Status of the SU3 Lambda Lattice Scale

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Introduction

Sommer [NPB 411 (1994) 839, hep-lat/9310022] proposed to set a hadronic scale r_c through the force $F(r)$ between static quarks at intermediate distances r: $r_c^2 F(r_c) = c$ (Sommer scale). Necco and Sommer (NS) [NPB 622 (2002) 328, hep-lat/0108008] calculated in this ways the lambda lattice scale of pure SU3 gauge theory with the Wilson action using $r_0^2 F(r_0) = 1.65$ and $r_1^2 F(r_1) = 0.65$.

From MCMC data for the potential they found

$$r_1/r_0 = 0.5133\,(24)$$

and for the range $5.7 \leq \beta \leq 6.92$ the parametrization

$$[a\,\Lambda_L]^{NS} = f_\lambda^{NS}(\beta) = \exp\left[\,-1.6804 - 1.7331\,(\beta-6) + 0.7849\,(\beta-6)^2 - 0.4428\,(\beta-6)^3\,\right],\ \beta = \frac{6}{g^2}\,.$$

Later, but independently, Bazavov, Berg and Velytsky [PRD 74 (2006) 014501] extracted from work of the Bielefeld Group [Boyd et al. NPB 469 (1996) 419] on the deconfining transition a SU3 lambda lattice scale, which approaches the perturbative limit for $\beta \to \infty$:

$$[a\,\Lambda_L]^{BBV} = f_\lambda^{BBV}(\beta) = \lambda(g^2)\,\left(b_0\,g^2\right)^{-b_1/(2b_0^2)}\,e^{-1/(2b_0\,g^2)}\,.$$

The coefficients b_0 and b_1 are perturbatively obtained from the renormalization group equation,

$$b_0 = \frac{11}{3}\frac{3}{16\pi^2} \quad \text{and} \quad b_1 = \frac{34}{3}\left(\frac{3}{16\pi^2}\right)^2$$

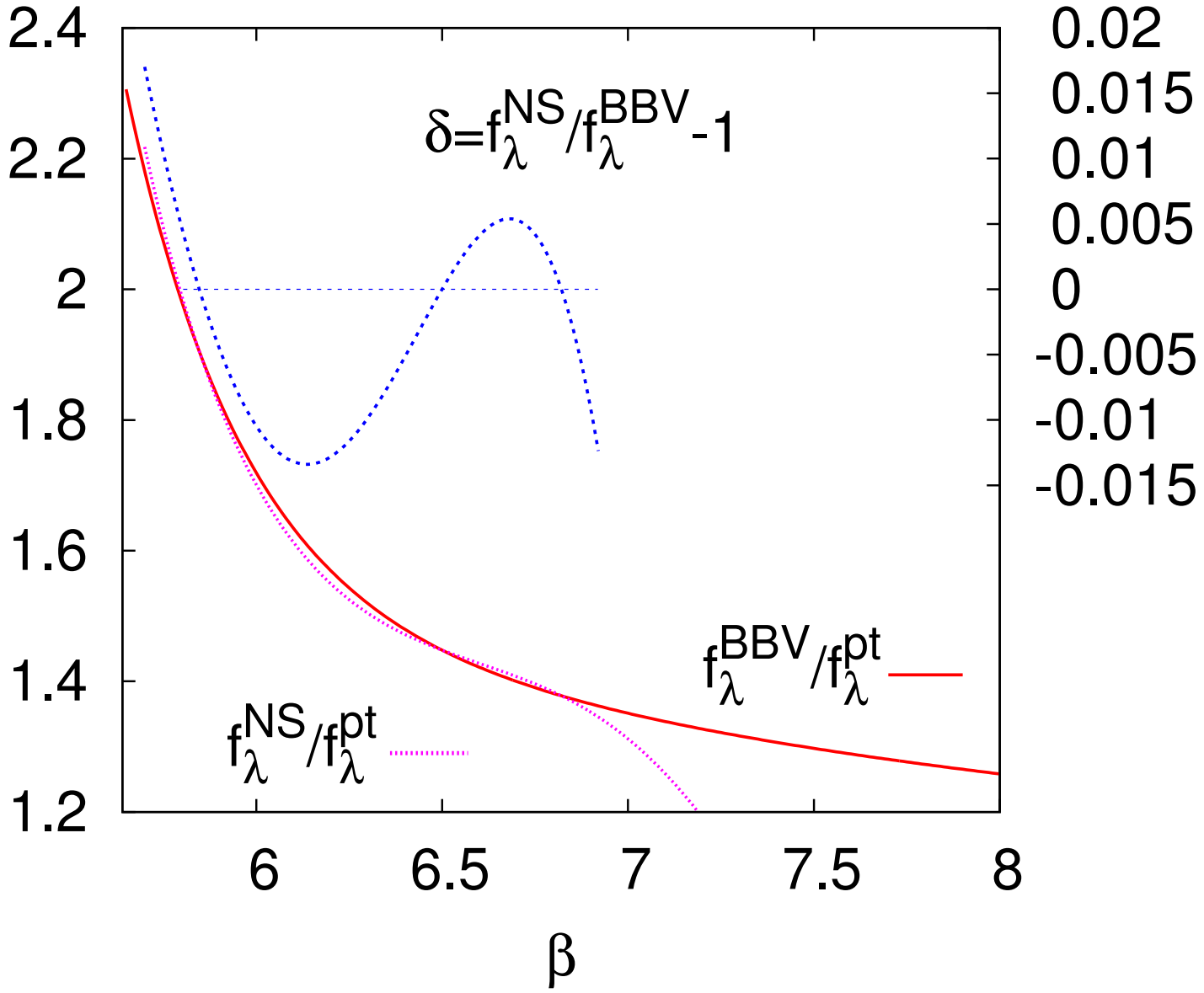
and higher perturbative and non-perturbative corrections were parametrized by

