QCD Thermodynamics With Continuum Extrapolated Wilson Fermions

S. Borsányi[1] S. Dürr[12] Z. Fodor[123] C. Hoelbling[1]
S. D. Katz[3] S. Krieg[12] D. Nógrádi[3] K. K. Szabó[1]
B. C. Tóth[1] **N. Trombitás**[3]

[1] Department of Physics, University of Wuppertal, Germany

[2] Jülich Supercomputing Center, Forschungszentrum Jülich, Germany

[3] Institute for Theoretical Physics, Eötvös University, Hungary

Lattice 2014,
The 32nd International Symposium on Lattice Field Theory,
Brookhaven National Laboratory and Columbia University

Motivation

- Continuum extrapolated staggered $N_f = 2 + 1$ QCD thermodynamics at the physical point. [Wuppertal-Budapest, HotQCD]
- Disadvantages: rooting trick, taste symmetry breaking.
- Wilson fermions do not have these problems, continuum extrapolation is possible.

The action

Gauge fields

Symanzik tree level improved action:

$$S_{\mathrm{G}}^{\mathrm{Sym}} = \beta \left[\frac{c_0}{3} \sum_{\mathrm{plaq}} \mathrm{Re}\,\mathrm{Tr}\,(1 - U_{\mathrm{plaq}}) + \frac{c_1}{3} \sum_{\mathrm{rect}} \mathrm{Re}\,\mathrm{Tr}\,(1 - U_{\mathrm{rect}}) \right],$$

where $c_0 = 5/3$ and $c_1 = -1/12$.

Fermionic fields, $N_f = 2 + 1$

$$S_{\mathrm{F}}^{\mathrm{SW}} = S_{\mathrm{F}}^{\mathrm{W}} - \frac{c_{\mathrm{SW}}}{4} \sum_x \sum_{\mu,\nu} \bar{\psi}_x \, \sigma_{\mu\nu} F_{\mu\nu,x} \, \psi_x \, ,$$

with six steps of stout smearing with smearing parameter $\varrho = 0.11$ and clover coefficient its tree level value $c_{\mathrm{SW}} = 1.0$, which leads to improved chiral properties.

Parameter tuning, LCP

- We used the fixed scale approach.
- Three sets of simulations each corresponding to a fixed m_π/m_Ω and m_K/m_Ω mass ratio.
- At each lattice spacing m_s is fixed at its physical value.
- The scale was set by $m_\Omega = 1672\,\mathrm{MeV}$.

β	am_{ud}	am_s	N_s	N_t
3.30	-0.0985	-0.0710	32	4 - 16, 32
3.57	-0.0260	-0.0115	32	4 - 16, 64
3.70	-0.0111	0.0	48	8 - 28, 48
3.85	-0.00336	0.0050	64	12 - 28, 64

β	am_{ud}	am_s	N_s	N_t
3.30	-0.1122	-0.0710	32	6 - 16, 32
3.57	-0.0347	-0.0115	48	6 - 16, 64
3.70	-0.0181	0.0	48	8 - 24, 48
3.85	-0.0100	0.0050	64	8 - 36, 64

β	am_{ud}	am_s	N_s	N_t
3.30	-0.1245	-0.0710	32	6 - 16, 32
3.57	-0.0443	-0.0115	48	8 - 24, 64
3.70	-0.0258	0.0	64	8 - 24, 96

Parameter tuning, LCP

$m_\pi \approx 545\,\mathrm{MeV}$
$m_\mathrm{K} \approx 614\,\mathrm{MeV}$
$m_\pi L \gtrsim 8$

β	m_π/m_Ω	m_K/m_Ω	am_PCAC	am_Ω	a [fm]
3.30	0.332(3)	0.373(3)	0.0428(2)	1.16(1)	0.139(1)
3.57	0.319(6)	0.359(4)	0.02649(4)	0.777(9)	0.093(1)
3.70	0.326(5)	0.369(5)	0.01994(4)	0.586(8)	0.070(1)
3.85	0.314(7)	0.358(6)	0.01559(2)	0.480(8)	0.057(1)

