Gauge and Higgs Boson Masses From an Extra Dimension

Graham Moir
Bergische Universität Wuppertal

Lattice 2014, Columbia University, 23 June

(With: F. Knechtli, N. Irges, K. Yoneyama, P. Dziennik)

Gauge-Higgs Unification in 5 dimensions

Perturbative regime

Higgs potential can break gauge symmetry via the Hosotani mechanism

- Components of the 5-d gauge field, A_M, for compactified directions can't be gauged away.
- They can develop vacuum expectation values: $\langle A_5 \rangle \neq 0$.
- Breaks gauge symmetry dynamically.
- Higgs mass and potential are finite at 1-loop: solves the hierarchy problem.
- Note: requires the presence of fermions.

Gauge-Higgs Unification in 5 dimensions

Pure gauge non-perturbative regime

Spontaneous symmetry breaking (SSB) observed on an orbifold without fermions. [Irges, Knechtli 2007]

- The orbifold has an additional global symmetry - 'stick symmetry'.

  [Ishiyama, Murata, So, Takenaga 2010]

- Spontaneous breaking of the stick symmetry triggers SSB in accordance with Elitzur's theorem.

  [Irges, Knechtli 2014]

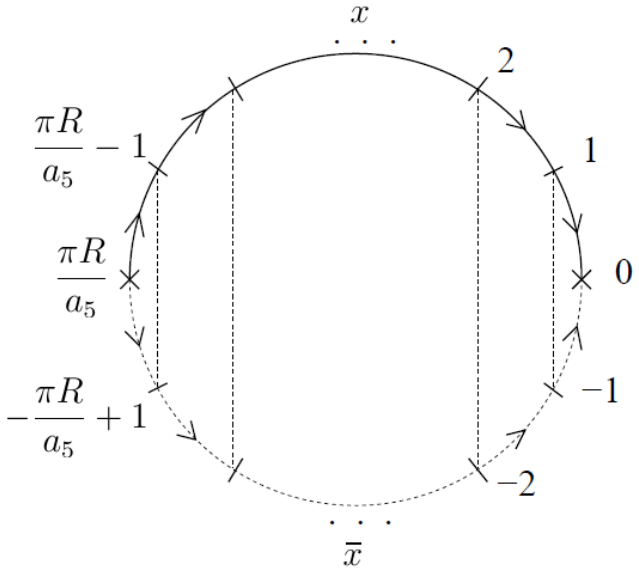
- Vector polyakov loop (cf. Z boson operator) is the order parameter for SSB: $\langle Z \rangle \neq 0 \Rightarrow$ breaks stick symmetry.
- $SU(N)$ with odd N does not have a stick-like symmetry $\Rightarrow$ no SSB. (Pure gauge $SU(3)$ - minimal theory for standard model is ruled out on the orbifold) [cf. P. De Forcrand, Fri 15.35]

Is it a viable theory for the Higgs mechanism in the standard model?

$S^1/\mathbb{Z}_2$ Orbifolded Extra Dimension

Orbifold projection on boundary:

- Reflection and group conjugation: $A_M = \alpha_M g A_M g^{-1}$ $\alpha_5 = -1, \alpha_\mu = 1$
- Interval with fixed end points: $x_5 = 0, \pi R$
- Constant $g \Rightarrow$ gauge symmetry is broken on the boundaries



SU(2) case:

- Choose $g = -i\sigma^3$ $\Rightarrow SU(2) \to U(1)$
- Only $A_5^1, A_5^2, A_\mu^3 \neq 0$ on the boundaries

