Nucleon observables and axial charges of other baryons

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with

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The XXXII International Symposium on Lattice Field Theory
New York, 23 -28 June 2014

Outline

Wilson twisted mass lattice QCD

- $N_f = 2$: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$
  Change of variables: $\psi = \frac{1}{\sqrt{2}}[\mathbf{1} + i\tau^3\gamma_5]\chi \qquad \bar{\psi} = \bar{\chi}\frac{1}{\sqrt{2}}[\mathbf{1} + i\tau^3\gamma_5]$
  $\Rightarrow$ mass term: $\bar{\psi}m\psi = \bar{\chi}i\gamma_5\tau^3 m\chi$

$$S = S_g + a^4 \sum_x \bar{\chi}(x)\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu + m_{\text{crit}} + i\gamma_5\tau^3\mu\right]\chi(x)$$

  Simulations by ETMC: Ph. Boucaud *et al.*, Comput.Phys.Commun. 179 (2008) 695; Phys.Lett. B650 (2007) 304

- $N_f = 2 + 1 + 1$

$$S_h = \sum_x \bar{\chi}_h(x)\left[D_W + m_{(0,h)} + i\gamma_5\tau^1\mu_\sigma + \tau^3\mu_\delta\right]\chi_h(x)$$

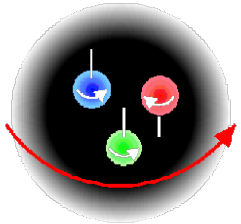
  Simulations by ETMC: R. Baron *et al.*, JHEP 1008 (2010) 097

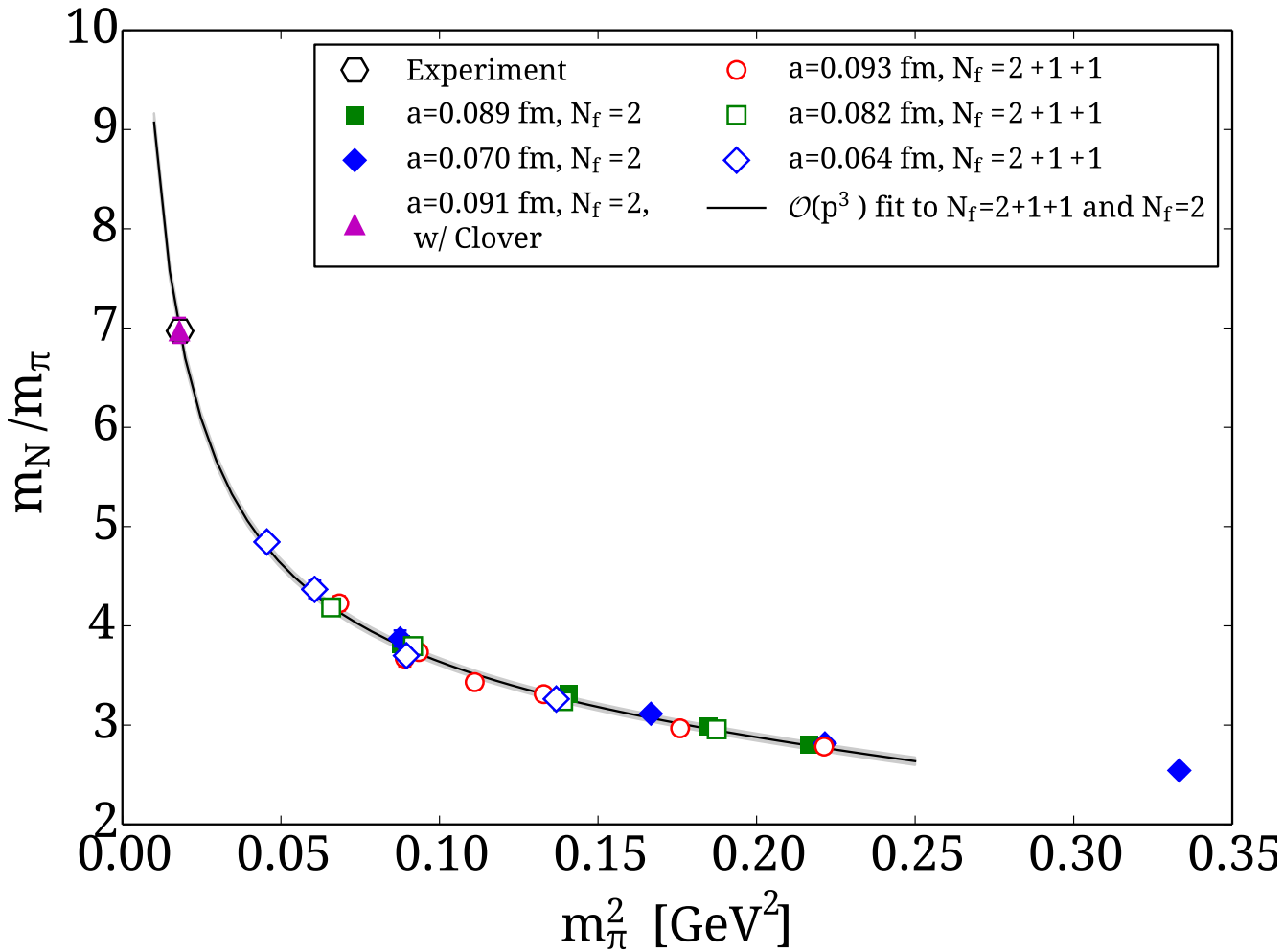
- $N_f = 2$ twisted mass plus clover
  $\rightarrow$ a good formulation for simulations at the physical point, see talk by B. Kostrzewa, (ETMC) A. Abdel-Rehim *et al.*, arXiv:1311.4522
  $\rightarrow$ preliminary results at physical point, see plenary talk by M. Constantinou, (ETMC) C. Alexandrou *et al.*, PoS LATTICE2013 (2013) 292

Wilson tmQCD at maximal twist, R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007

- Automatic $O(a)$ improvement
- No operator improvement needed, renormalization simplified $\rightarrow$ important for hadron structure

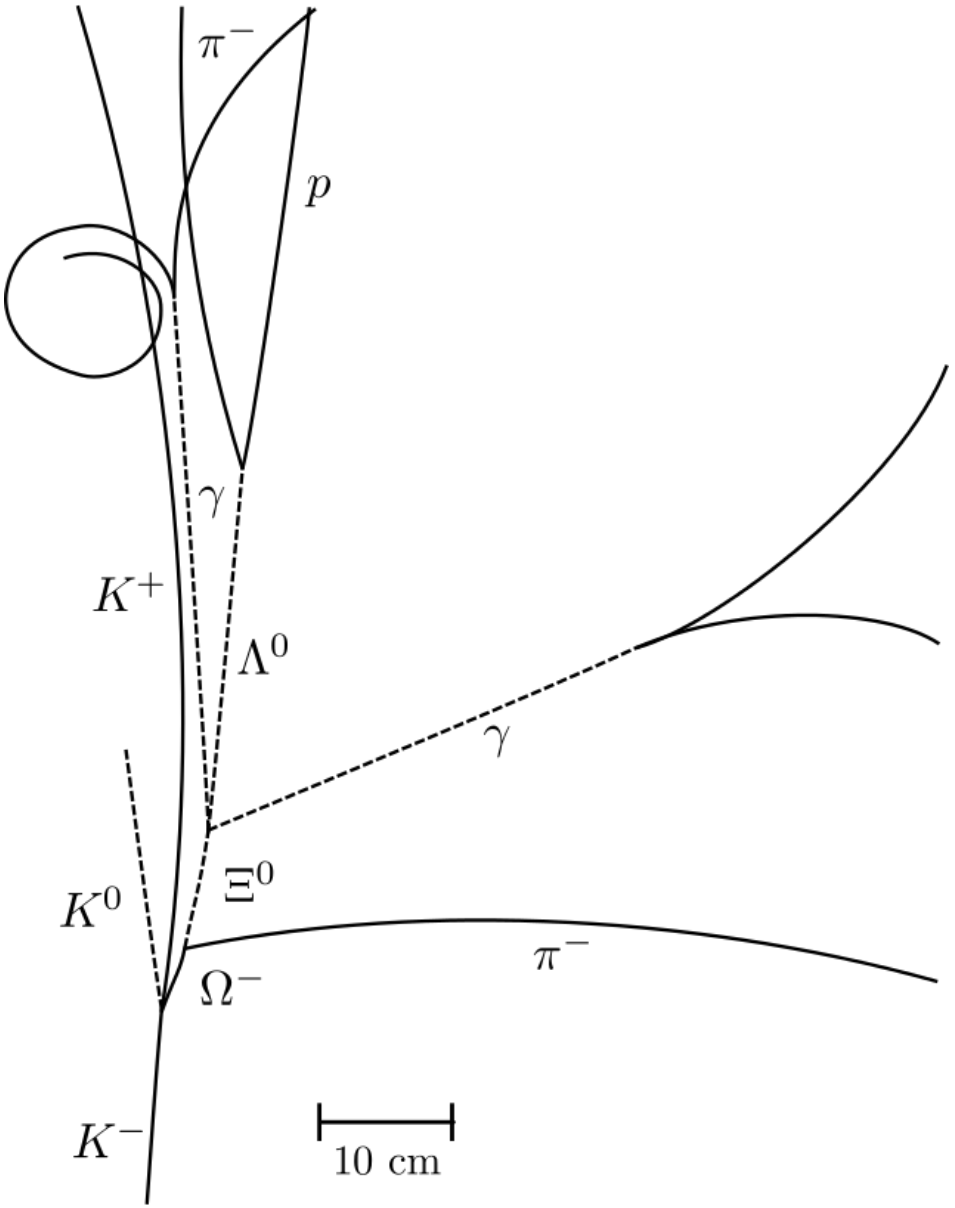
The nucleon



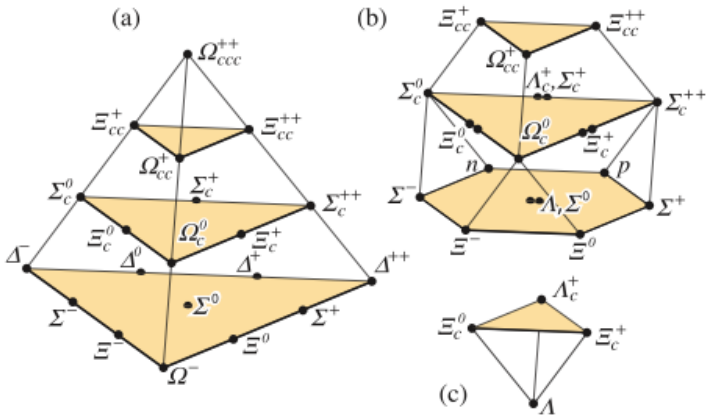


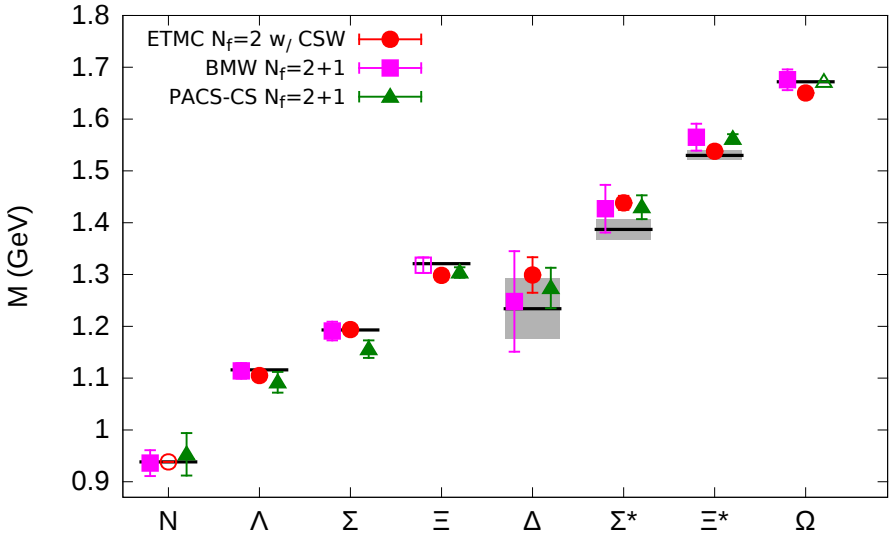
- Cut-off effects small for these lattice spacings
- LO fit with $m_\pi < 375$ MeV does not include the physical point
- Complete agreement with experimental value
- Determine lattice spacing using the $\mathcal{O}(p^3)$ result, see talk by Ch. Kallidonis
  $\rightarrow \sigma$-term from m_N using $\mathcal{O}(p^3)$: $\sigma_{\pi N} = 65(2)(20)$ MeV and $r_0 \sim 0.479(4)$ fm in the continuum limit

Hadron spectrum



Ω and Ξ discovery, Brookhaven 1964: V. E. Barnes et al., Phys. Rev. Lett. 12, 204 (1964)