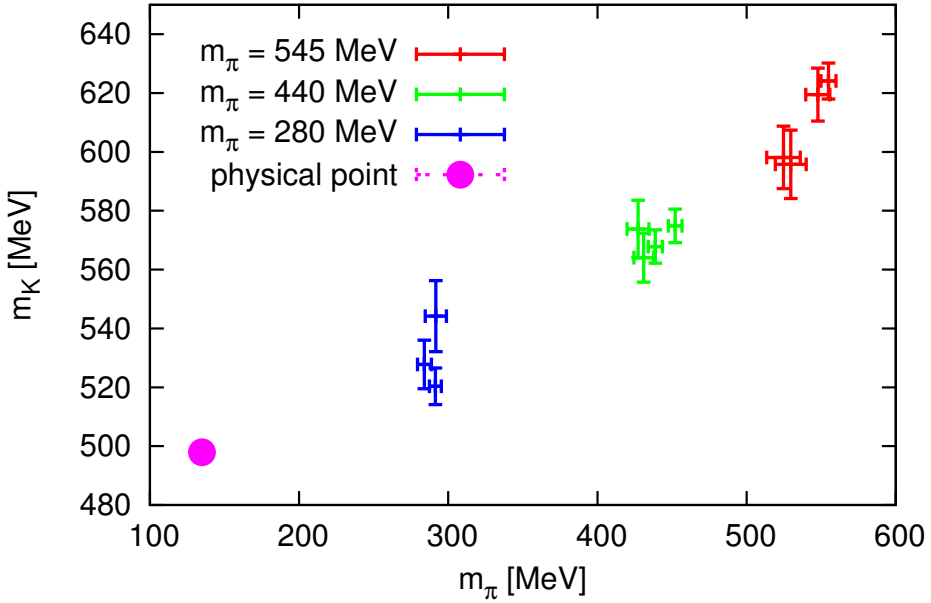
$m_\pi \approx 440\,\mathrm{MeV}$
$m_\mathrm{K} \approx 570\,\mathrm{MeV}$
$m_\pi L > 7$

β	m_π/m_Ω	m_K/m_Ω	am_PCAC	am_Ω	a [fm]
3.30	0.262(3)	0.340(3)	0.0248(2)	1.11(1)	0.133(1)
3.57	0.270(3)	0.344(3)	0.01710(5)	0.737(7)	0.088(1)
3.70	0.258(4)	0.337(5)	0.01266(3)	0.578(8)	0.069(1)
3.85	0.256(4)	0.343(6)	0.00890(1)	0.446(7)	0.053(1)

$m_\pi \approx 280\,\mathrm{MeV}$
$m_\mathrm{K} \approx 520\,\mathrm{MeV}$
$m_\pi L > 5.4$

β	m_π/m_Ω	m_K/m_Ω	am_PCAC	am_Ω	a [fm]
3.30	0.174(4)	0.325(7)	0.0084(2)	0.97(2)	0.117(3)
3.57	0.174(2)	0.311(4)	0.00693(4)	0.723(8)	0.087(1)
3.70	0.170(1)	0.316(5)	0.00481(2)	0.560(9)	0.067(1)

- At each finite temperature point around 1000-1500 equilibrated trajectories were generated while around 1000 at zero temperature.
- m_Ω and hence the lattice spacing depends rather mildly on the light quark masses.



Estimating uncertainties

- Statistical error: jackknife analysis
- Systematic error: histogram method [BMW, 2008]
- Different T interpolations and continuum limits (a^2, αa)
- Various weights: goodness of fit, flat, or Akaike Information Criterion (AIC):

$$w = \exp(-AIC/2) \qquad \text{with} \quad AIC = \chi^2 + 2 \times (\#\,\text{of parameters})$$

Chiral condensate

The bare chiral condensate requires both additive and multiplicative renormalization. [S. Borsanyi *et al.* 1205.0440]

$$m_{\mathrm{R}}\langle\bar{\psi}\psi\rangle_{\mathrm{R}}(T) = 2\,N_f\,m_{\mathrm{PCAC}}^2\,Z_{\mathrm{A}}^2\,\Delta_{PP}(T),$$