- $A_5^{1,2}$: Complex 'Higgs'
- A_μ^3: 'Z' boson

Lattice Set-Up

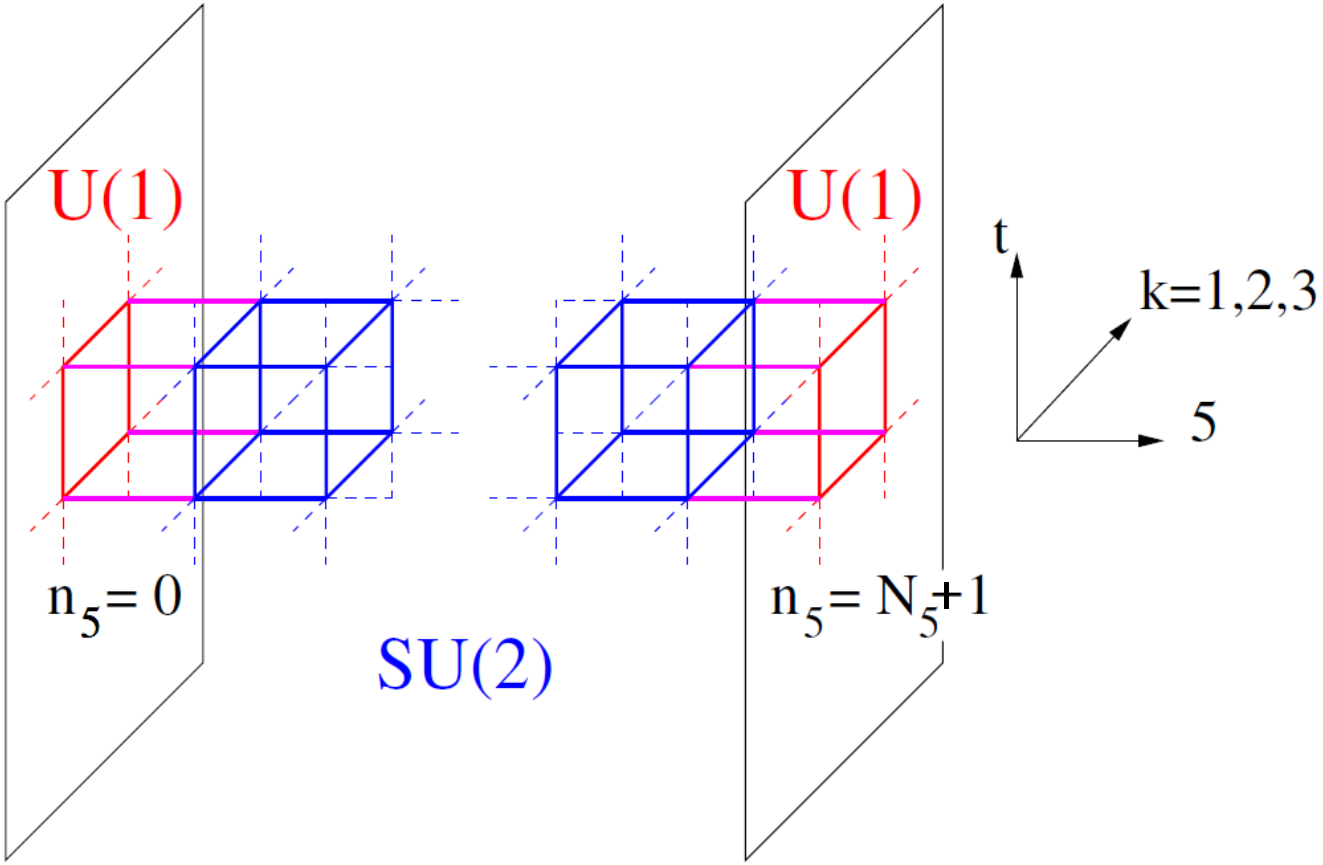
Anisotropic Wilson Action on a 5-d Euclidean orbifold

$$S_W^{orb} = \frac{\beta}{2} \sum_x \left[\frac{1}{\gamma} \sum_{\mu<\nu} w \, \mathrm{tr} \left\{ \mathbb{1} - U_{\mu\nu}(x) \right\} + \gamma \sum_\mu \mathrm{tr} \left\{ \mathbb{1} - U_{\mu 5}(x) \right\} \right]$$

$$\mathrm{w} = \begin{cases} \frac{1}{2} & \text{plaquette on boundary} \\ 1 & \text{otherwise} \end{cases}$$

- The gauge couplings in the four and fifth dimensions are related via the anisotropy parameter $\gamma = \sqrt{\beta_5/\beta_4}$.
- Lattice volume given by: $T \times L^3 \times (N_5 + 1)$ where N_5 is the number of links in the fifth dimension.
- Three types of links: bulk, boundary and hybrid.

Lattice Set-Up



Boundary:

$U \to \Omega^{U(1)} U \Omega^{\dagger U(1)}$

Bulk:

$U \to \Omega^{SU(2)} U \Omega^{\dagger SU(2)}$

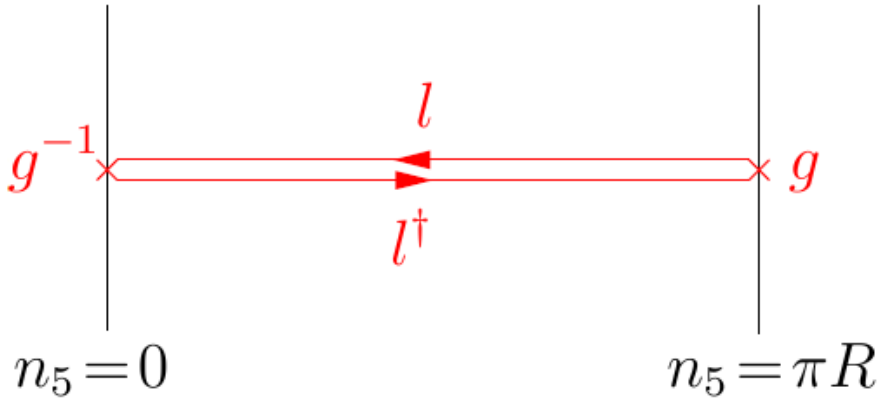
Hybrid:

$U \to \Omega^{U(1)} U \Omega^{\dagger SU(2)}$

Higgs Operator

Higgs d.o.f come from **Polyakov loops** winding around extra dimension.