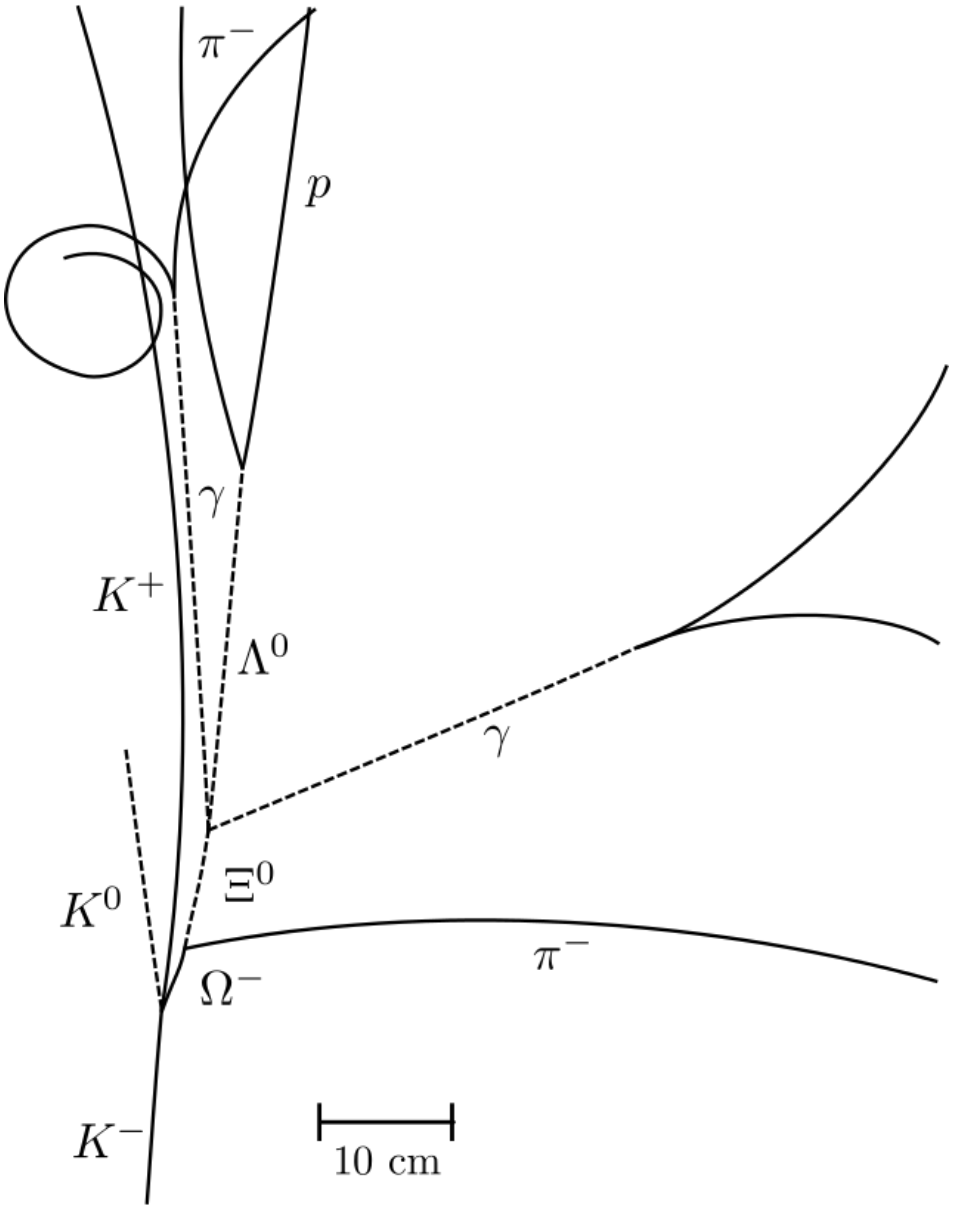




Results by ETMC using simulations at the physical pion mass

Interpolating fields the same as those used for the $N_f = 2+1+1$ ensembles; see talk by Ch. Kallidonis

Hadron spectrum



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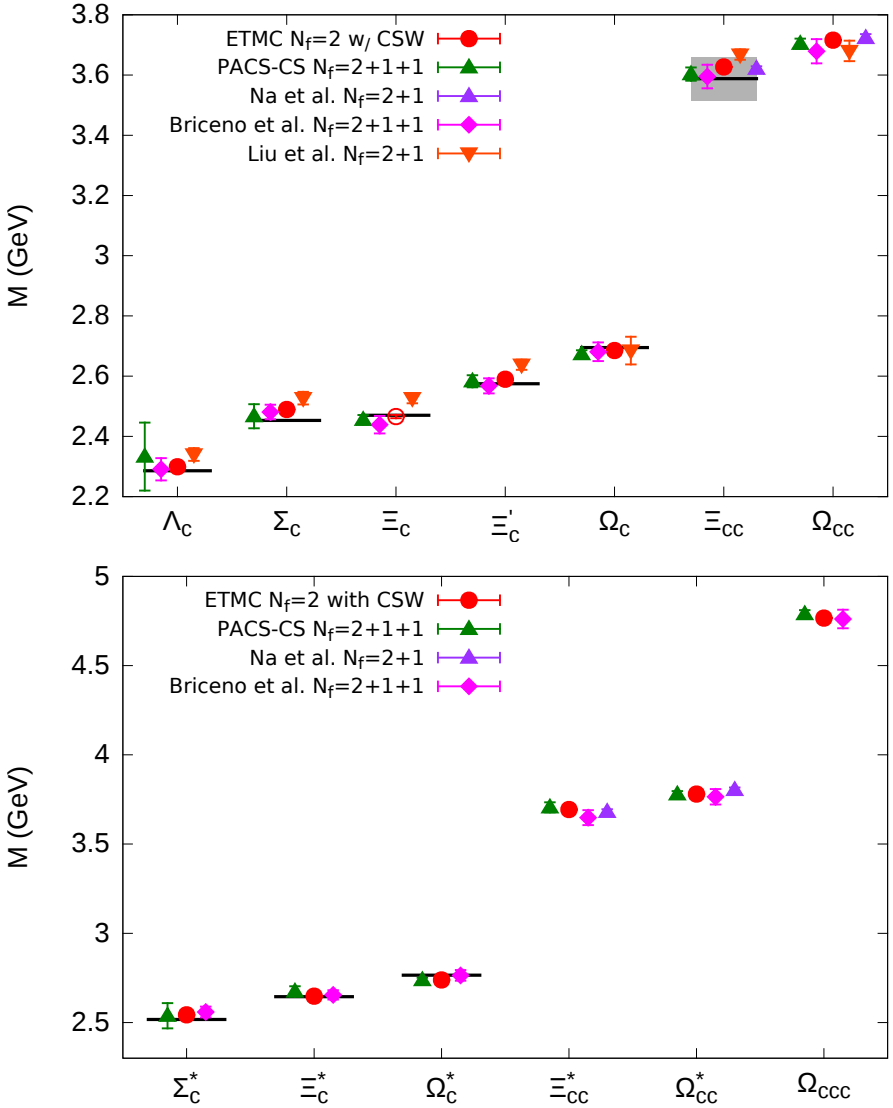


Results by ETMC using simulations at the physical pion mass

Interpolating fields the same as those used for the $N_f = 2+1+1$ ensembles; see talk by Ch. Kallidonis

Nucleon structure

- Axial charge
- $\langle x \rangle_q$

- Scalar charge
- $\langle x \rangle_{\Delta q}$

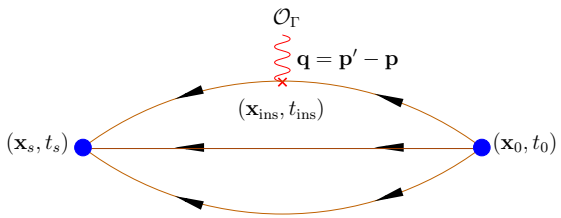
- Tensor charge
- $\langle x \rangle_{\delta q}$

- $N_f = 2 + 1 + 1$ twisted mass, $32^3 \times 64$, a=0.082 fm, $m_\pi = 373$ MeV-high statistics analysis including disconnected contributions, 7 sink-source time separations ranging from 0.5 fm to 1.5 fm
- $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, a = 0.091 fm, m_π = 134 MeV, ~ 1000 confs, 4 sink-source time separations ranging from 0.9 fm to 1.5 fm

- Ground state dominance
  - nucleon axial charge g_A, tensor charge - weak, S. Dinter, C.A., M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076
  - momentum fraction $\langle x \rangle_{u-d}$, electromagnetic form factors, see talk by G. Koutsou - intermediate
  - scalar charge (equivalently σ-terms) - severe
- Disconnected contributions - see talk by A. Vaquero
  - scalar charge, axial charge - need to be taken into account C. Alexandrou *et al.*, arXiv:1309.2256; A. A. Rehim *et al.*, arXiv:1310.6339
  - small for EM form factors

Extracting nucleon matrix elements

Form ratio by dividing the three-point correlator by an appropriate combination of two-point functions:

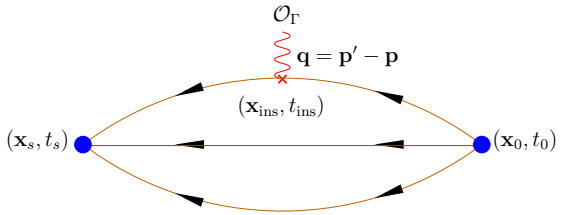


- Plateau method:

$$R(t_s, t_{\rm ins}, t_0) \xrightarrow[(t_s - t_{\rm ins})\Delta \gg 1]{(t_{\rm ins} - t_0)\Delta \gg 1} \mathcal{M}[1 + \ldots e^{-\Delta(\mathbf{p})(t_{\rm ins} - t_0)} + \ldots e^{-\Delta(\mathbf{p}')(t_s - t_{\rm ins})}]$$

  - $\mathcal{M}$ the desired matrix element
  - $t_s, t_{\rm ins}, t_0$ the sink, insertion and source time-slices
  - $\Delta(\mathbf{p})$ the energy gap with the first excited state

Extracting nucleon matrix elements



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- Plateau method:

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  - $\mathcal{M}$ the desired matrix element
  - $t_s, t_{\rm ins}, t_0$ the sink, insertion and source time-slices
  - $\Delta(\mathbf{p})$ the energy gap with the first excited state

- Summation method: Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s, t_{\rm ins}, t_0) = \text{Const.} + \mathcal{M}[(t_s - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_s - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s - t_0)})].$$

  - Excited state contributions are suppressed by exponentials decaying with $t_s - t_0$, rather than $t_s - t_{\rm ins}$ and/or $t_{\rm ins} - t_0$
  - Also works if one does not include t_0 and t_s in the sum $\rightarrow$ used for the results shown here
  - However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant

L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

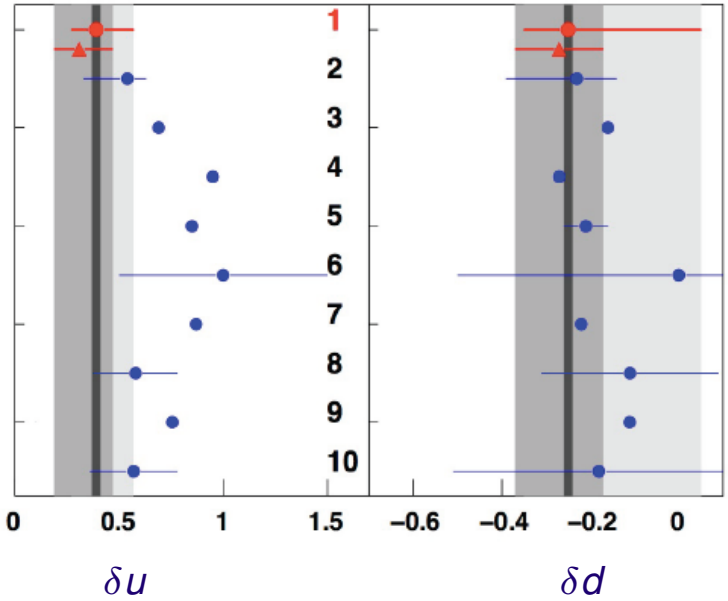
All should yield the same answer in the end of the day!

Nucleon charges: g_A, g_s, g_T

- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
- tensor operator: $\mathcal{O}_T^a = \bar{\psi}(x)\sigma^{\mu\nu}\frac{\tau^a}{2}\psi(x)$

$\Longrightarrow$ extract from ratio: $\langle N(\vec{p'})\mathcal{O}_X N(\vec{p})\rangle|_{q^2=0}$ to obtain $g_s,\ g_A,\ g_T$
(i) isovector combination has no disconnect contributions; (ii) g_A well known experimentally, g_T to be measured at JLab

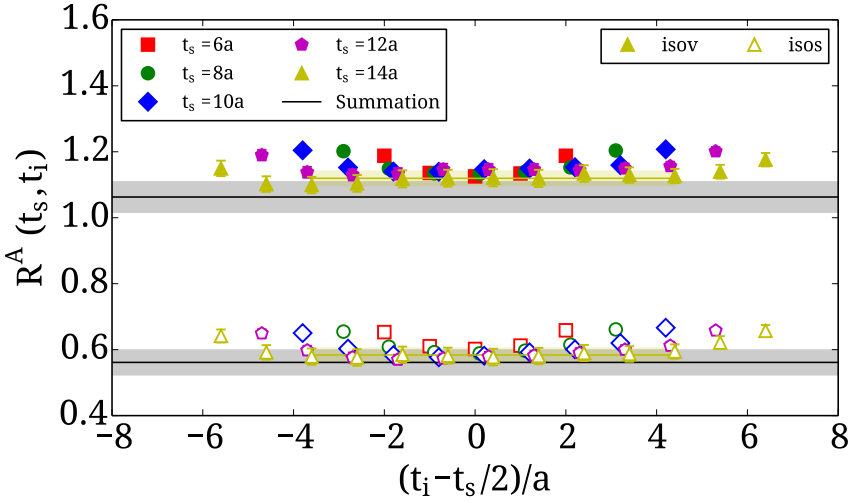
Planned experiment at JLab, SIDIS on ^{3}He/Proton at 11 GeV:

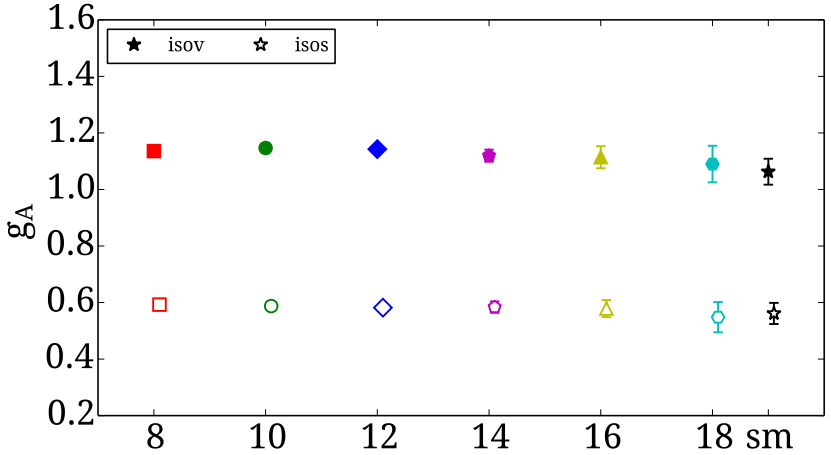


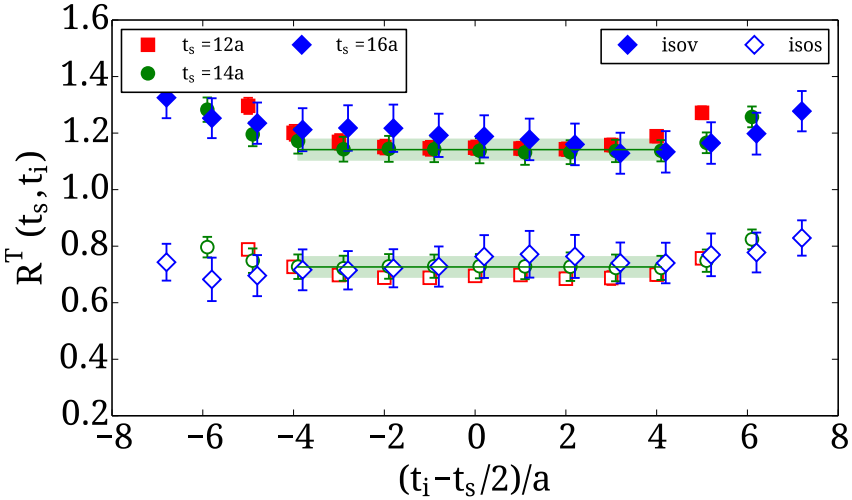
Experimental values: $\delta u = 0.39^{+0.18}_{-0.12}$ and $\delta d = -0.25^{+0.3}_{-0.1}$

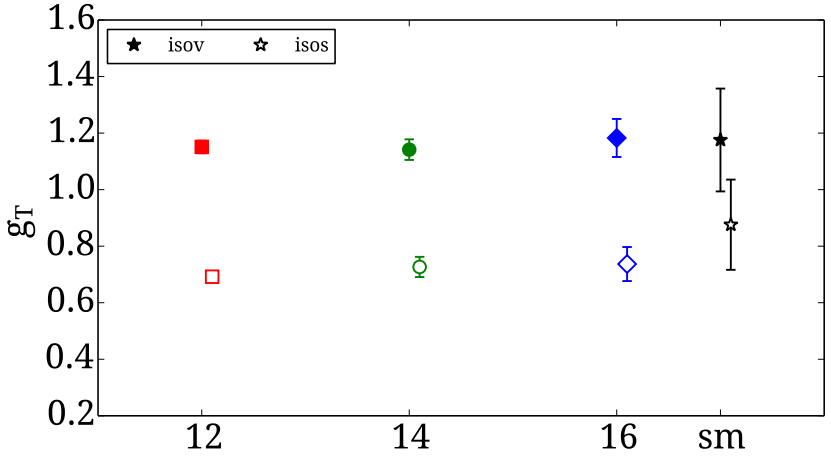
Nucleon charges: g_A, g_s, g_T

- High statistics analysis with $N_f = 2+1+1$ TMF, $a = 0.082$ fm, $m_\pi = 373$ MeV
- Connected part with 1200 statistics









Nucleon charges: g_A, g_s, g_T

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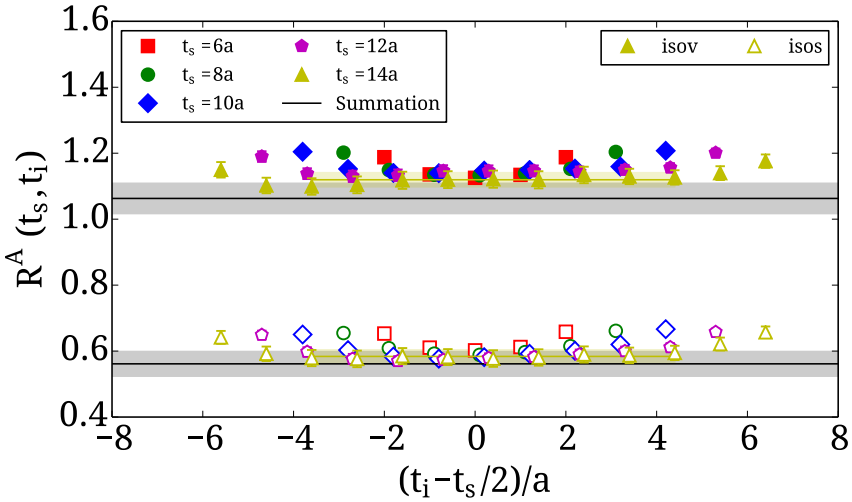
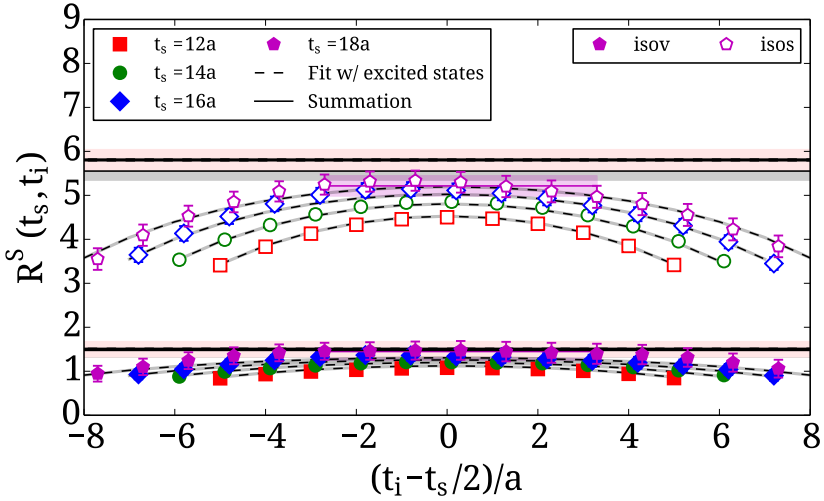
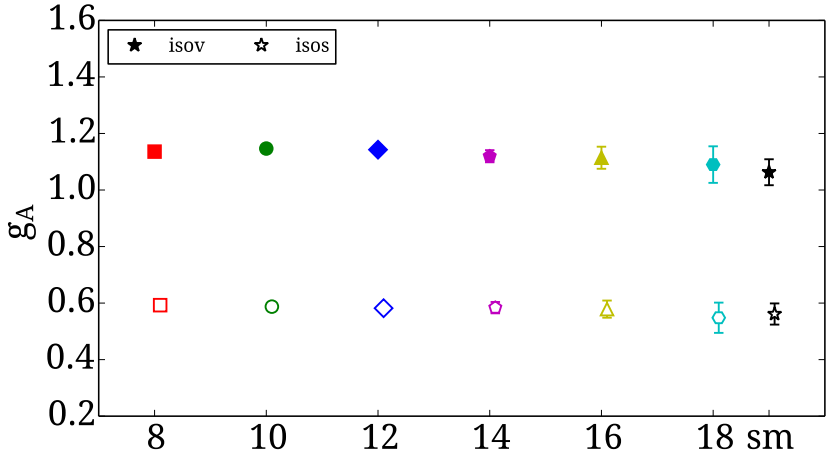
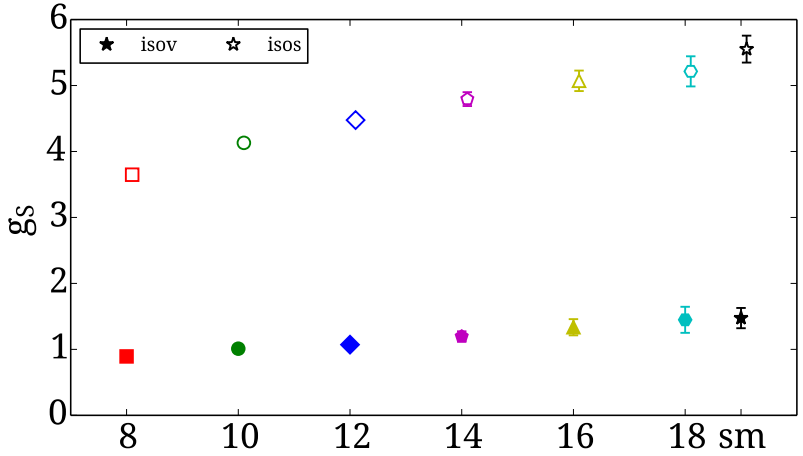




Agreement of summation, plateau and two-states fits give confidence to the correctness of the final result

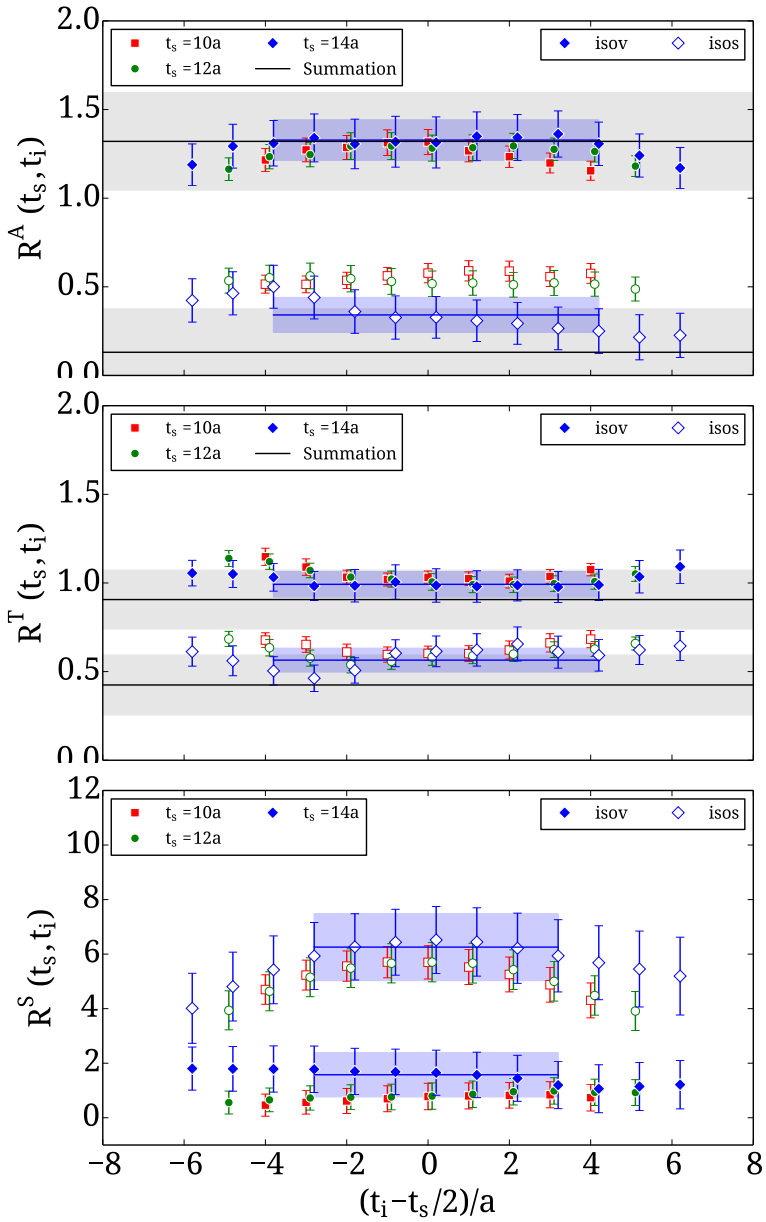




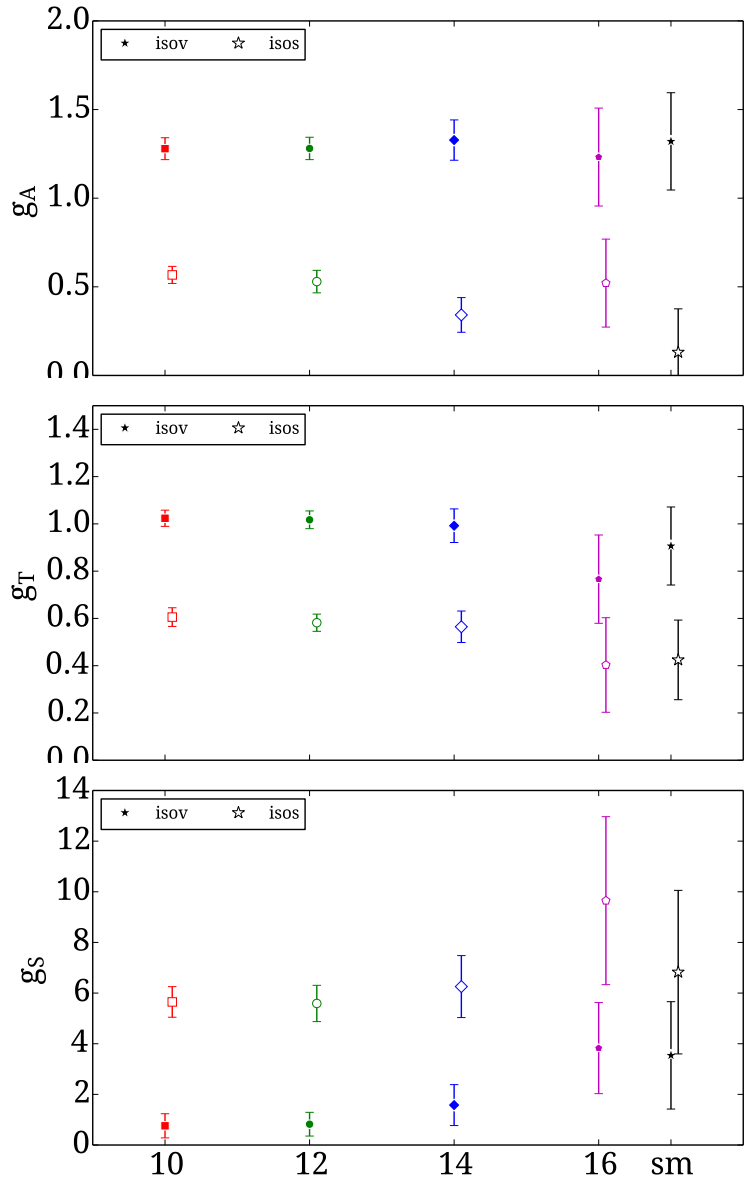
- g_A: No detectable excited states
- g_T: similar to g_A
- g_s: severe contamination from excited states

