Charmonium spectral functions from 2+1 flavour lattice QCD

Attila Pásztor [12]
apasztor@bodri.elte.hu
In collaboriation with
S. Borsányi[3] S. Dürr[34] Z. Fodor[134] C. Hoelbling[3]
S. D. Katz[12] S. Krieg[34] S. Mages[5] D. Nógrádi[1] A. Schäfer[5]
B. C. Tóth[3] N. Trombitás[1] K. K. Szabó[34]

[1]Eötvös University

[2]MTA-ELTE Lattice Gauge Theory Group

[3]University of Wuppertal

[4]Jülich Supercomputing Center

[5]University of Regensburg

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Mesonic spectral functions

- The spectral function (SF) is the Fourier-transform of the imaginary part of the retarded correlator
- We will consider correlators of charmonium currents in the pseudoscalar(PS) and vector(V) channels
- These correspond to η_c and J/Ψ
- J/Ψ suppression is regarded as an important signal of QGP formation
- The low frequency behaviour of the SF is related to transport coefficients

The talk is based on: *JHEP 1404 (2014) 132*

Spectral function = im part of real-time retarded correlator

$$A(\omega) = \frac{(2\pi)^2}{Z} \sum_{m,n} \left(e^{-E_n/T} - e^{-E_m/T} \right) |\langle n|J_H(0)|m\rangle|^2 \, \delta(p - k^n + k^m)$$

Relation to the Euclidean time correlator

$$G(\tau, \vec{p}) = \int_0^\infty d\omega A(\omega, \vec{p}) K(\omega, \tau) \text{ where } \mathrm{K}(\omega, \tau) = \frac{\cosh(\omega(\tau - 1/2\mathrm{T}))}{\sinh(\omega/2\mathrm{T})}$$

The inversion of this equation is and ill-posed problem.

The method in a nutshell

$$Q = \alpha S - \frac{1}{2}\chi^2$$

$$S = \int d\omega \left(A(\omega) - m(\omega) - A(\omega) \log\left(\frac{A(\omega)}{m(\omega)}\right)\right)$$

$$\chi^2 = \sum_{i,j} (G_i^{\mathrm{fit}} - G_i^{\mathrm{data}}) C_{ij}^{-1} (G_j^{\mathrm{fit}} - G_j^{\mathrm{data}})$$

$$G_i = \int A(\omega) K(\omega, \tau_i) \mathrm{d}\omega$$

$m(\omega)$ is a function, summarizing our prior knowledge of the solution. Then we average over α. The conditional probability $P[\alpha|\mathrm{data}, \mathrm{m}]$ is given by Bayes' theorem.

Lattice details

Action of BMW collaboration in 2008 (talk tomorrow: Trombitas).
Gauge action = Symanzik tree-level improved gauge action
Fermion action = 2+1 dynamical Wilson fermions with 6 step stout smearing ($\rho = 0.11$) and tree-level clover improvement
Same configurations as in *JHEP 1208 (2012) 126*
$a = 0.057(1)$fm
$m_\pi = 545$MeV
$N_s = 64$
$N_t = 28...12$
$T = 123...288$MeV
We measured the charm meson correlators on these lattices.