- Orbifold Higgs field:
  $\phi = [P - P^\dagger, g]/4N_5$
- Orbifold Higgs operator:
  $H = \mathrm{tr}\{\phi\phi^\dagger\}$
- Expect this operator to overlap strongly onto Higgs-like states.

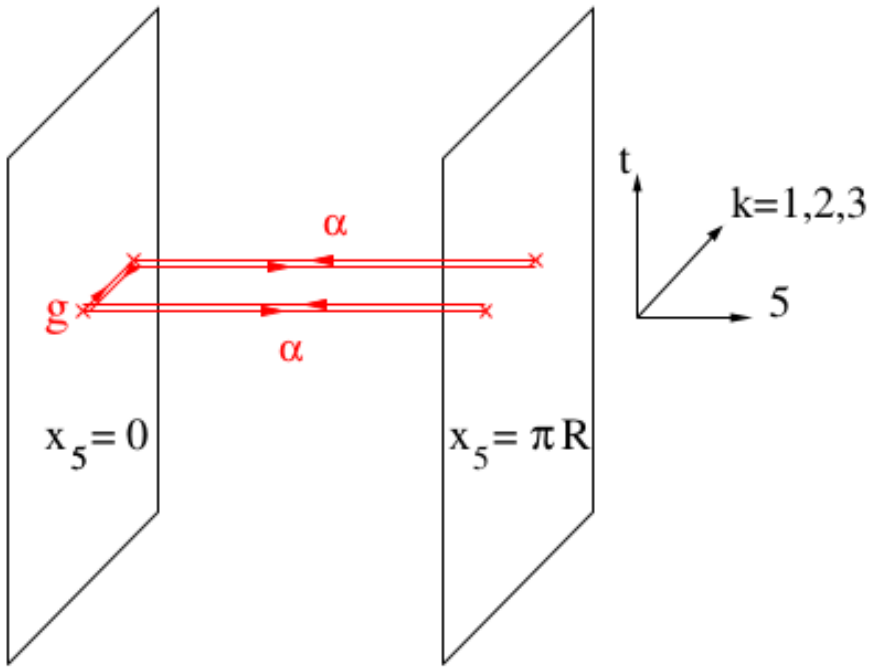


- $P(n_\mu) = l(n_\mu) g l(n_\mu)^\dagger g^{-1}$
- $l(n_\mu) = U(n_\mu, 0) \ldots U(n_\mu, N_5)$

Vector Boson Operators

Can build gauge boson operators from **vector** polyakov loops.

- Orbifold Z boson operator: $\mathrm{tr}\{Z_k\} = \mathrm{tr}\{gU_k\alpha U_k^\dagger\alpha\}$
- $\alpha = \phi/\sqrt{\det(\phi)}$
- $\mathrm{tr}\{Z_k\}$ changes sign under stick symmetry $\Rightarrow$ order parameter for SSB
- Can build a similar operator using polyakov lines: $\mathrm{tr}\{\tilde{Z}_k\} = \mathrm{tr}\{U_k l U_k^\dagger g l^\dagger\}$



Extraction of Spectrum

Construct a large basis of operators in the scalar and vector channels by using two types of smearing:

- Hypercubic smearing of gauge fields.
- Ape-like smearing of polyakov loops.