Nucleon charges: g_A, g_s, g_T

- $N_f = 2$ TMF with clover term $a = 0.091$ fm with $m_\pi = 134$ MeV; Connected part with 1018 statistics

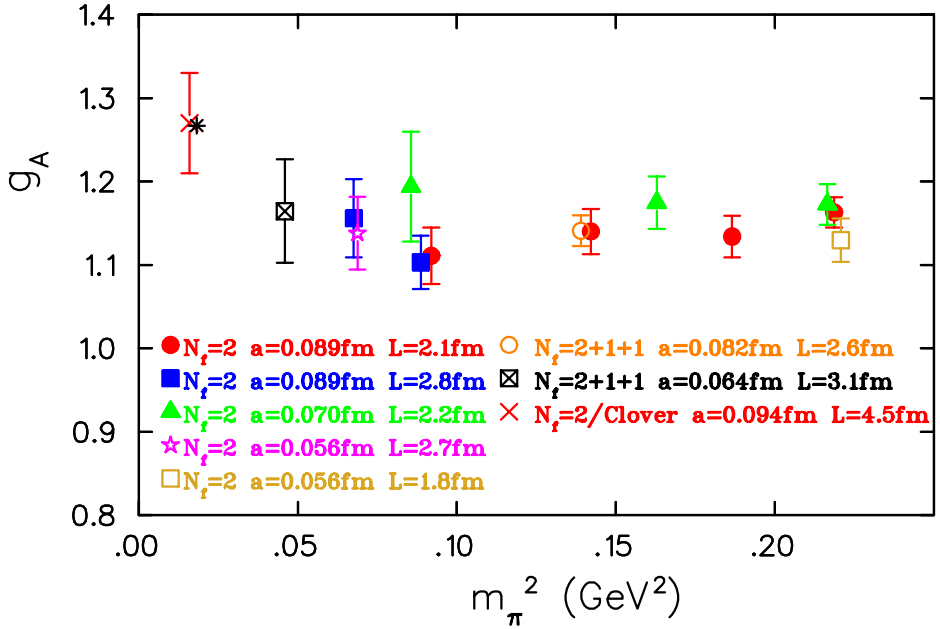


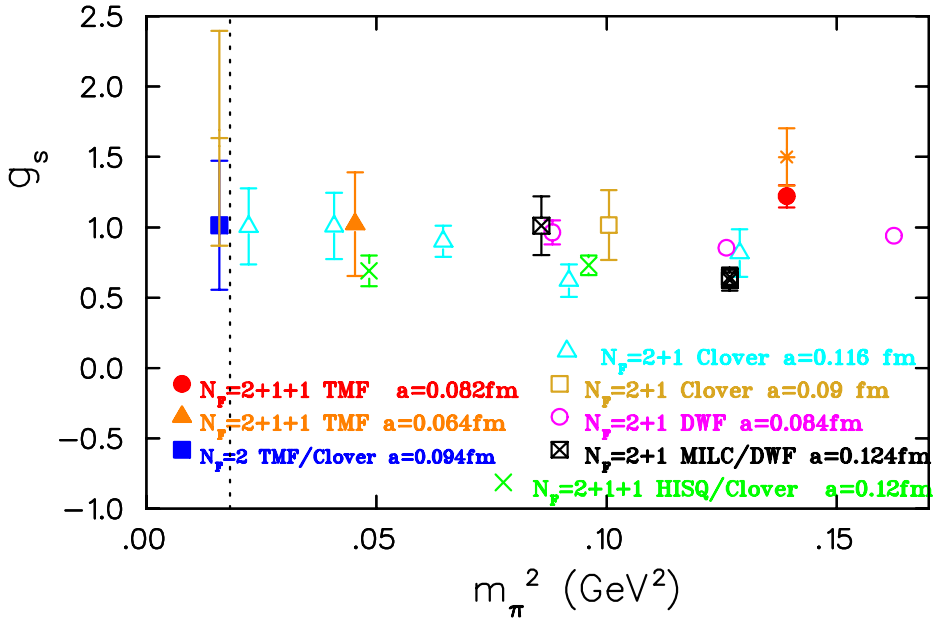
Need more statistics

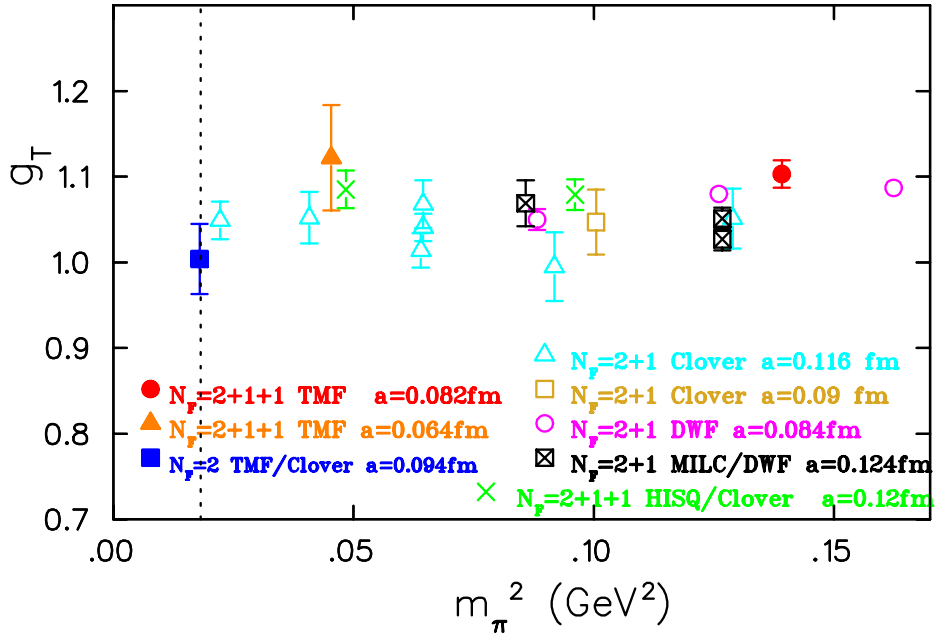


Summary of results on nucleon charges: g_A, g_s, g_T

Isovector







- g_A at the physical point mass indicates agreement with the physical value → important to reduce error - many results from other collaborations, see plenary by M. Constantinou
- Experimental value of $g_T \sim 0.54^{+0.30}_{-0.13}$ from global analysis of HERMES, COMPASS and Belle e^+e^- data, M. Anselmino *et al.* (2013)
- For g_s increasing the sink-source time separation to ~ 1.5 fm is crucial

Isoscalar nucleon charges: g_A, g_s, g_T

- scalar operator: $\mathcal{O}_S^a = \bar{\psi}(x)\frac{\tau^a}{2}\psi(x)$
- axial-vector operator: $\mathcal{O}_A^a = \bar{\psi}(x)\gamma^\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
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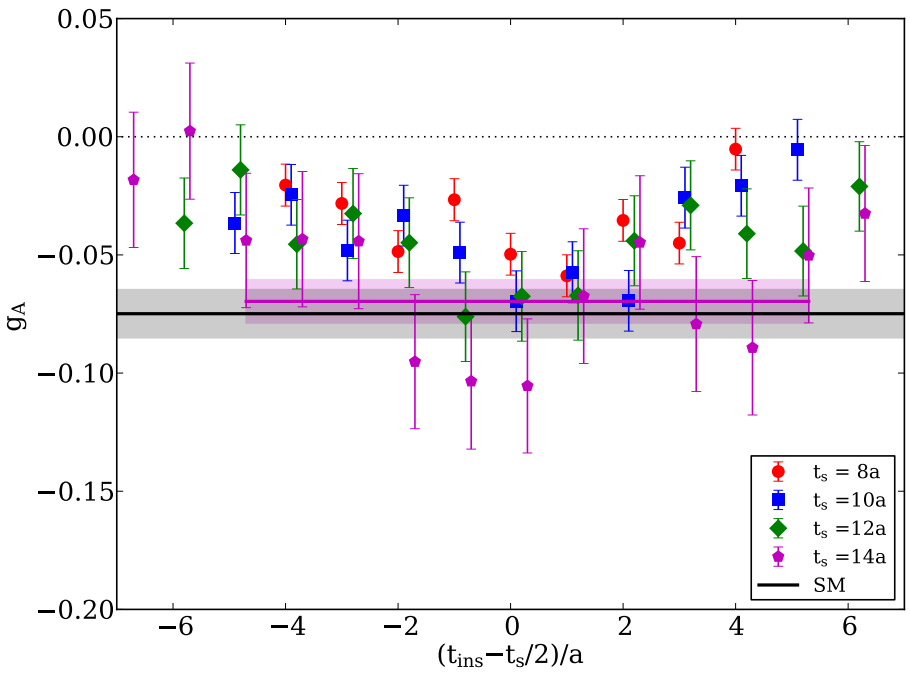
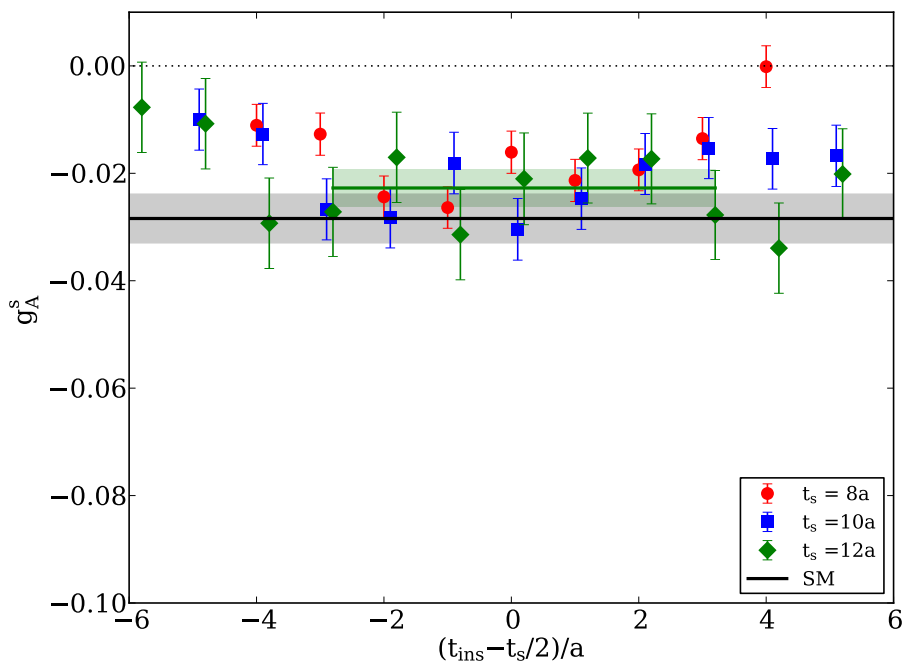
- $N_f = 2+1+1$ twisted mass, a = 0.082 fm, m_π = 373 MeV
- Disconnected part, $\sim$ 150 000 statistics using GPUs, see talk by A. Vaquero



Disconnected isoscalar, agrees with Bali *et al.* (QCDSF), Phys.Rev.Lett. 108 (2012) 222001

Analysis at the physical point still preliminary



Strange quark loop

Isoscalar nucleon charges: g_A, g_s, g_T

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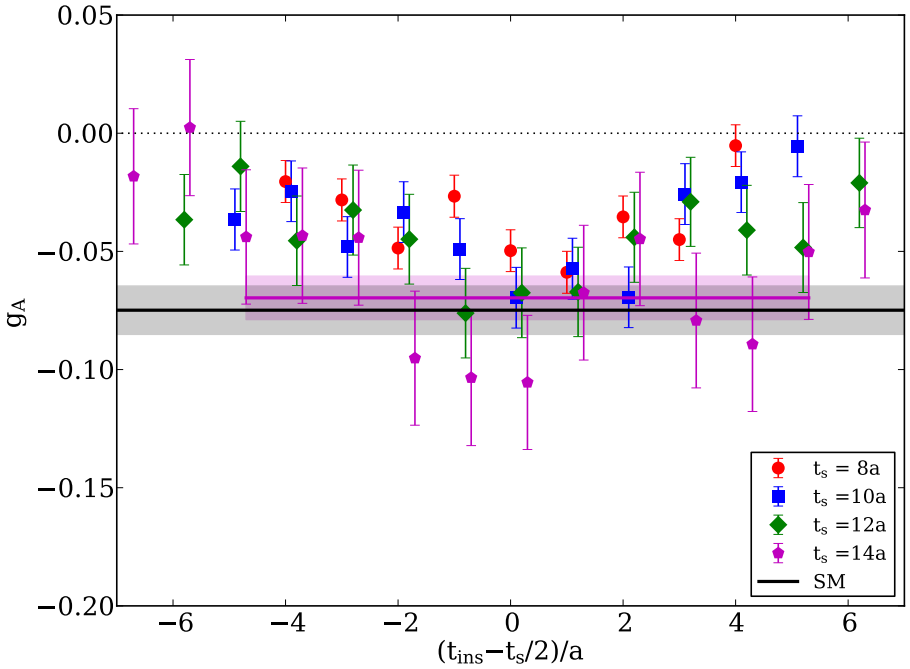

