Tensor network states for gauge theories

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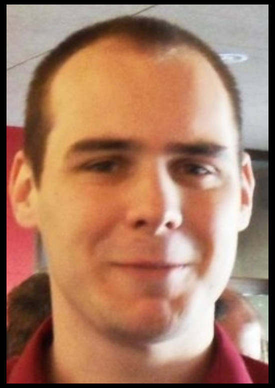
with: Boye Buyens, Jutho Haegeman, Norbert Schuch, Henri Verschelde, Frank Verstraete



Lattice 2014, Columbia NY

Motivation

- Hamiltonian simulations, working with wave-functions, real-time physics

- No sign problem, finite fermionic chemical potential

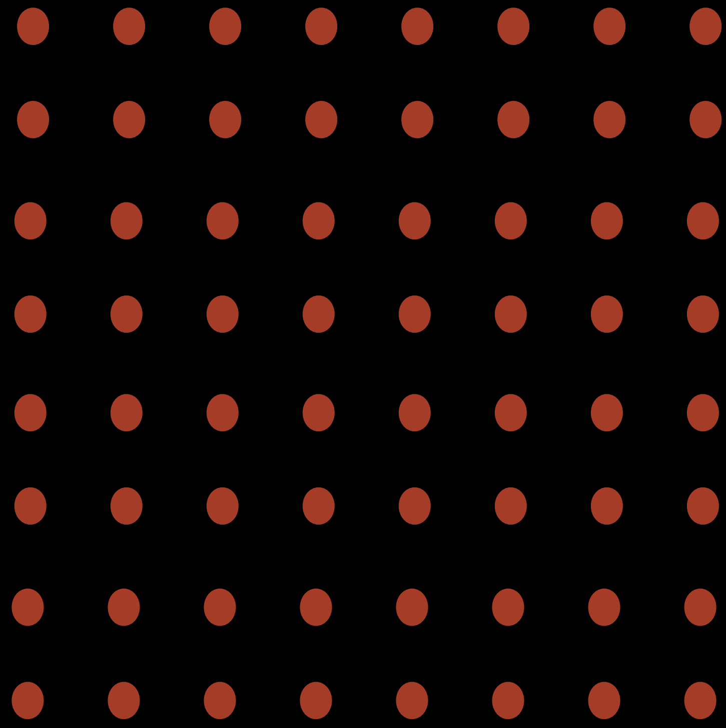
- Understand gauge theories in the tensor network language i.e. in terms of their entanglement structure

Related work:

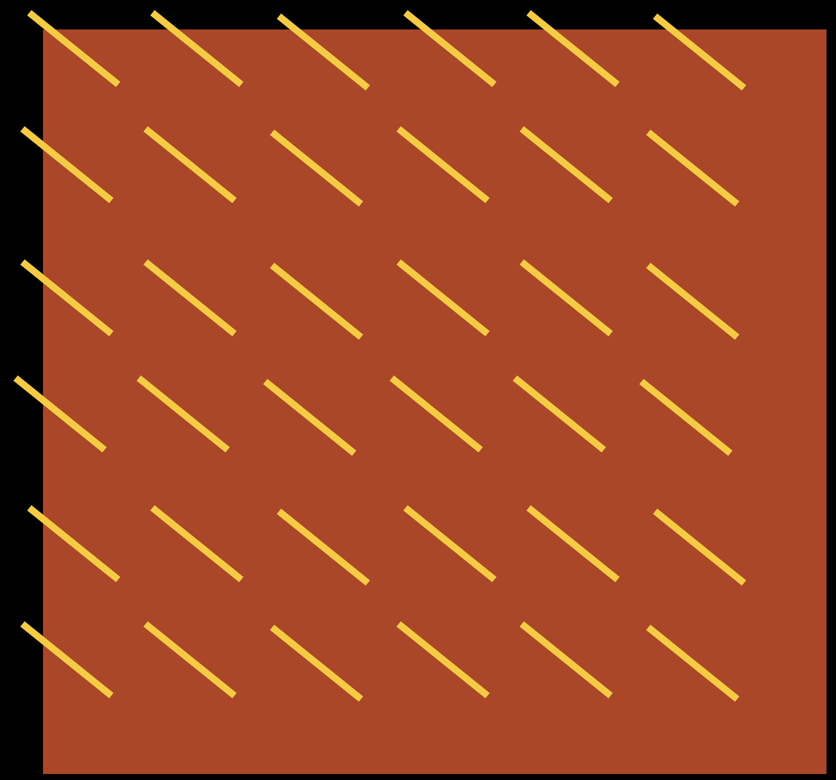
- T. M. Byrnes, P. Sriganesh, R.J. Bursill, C.J. Hamer Phys. Rev. D66, 13002 (2002)
- T. Sugihara, JHEP 07, 022 (2005)
- L. Tagliacozzo and G. Vidal, Phys. Rev. B 83, 115127 (2011)
- M.C. Bañuls, K. Cichy, K. Jansen, and J.I. Cirac, JHEP 11, 158 (2013)
- M.C Bañuls, K. Cichy, J.I. Cirac, K. Jansen, H. Saito, PoS (Lattice 2013) 332
- E.Rico, T. Pichler, M.Dalmonte, P. Zoller, S.Montangero, PRL 112, 201601 (2014)
- P. Silvi, E. Rico, T. Calarco and S. Montangero, arXiv: 1404.7439 (2014)
- L. Tagliacozzo, A. Celi and M. Lewenstein, arXiv: 1405.7439 (2014)

Tensor network states

taming the humongous Hilbert space



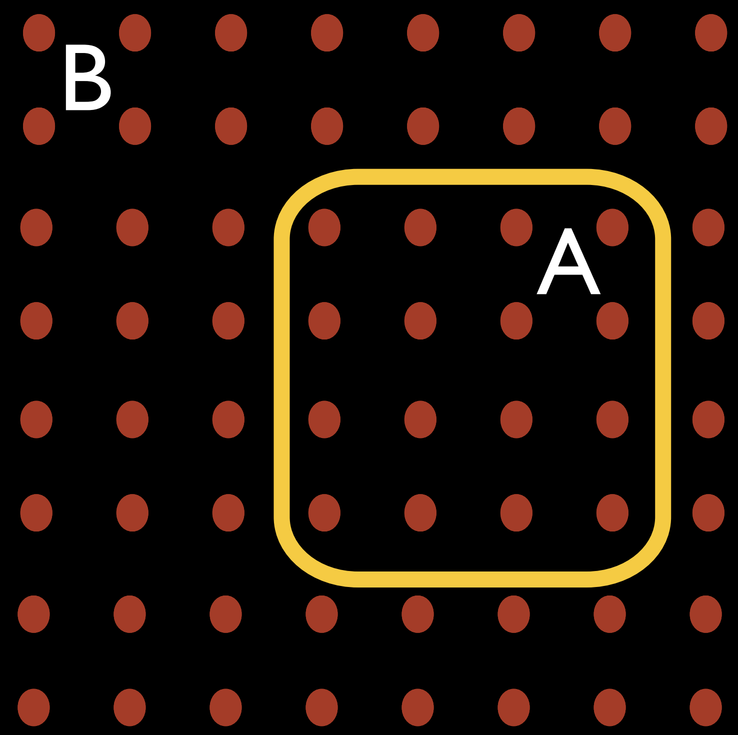
(spins, fermions, bosons, QFT)