$$\lambda(g^2) \;=\; 1 + e^{\ln a_1}\, e^{-a_2/g^2} + a_3\, g^2 + a_4\, g^4$$

with $\ln a_1 = 18.08596$, $a_2 = 19.48099$, $a_3 = -0.03772473$ and $a_4 = 0.5089052$.

Comparison (overall constants adjusted):

Less than 2% relative error.



Error propagation

Berg and Wu: "SU3 deconfining phase transition with finite volume corrections due to a confined exterior" [PRD 88 (2013) 074507]. Referee (we complied):

1. Present results in terms of the reduced temperature.
2. Use Sommer scale instead of BBV scale.

Reduced temperature:

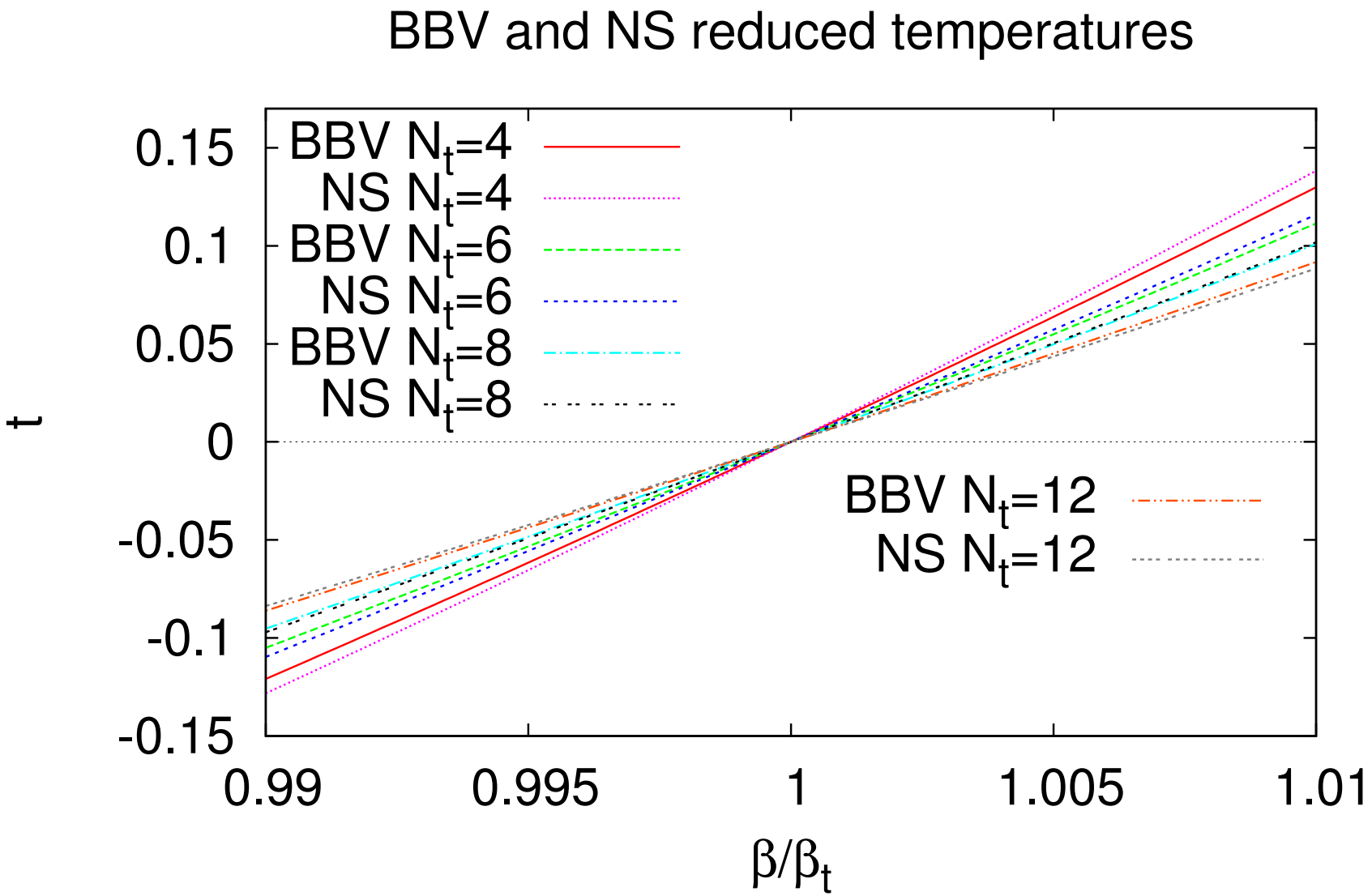
$$t = \frac{T - T_t}{T_t} = \frac{T(\beta) - T(\beta_t)}{T(\beta_t)} \quad \text{with} \quad \beta_t = \beta_t[N_t]\,.$$