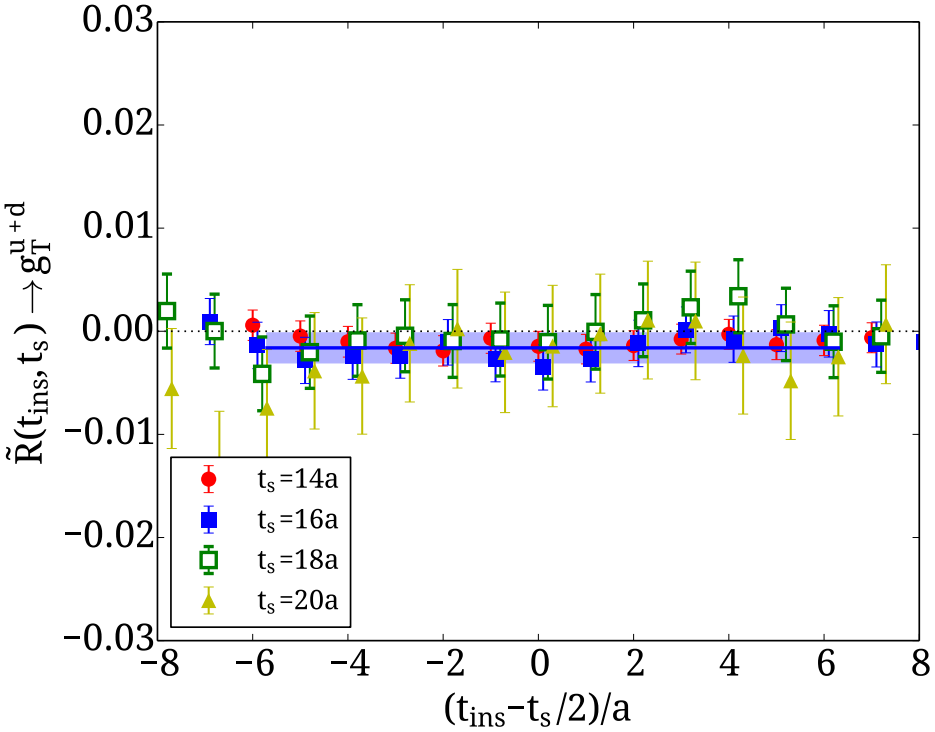

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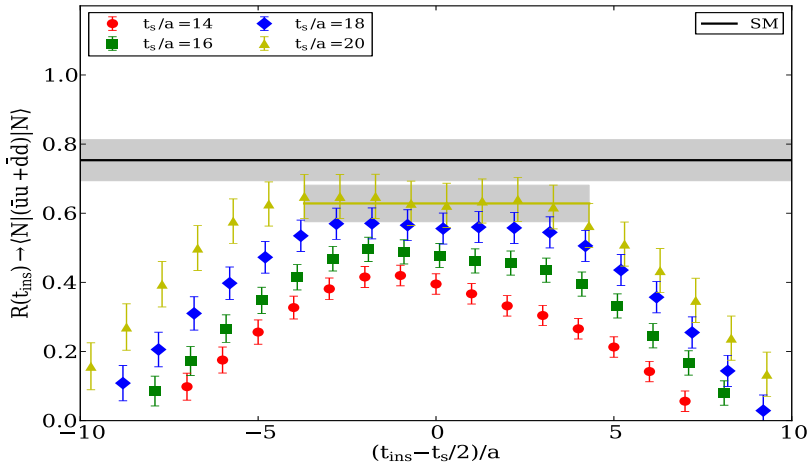
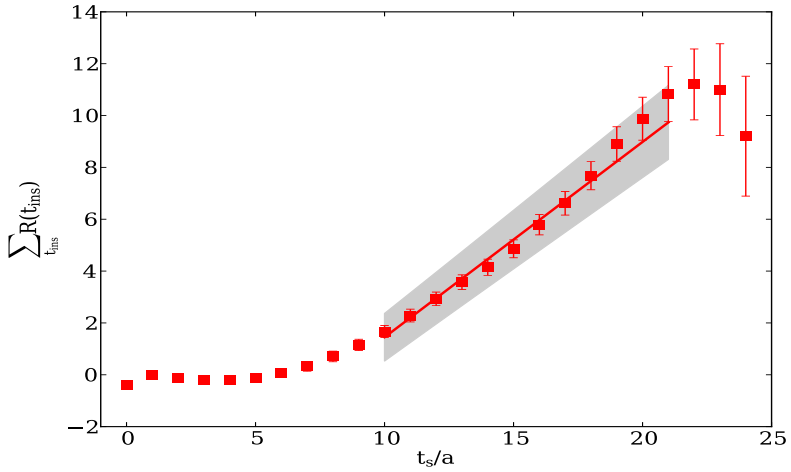
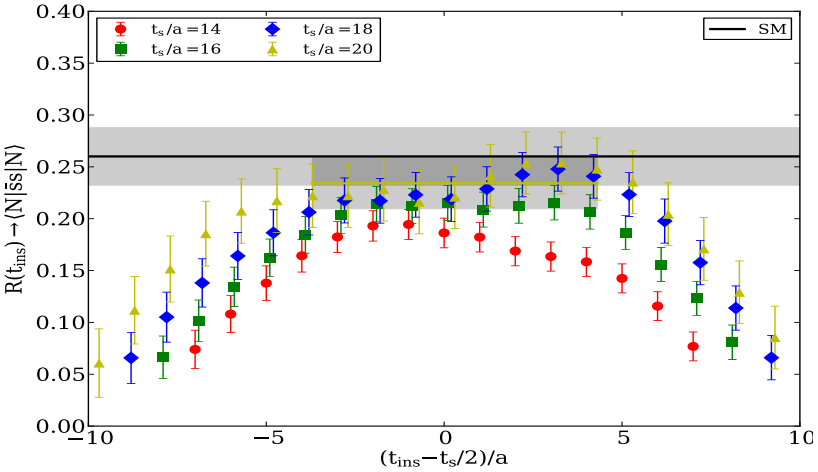
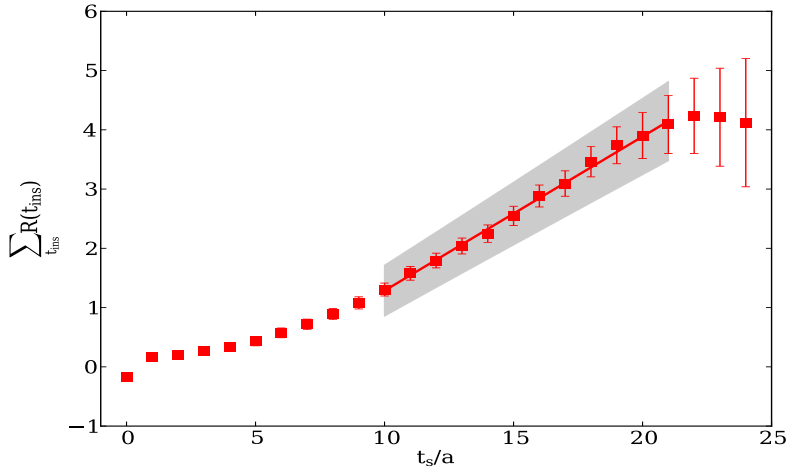






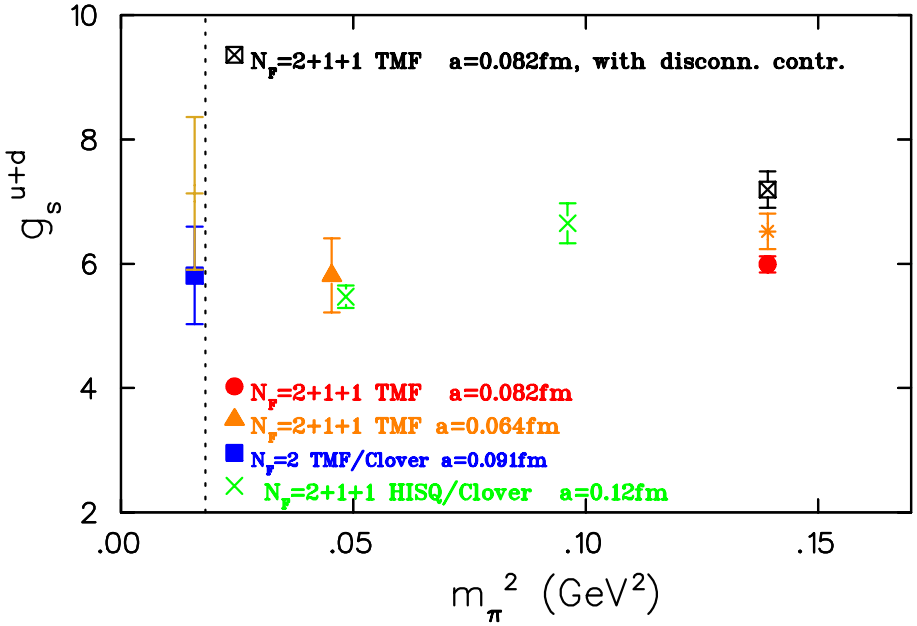


- Large contamination from excited states
- Compute perturbatively the difference between isovector and isoscalar renormalization constants at two-loop, see talk by H. Panagopoulos

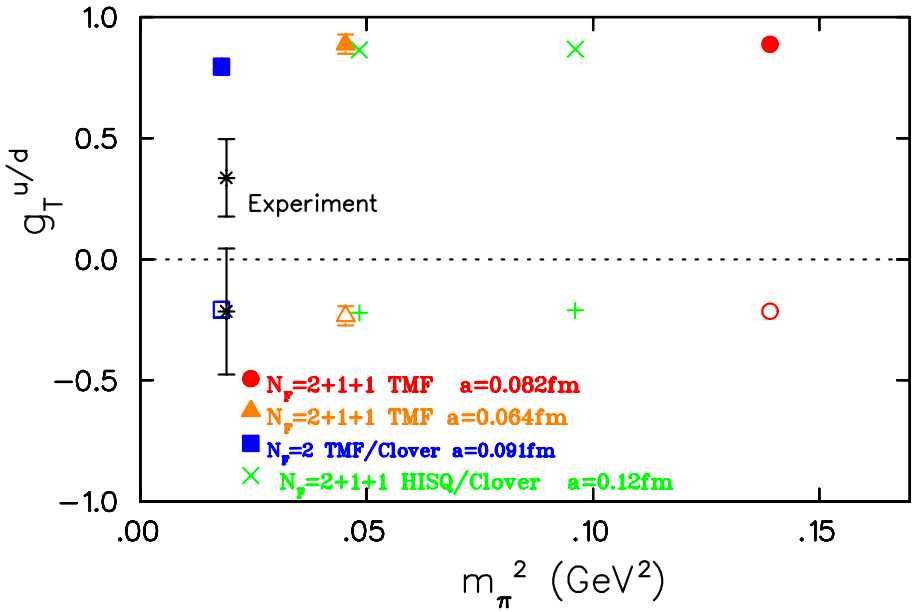
Isoscalar nucleon charges: g_A, g_s, g_T

- $N_f = 2 + 1 + 1$ twisted mass, a = 0.082 fm, m_π = 373 MeV
- Disconnected part, $\sim$ 150 000 statistics using GPUs,

Results shown in $\overline{MS}$ at 4 GeV^2
Analysis at the physical point still preliminary



Large source-sink separation and inclusion of disconnected is required

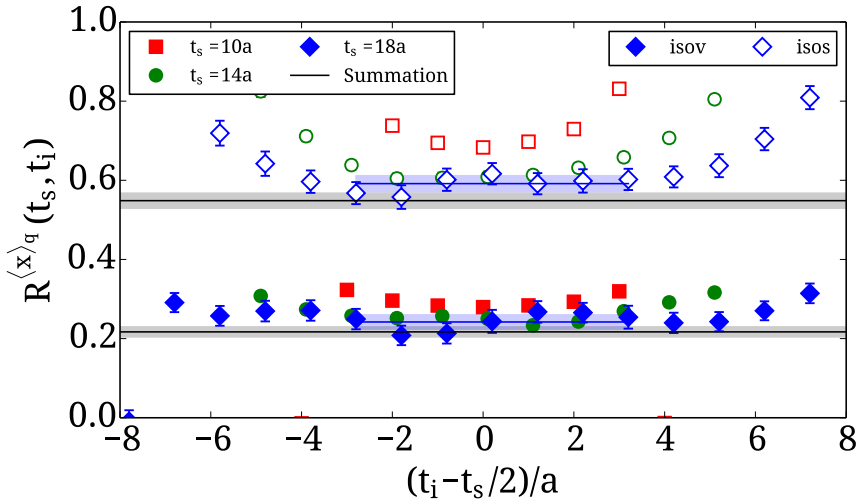


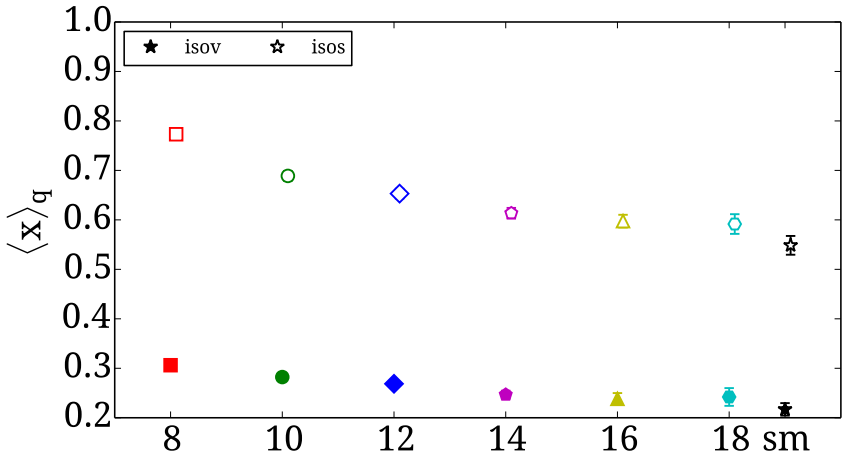
Experimental values from global analysis of HERMES, COMPASS and Belle e^+e^- data, M. Anselmino *et al.* (2013)

First moments of PDFs for the nucleon

- Unpolarized moment: $\langle x \rangle_q = \int_0^1 dx\, x\, [q(x) + \bar{q}(x)]$ $\quad q(x) = q(x)_\downarrow + q(x)_\uparrow$
- Helicity moment: $\langle x \rangle_{\Delta q} = \int_0^1 dx\, x\, [\Delta q(x) - \Delta\bar{q}(x)]$ $\quad \Delta q(x) = q(x)_\downarrow - q(x)_\uparrow$
- Transversity moment: $\langle x \rangle_{\delta q} = \int_0^1 dx x\, [\delta q(x) + \delta\bar{q}(x)]$, $\quad \delta q(x) = q(x)_\perp + q(x)_\top$

