The Hadronic Spectrum and Confined Phase in (1+1)-Dimensional Massive Yang-Mills Theory

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Including joint work with Peter Orland
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The Principal Chiral Sigma Model (PCSM)

$$\text{Action}: S = \frac{N}{2g^2}\int d^2x\, \mathrm{Tr}\partial_\mu U^\dagger(x)\partial^\mu U(x),$$

$$U(x) \in SU(N):$$

$$SU(N)\times SU(N)\,\text{symmetry}: \quad U(x) \to V_L U(x) V_R, \;\; V_{L,R} \in SU(N).$$

Associated Noether currents:

$$j^L_\mu(x)^c_a = \frac{-iN}{2g^2}\partial_\mu U_{ab}(x)\, U^{\dagger\, bc}(x),$$

$$j^R_\mu(x)^d_b = \frac{-iN}{2g^2} U^{\dagger\, da}(x)\partial_\mu U_{ab}(x)$$

Asymptotically free theory of massive particles, with left and right color.

The S-Matrix

Particles and antiparticles have two color charges (color dipoles).

$$_{\text{out}}\langle P, \theta_1', c_1, d_1; P, \theta_2', c_2, d_2 | P, \theta_1, a_1, b_1; P, \theta_2, a_2, b_2 \rangle_{\text{in}}$$

$$= \frac{\sinh(\frac{\theta}{2} - \frac{\pi i}{N})}{\sinh(\frac{\theta}{2} + \frac{\pi i}{N})} \left[\frac{\Gamma(i\theta/2\pi + 1)\Gamma(-i\theta/2\pi - \frac{1}{N})}{\Gamma(i\theta/2\pi + 1 - \frac{1}{N})\Gamma(-i\theta/2\pi)} \right]^2$$

$$\times \left(\delta_{a_1}^{c_1} \delta_{a_2}^{c_2} - \frac{2\pi i}{N\theta} \delta_{a_1}^{c_2} \delta_{a_2}^{c_1} \right) \times \left(\delta_{b_1}^{d_1} \delta_{b_2}^{d_2} - \frac{2\pi i}{N\theta} \delta_{b_1}^{d_2} \delta_{b_2}^{d_2} \right) \langle \theta_1' | \theta_1 \rangle \langle \theta_2' | \theta_2 \rangle$$

$$\theta_j = \text{rapidity} : E_j = m \cosh \theta_j, \;\; p_j = m \sinh \theta_j, \;\; E^2 = p^2 + m^2$$

$$\text{rapidity difference}\, \theta = \theta_1 - \theta_2$$

P. B. Wiegmann, Phys. Lett. 142 B (1984)

Two-particle form factor

For $N > 2$

$$\langle 0|j^L_\mu(0)_{a_0c_0}|A,\theta_1,b_1,a_1;P,\theta_2,a_2,b_2\rangle$$
$$= (p_1-p_2)_\mu\left(\delta_{a_0a_2}\delta_{c_0a_1}-\frac{1}{N}\delta_{a_0c_0}\delta_{a_1a_2}\delta_{b_1b_2}\right)$$
$$\times\frac{2\pi i}{(\theta+\pi i)}\exp\int_0^\infty\frac{dx}{x}\left[\frac{-2\sinh\left(\frac{2x}{N}\right)}{\sinh x}+\frac{4e^{-x}\left(e^{2x/N}-1\right)}{1-e^{-2x}}\right]\frac{\sin^2[x(\pi i-\theta)/2\pi]}{\sinh x}$$

A. C. C., Phys. Rev. D 86 (2012) 025025

For $N = 2$, the form factors are known from the $O(4)$ sigma model, by

$$SU(2)\times SU(2)\simeq O(4),$$

M. Karowski and P. Weisz, Nucl. Phys. B 139 (1978) 455

Gauged PCSM

Not integrable anymore

$$S \int d^2x - \frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2g_0^2}\mathrm{Tr}D_\mu U^\dagger D^\mu U$$

with

$$D_\mu = \partial_\mu + ieA_\mu^L$$

The left $SU(N)$ symmetry is now a local gauge symmetry.

There is a "Gauss Law" that requires the left color indices of sigma-model particles to contract into singlets.

What is the mass spectrum?

Unitary gauge $U = 1$

$$S = \int d^2x - \frac{1}{4}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{e^2}{2g_0^2}\mathrm{Tr}A_\mu A^\mu$$

In unitary gauge, the PCSM works as a Higgs field, giving mass e/g_0 to the gluon.

Asymptotic freedom forces $g_0 \to 0$! The gluon would have a huge mass, not visible at low energies.

Is there more to life than this?