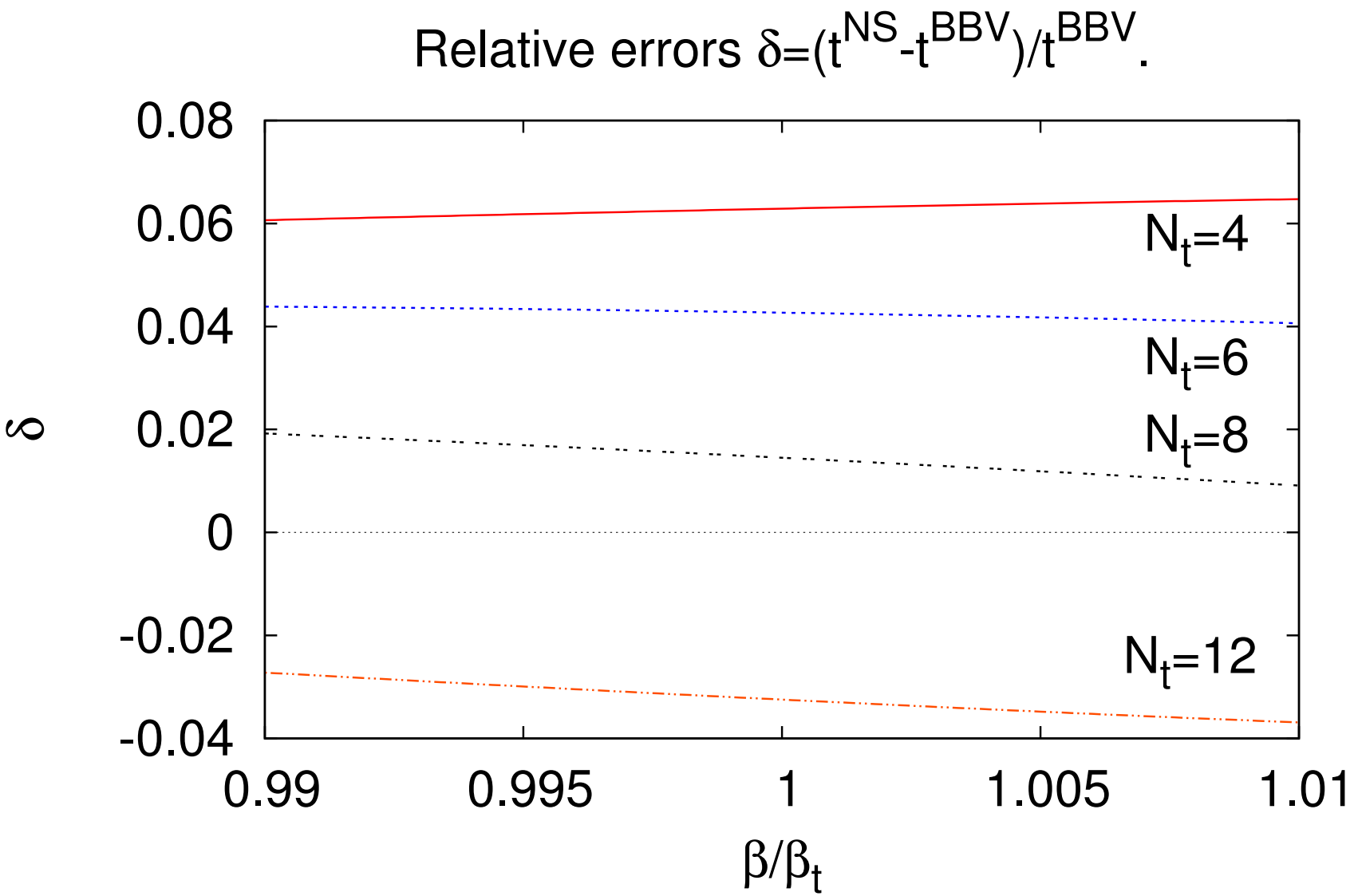
$$t = \left[\frac{1}{a(\beta)\,N_t} - \frac{1}{a(\beta_t)\,N_t}\right] a(\beta_t)\,N_t = \frac{a(\beta_c)}{a(\beta)} - 1$$

