Update on the critical endpoint of the finite temperature phase transition for three flavor QCD with clover type fermions

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in collaboration with

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m_π at the endpoint at $\mu = 0$ (bottom-left corner of Columbia plot)

N_t	action	m_π^E [MeV]
4	unimproved staggered	260
6	unimproved staggered	150
4	p4-improved staggered	70
6	stout-improved staggered	$\lesssim 50$
6	HISQ	$\lesssim 45$
4	unimproved Wilson	~ 1100

- staggered type:[de Forcrand, Philipsen '07, Karsch, et. al. '03, Endrődi, et. al. '07, Ding, et. al. '11]
  - m_π^E decreases with decreasing lattice spacing
  - the crossover may persist down to $\sim 0.1 m^{phy}$
- Wilson tyep: [Iwasaki, et. al. '96]
  - 1st order at rather heavy m_q

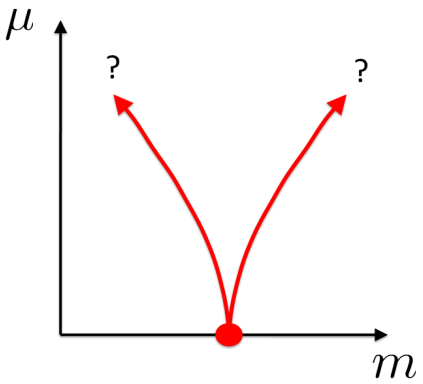
Motivation

- Critical endpoint (CEP) obtained with staggered and Wilson type fermions is inconsistent
- Results in the continuum limit is necessary

We determine CEP on $m_l = m_s$ line with clover fermions

$N_f = 3$ study is a stepping stone

- to the physical point
- curvature of critical surface
  → talk by S. Takeda [15:15 Tue]



Distinguishing between 1st, 2nd and crossover

criterion	first order	second order	crossover
distribution	double peak	single peak	singe peak
χ_{peak}	$\propto N_l^d$	$\propto N_l^{\gamma/\nu}$	-
$\beta(\chi_{\mathrm{peak}}) - \beta_c$	$\propto N_l^{-d}$	$\propto N_l^{-1/\nu}$	-
kurtosis at $N_l \to \infty$	K= -2	-2 < K < 0	-

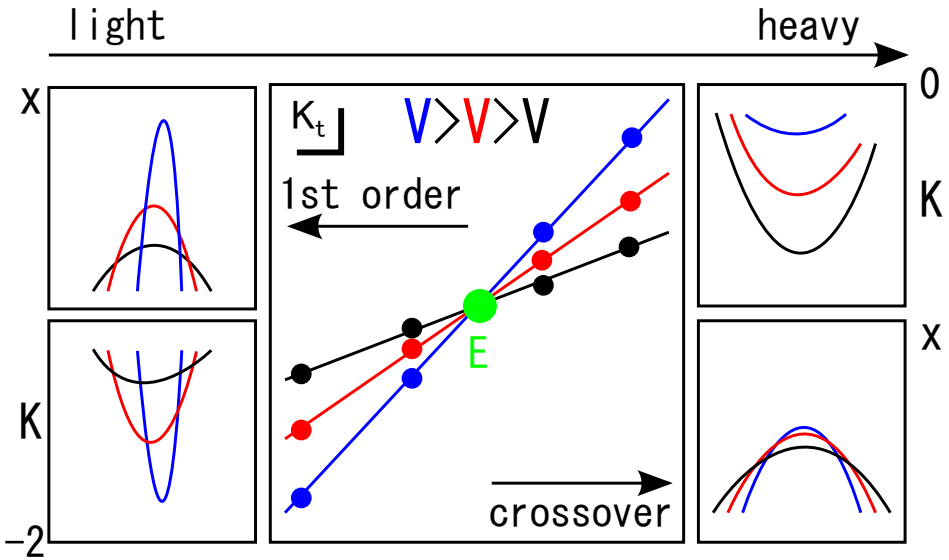
- scaling might work with wrong exponents near CEP
- peaks in histgram might emerge only at large N_l on weak 1st order
- K does not depend on volume at 2nd order phase transition point

$$M = N_l^{-\beta/\nu} f_M(tN_l^{1/\nu})$$

$$K + 3 = B_4(M) = \frac{N_l^{-4\beta/\nu} f_{M^4}(tN_l^{1/\nu})}{[N_l^{-2\beta/\nu} f_{M^2}(tN_l^{1/\nu})]^2} = f_B(tN_l^{1/\nu})$$

Method to determine CEP (kurtosis intersection)

- determine the transition point (peak position of susceptibility)
- determine kurtosis at transition point at each spatial lattice size
- find intersection point of kurtosis by fit, $K_E + aN_l^{1/\nu}(\beta - \beta_E)$
  → other method (gap of masses), talk by X.-Y. Jin [14:55 Tue]