Outline of MEM procedure

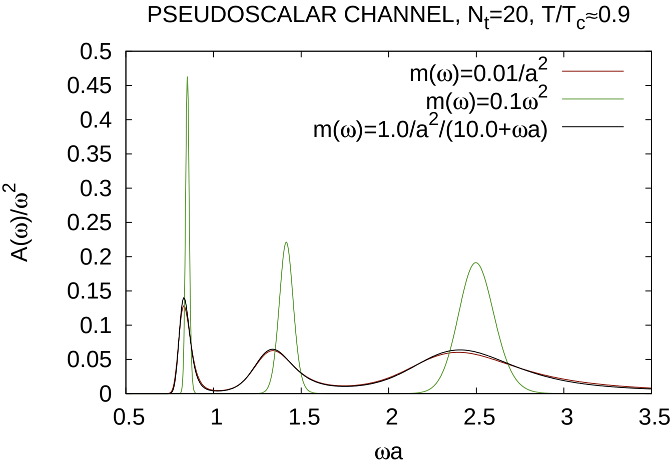
Stability test at the lowest temp

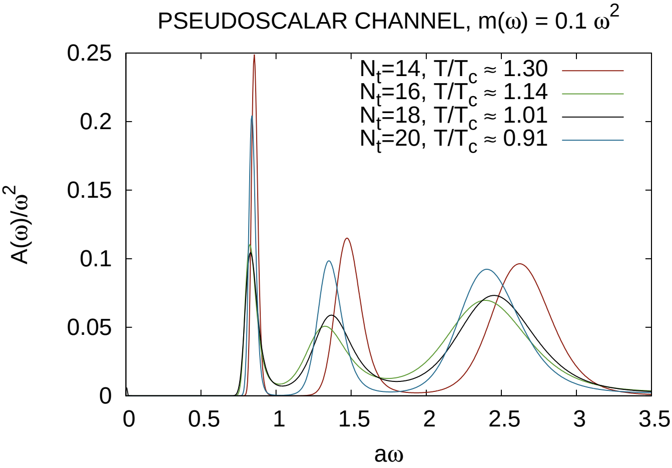
- Drop data points, emulating the number of data points available at the lowest temperature ($N_t = 28$)
- Do the same analysis as with the higher temperature correlators. If the ground state peak cannot be reconstructed, the given number of data points is not reliable
- RESULT: N_t=12 NOT OK, N_t=14,16,18,20 OK

Error analysis

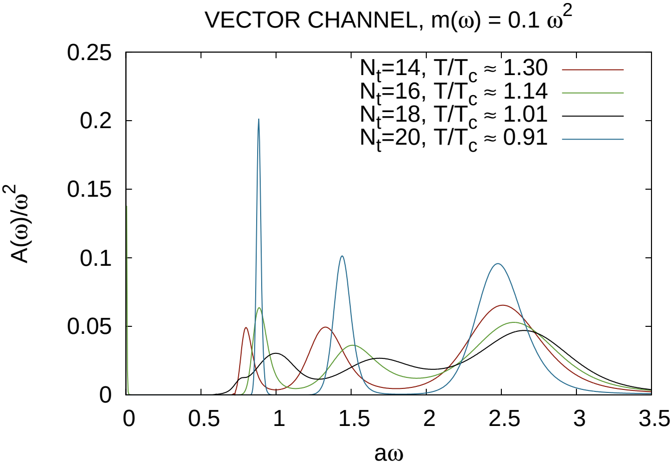
- Systematic error analysis: vary $\Delta\omega$, N_ω, the shape of the prior function: m_0, $m_0\omega^2$, $1/(m_0 + \omega)$, $m_0\omega$ and m_0=0.01, 0.1, 1.0, 10.0.
- Statistical error analysis: given set of parameters, 20 jackknife samples

This is the PS channel, but V looks similar

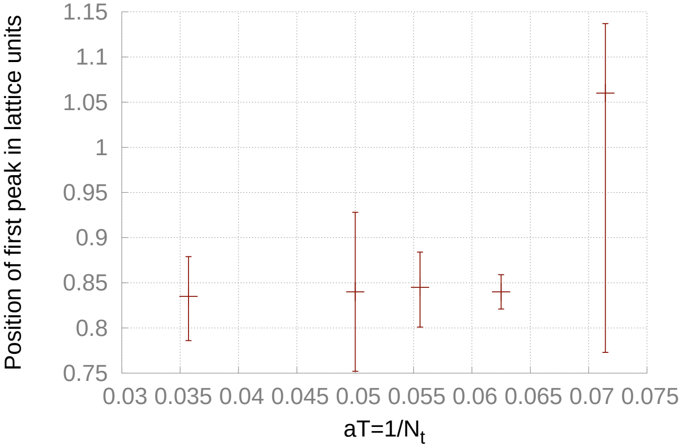



PSEUDOSCALAR CHANNEL, m(ω) = 0.1 ω^2
A(ω)/ω^2
aω
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
3
3.5
N_t=14, T/T_c ≈ 1.30
N_t=16, T/T_c ≈ 1.14
N_t=18, T/T_c ≈ 1.01
N_t=20, T/T_c ≈ 0.91

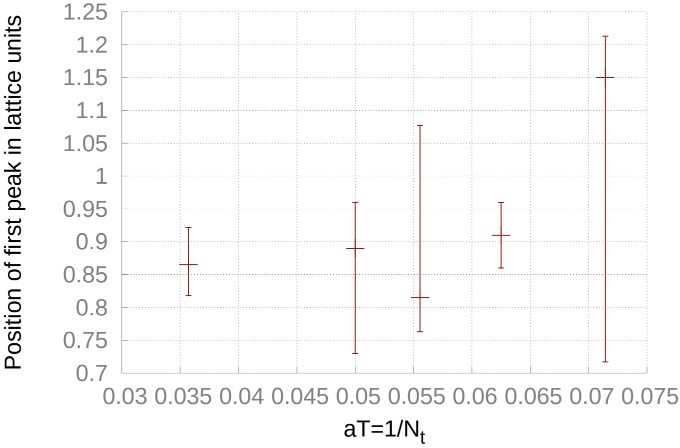
Vector channel



Pseudoscalar channel - position of 1st peak



Vector channel - position of 1st peak



Kubo-formula

$$D = \frac{1}{6\chi} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{A_{ii}(\omega, T)}{\omega},$$