$$t = \frac{f_\lambda(\beta_t)}{f_\lambda(\beta)} - 1 \quad \text{with} \quad \beta_t = \beta_t[N_t]\,.$$

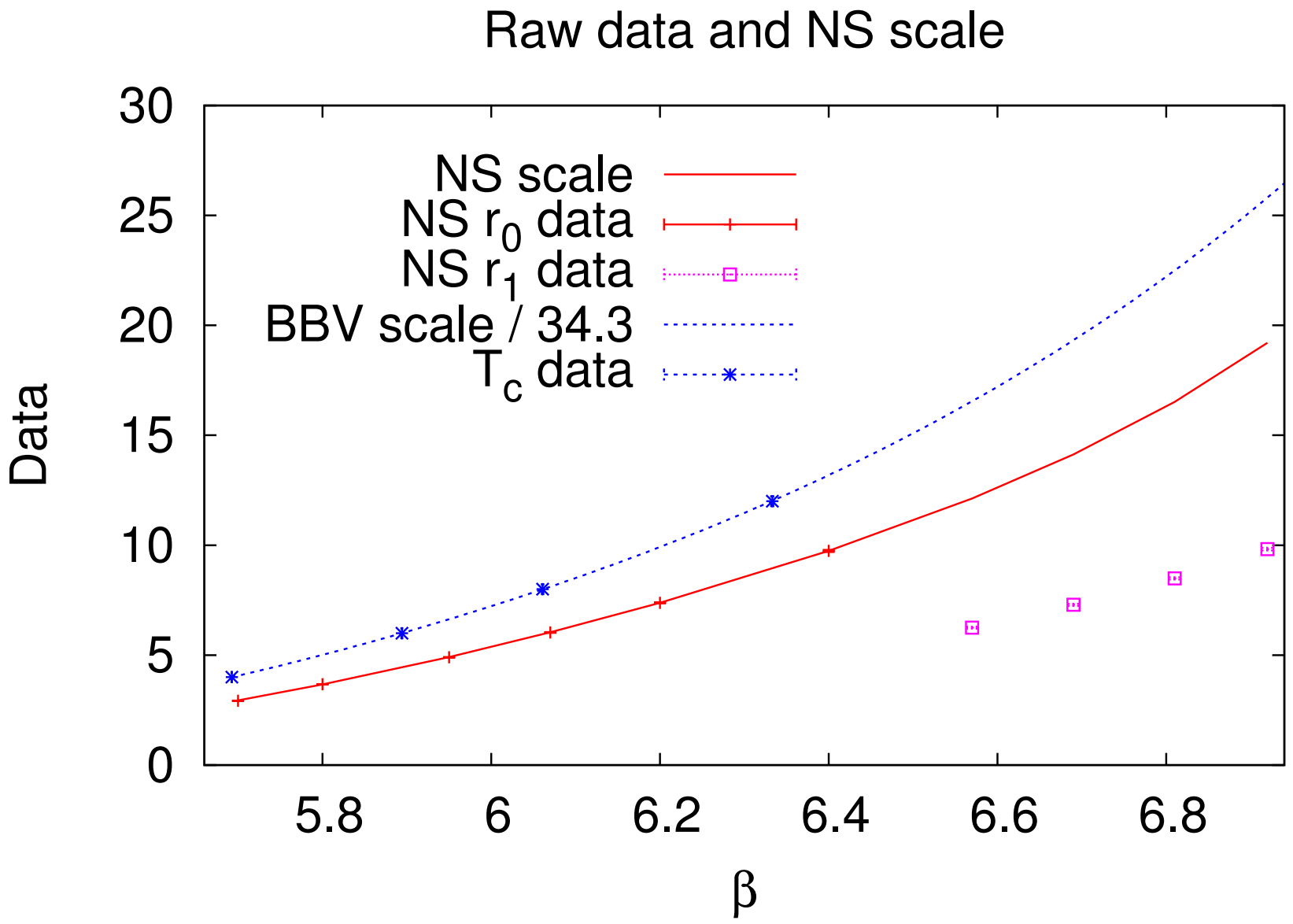
Fitting pseudo-transition β values [Bielefeld paper] on $N_t\, N_s^3$ lattices with N_t fixed and $N_s \to \infty$ gives the infinite volume estimates