$N_f = 2 + 1 + 1$ twisted mass, a = 0.082 fm, m_π = 373 MeV, 1200 statistics; Isovector

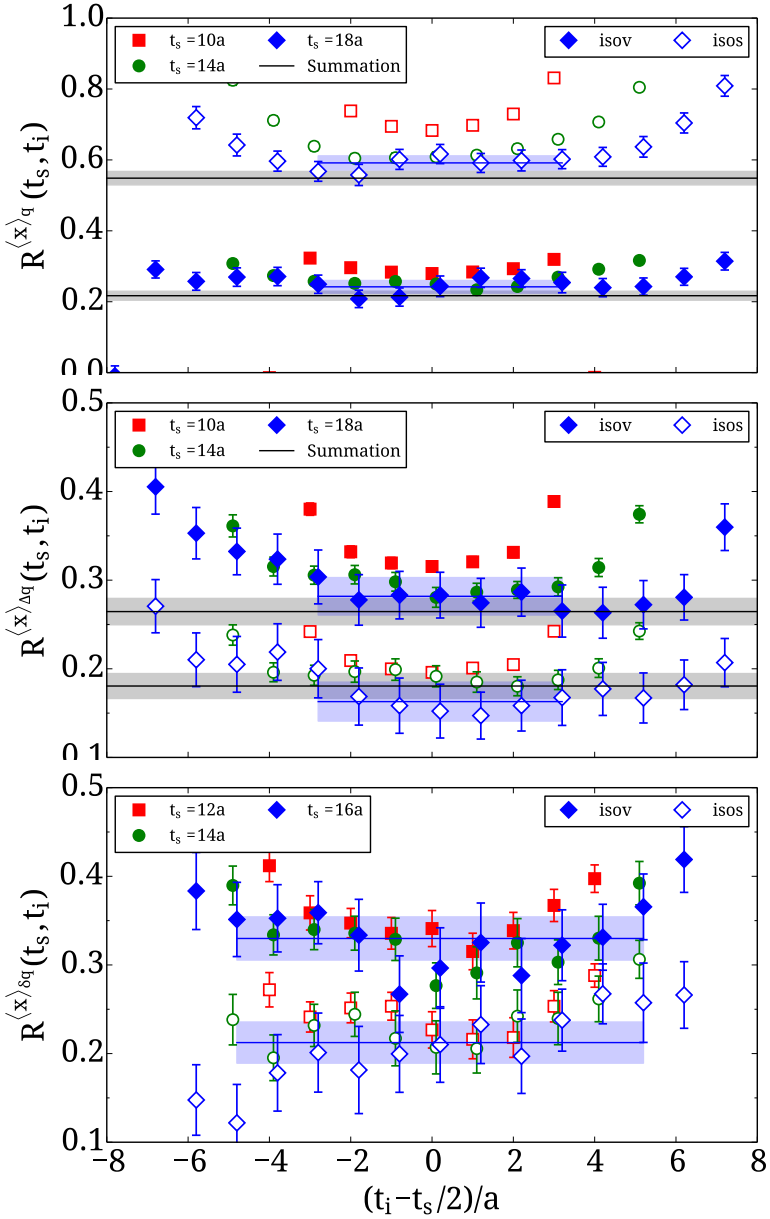


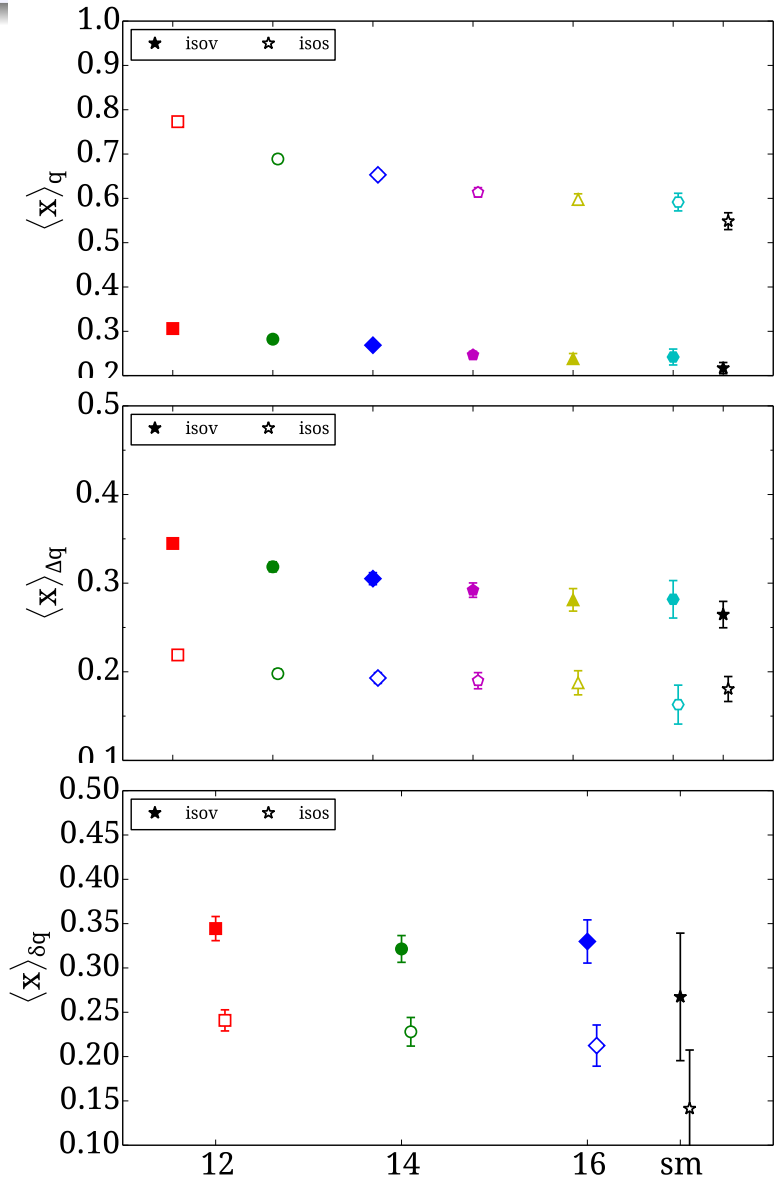


- Noticeable excited state contamination, especially for the iso-scalar
- For the plateau method one needs to show convergence by varying the sink-source time separation $\rightarrow$ also requires a number of sequential inversions $\Longrightarrow$ consistency of plateau and summation method gives confidence in the results

First moments of PDFs for the nucleon

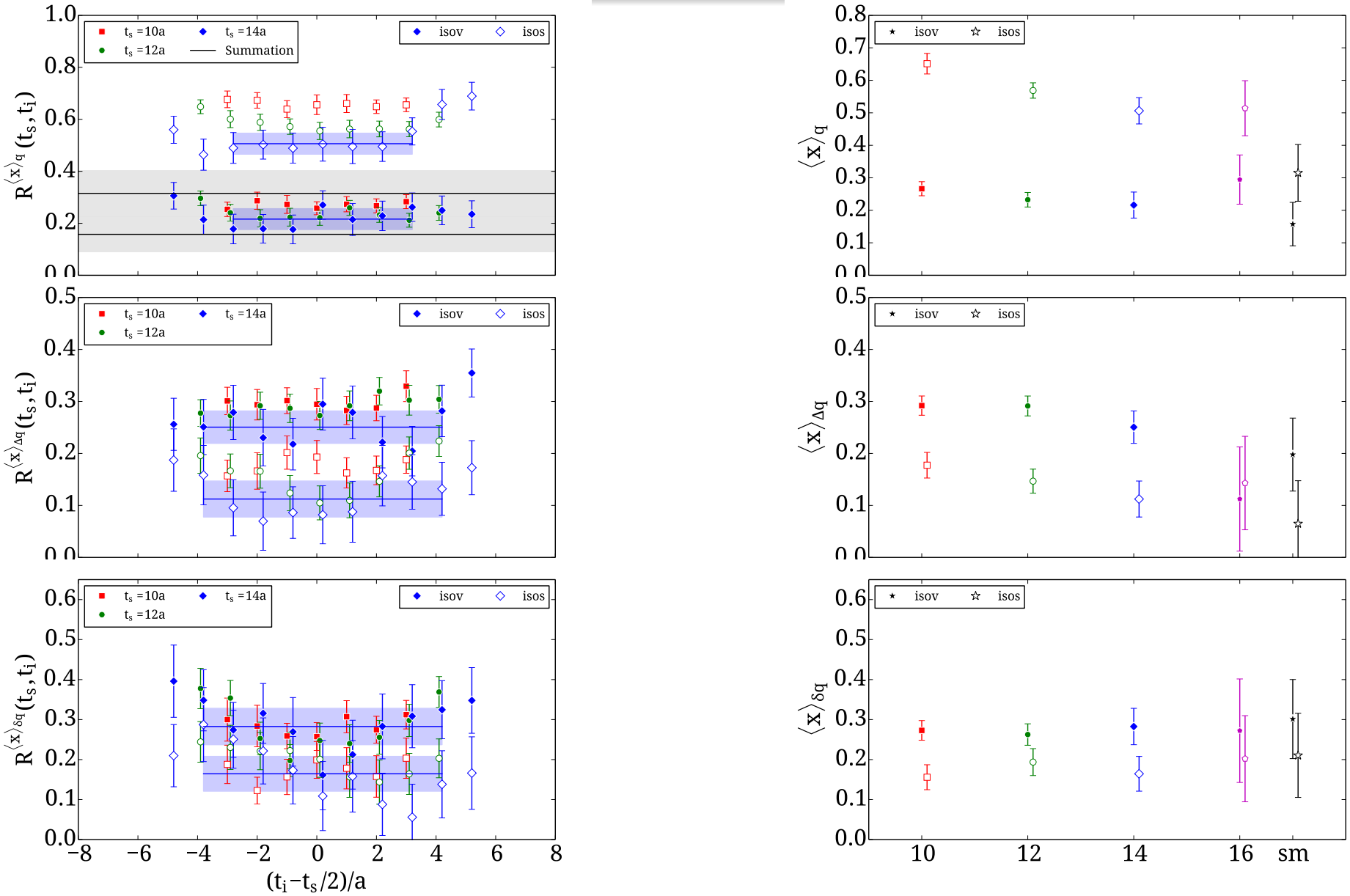
$N_f = 2 + 1 + 1$ twisted mass, a = 0.082 fm, m_π = 373 MeV, 1200 statistics; Isovector





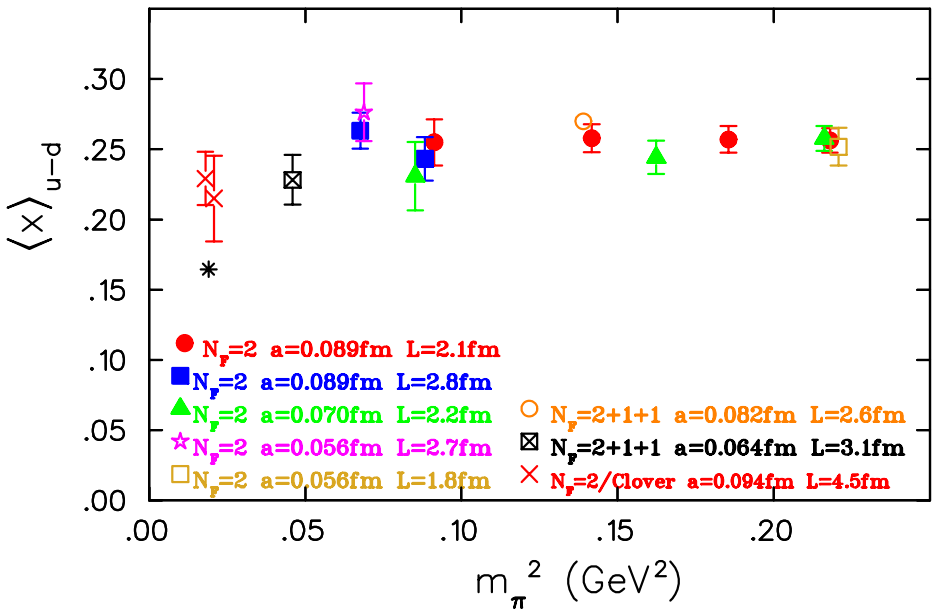
First moments of PDFs for the nucleon

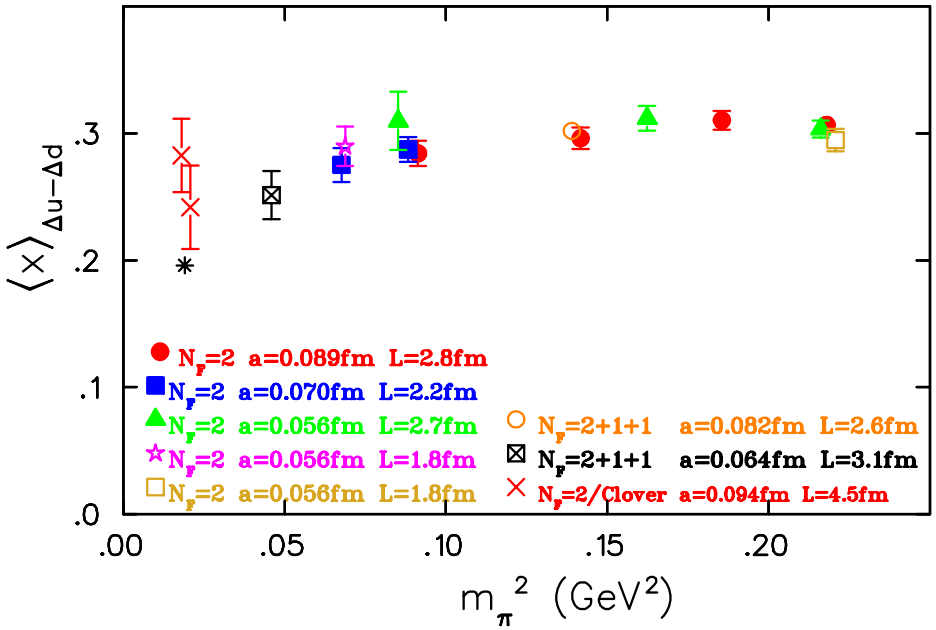
$N_f = 2$ TMF with a clover term , a = 0.091 fm, m_π = 134 MeV, 1018 statistics; Isovector