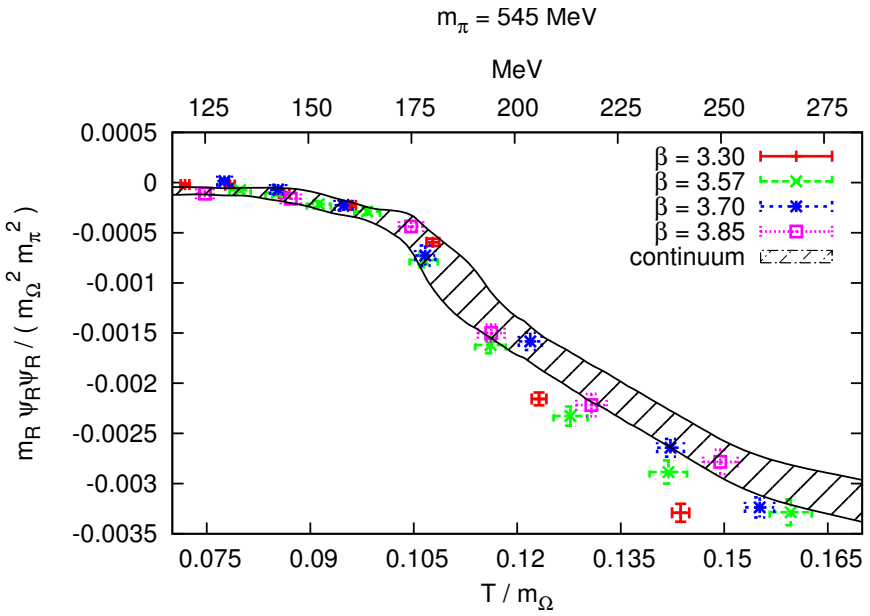
where

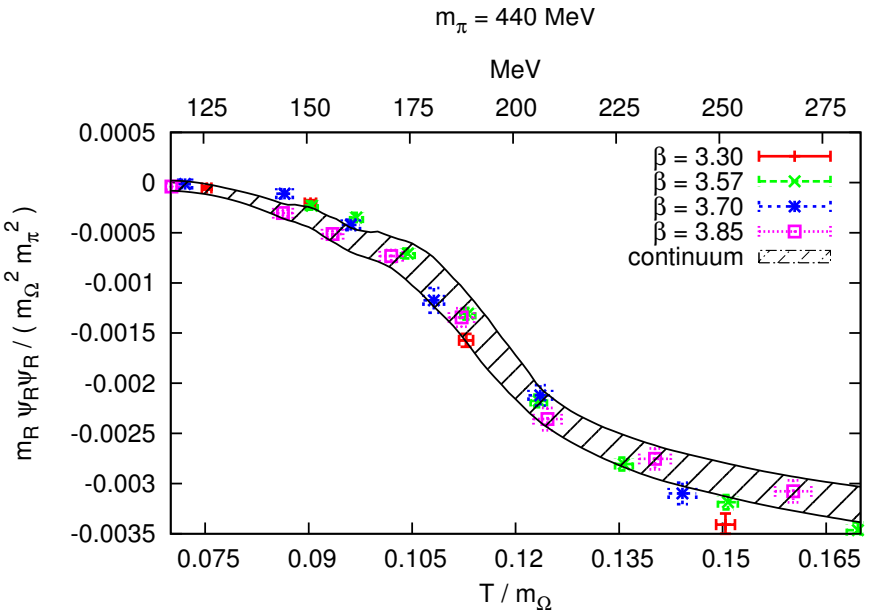
$$\Delta_{PP}(T) = \int \mathrm{d}^4x\,\langle P_0(x)P_0(0)\rangle(T) - \int \mathrm{d}^4x\,\langle P_0(x)P_0(0)\rangle(T=0).$$