$$\begin{aligned}
\beta_t[4] &= 5.6925\ (2)\,, \\
\beta_t[6] &= 5.8941\ (5)\,, \\
\beta_t[8] &= 6.0609\ (9)\,, \\
\beta_t[12] &= 6.3331\ (13)\,.
\end{aligned}$$

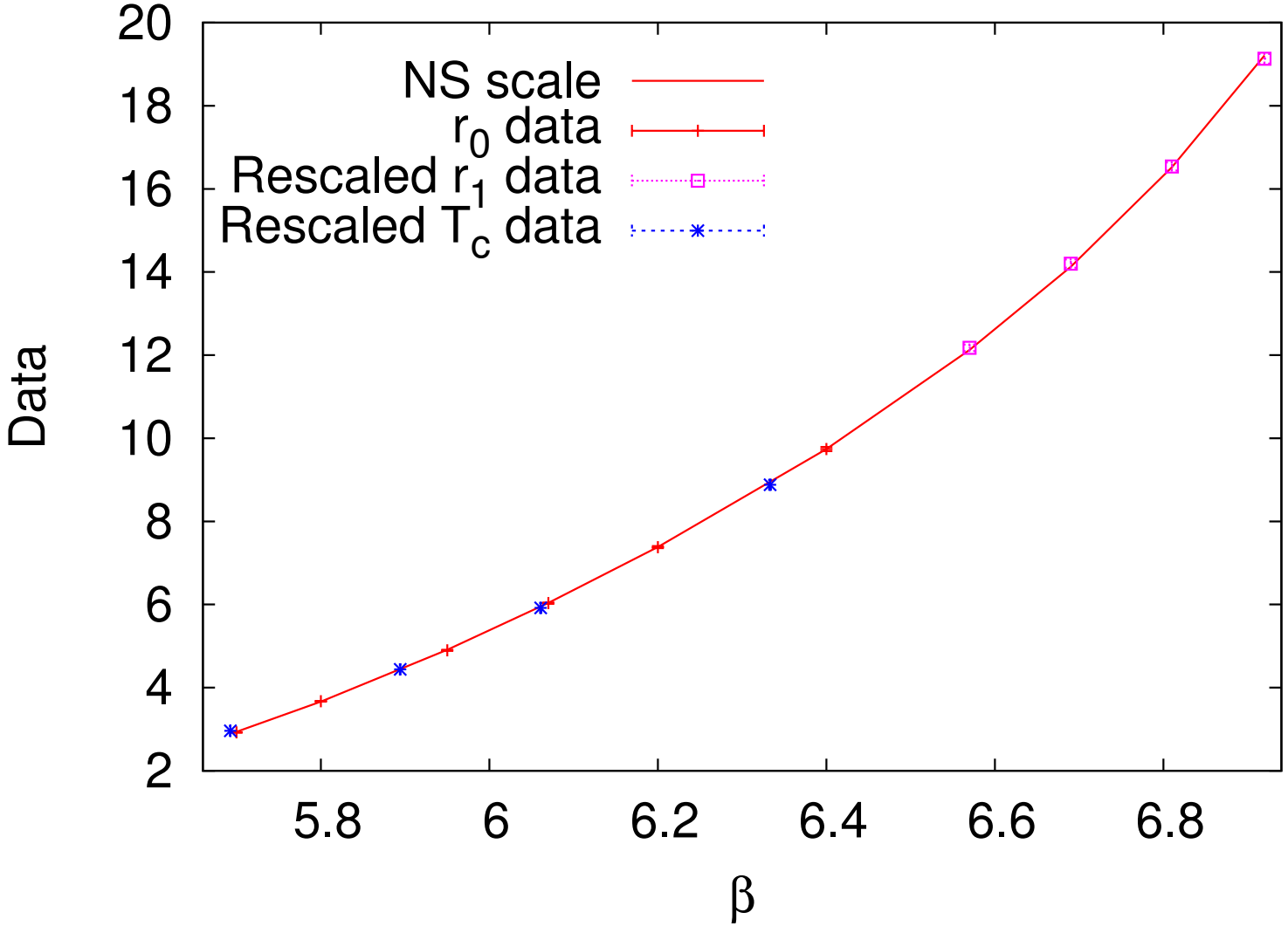

BBV and NS reduced temperatures
BBV N_t=4
NS N_t=4
BBV N_t=6
NS N_t=6
BBV N_t=8
NS N_t=8
BBV N_t=12
NS N_t=12
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
t
0.99
0.995
1
1.005
1.01
β/β_t