If $D > 0$ we have $\rho/\omega > 0$ for small ω implying a transport peak

Narrow transport peak

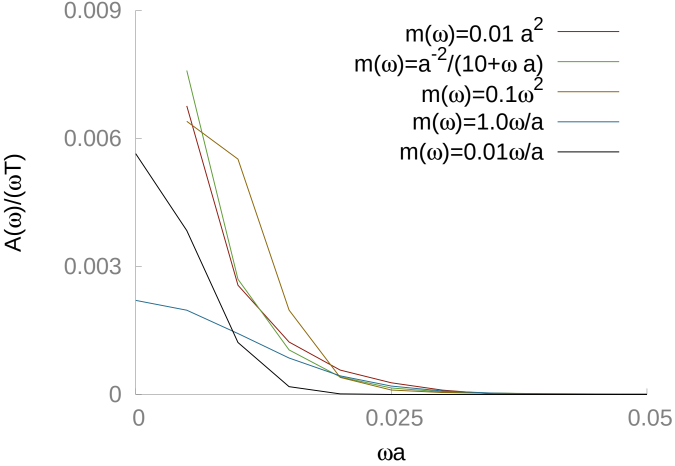
In the case of a narrow transport peak, we can use the ansatz:

$$A_{\mathrm{transport}}(\omega, T) = f(T)\omega\delta(\omega - 0^{+})$$

This does not mean, that the diffusion coefficient is infinite. But in case of a narrow transport peak, the Euclidean correlator $G(\tau, T) = \int K(\omega, \tau) A(\omega, T)$ is not sensitive to the full shape of the peak, only the area. The contrubtion of the transport peak will be a temperature dependent constant (zero mode).

Some indication of a transport peak

$N_t = 16$ not conclusive



Definition of G/G_{rec}

Jakovac, Petreczky, Petrov, Velytsky: Phys.Rev. D75 014506 (2007)

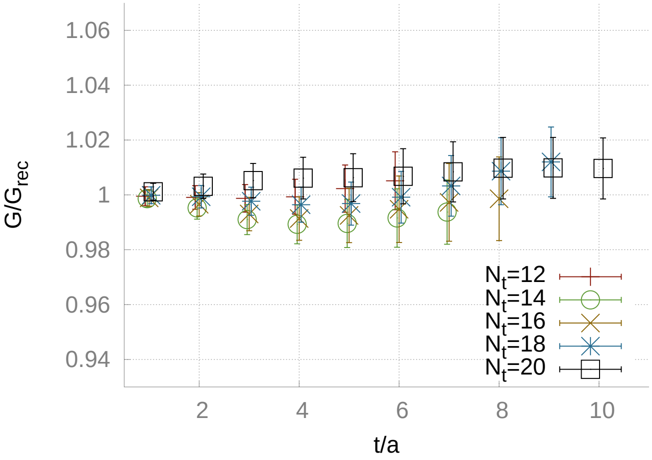
$$\frac{G\left(t,T\right)}{G_{\mathrm{rec}}\left(t,T\right)}=\frac{G(t,T)}{\int A(\omega,T_{\mathrm{ref}})K(\omega,t,T)\mathrm{d}\omega}$$

Midpoint subtracted version

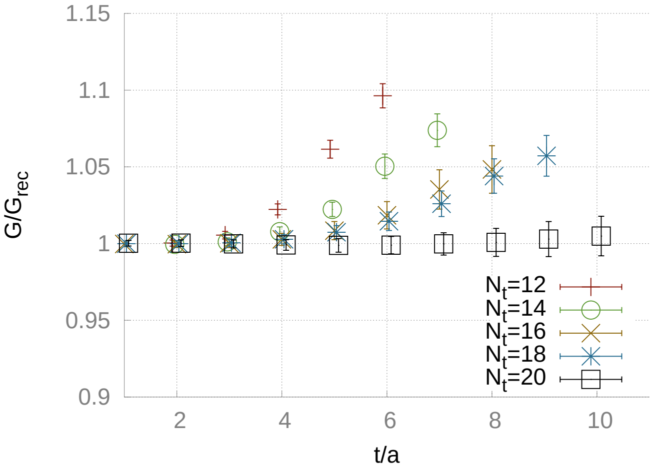
$$\frac{G^{-}}{G^{-}_{\mathrm{rec}}}=\frac{G\left(t,T\right)-G\left(N_t/2,T\right)}{G_{\mathrm{rec}}\left(t,T\right)-G_{\mathrm{rec}}\left(N_t/2,T\right)}=$$
$$\frac{G\left(t,T\right)-G\left(N_t/2,T\right)}{\int A(\omega,T_{\mathrm{ref}})\left[K(\omega,t,T)-K(\omega,N_t/2,T)\right]\mathrm{d}\omega}$$