Hamiltonian in the (completely fixed) Axial gauge

Find Hamiltonian in the axial gauge $A_0 = 0, \;\; A_1(t=0) = 0$.

$$H = H_{\rm PCSM} - \frac{e^2}{2g_0^4} \int dx^1 \int dy^1 |x^1 - y^1| j_0^L(x^1) j_0^L(y^1).$$

The system is in a confined phase. The physical particles are hadron-like bound states of sigma model particles

Mesons: one sigma model particle and one antiparticle, with string tension $\sigma = e^2 C_N$.

The meson spectrum is of the form $M = 2m + E$.

Nonrelativistic meson wave function

$(x = x^1 - y^1)$

$$-\frac{1}{m}\frac{d^2}{dx^2}\Psi(x) + \sigma\,|x|\;\Psi(x) = E\Psi(x)$$

The wave function for particles confined by the potential $V(x^1, y^1) = \sigma|x^1 - y^1|$ is

$$\Psi(x) = CAi\left\{(m\sigma)^{\frac{1}{3}}\left[|x| - \frac{E}{\sigma}\right]\right\}$$

The N-particle hadron spectrum can be computed in principle by solving the N-body problem with potential

$$V(x_1, \ldots, x_N) = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\sigma|x_i^1 - x_j^1|$$

The particle-antiparticle wave function

For a free sigma model particle and antiparticle, the wave function is

$$\Psi(x^1, y^1) = \begin{cases} e^{ip_1 x^1 + ip_2 y^1}, & \text{for } x^1 < y^1 \\ e^{ip_2 x^1 + ip_1 y^1} S(\theta), & \text{for } x^1 > y^1 \end{cases}$$

The free and confined wave functions must agree at $x^1 \approx y^1$. Quantization condition for the binding energies E!

The meson spectrum

$$M_n = 2m + E_n, \quad n = 0, 1, 2, \ldots$$

$$E_n = \left\{ \left[\epsilon_n + \left(\epsilon_n^2 + \beta_N^3 \right)^{\frac{1}{2}} \right]^{\frac{1}{3}} + \left[\epsilon_n - \left(\epsilon_n^2 + \beta_N^3 \right)^{\frac{1}{2}} \right]^{\frac{1}{3}} \right\}^{\frac{1}{2}},$$

where

$$\epsilon_n = \frac{3\pi}{4} \left(\frac{\sigma}{m} \right)^{\frac{1}{2}} \left(n + \frac{1}{2} \pm \frac{1}{4} \right),$$

$$\beta_N = \frac{\sigma^{\frac{1}{2}}}{2\pi m} \int_0^\infty \frac{d\xi}{\sinh \xi} \left[2(e^{2\xi/N} - 1) - \sinh(2\xi/N) \right],$$

where $\pm = +$ for the $SU(N)^R$ $(N^2 - 1)$-plet, and $\pm = -$ for the singlet.

Form factors and correlation functions

Two-quark approximation

P. Fonseca, A. Zamolodchikov, RUNHETC-2001-37

$$|B,\phi,n\rangle \approx e^{ix^1 M_n \sinh\phi}\frac{1}{\sqrt{m}}\int_{-\infty}^{\infty}\frac{d\theta}{4\pi}\Psi_n(\theta)\,|A,\theta,a_1,b_1;P,-\theta,a_1,b_1\rangle$$