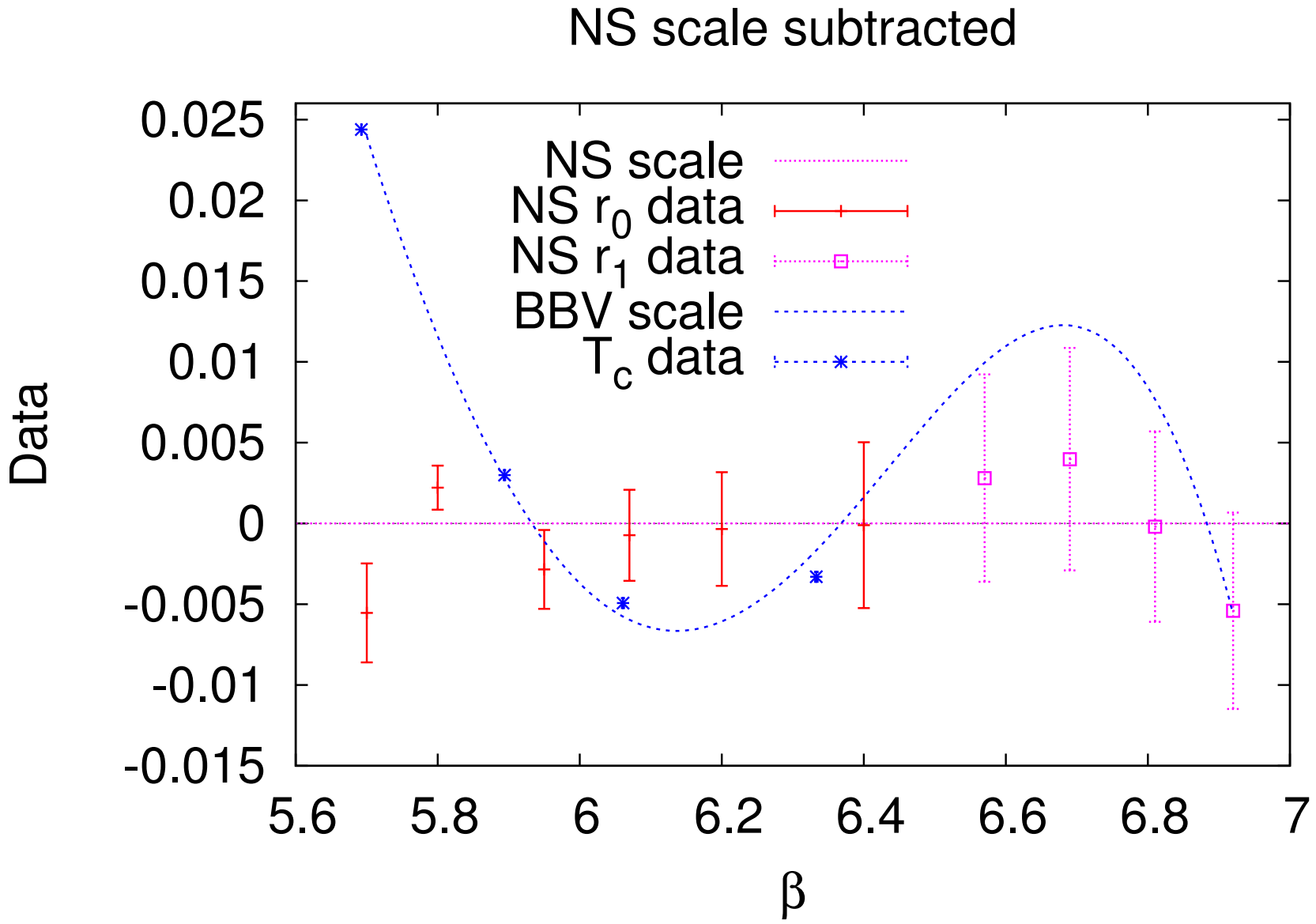

Relative errors δ=(t^NS-t^BBV)/t^BBV.
0.08
0.06
0.04
0.02
0
-0.02
-0.04
δ
N_t=4
N_t=6
N_t=8
N_t=12
0.99
0.995
1
1.005
1.01
β/β_t

Tracking the Differences (const / scale functions used)