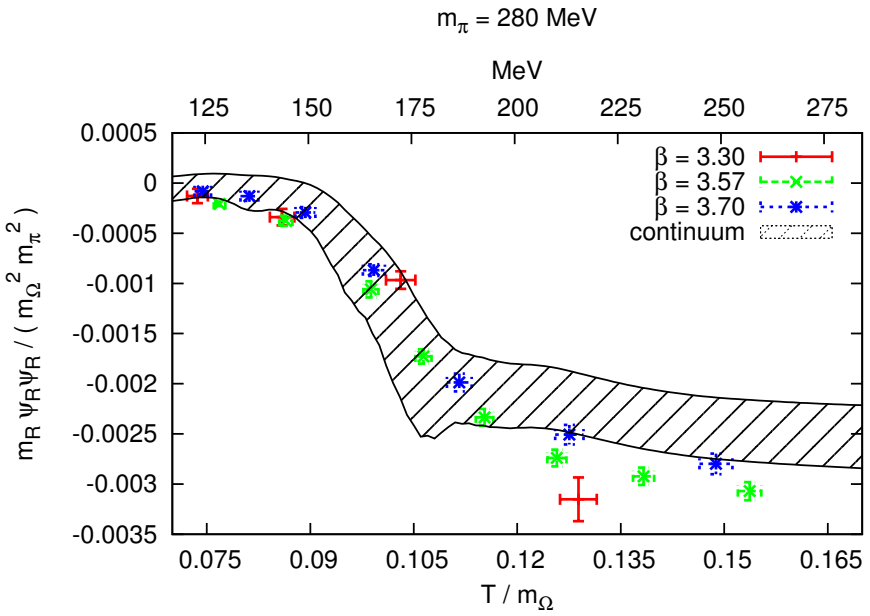
To avoid a strong pion mass dependence the following dimensionless combination is convenient when comparing different pion masses:

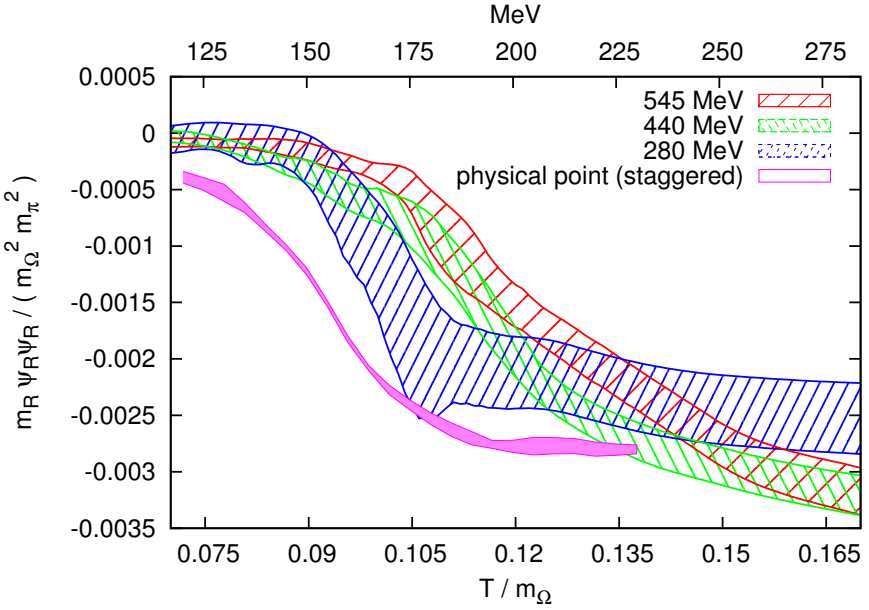
$$m_{\mathrm{R}}\langle\bar{\psi}\psi\rangle_{\mathrm{R}}(T)/m_\pi^2/m_\Omega^2.$$