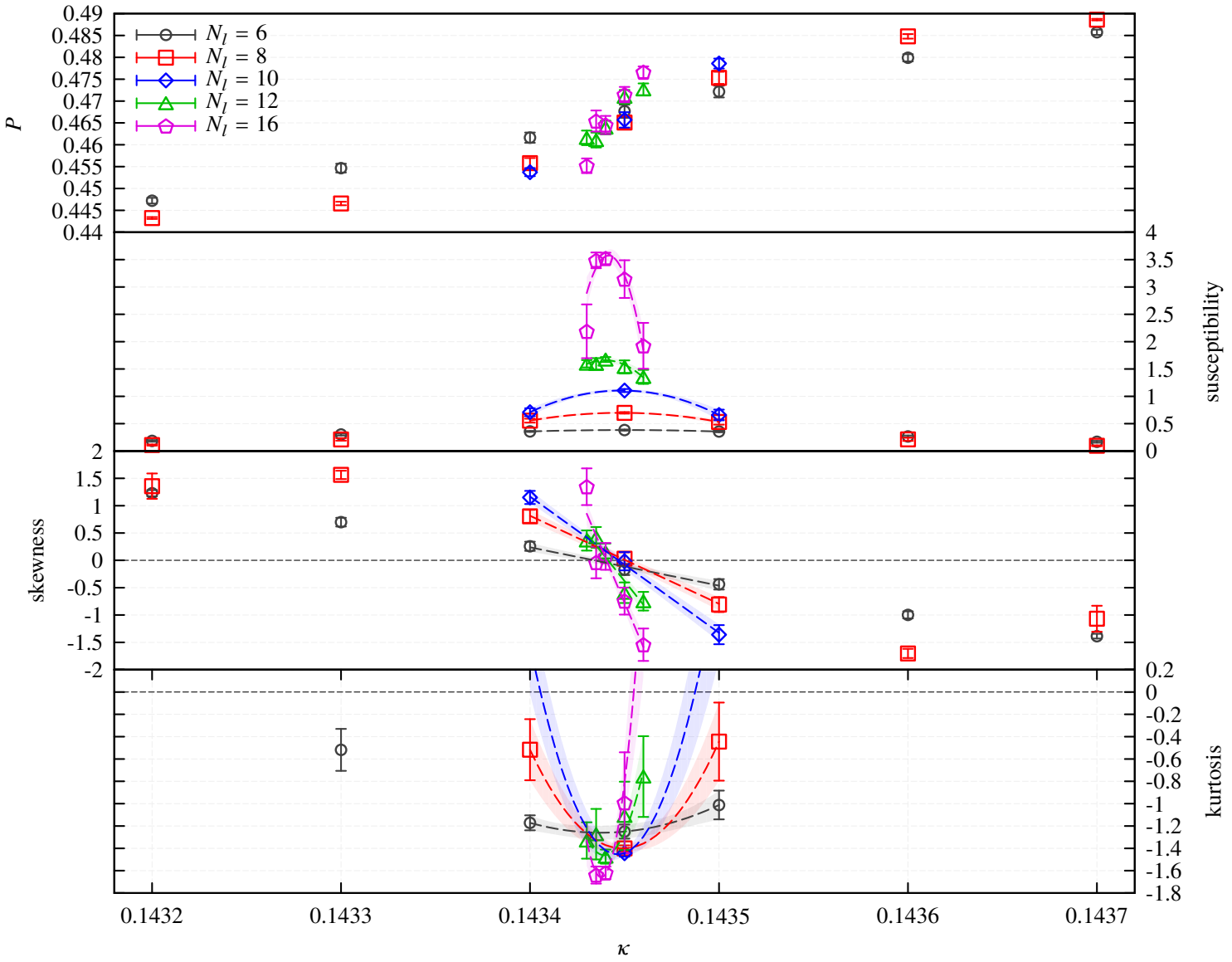


- interpolate/extrapolate $(m_{PS}/m_V)_t$ measured at transition point to β_E
- extrapolate $(m_{PS}/m_V)_E$ to the continuum limit

Simulations

- action: Iwasaki gluon + $N_f = 3$ clover
  (non perturbative c_{SW}, degenerate)
- temporal lattice size $N_t = 4, 6, 8$ for continuum extrapolation
