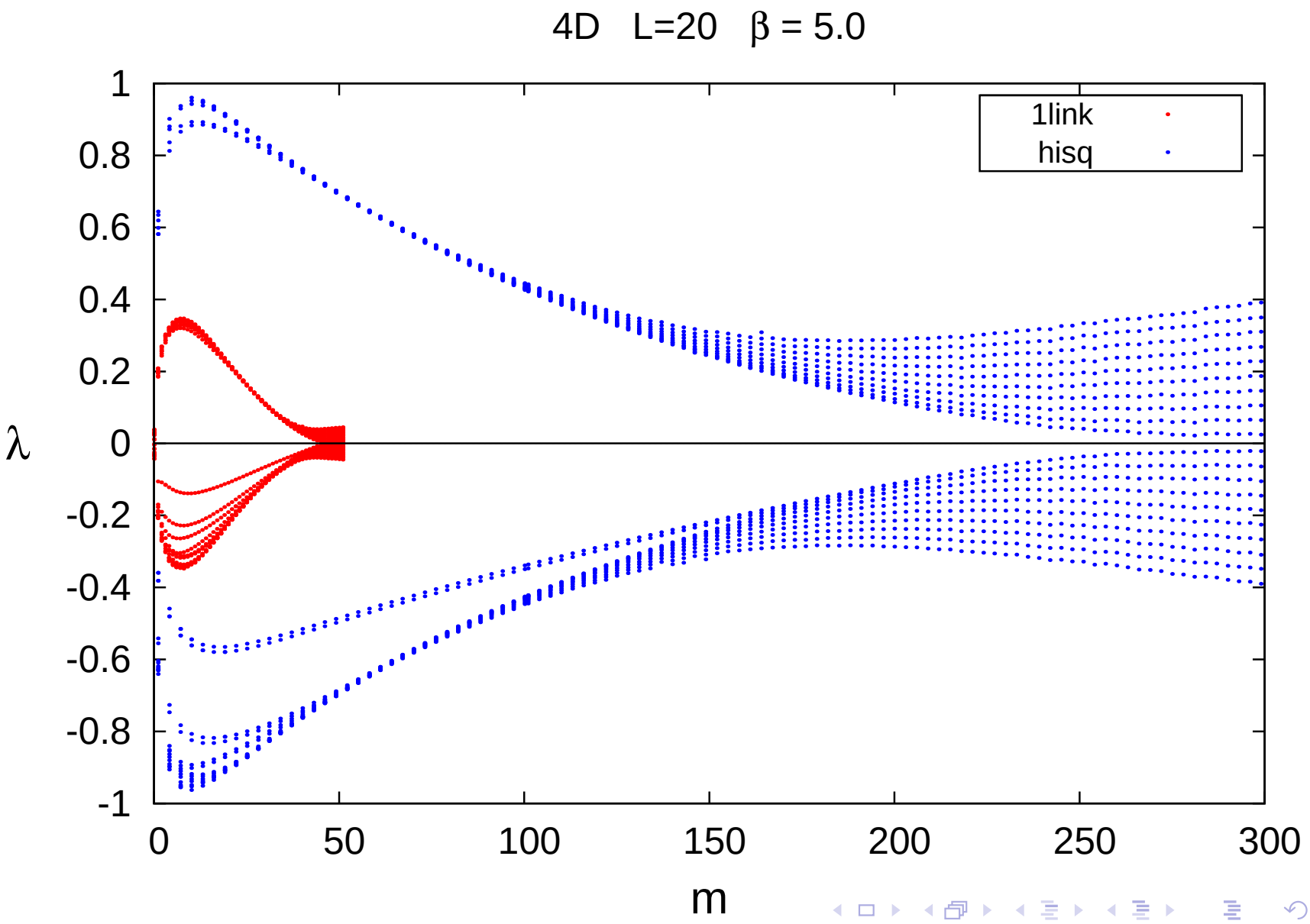

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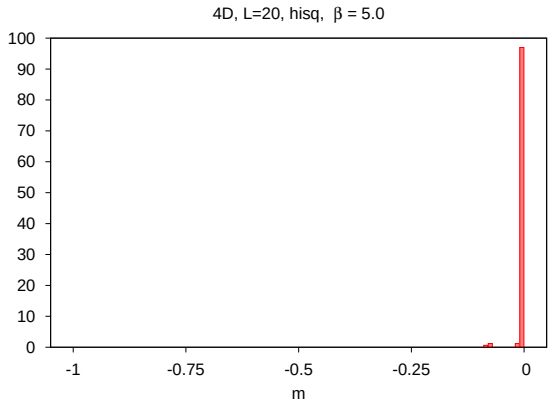