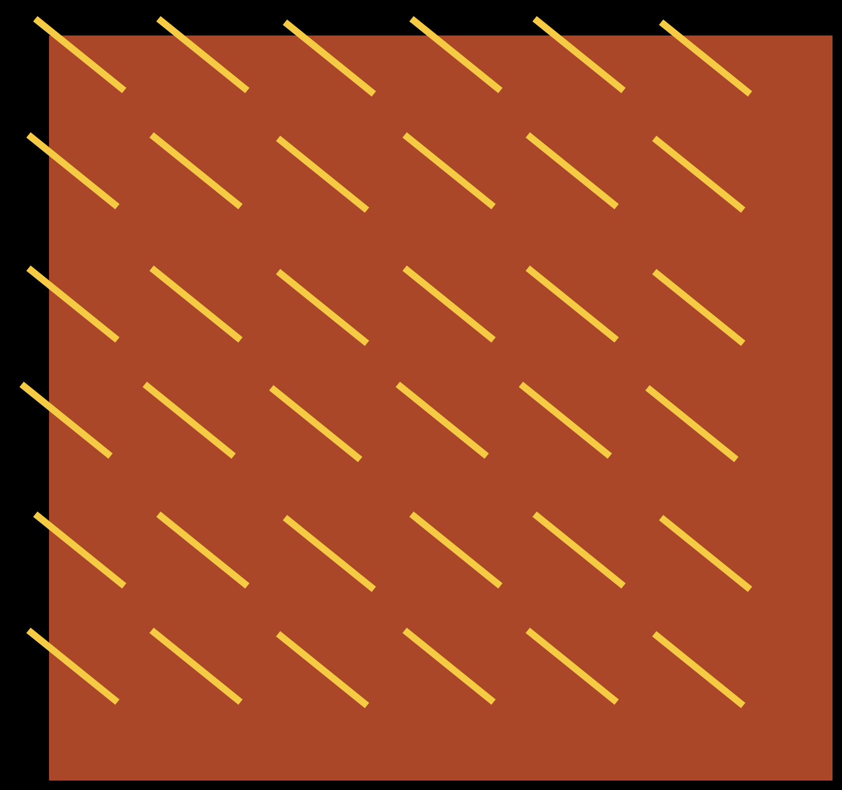


$$|\Psi> = c_{s_1,s_2,\ldots,s_N}|s_1,s_2,\ldots,s_N>$$

dimension: p^N

The tiny corner of Hilbert space





area law for entanglement entropy of low-energy states:

$$S_A = -Tr_A \rho_A \log \rho_A \sim \partial A$$

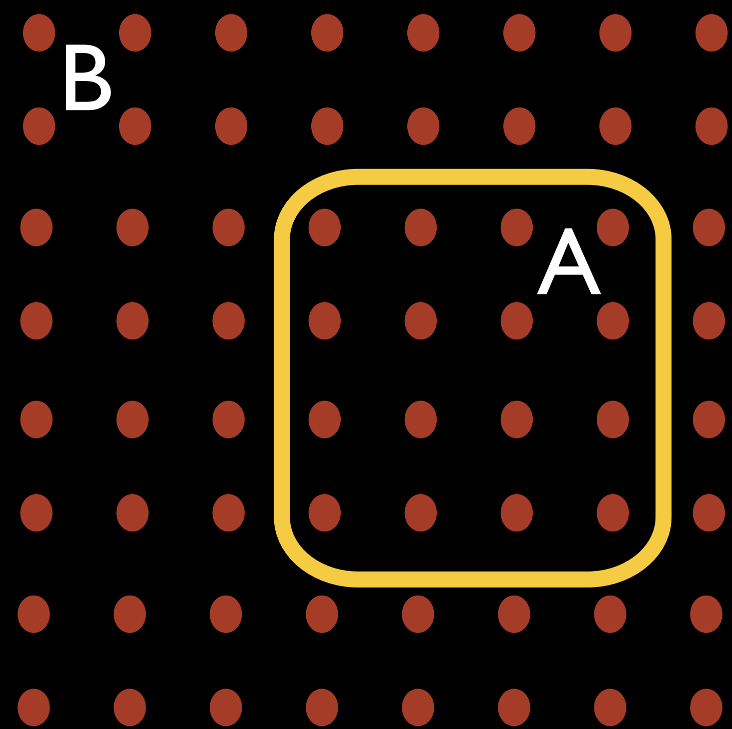
$$\rho_A = Tr_B |\Psi >< \Psi|$$

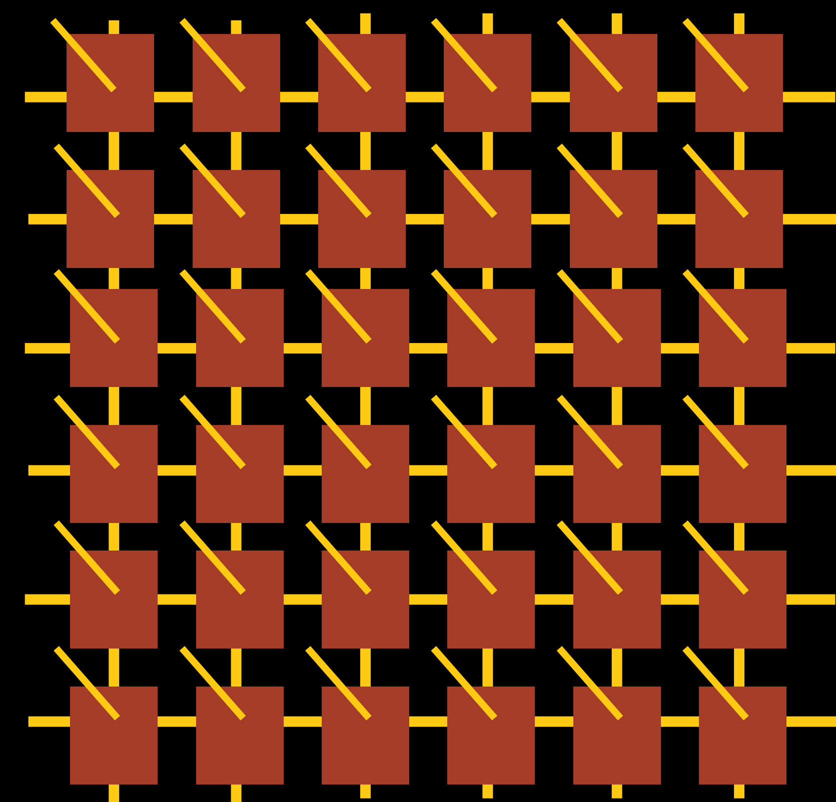
(proven by Hastings '07 for d=1)

$$|\Psi >= c_{s_1, s_2, \ldots, s_N} |s_1, s_2, \ldots, s_N >$$

dimension: p^N

The tiny corner of Hilbert space





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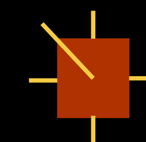
tensor network:



$$= A^s_{\alpha_L \alpha_R \alpha_U \alpha_D}$$

$$S_A \leq \log D \, \partial A$$

two reviews

(with the proper references)