Continuum limit of the chiral condensate









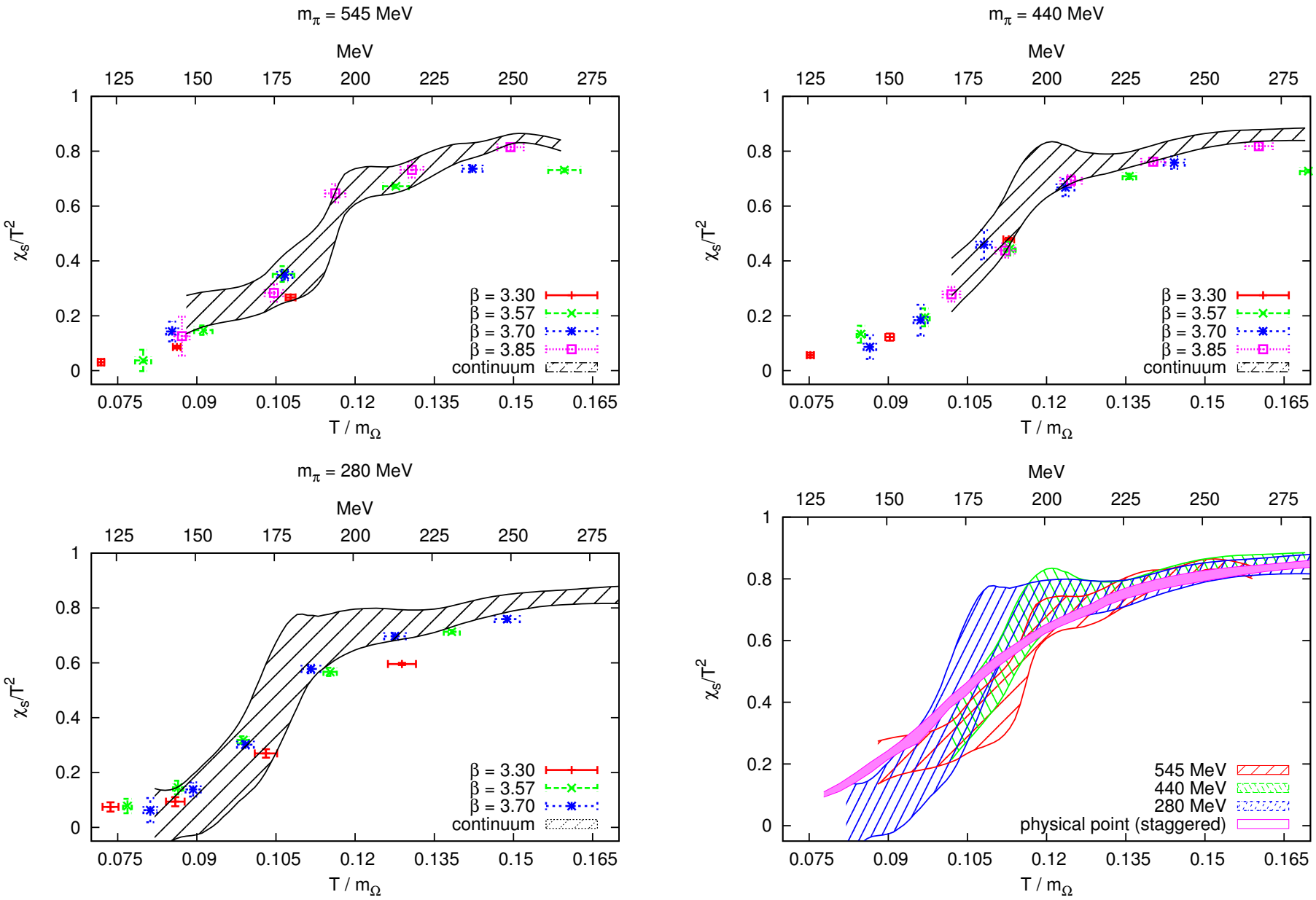
Strange quark number susceptibility

$$\chi_s = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_s^2}\bigg|_{\mu_s=0}$$

- It characterizes strangeness fluctuation.
- There is no need for renormalization, the continuum limit is straightforward.
- Tree level improved $\frac{\chi_s}{T^2}$ dimensionless combination.