- statistics: O(200,000) traj.
- observables: gauge action density, plaquette, Polyakov loop, chiral condensate and their higher moments

plaquette at $\beta = 1.60$, $N_t = 4$



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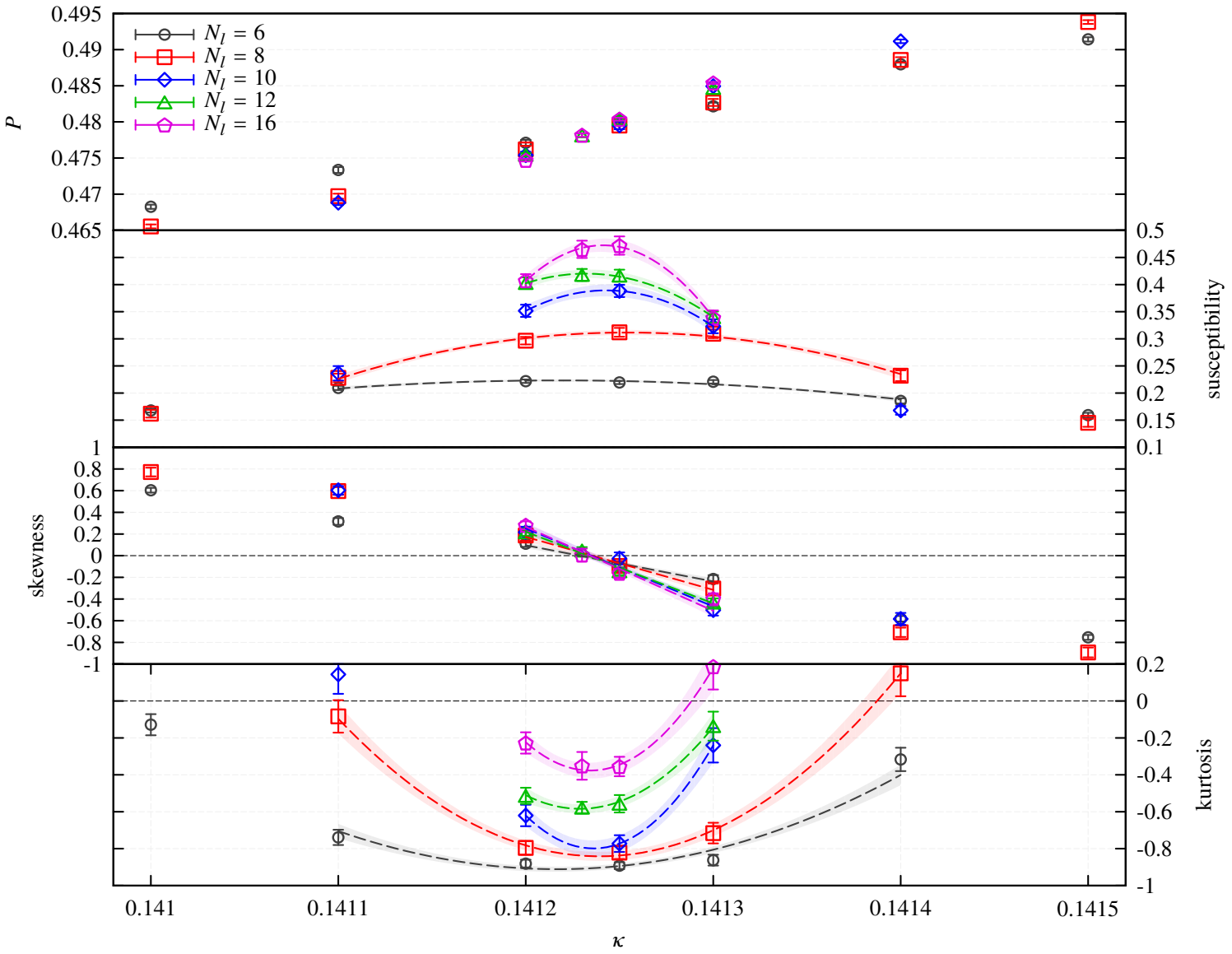
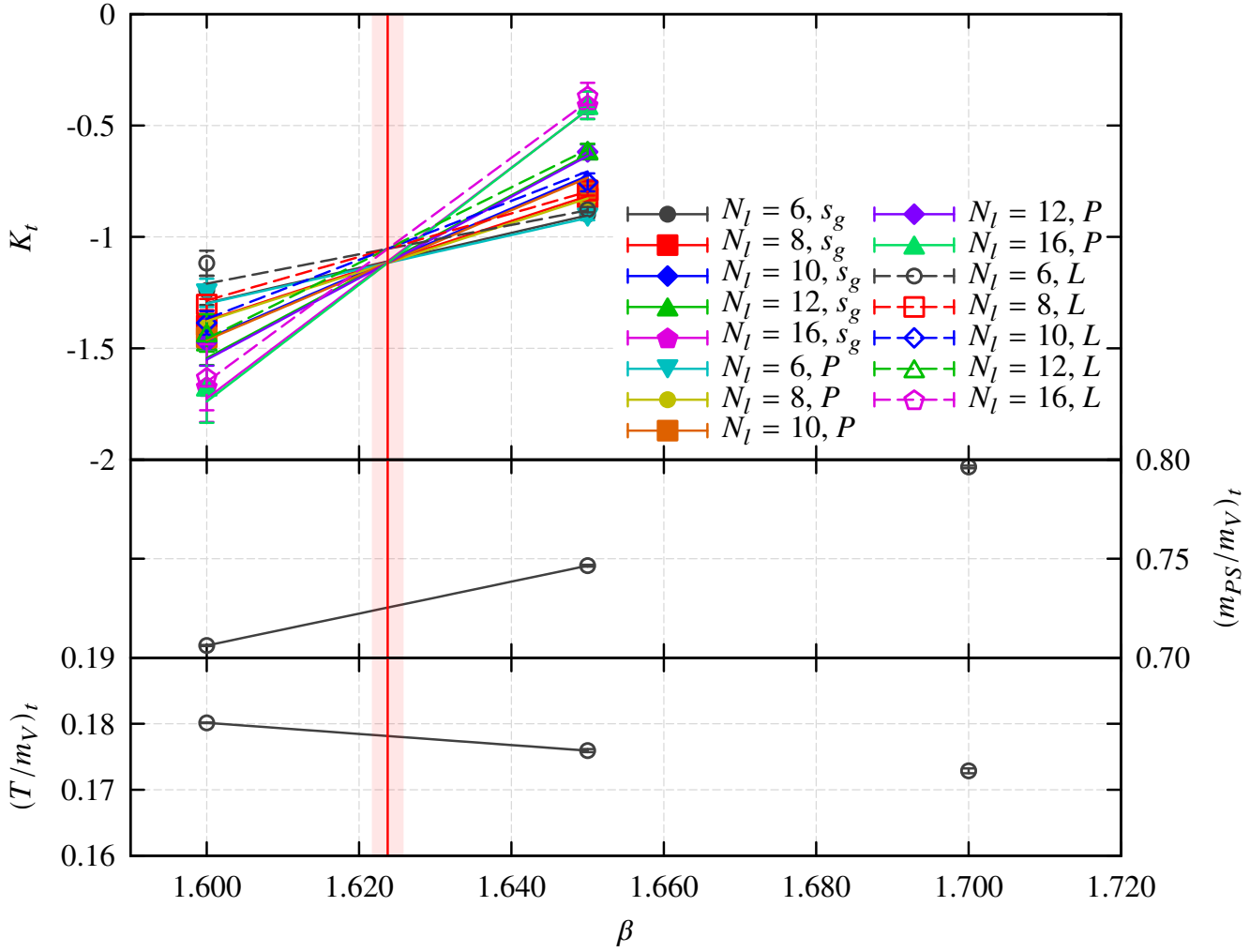