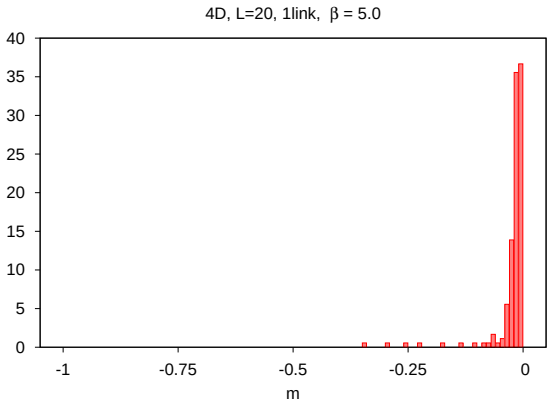

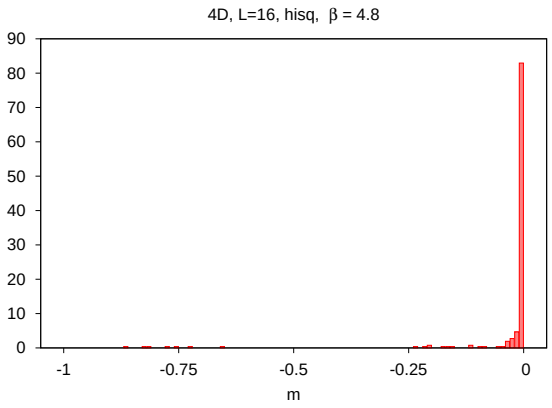

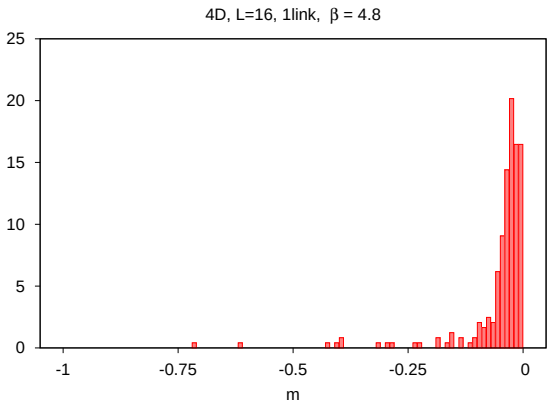

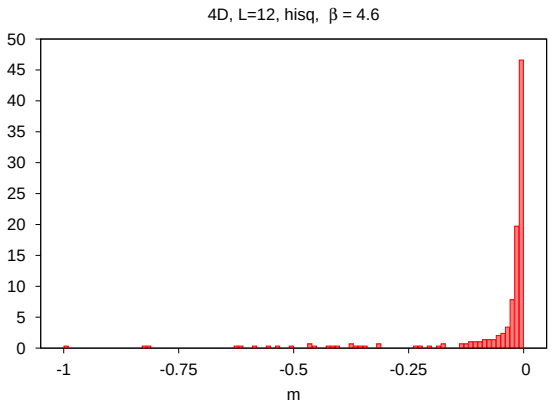

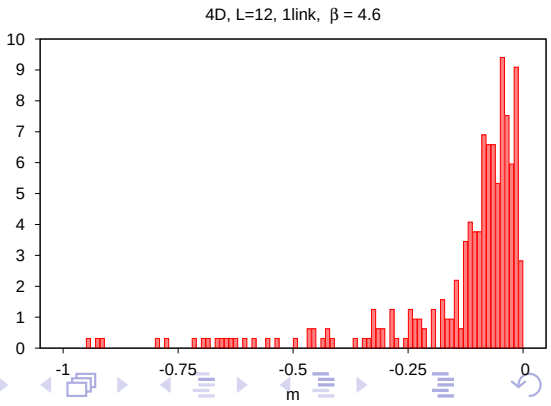

Conclusions and outlook

- The topology as determined by the spectral flow defined by the staggered Dirac operator works very well for realistic gauge configurations. It is a perfectly good definition, on the same footing as the one coming from the Wilson flow.
- For most configurations, the index determined through the spectral flow and through the chiral modes of HISQ coincides.
- Much harder numerically.
- Study the flow for full QCD configurations.