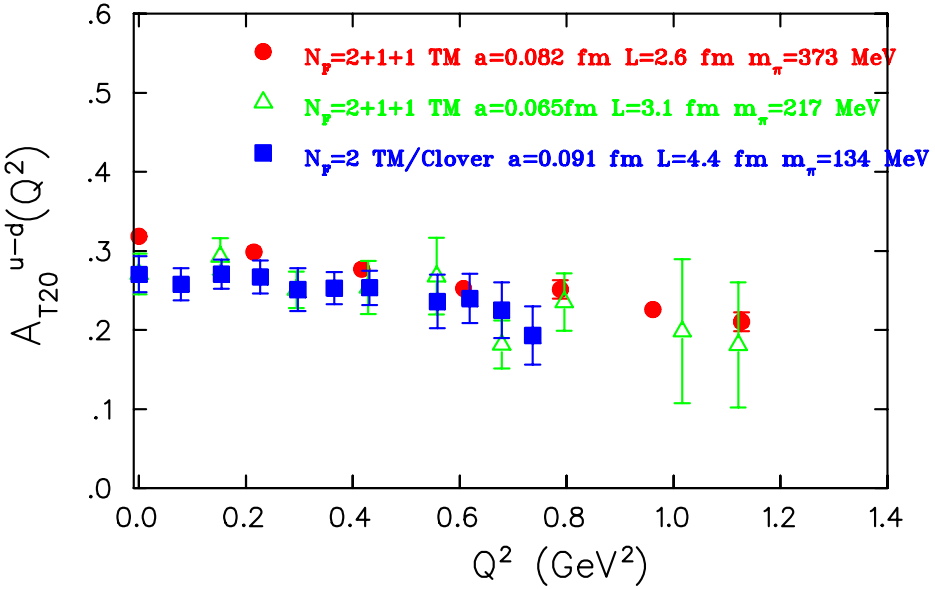


Summary of results on the lowest moments of the nucleon

Isovector in $\overline{MS}$ at 2 GeV







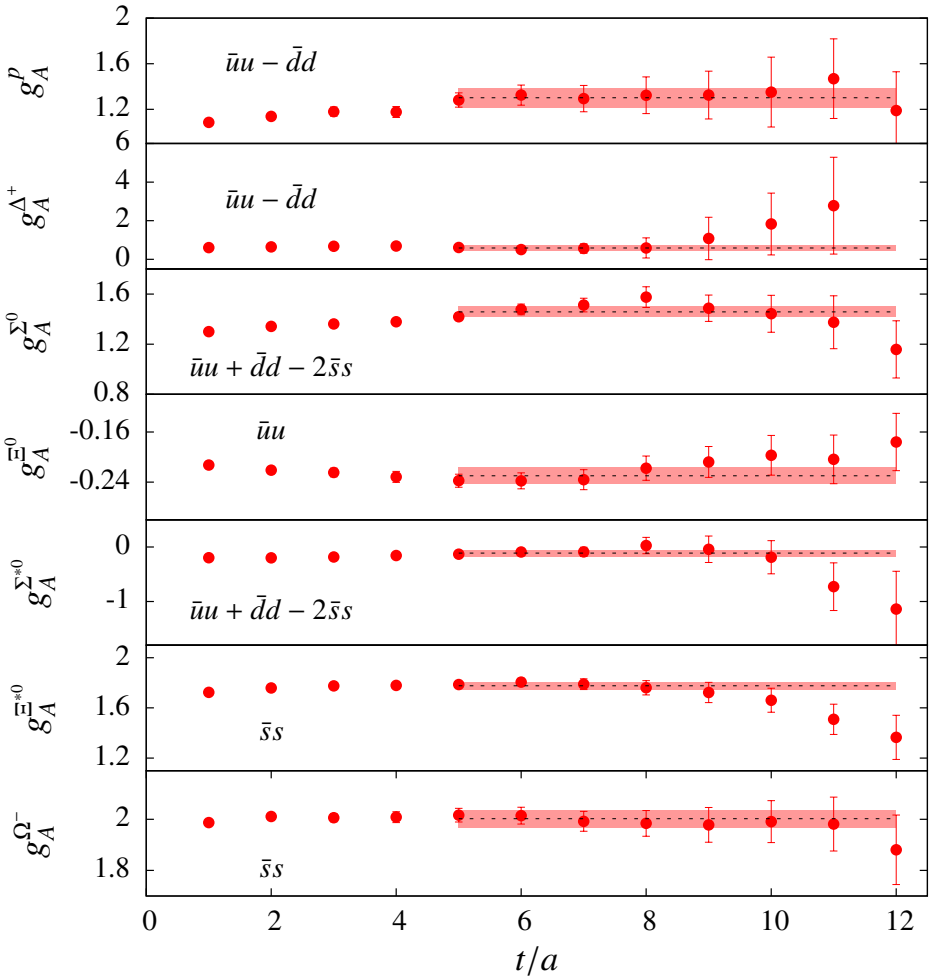
- $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u-\Delta d}$ approach physical value for bigger source-sink separations $\rightarrow$ need an equivalent high statistics study as at $m_\pi = 373$ MeV
- Can provide a prediction for $\langle x \rangle_{\delta u-\delta d}$

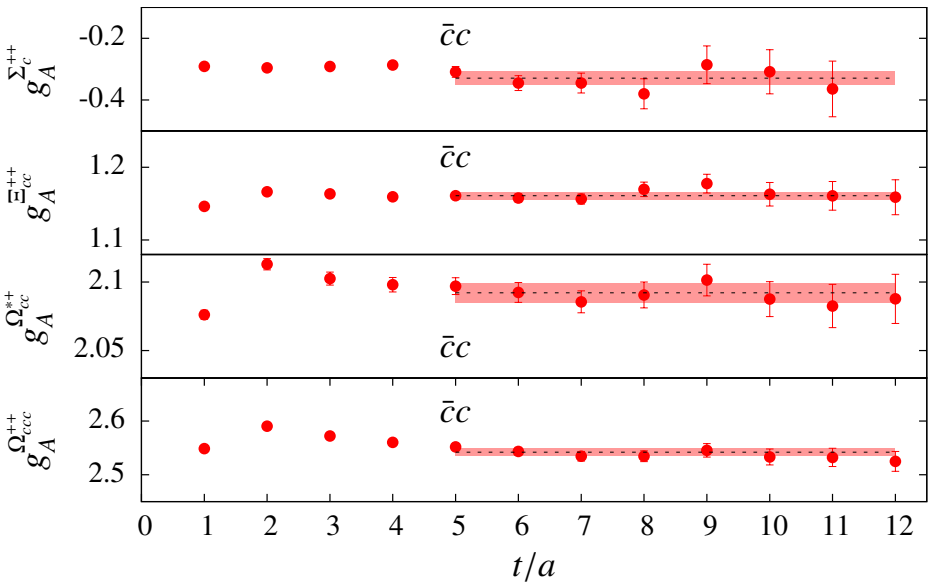
Axial charges of hyperons and charmed baryons

Axial matrix element: $\langle B(\vec{p}\,')|\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)|B(\vec{p})\rangle|_{q^2=0}$

Preliminary

- Only connected
- Use fixed current method





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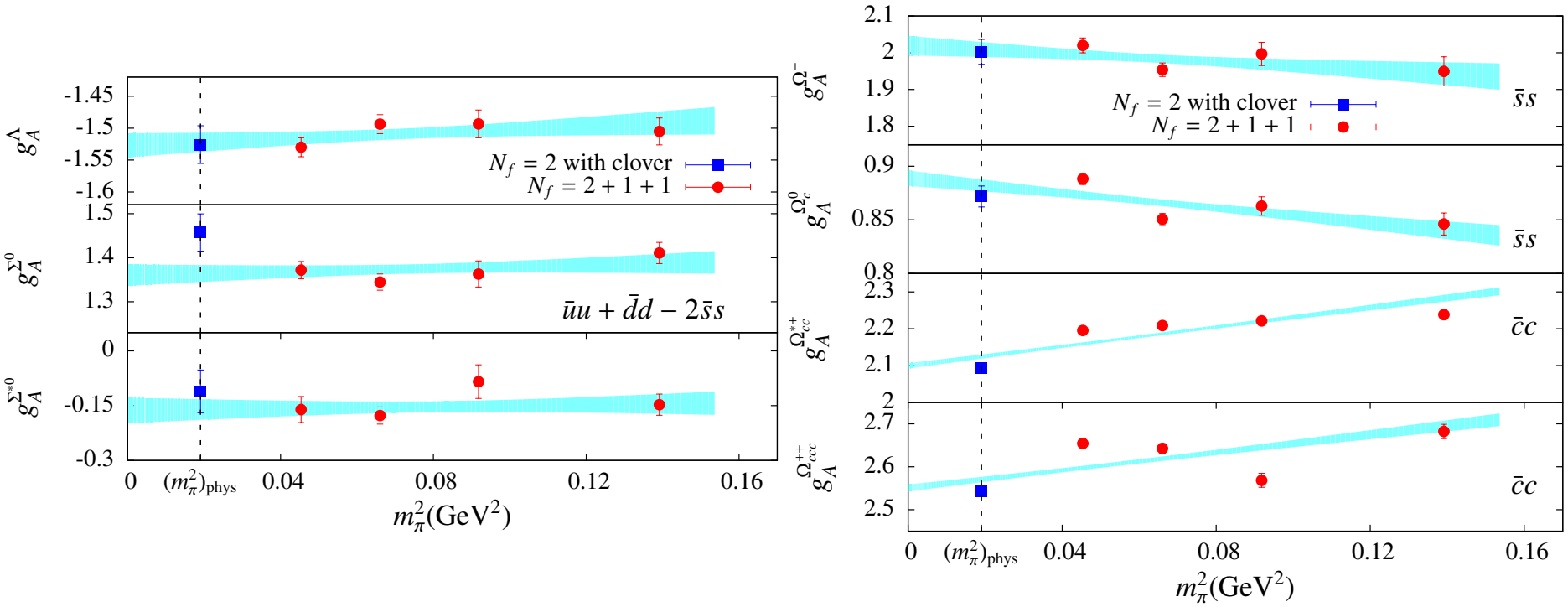
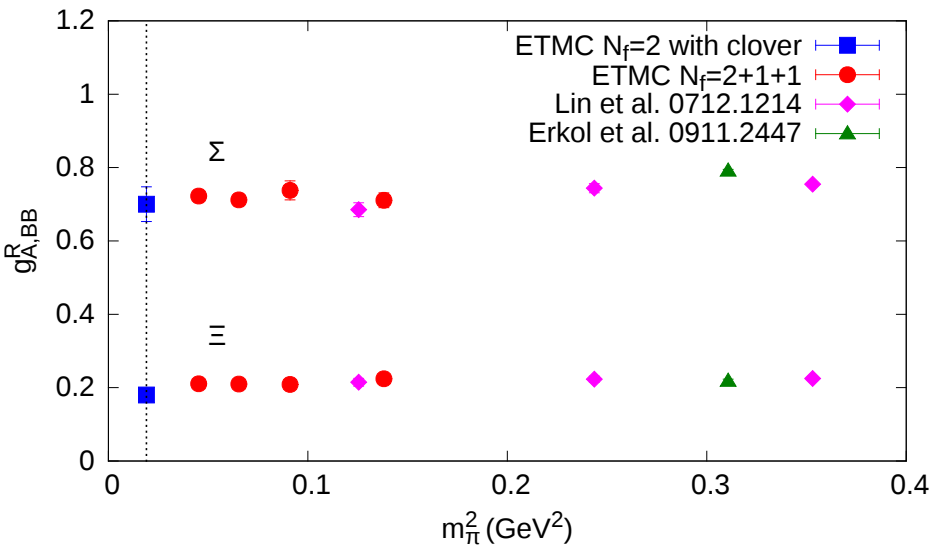
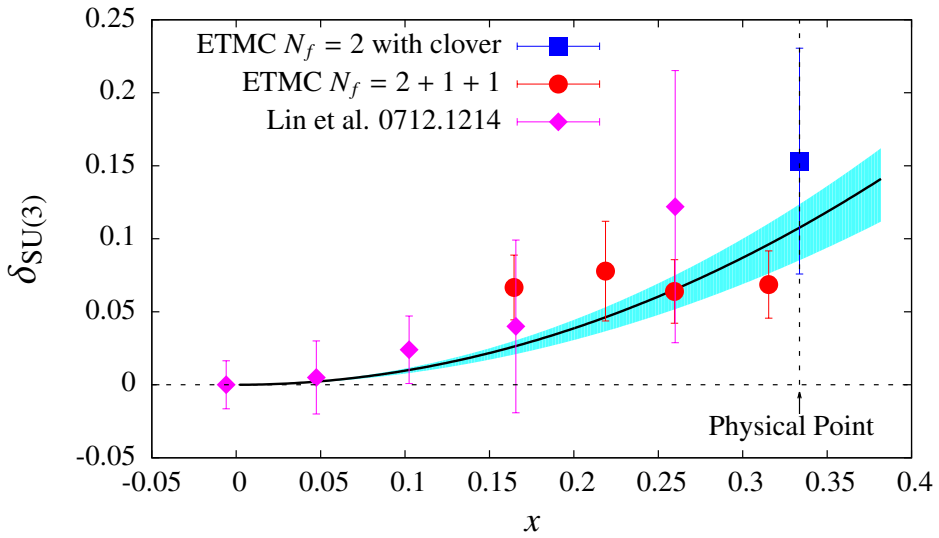