Kurtosis intersection at $N_t = 4$



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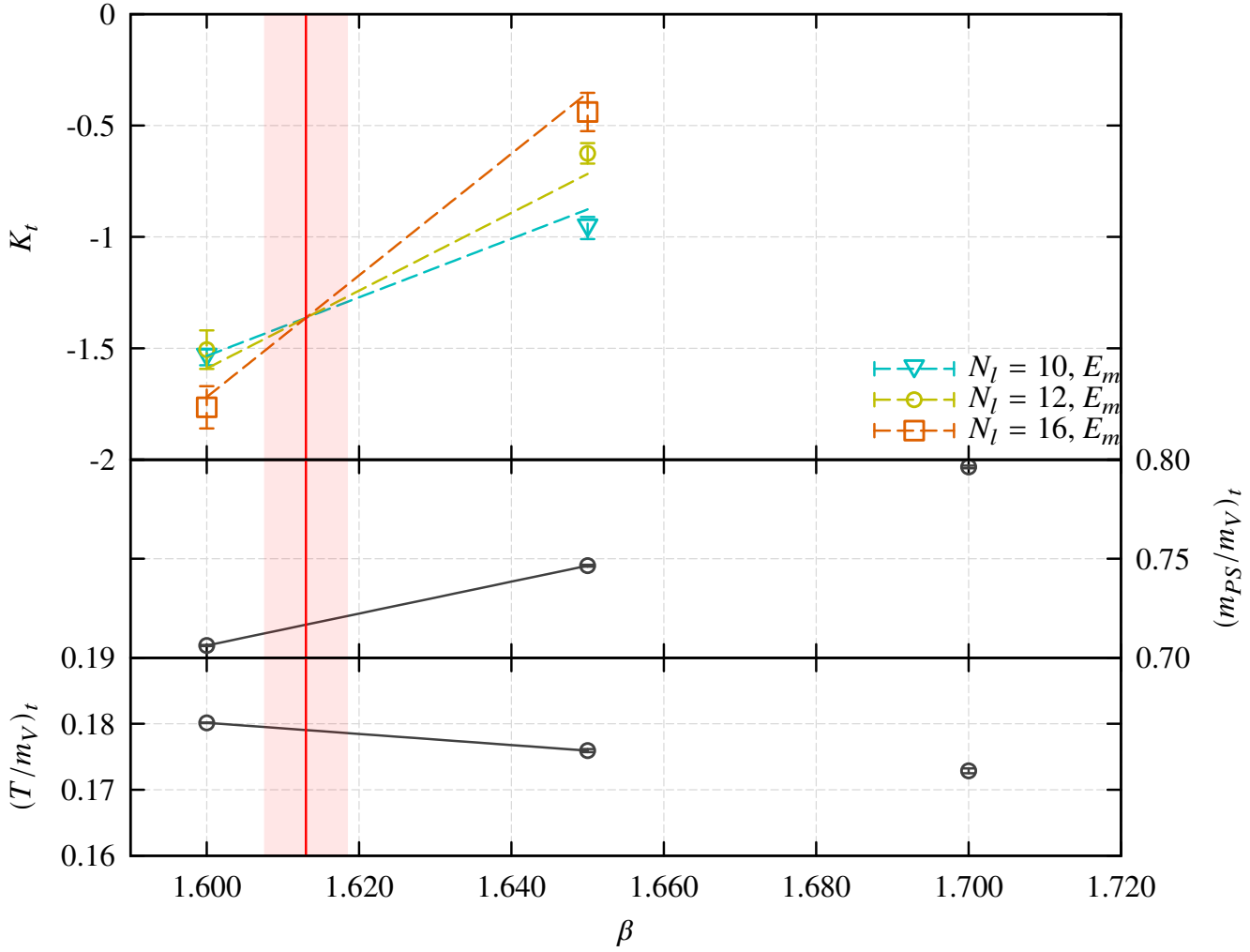
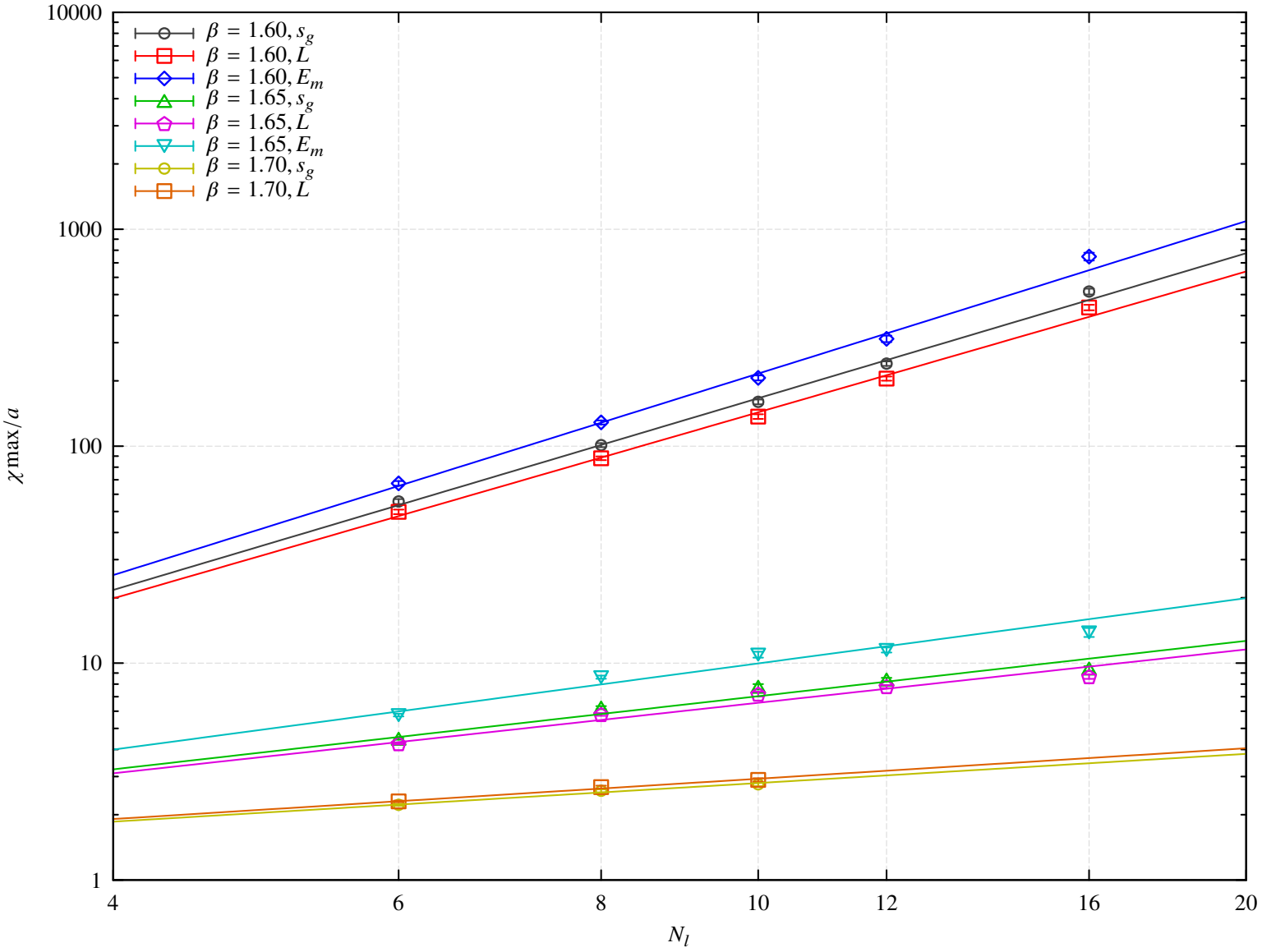
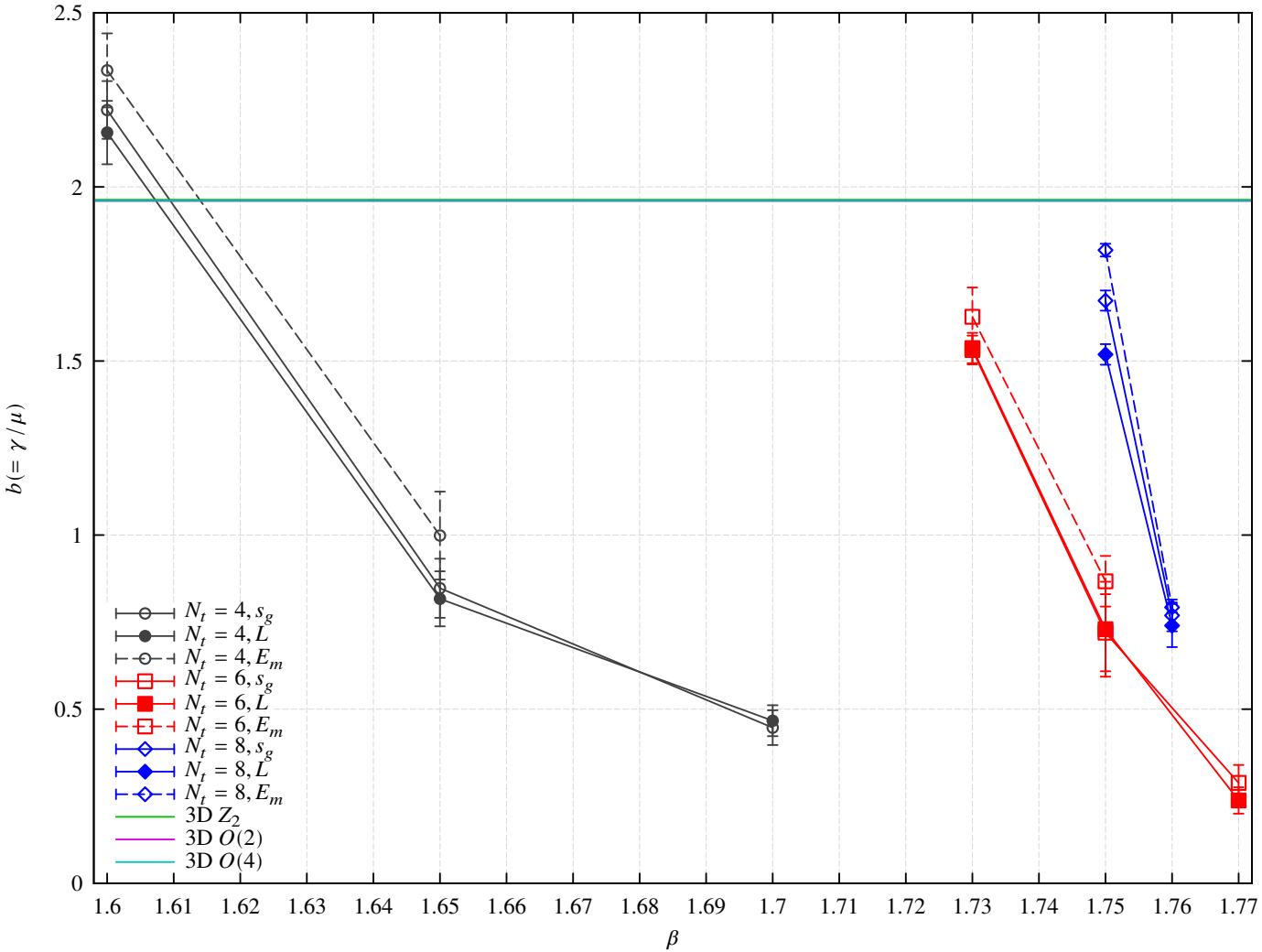