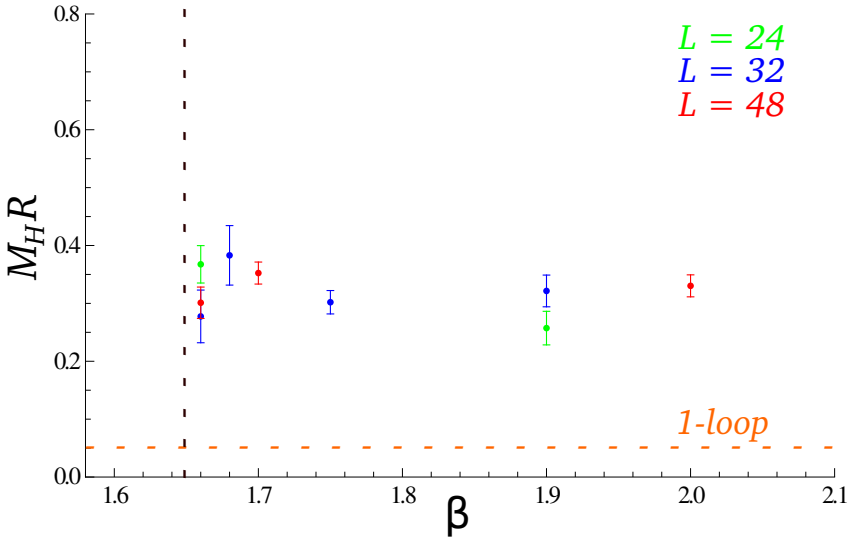
Extract spectrum by solving generalised eigenvalue problem:

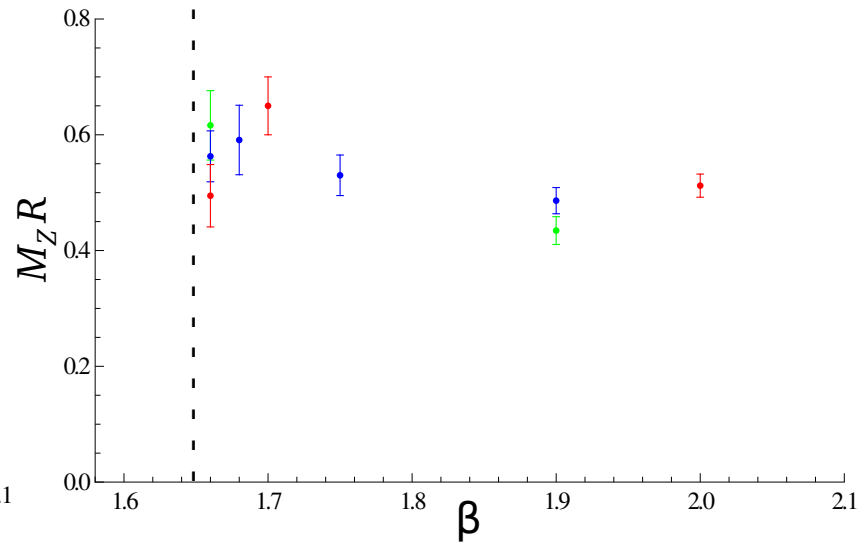
$$C_{ij} v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

- Eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t} \left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- Eigenvectors: Relate to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

Note: $\mathrm{tr}\{P\}$ found to be noisy and has negligible overlap onto states in the scalar channel $\Rightarrow$ removed from basis.

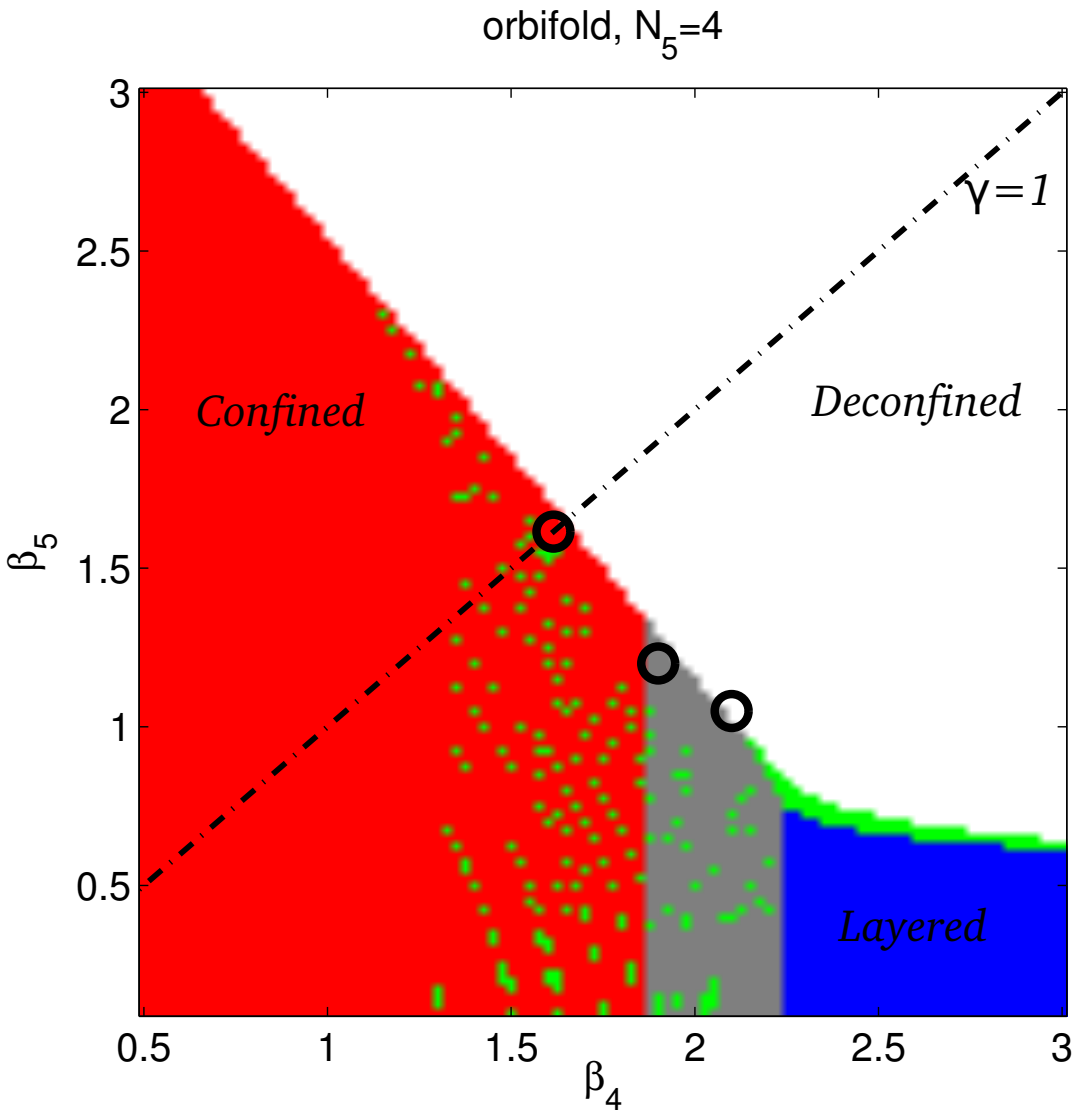
Spectrum Along Isotropic $\gamma = 1$ Line (Preliminary)