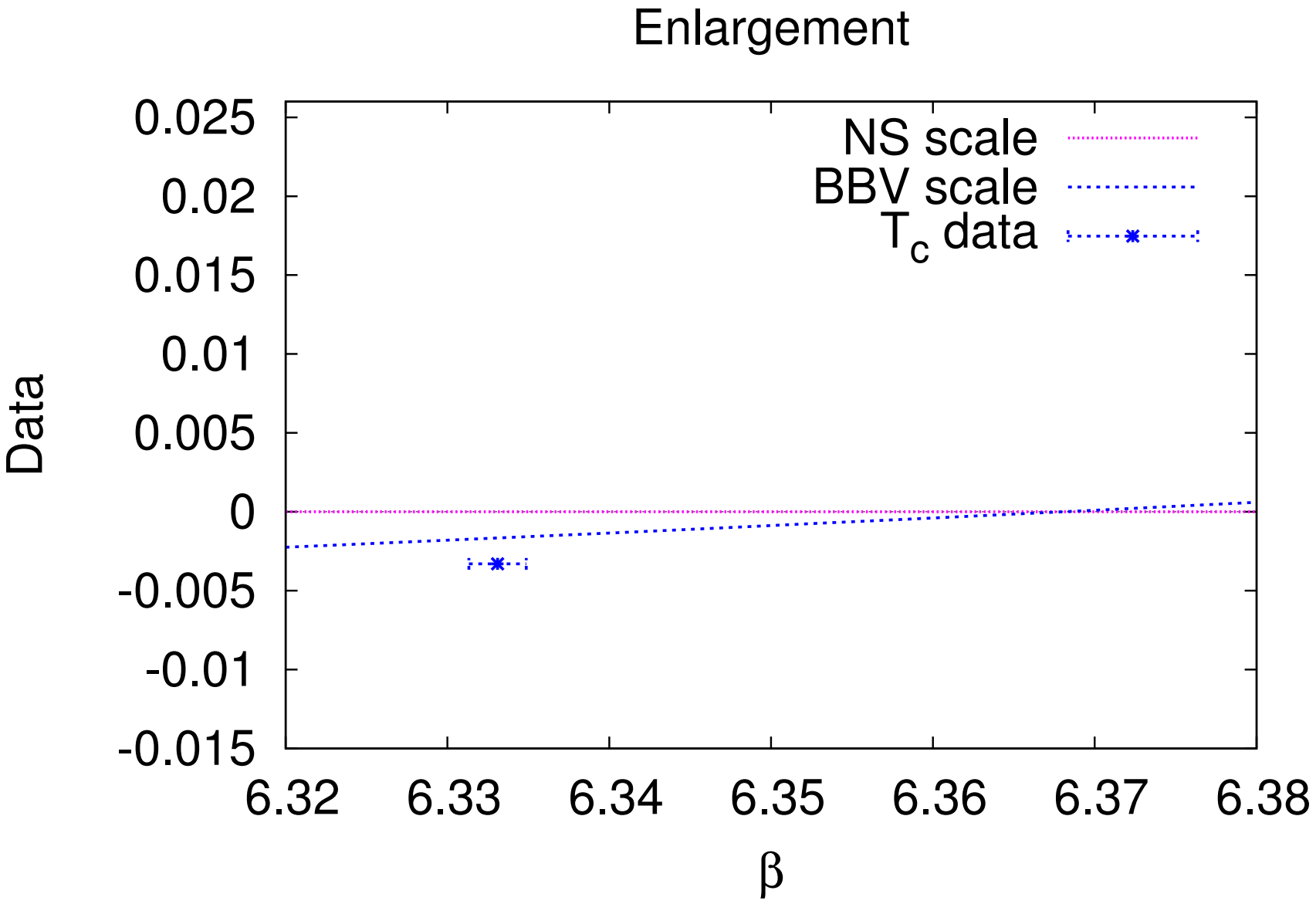


Chi2 fits of free constants: $1.9454\,(62)$, $\chi^2_{pdf} = 0.44$ for r_1 data; $0.73991\,(54)$, $\chi^2_{pdf} = 12.2$ for T_c data.





Note: T_c data are very accurate (error in β).