J.I. Cirac and F. Verstraete: J. Phys. A: Math. Theor. 42, 504004 (2009), arXiv: 0910.1130

R. Orus, Anals of Physics (2013) arXiv:1306.2164

d=1+1 QED a.k.a. the Schwinger model

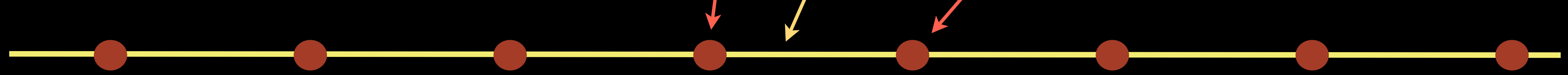
(B. Buyens, J. Haegeman, K.V.A., H. Verschelde, F. Verstraete, arXiv: 1312.6654)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^{\mu}(\partial_{\mu} - ieA_{\mu})\psi + m\bar{\psi}\psi$$

- Can be solved exactly for $g \rightarrow \infty$ (Schwinger '62, Coleman '76)
- Non-trivial physics, similar to QCD: e.g. confinement

Kogut-Susskind ($A_0 = 0$ + staggered fermions) + Jordan-Wigner:

$$H = \frac{g}{2\sqrt{x}} \left(\sum_{n\in\mathbb{Z}} L(n)^2 + \frac{\mu}{2} \sum_{n\in\mathbb{Z}} (-1)^n (\sigma_z(n) + (-1)^n) + x \sum_{n\in\mathbb{Z}} (\sigma^+(n) e^{i\theta(n)} \sigma^-(n+1) + h.c.) \right).$$



fermions: $\sigma_z(n)|s_n> = s_n|s_n>$ $\quad (s_n = \pm 1)$

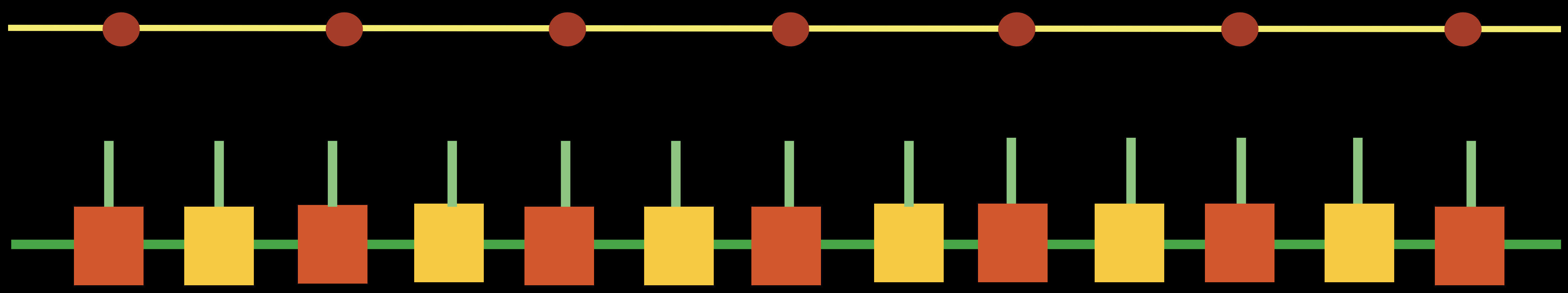
gauge-fields: $L_n|p_n> = p_n|p_n>$ $\quad p_n \in Z$ $\qquad [\theta(n), L(m)] = i\delta_{nm}$

Extra ingredient: gauge invariance/Gauss law

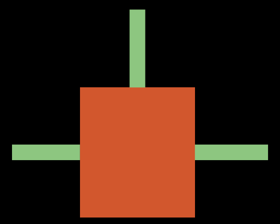
$$G_n|\Psi>_{phys} = 0$$

$$G_n = L(n) - L(n-1) - \frac{1}{2}(\sigma_z(n) + (-1)^n) \qquad (\nabla . E = \rho)$$

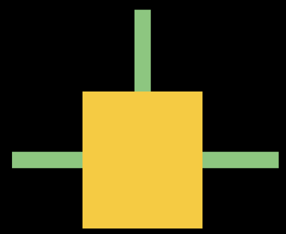
gauge-invariant Matrix Product State



$$|\Psi> = \sum_{s_n, p_n} (v_L^+ B_1^{s_1} C_1^{p_1} B_2^{s_2} C_2^{p_2} \dots C_{2N}^{p_{2N}} v_R) \, |s_1, p_1, s_2, p_2 \dots, p_{2N}>$$

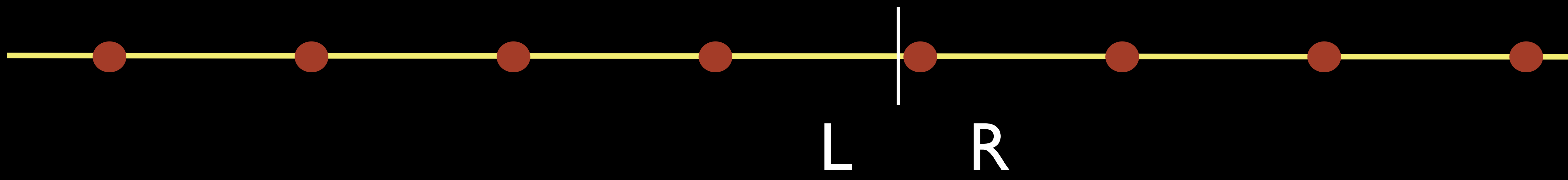


$$= [B_n^{s_n}]_{(q,\alpha_q),(r,\beta_r)} = [b_{n,q}^{s_n}]_{\alpha_q,\beta_r} \delta_{q+(s_n+(-1)^n)/2,r}$$