This removes the zero mode.

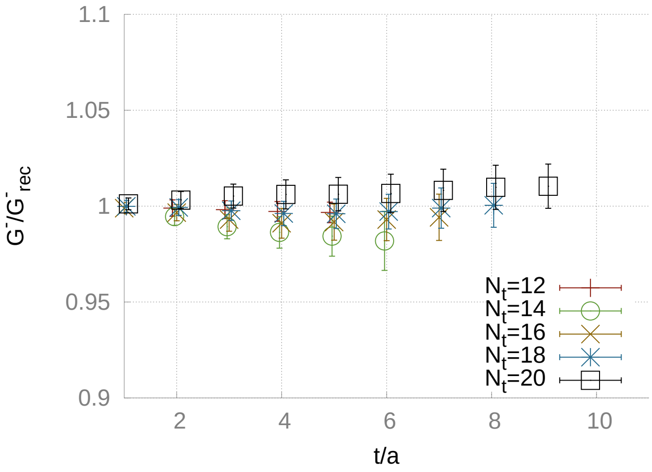
Pseudoscalar channel



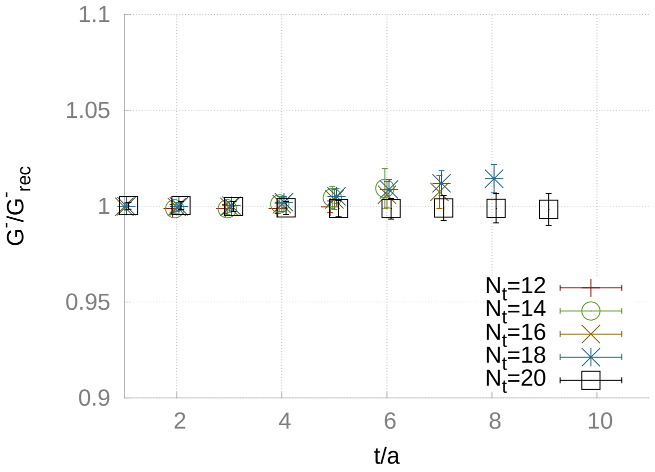
Vector channel



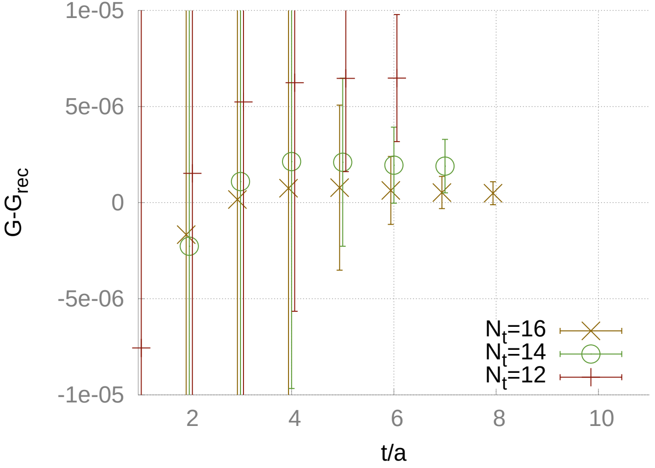
Pseudoscalar channel, midpoint subtracted version



Vector channel, midpoint subtracted version



Vector channel



Conclusions

MEM analysis

- There seems to be no change in the SF in the PS channel up to $1.4T_c$
- There seems to be some change in SF in the V channel
- Indications of a transport peak in the V channel

G/G_{rec} analyis

- No change in the PS channel
- In the V channel, results are consistent with a temperature independent $\omega > 0$ part and a temperature dependent zero mode (narrow transport peak), described by the ansatz $A(\omega) = f(T)\omega\delta(\omega - 0^+) + A(\omega, T = 0)$.

MEM continued...