$\chi_{\max}$ fit: aN_l^b

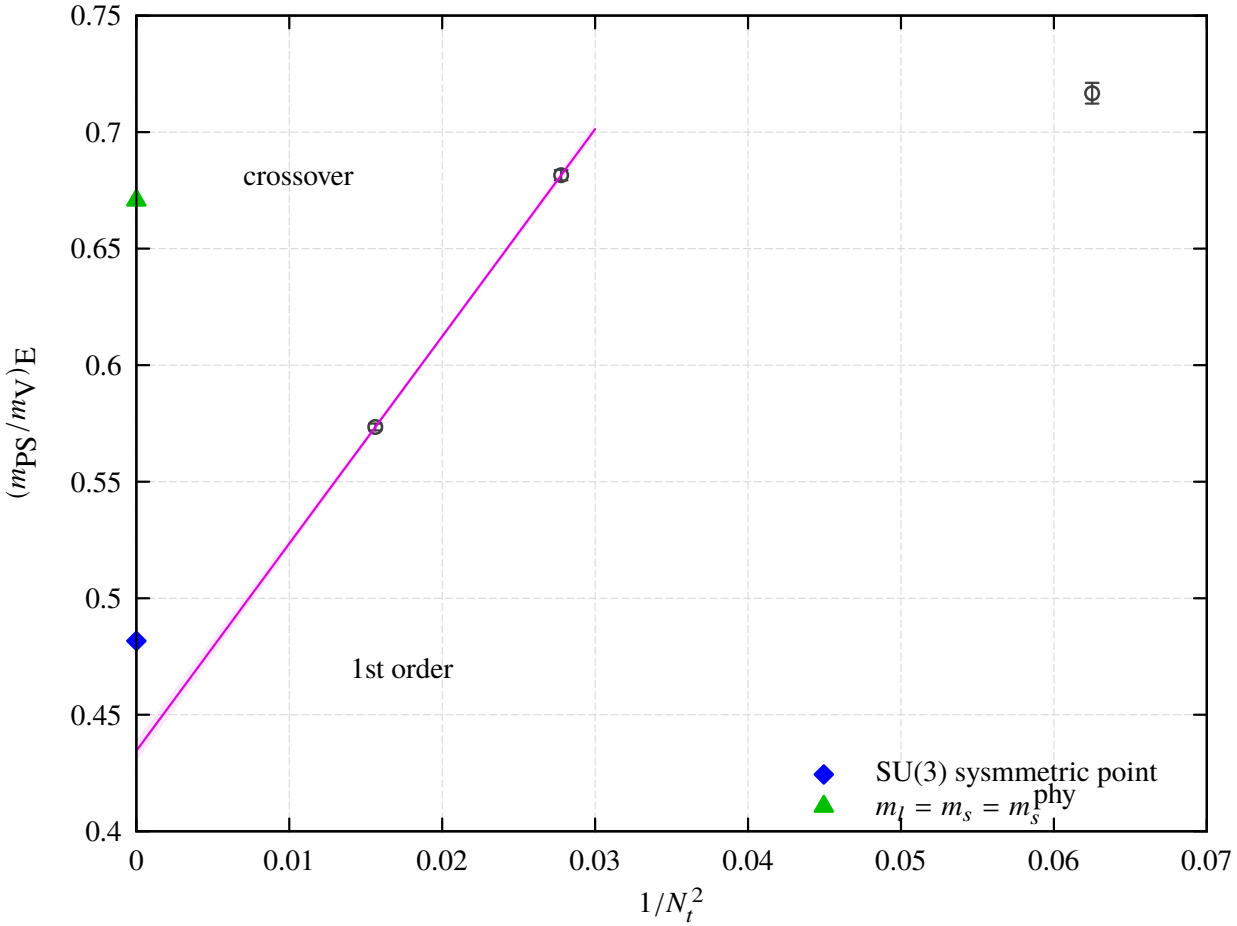


γ/ν v.s. β



N_t	4	6	8
$\beta_{\mathrm{E}}(E_m)$	1.6130(55)	1.7269(42)	1.7505(19)
$\beta_{\mathrm{E}}(P, s_g, L)$	1.6238(21)	1.7361(16)	1.75491(92)

continuum extrapolation for $(m_{\mathrm{PS}}/m_{\mathrm{V}})_{\mathrm{E}}$



◆ : $m_{PS}^{phy;sym}/m_V^{phy;sym} = \sqrt{(m_\pi^2 + 2m_K^2)/3}/[(m_\rho + 2m_{K^*})/3] \sim 0.4817$

▲ : $m_{\eta_{ss}}/m_\phi \sim 0.6719$

Summary

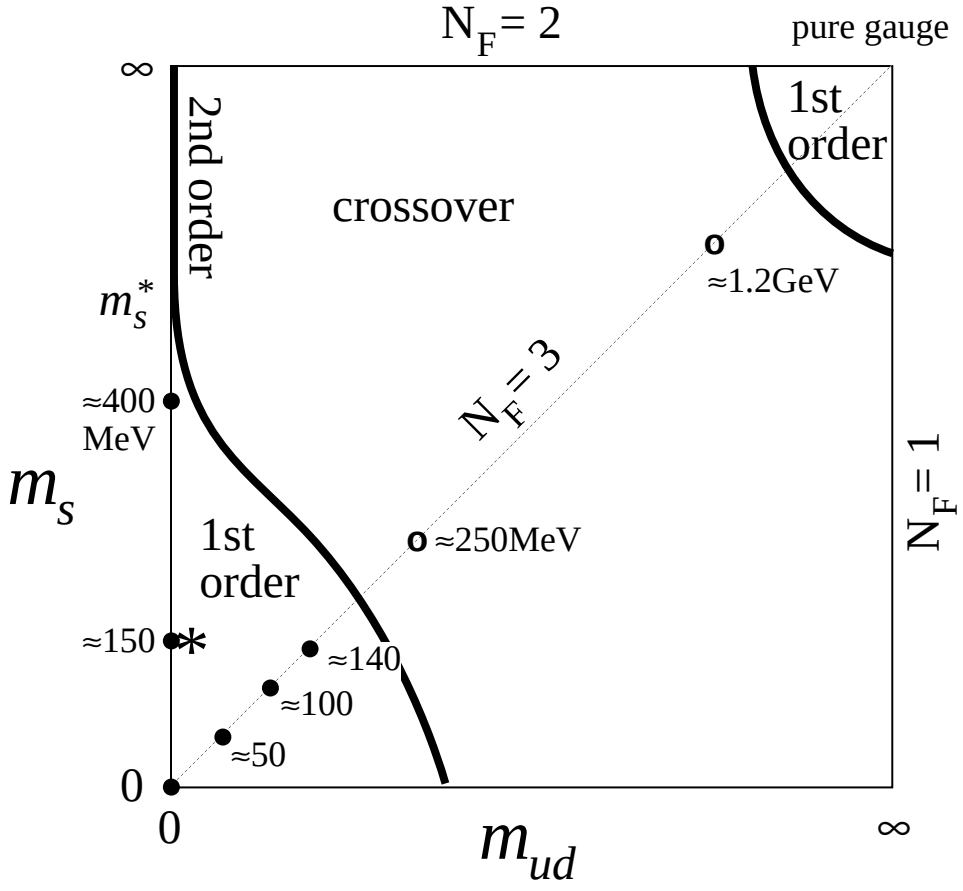
We have investigated the critical endpoint of QCD with clover fermions and determined the critical endpoint by using the intersection points of the Binder cumulants at $N_t = 4, 6, 8$ and extrapolated to the continuum limit

- kurtosis intersection analysis is consistent with χ_{max} analysis
- $(m_{\mathrm{PS}}/m_{\mathrm{V}})_{\mathrm{E}}$ at $N_t = 4$ is out of scaling region
- $(m_{\mathrm{PS}}/m_{\mathrm{V}})_{\mathrm{E}}$ in the continuum limit is smaller than the SU(3) sysmmetric point, not so small as staggered type fermions at $N_t = 6$ and it will be controlled by values at larger N_t