Bound state form factor

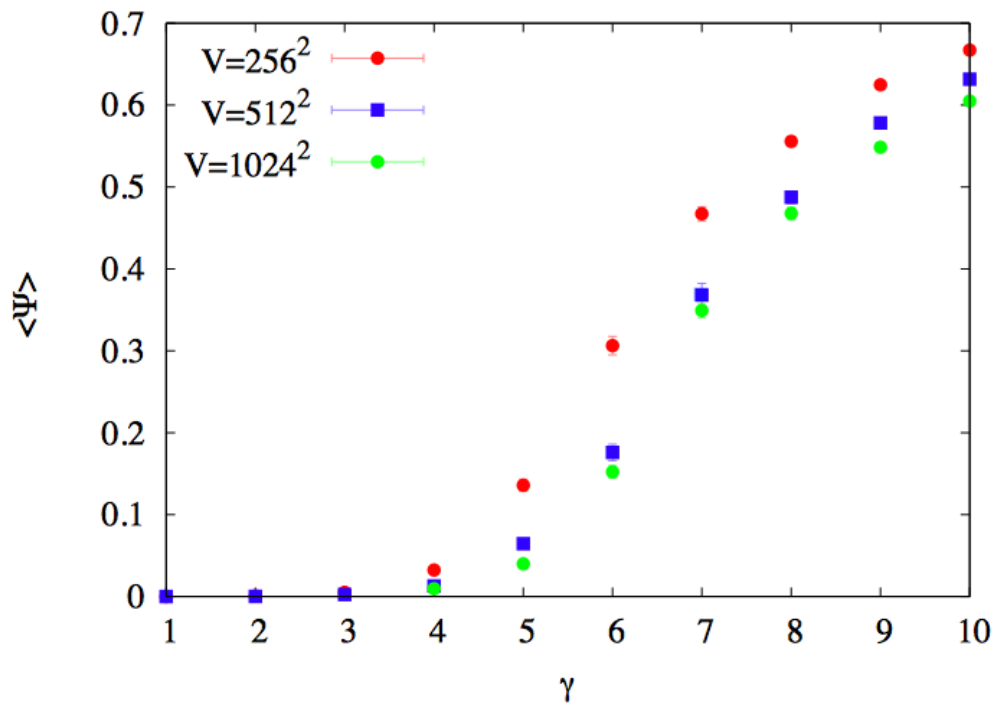
$$\langle 0|\mathcal{A}(x^1,x^2)|B,\phi,n\rangle$$
$$= e^{s\phi}e^{ix^1 M_n\sinh\phi}\int dz\int\frac{d\theta}{4\pi}e^{izm\sinh\theta}\frac{1}{\sqrt{m}}\left(\frac{E_n}{\sigma^H}\right)^{\frac{1}{4}}\mathrm{Ai}\left[(m\sigma^H)^{\frac{1}{3}}\left(|z|-\frac{E_n}{\sigma^H}\right)\right]$$
$$\times\langle 0|\mathcal{A}(0,x^2)|A,\theta,a_1,b_1;P,-\theta,a_1,b_1\rangle$$

Correlation functions

$$\langle 0|\mathcal{A}(x^1,x^2)\mathcal{A}(0,x^2)|0\rangle = \sum_{\Psi}\langle 0|\mathcal{A}(x^1,x^2)|\Psi\rangle\langle\Psi|\mathcal{A}(0,x^2)|0\rangle$$

Lattice results by Gongyo and Zwanziger ($SU(2)$)

Order parameter: $\Psi = \frac{1}{2}\mathrm{Tr}[\bar{U}^\dagger \bar{U}]$



S. Gongyo and D. Zwanziger, arXiv:1402.7124

Also quark-antiquark potential from Wilson loop, and W-boson propagator suggest a confined phase, and a Higgs-like (Kosterlitz-Thouless) phase that seems to go away at large volume.

Where are the W bosons, hiding at finite volume?

Look at the action in axial gauge $A_1 = 0$

$$S = \int d^2x \left[\frac{1}{2} \mathrm{Tr}\,(\partial_1 A_0)^2 + \frac{1}{2g_0^2} \mathrm{Tr}\,(\partial_0 U^\dagger + \mathrm{i}eU^\dagger A_0)(\partial_0 U - \mathrm{i}eA_0 U) \right.$$
$$\left. - \frac{1}{2g_0^2} \mathrm{Tr}\,\partial_1 U^\dagger \partial_1 U \right]$$

Integrate out A_0:

$$S = \int d^2x \left(\frac{1}{2g_0^2} \mathrm{Tr}\,\partial_\mu U^\dagger \partial^\mu U + \frac{1}{2}\, j^L_{0\,a} \frac{1}{-\partial_1^2 + e^2/g_0^2 \mathbf{U}^\dagger \mathbf{U}}\, j^L_{0\,a} \right)$$

Problem! The renormalized field, $\Phi(x) \sim Z(g_0, \Lambda)^{-1/2} U(x)$, is not unitary, and in fact $Z^{-1/2} \to \infty$ as $\Lambda \to \infty$.

Screening at finite volume?

Expectation values at finite volume (Mussardo-LeClair formula):

$$\langle \Phi^\dagger(0)\Phi(0)\rangle_V = Z_V^{-1}$$

$$= \sum_n^\infty \frac{1}{n!}\frac{1}{(2\pi)^n}\int \left[\prod_{i=1}^n d\theta_i \frac{e^{-\epsilon(\theta_i)}}{1+e^{-\epsilon(\theta_i)}}\right] \mathrm{F.P.}\,\langle \theta_n,\ldots,\theta_1|\Phi^\dagger(0)\Phi(0)|\theta_1,\ldots,\theta_n\rangle$$

where the pseudo energies, ϵ are obtained from the thermodynamic Bethe ansatz

$$\epsilon(\theta) = mL\cosh(\theta) - \int \frac{d\theta'}{2\pi}\left[i\log\frac{d}{dx}S(x)\right]|_{x=\theta-\theta'}\log(1+e^{-\epsilon(\theta')})$$

An anticlimactic Lattice '14 picture for an anticlimactic Lattice '14 talk