  [S. Borsanyi *et al.* 1205.0440]

Cont. limit of the strange quark number susceptibility



Polyakov loop

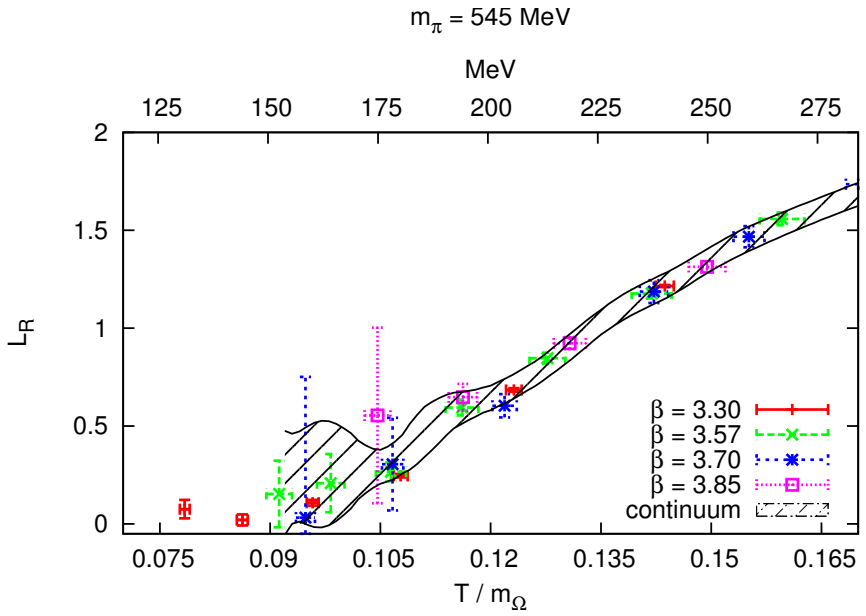
Multiplicative divergence has to be removed.

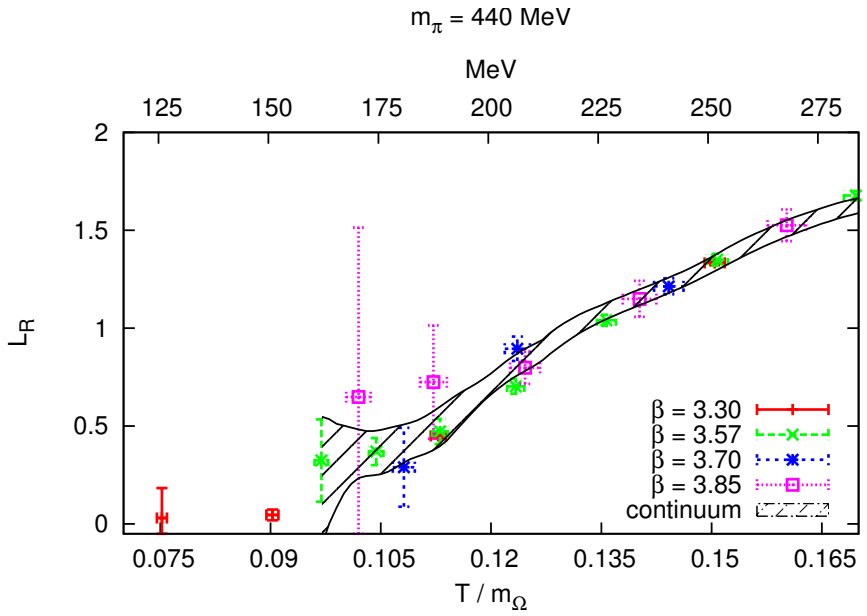
- A value L_* can be fixed for the renormalized Polyakov loop at a fixed but arbitrary temperature $T_* > T_\mathrm{c}$:

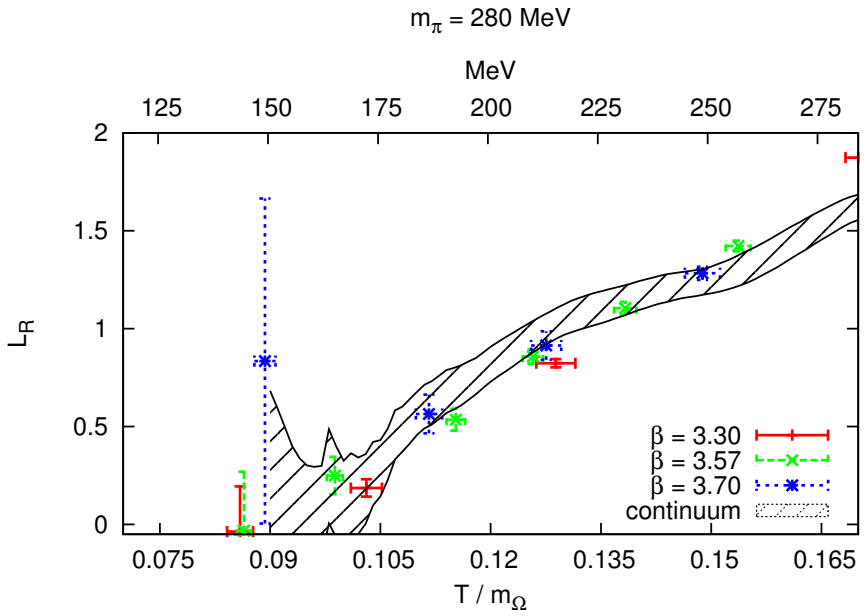
$$L_\mathrm{R}(T) = \left(\frac{L_*}{L_0(T_*)}\right)^{\frac{T_*}{T}} L_0(T)$$

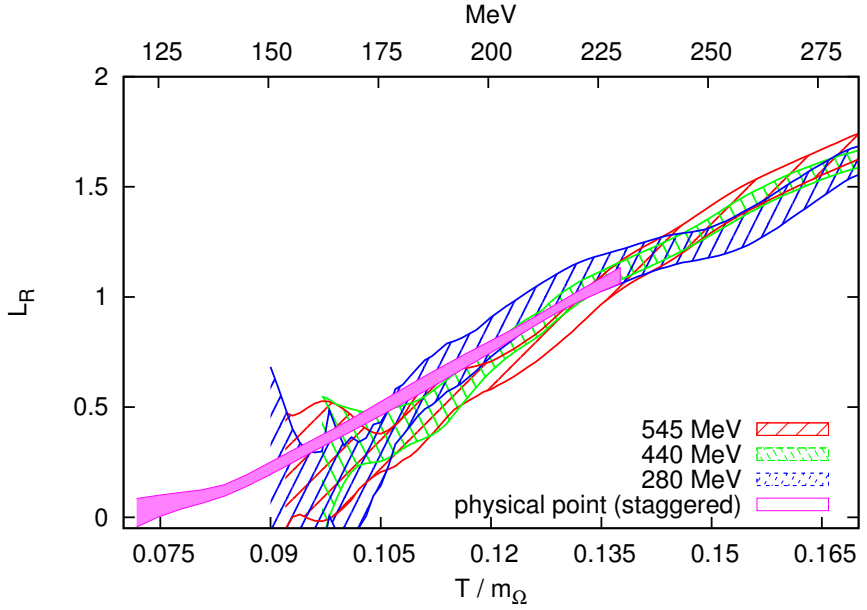
We choose $T_* = 0.143\, m_\Omega$ and $L_* = 1.2$

Continuum limit of the Polyakov loop









Summary

- We investigated the pion mass dependence of several observables which may be used to define a pseudo-critical temperature.
- Continuum extrapolation was fully under control for $m_\pi = 545\,\mathrm{MeV}$ and $440\,\mathrm{MeV}$, only continuum estimates for $m_\pi = 280\,\mathrm{MeV}$ (except for the Polyakov loop).
- Chiral condensate shows much stronger pion mass dependence than the other two observables.
- Smaller (close to physical) pion mass is needed for a full result at the physical point.