Axial charges of hyperons and charmed baryons

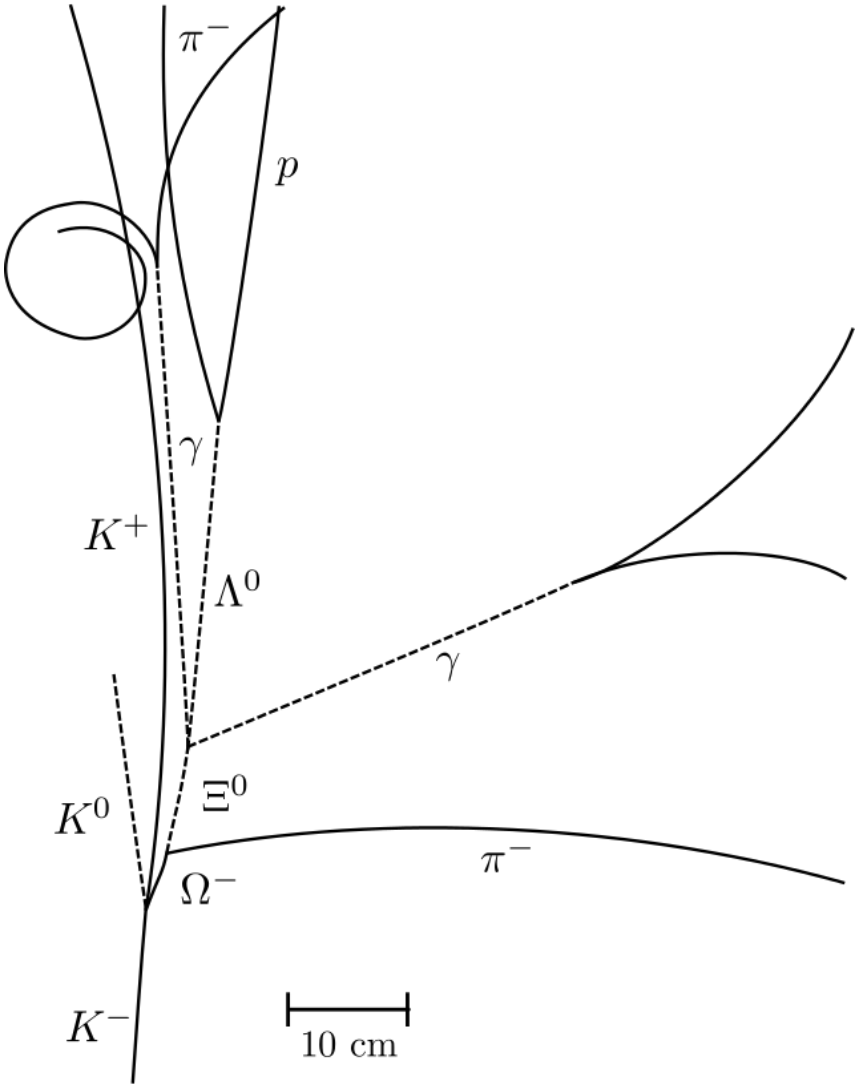
Preliminary





- First promising results at the physical point
- SU(3) breaking $\delta_{SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ versus $x = \left(m_K^2 - m_\pi^2\right)/(4\pi^2 f_\pi^2)$

Conclusions



50 years from the discovery of Ω and Ξ at Brookhaven!

- Simulations at the physical point $\rightarrow$ that's where we always wanted to be!
  $\Longrightarrow$ Results on g_A, $\langle x \rangle_{u-d}$ etc at the physical point are now directly accessible
  But will need high statistics and careful cross-checks $\rightarrow$ noise reduction techniques are crucial e.g. AMA, TSM, smearing etc
- Evaluation of quark loop diagrams has become feasible - need to make our methods work at the physical point
- Predictions for other hadron observables are emerging e.g. axial charge of hyperons and charmed baryons
- Confirmation of experimentally known quantities such as g_A will enable reliable predictions of others $\rightarrow$ provide insight into the structure of hadrons and input that is crucial for new physics such as the nucleon σ-terms, g_s and g_T

Thank you for your attention

REPUBLIC OF CYPRUS

EUROPEAN UNION

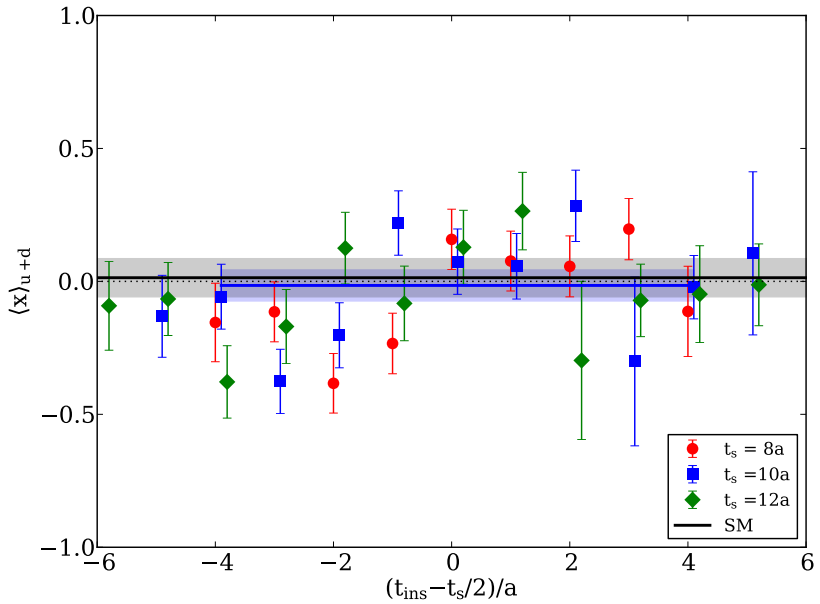
STRUCTURAL FUNDS
of the European Union in Cyprus
our ideas, actions for development

The Project Cy-Tera (NEA ΥΠΟΔΟΜΗ/ΣΤΡΑΤΗ/0308/31) is co-financed by the European Regional Development Fund and the Republic of Cyprus through the Research Promotion Foundation

Backup slides

First moments of PDFs for the nucleon

Twisted Mass, $a = 0.082$ fm, $32^3 \times 64$, $m_\pi = 373$ MeV, $\sim$150 0000 statistics (on 4700 confs; Disconnected)

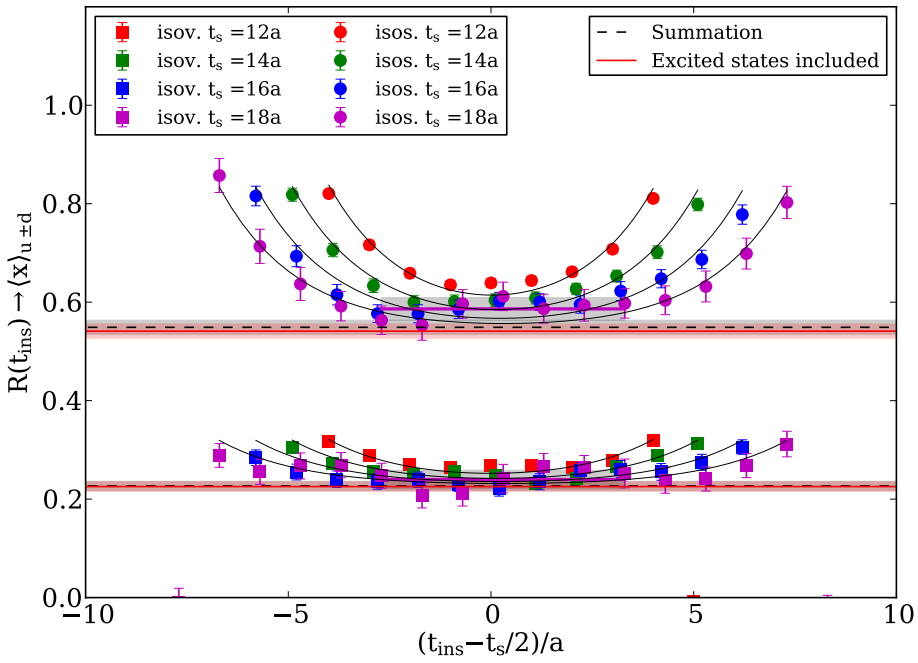


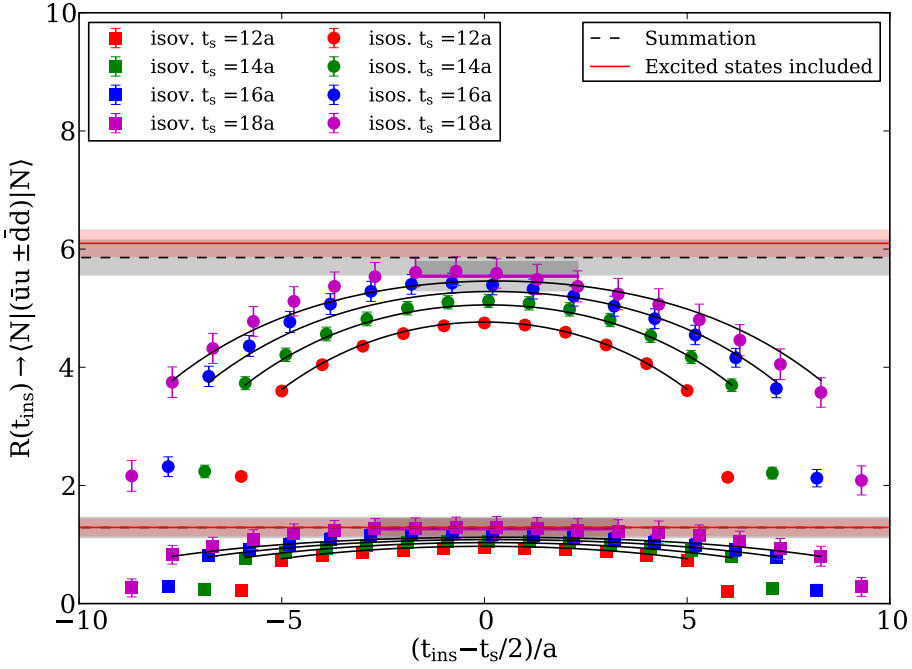
- Can put bound on its value
- Including momentum in the sink/source improves statistical accuracy

A. Abdel-Rehim, C. A., M. Constantinou, S. Dinter, V. Drach, K. Hadjiyiannakou, K. Jansen, Ch. Kallidonis, G. Koutsou

Two-state fits

Fitting the ratio to two-states simultaneous for several sink-source separations works for the scalar charge and momentum fraction. As stressed g_A does not pick up contributions from excited states.





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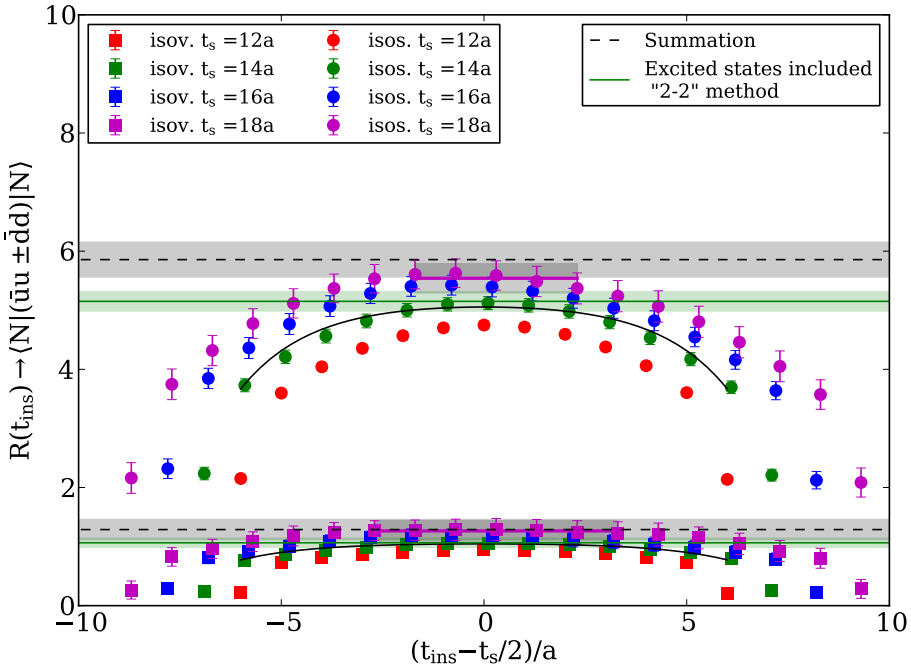

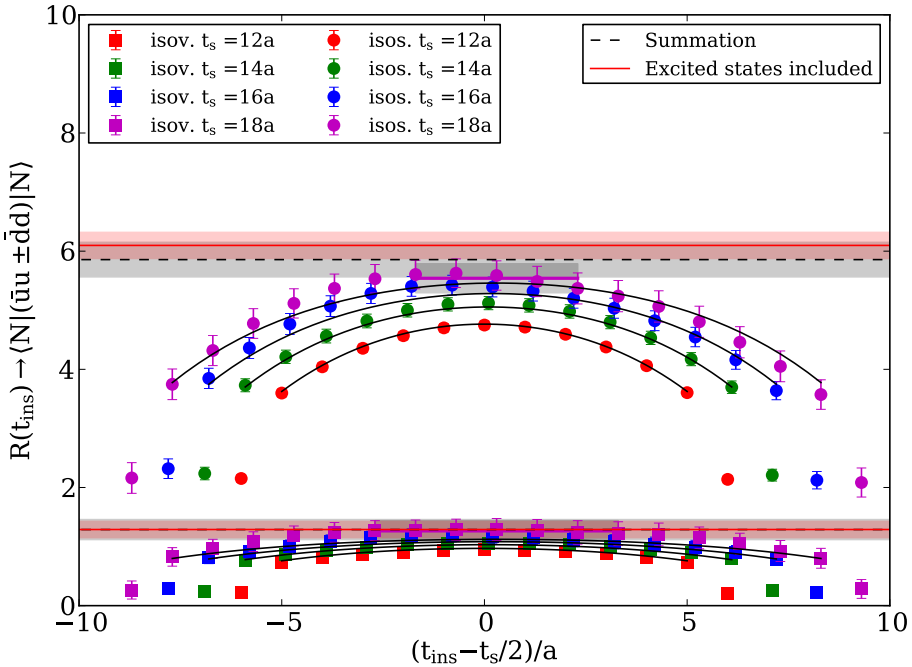

Not useful for predicting the large time dependence