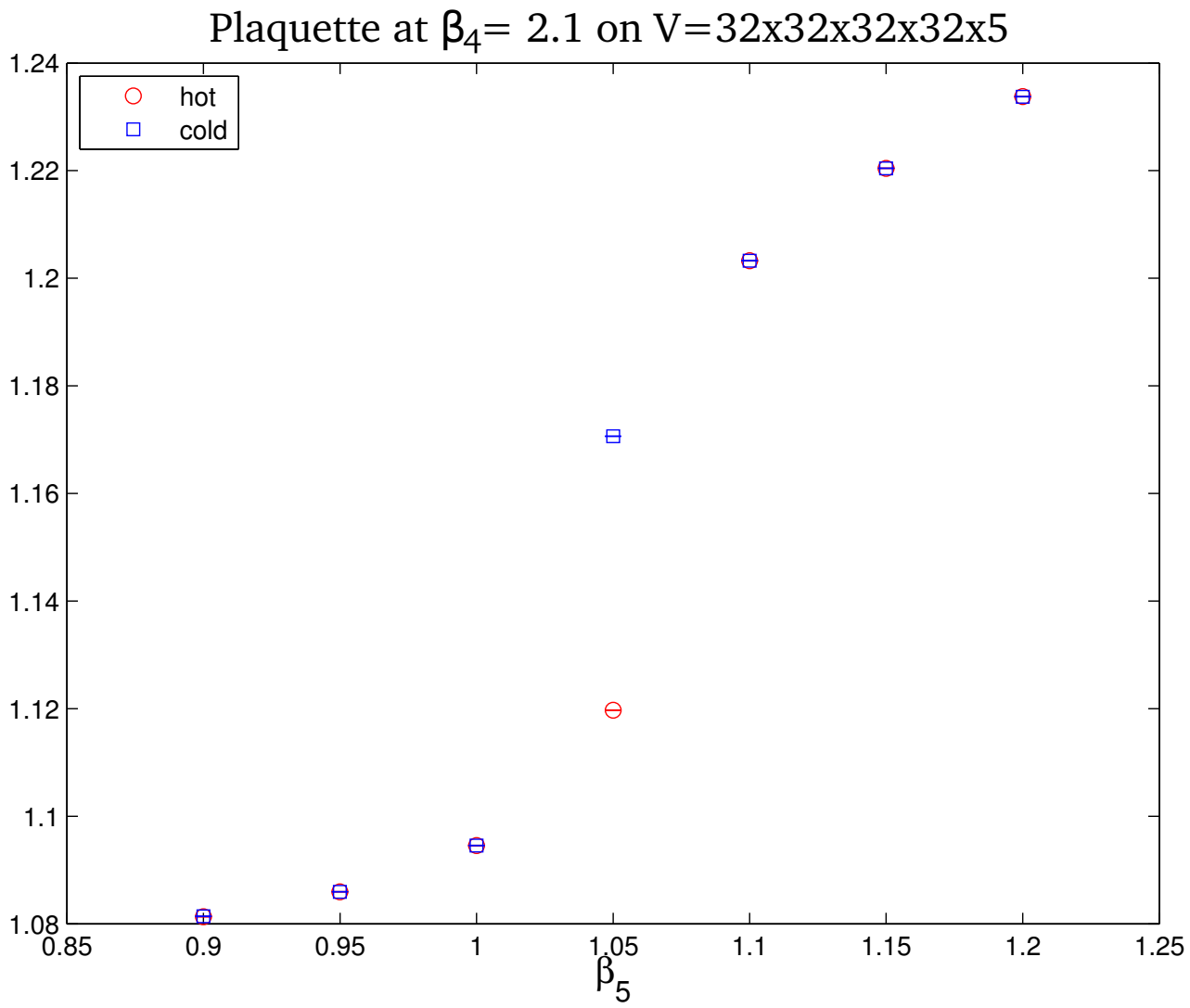


- Do not see correct hierarchy of masses.
- Is there a region of phase space with the correct hierarchy?
- Mean-field predicts correct hierarchy in the deconfined phase for $\gamma < 1$ close to the bulk phase transition. [Irges, Knechtli, Yoneyama 2012]