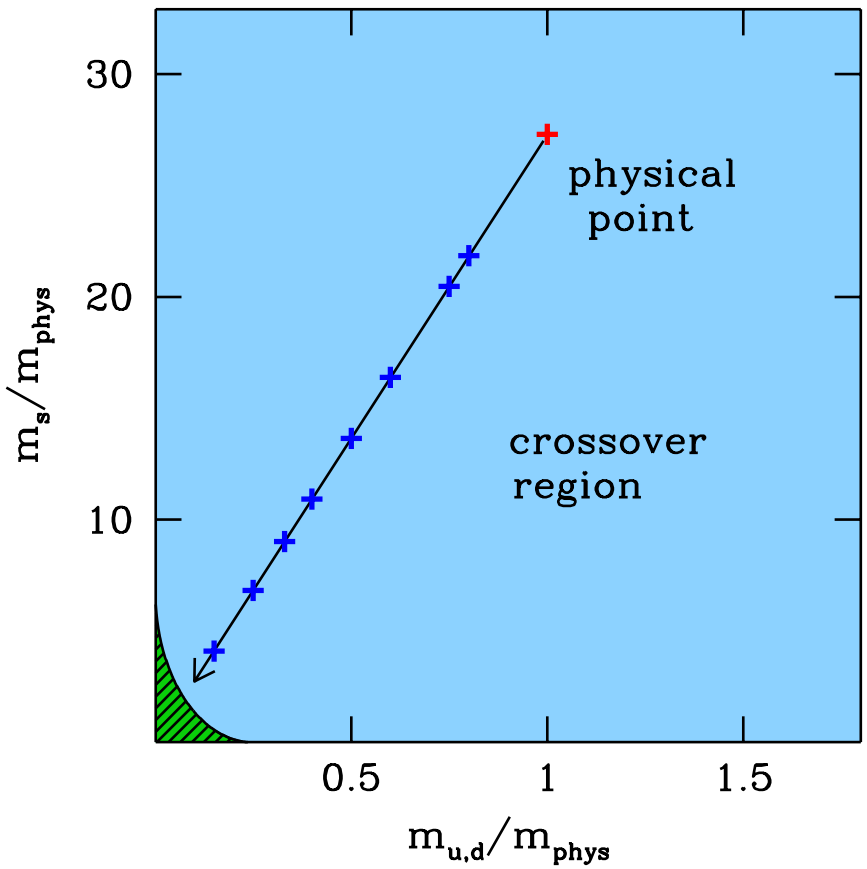
Backup slides

Columbia plot

- inconsistent results: Wilson and staggered type fermion



Wilson



staggered

Higher moments

i-th derivative of $\ln Z$ with respect to control parameter c:

$$E = \frac{\partial \ln Z}{\partial c}$$

- Variance

$$V = \frac{\partial^2 \ln Z}{\partial c^2} = \sigma^2$$

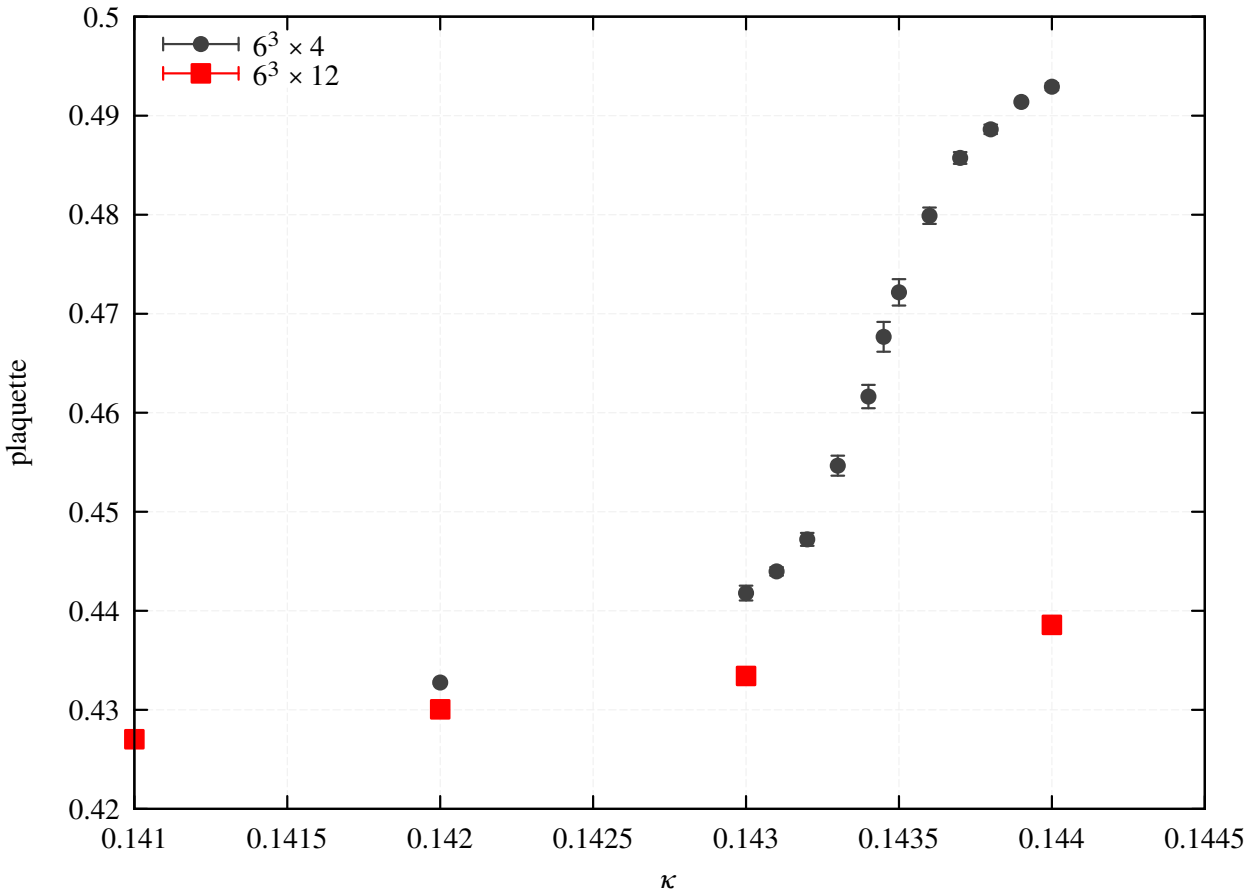
- Skewness (e.g. right-skewed $\rightarrow S > 0$, left-skewed $\rightarrow S < 0$)

$$S = \frac{1}{\sigma^3}\frac{\partial^3 \ln Z}{\partial c^3}$$

- Kurtosis(e.g. Gaussian $\rightarrow K = 0$, 2δ func. $\rightarrow K = -2$)