Enlargement
NS scale
BBV scale
T_c data
Data
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
6.32
6.33
6.34
6.35
6.36
6.37
6.38
β

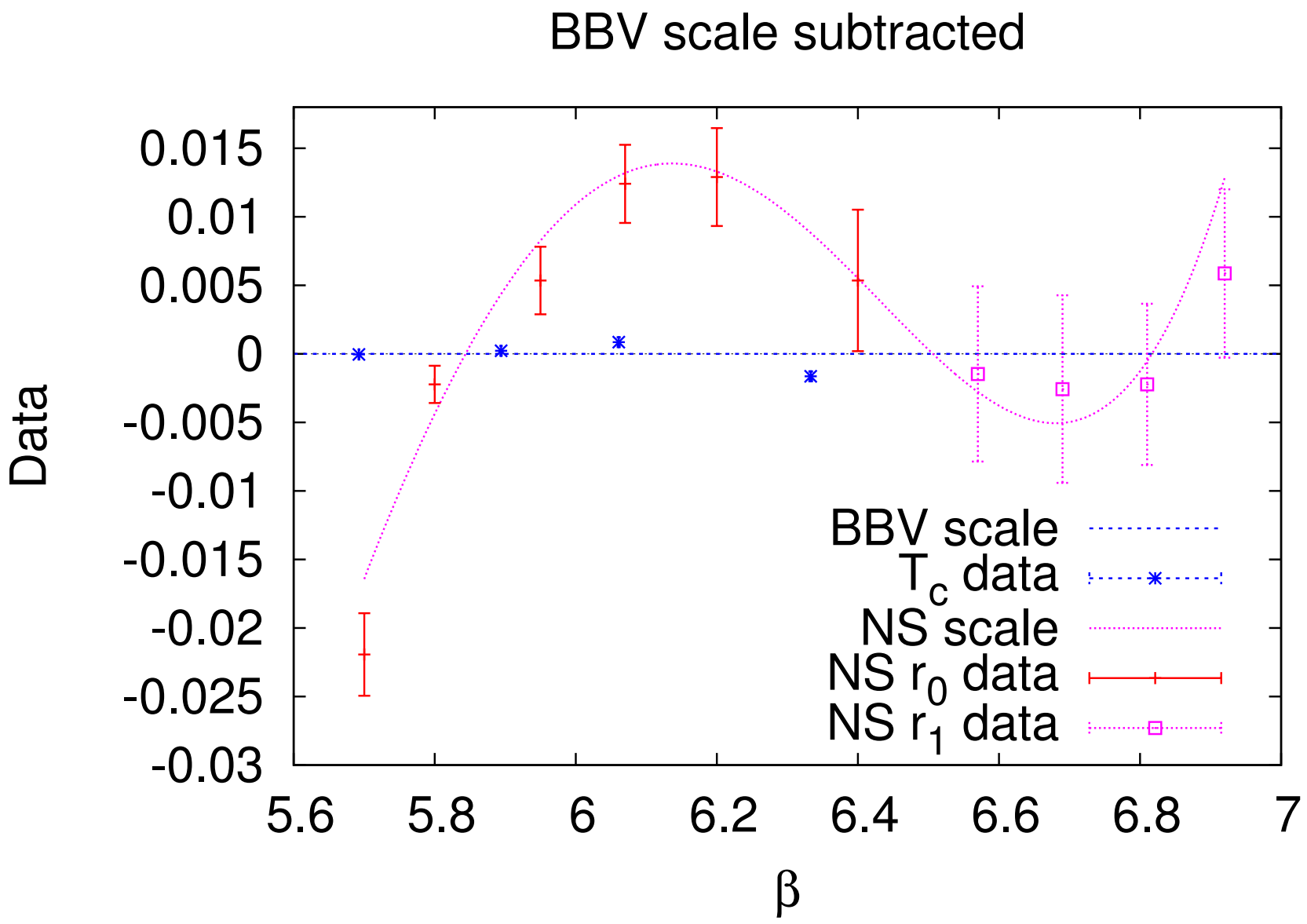
The entire difference between the NS and BBV SU3 scales comes from the small lattice data in the range $\beta \leq 6.4$.

To show this we optimize the free constants of the r_0 and r_1 data sets to best fit the BBV scale. This results in:

$$46.697\,(46),\ \chi^2_{pdf} = 18.7 \text{ for } r_0 \text{ data;}$$

$$90.71\,(29),\ \chi^2_{pdf} = 0.42 \text{ for } r_1 \text{ data.}$$

The large lattice NS r_1 data are well consistent with the BBV scale (may also have too larger error bars)!


BBV scale subtracted
Data
β
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
5.6
5.8
6
6.2
6.4
6.6
6.8
7
BBV scale
T_c data
NS scale
NS r_0 data
NS r_1 data

Conclusions

1. On the basis of the existing data is is impossible to say whether the NS or the BBV SU3 scale is more accurate in the $5.7 \leq \beta \leq 6.92$ range.
2. The BBV scale has advantage to approach the rigorously known perturbative limit for $\beta \to \infty$.
3. It appears that for pure lattice gauge theory accurate data for the deconfining phase transition points $\beta_t[N_t]$ are easier to calculate than similarly accurate data for the force between static quarks at intermediate distances.
4. $\beta_t[N_t]$ data for $N_t > 12$ would be of interest (I did not find any in the literature).