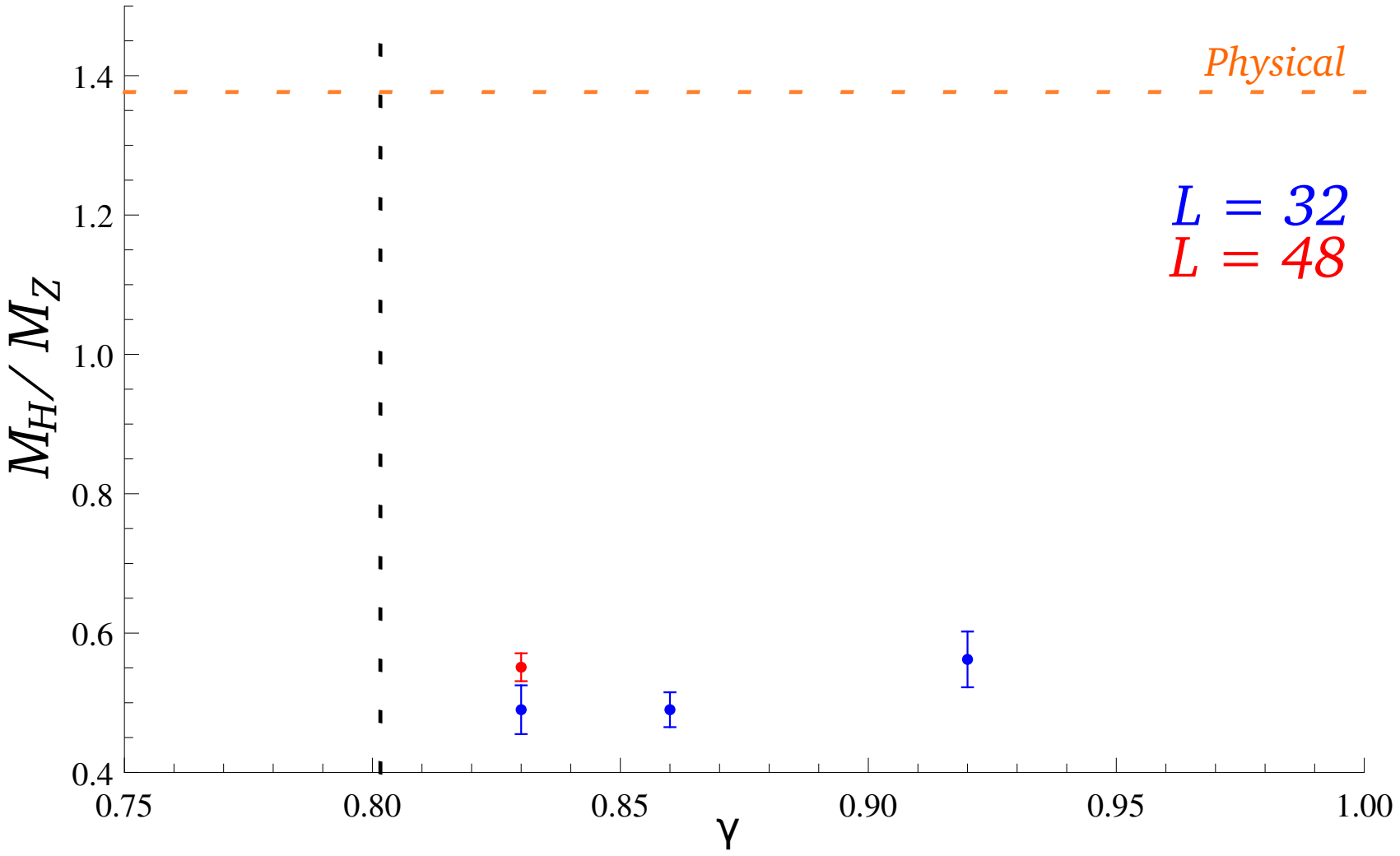
Phase Diagram



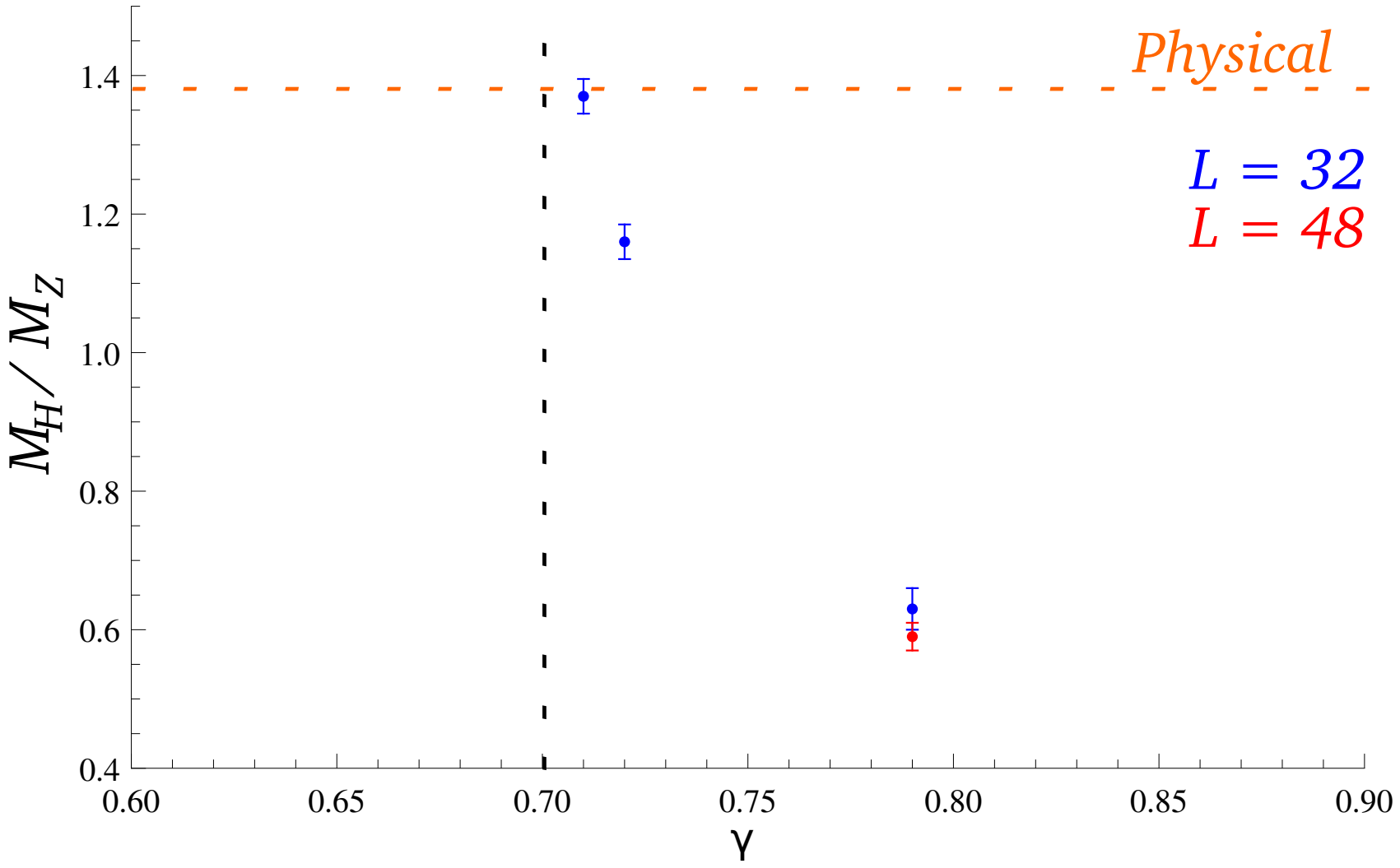
Bulk Phase Transition



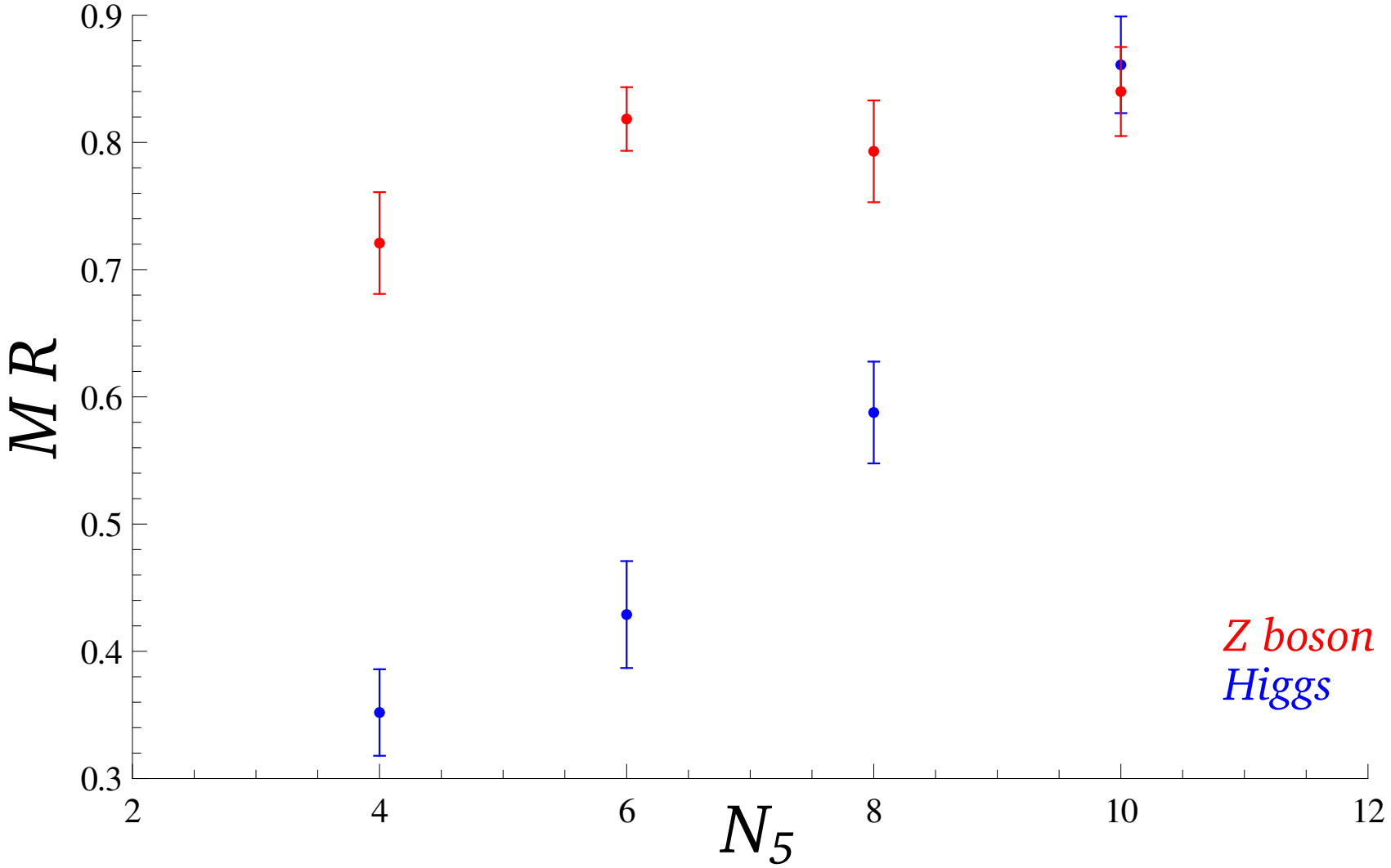
Mass Hierarchy at $\beta_4 = 1.9$ for $\gamma < 1$ (Preliminary)



Mass Hierarchy at $\beta_4 = 2.1$ for $\gamma < 1$ (Preliminary)



Spectrum Vs. Radius of Extra Dimension (Preliminary)



Outlook and Conclusions

Conclusions:

- Found a region of phase space where the correct hierarchy of the standard model can be reproduced.
- Observed that $M_Z \approx 1/R$: Expected from perturbative calculations.
- Observed that $M_H \approx$ constant as R grows: Unexpected from perturbative calculations.

Open Questions:

- How large are the regions where the correct hierarchy is observed?
- What will be the fate of M_H as R is increased further?
- What effect does mixing of other scalar particles (e.g. glueballs) have on the spectrum?
- Can we predict the existance of excited Higgs and gauge bosons?