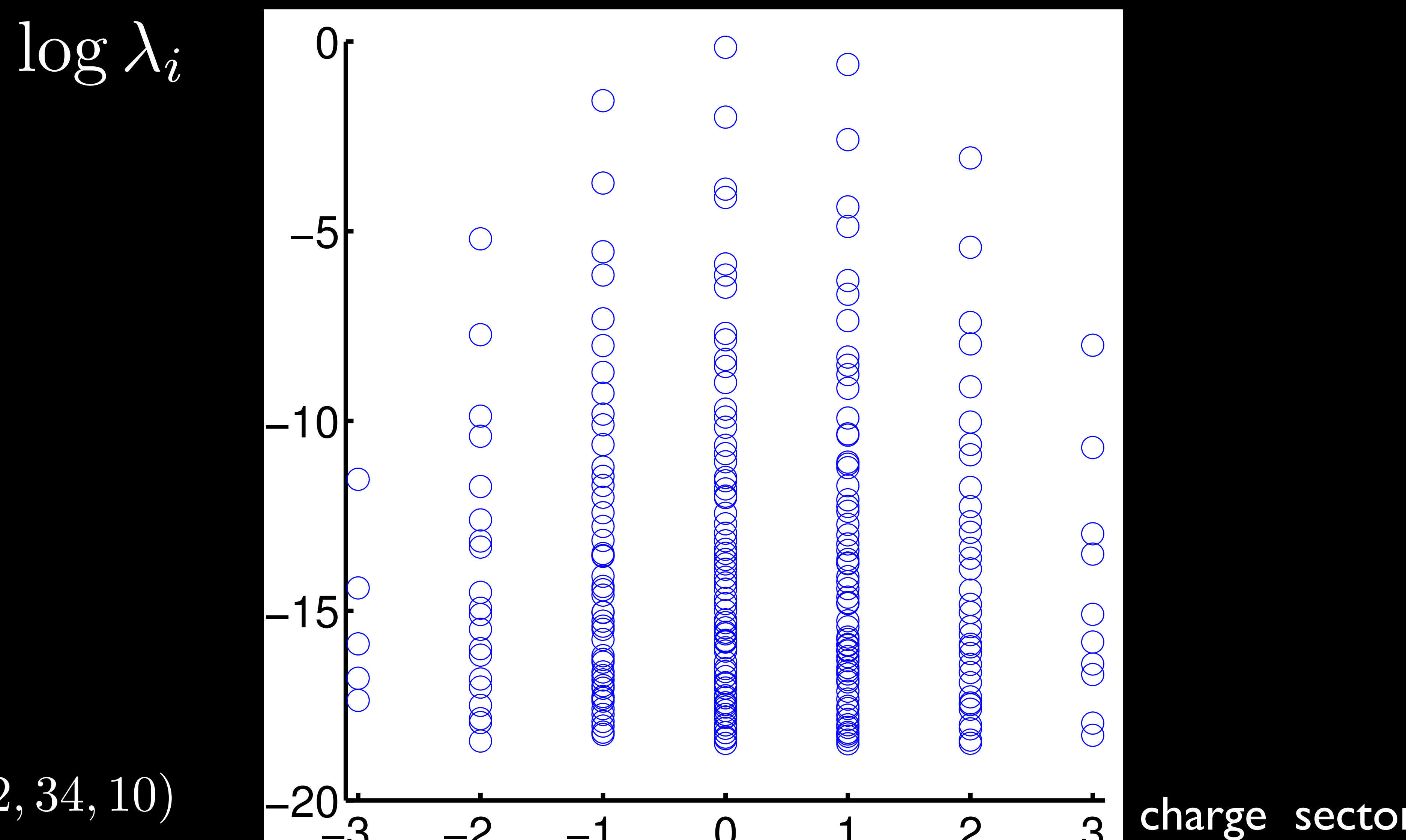


$$= [C_n^{p_n}]_{(q,\alpha_q),(r,\beta_r)} = [c_n^{p_n}]_{\alpha_q,\beta_r} \delta_{q,p_n} \delta_{r,p_n}$$

Effective truncation local Hilbert space



Schmidt-decomposition: $$|\Psi> = \sum_i \sqrt{\lambda_i} |\Psi_i>_L \, |\Psi_i>_R$$