$$K = \frac{1}{\sigma^4}\frac{\partial^4 \ln Z}{\partial c^4} = B_4 - 3$$

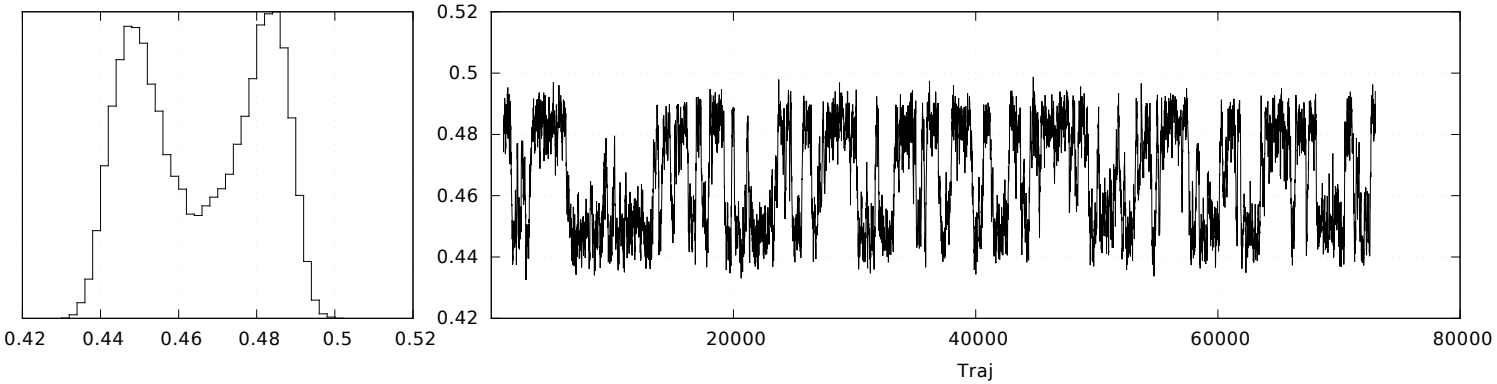
Finite temperature phase transition



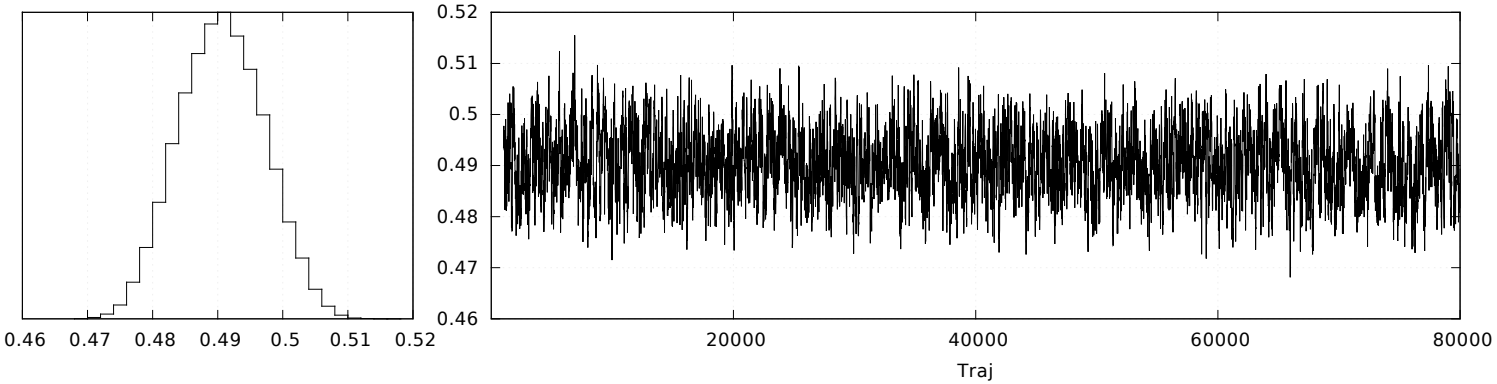
- Plaquette v.s. κ at lowest β **(= 1.60)**
- no bulk phase transition

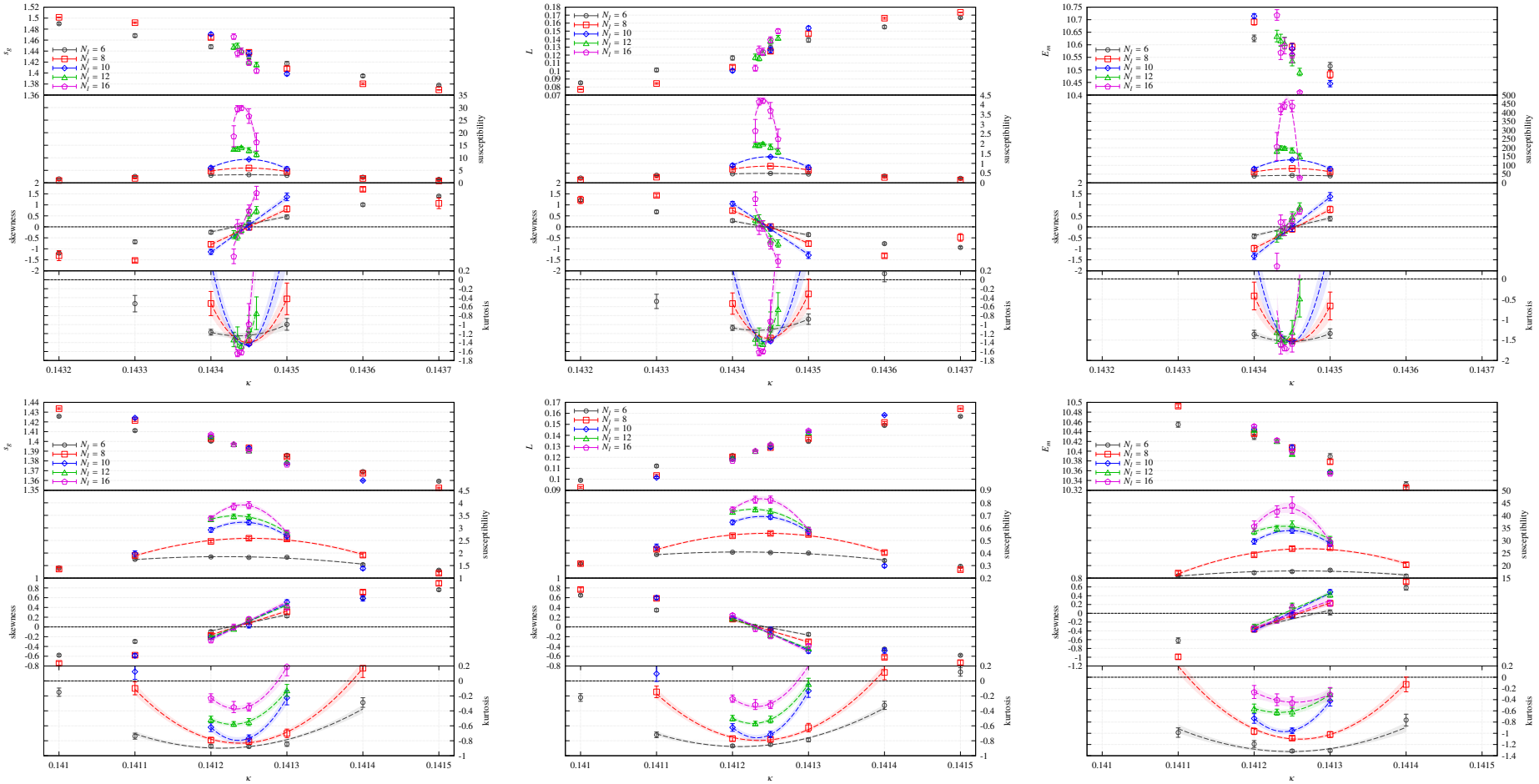
1st order phase transition and crossover (like)

$\beta = 1.60$ and $\kappa = 0.14345$ on $10^3 \times 4$, clear two states, $K \sim -1.5$