It can be shown, that the maximum of Q is in an N_{data} dimensional subspace:

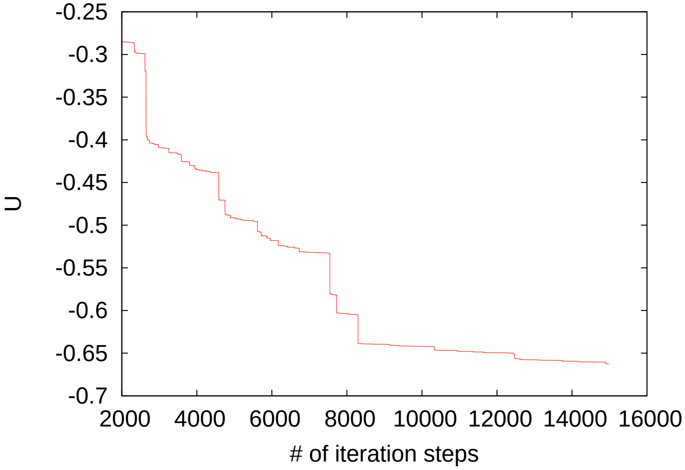
$$A(\omega) = m(\omega) \exp \left(\sum_{i=1}^{N_{data}} s_i f_i(\omega) \right)$$

Two choice for basis functions: Bryan (Eur. Biophys J. 18, 165 (1990)) or Jakovác et al (Phys.Rev. D75 014506 (2007). We use the latter. In this case the maximization of Q is equivalent to the minimization of

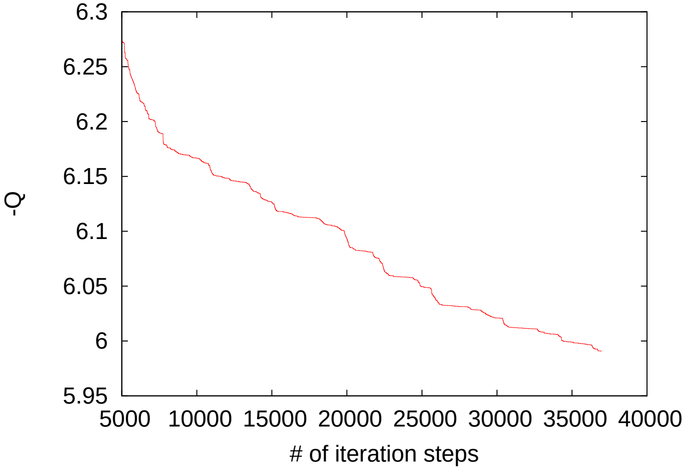
$$U = \frac{\alpha}{2} \sum_{i,j=1}^{N_{\mathrm{data}}} s_i C_{ij} s_j + \int_0^{\omega_{\mathrm{max}}} \mathrm{d}\omega \mathrm{A}(\omega) - \sum_{\mathrm{i}=1}^{\mathrm{N_{data}}} \mathrm{G_i^{data}} \mathrm{s_i}.$$

Comment: Have to use arbitrary precision arithmetics with both methods.

Problem: stopping criterion



Solution: going back to the N_ω dimensions



Conclusions from mock data analysis

- MEM gives the correct qualitative features of the spectral function, but it is not a precise quantitative method.
- The peak positions agree well with the input, the shapes do not
- As long as the data points are not too noisy, O(10) point are enough for reconstruction.
- Features that remain unchanged by varying the prior are restricted by the data.
- Peaks close in position can be merged into one broader peak.

Backup - charm mass tuning

From Davies et al PRL 104, 132003 (2010) $m_c/m_s = 11.85$. Because of additive renormalization, it is impossible to use this directly. We use $(m_c - m_s)/(m_s - m_{ud})$ where the additive renormalization constant cancels. We know that for ud and s the masses used in the simulation correspond to a mass ratio of 1.5 (Durr et al. Phys. Lett. B701 (2011) 265), from this we get $(m_c - m_s)/(m_s - m_{ud}) = 32.55$ We check if the meson masses are indeed in the right ballpark:

J^P	m_i		ma	$ma/m_{D_s^*}a$	$m_{exp}/m_{D_s^*}$
0^-	m_s, m_c	D_s	0.54(1)	0.95(2)	0.932
0^-	m_c, m_c	η_c	0.8192(7)	1.437(4)	1.411
1^-	m_s, m_c	D_s^*	0.570(1)	1	1
1^-	m_c, m_c	J/Ψ	0.8388(8)	1.472(2)	1.466
$3/2^+$	$3m_s$	Ω	0.478(8)	0.84(2)	0.791