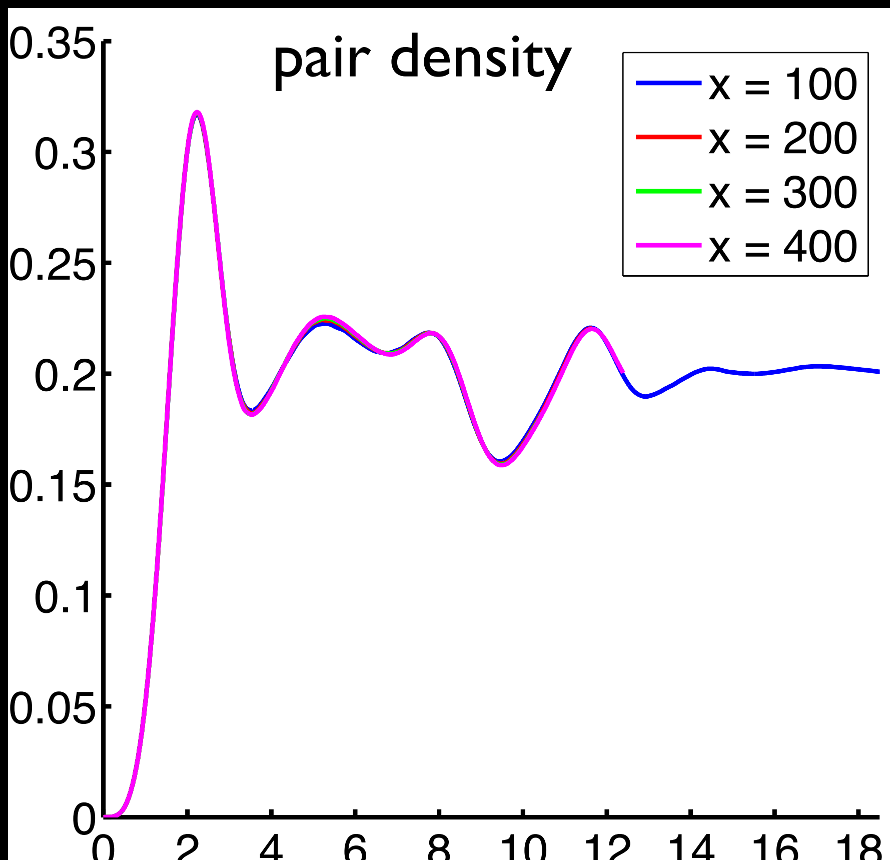


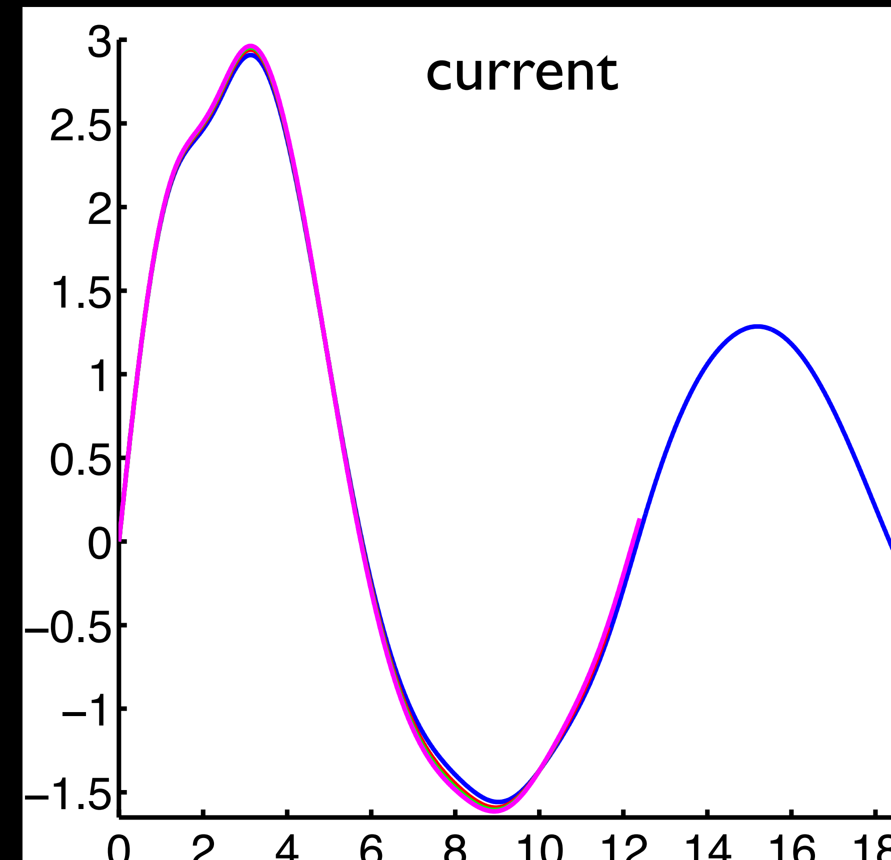
$$D = (5, 20, 48, 70, 62, 34, 10)$$

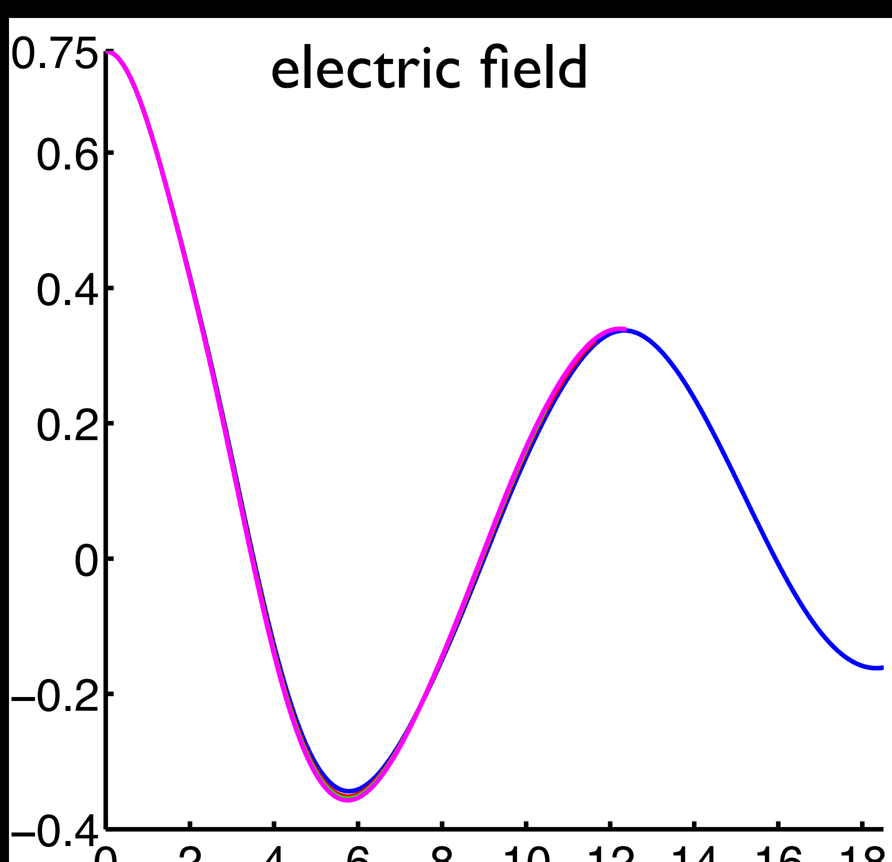
Groundstate energy+excitations

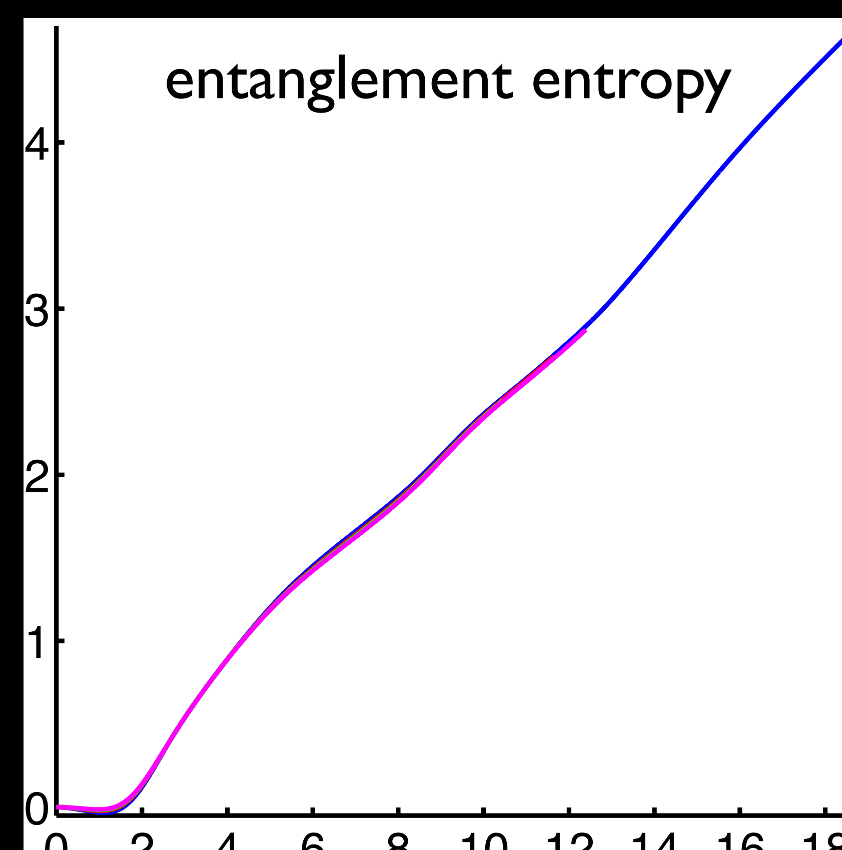
m/g	ω_0	$M_{v,1}$	$M_{s,1}$	$M_{v,2}$
0	-0.318320(4)	0.56418(2)		
0.125	-0.318319(4)	0.789491(8)	1.472(4)	2.10 (2)
0.25	-0.318316(3)	1.01917 (2)	1.7282(4)	2.339(3)
0.5	-0.318305(2)	1.487473(7)	2.2004 (1)	2.778 (2)
0.75	-0.318285(9)	1.96347(3)	2.658943(6)	3.2043(2)
1	-0.31826(2)	2.44441(1)	3.1182 (1)	3.640(4)