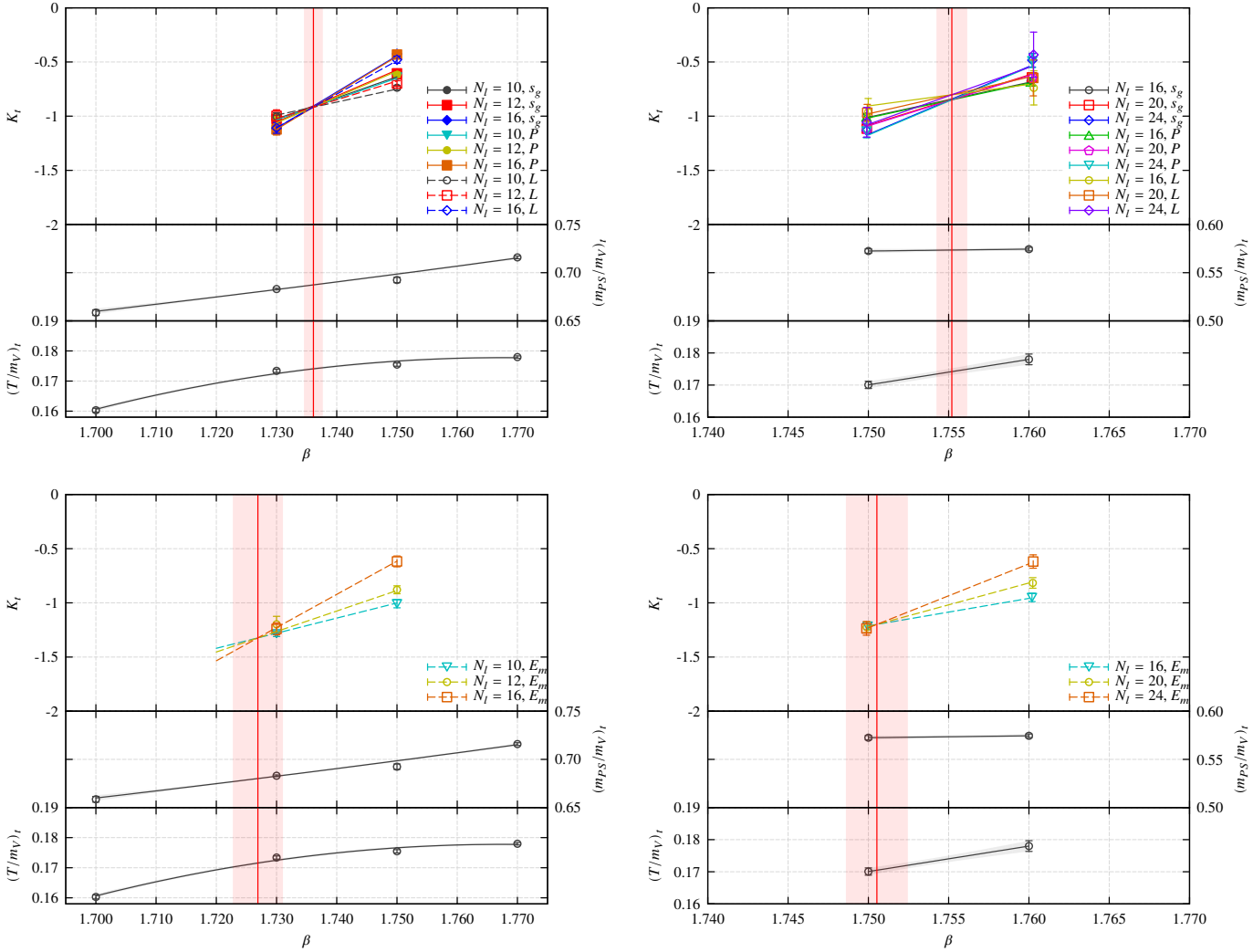


$\beta = 1.70$ and $\kappa = 0.13860$ on $10^3 \times 4$, one state, $K \sim -0.5$



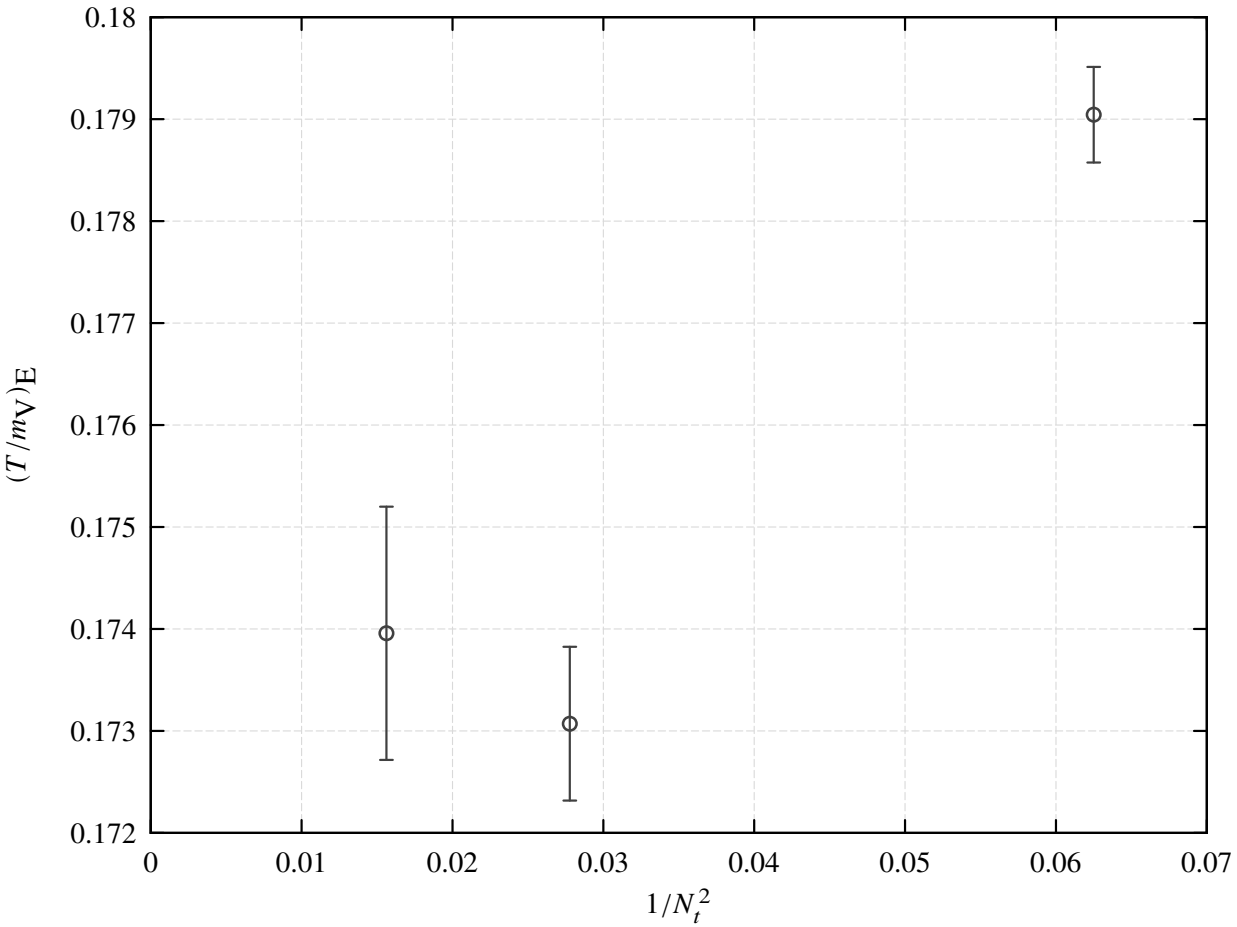

$N_f = 6$
$N_f = 8$
$N_f = 10$
$N_f = 12$
$N_f = 16$
s_g
L
E_m
susceptibility
skewness
kurtosis
κ

Critical endpoint at $N_t = 6, 8$



$N_t = 6$ $N_t = 8$

continuum extrapolation for $(T/m_V)_{\mathrm{E}}$



K_{E} and critical exponent ν

N_t	K_{E}	ν	class
4	-1.363(88)	0.64(11)	
6	-1.323(76)	0.60(14)	
8	-1.199(72)	0.48(14)	
	-1.396	0.63	3D Z2
	-1.758	0.67	3D O(2)
	-1.908	0.75	3D O(4)

K_E and ν are consistent with values of 3D Z2.