Real-time simulation Schwinger mechanism (new results)









Going to higher dimensions, d=2+1

Some facts:

1. The exact contraction of a 2dim.-tensor network (PEPS) is an exponentially hard problem. (equivalent to solving for the groundstate of a d=1+1 system)

2. But one can approximate this contraction, best algorithm so far has number of steps $\mathcal{O}(\chi^3 D^4 + p\chi^2 D^6)$. Therefore at present, we can only perform PEPS simulations with relatively low bond dimension.

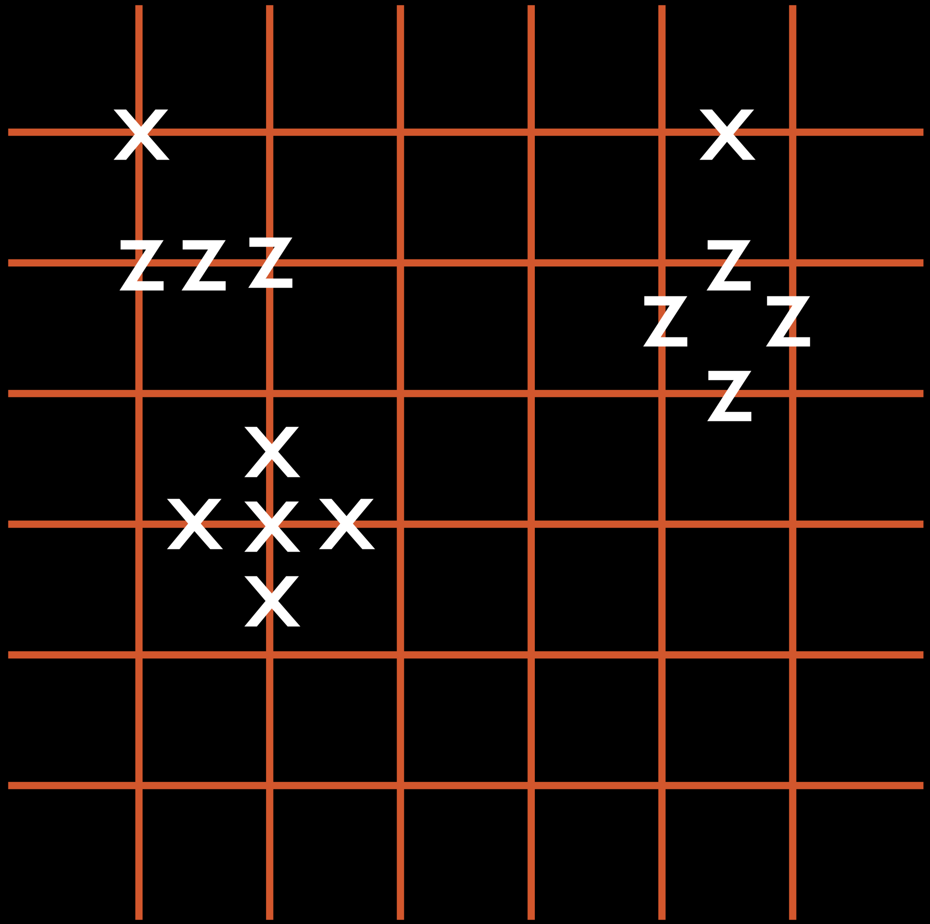
3. A PEPS is a groundstate of some local parent Hamiltonian (unique groundstate if the PEPS is *injective*)

So already from the study of low bond-dimension PEPS, one can probe the phase space of certain local parent Hamiltonians. IR universality?

Probing phase diagram of gauge theories with parent Hamiltonians

J. Haegeman, K.V.A. , N. Schuch, F. Verstraete (coming soon)

d=2+1 Z2 lattice gauge theory:

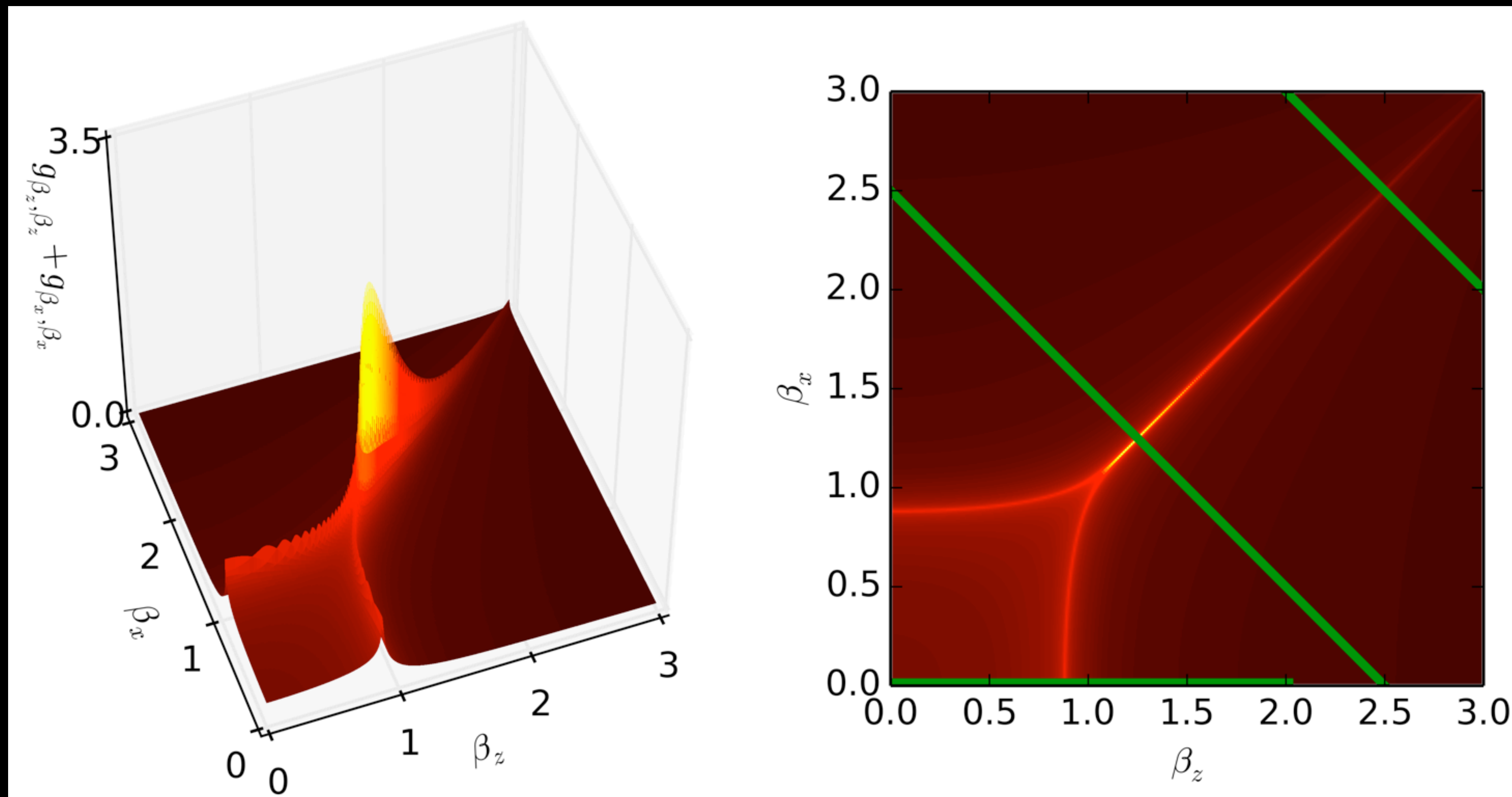


$$H = -\sigma_x - h_\square - \beta_x \tau_x - \beta_z \sigma_z \tau_z \sigma_z$$

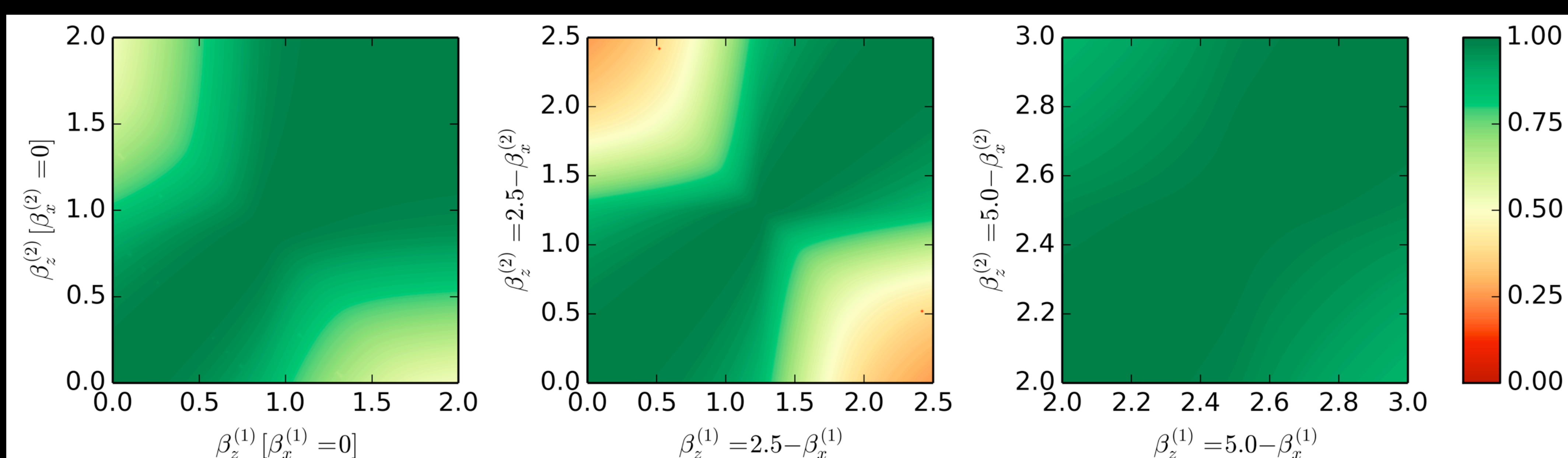
D=2 PEPS, with parent Hamiltonian:

$$H_p = e^{-\frac{\beta_x}{2}\tau_x - \frac{\beta_z}{2}\sigma_z\tau_z\sigma_z}(-\sigma_x - h_\square)e^{-\frac{\beta_x}{2}\tau_x - \frac{\beta_z}{2}\sigma_z\tau_z\sigma_z}$$

Phase-diagram parent Hamiltonian:



$$ds^2 = <\delta\psi|\delta\psi> - <\delta\psi|\psi><\psi|\delta\psi>$$



Conclusions/Outlook

For d=1+1 TNS formalism can simulate gauge theories with high precision. Specifically in those regimes (real-time, non-zero chemical potential) that are difficult/impossible for lattice Monte-Carlo. (also works for thermal states, see talk by H. Saito)

For higher dimensions we need better algorithms!

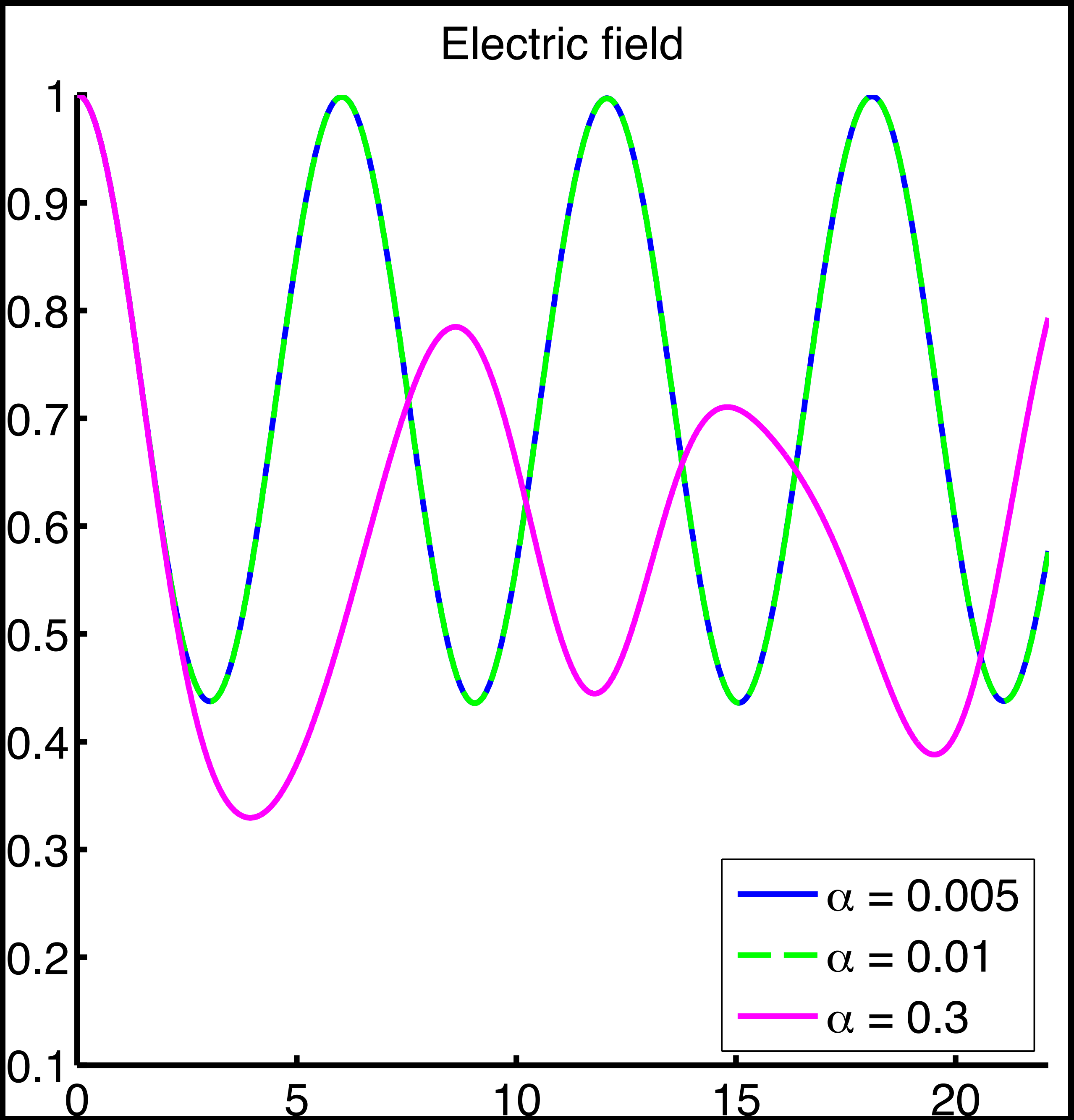
Still one can make already progress, by studying PEPS parent model Hamiltonians.

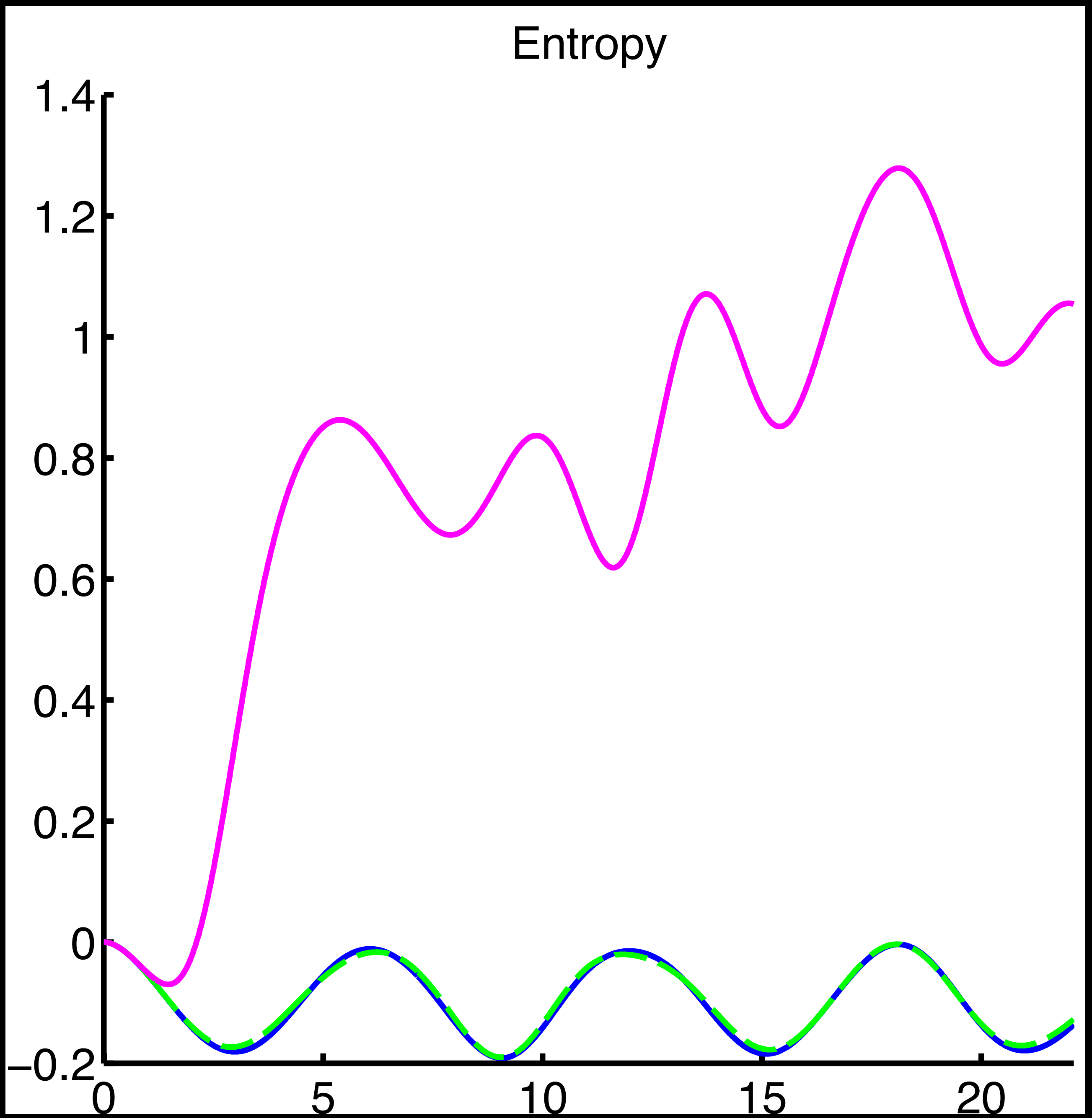
It should be possible to include fermions in this approach and study gauge theory phases at non-zero chemical potential, for d=2+1, d=3+1

Lots of things to do!!

Extra slides

Other values of the electric field background, linear response +beyond linear response, but no sign of thermalization





Exponential growth bond-dimension during linear growth entropy (orange line):