A Feynman-Hellmann approach to the spin structure of hadrons

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also M. Constantinou for renormalisation

Outline

- Motivation: Spin of the proton
- Feynman-Hellmann
- Quark-line connected spin fractions [arXiv:1405.3019]
- Disconnected quark contributions
- Tensor charge
- Summary and Future work

Motivation for Investigation of Hadron Spin

- Proton "Spin Crisis": only 33(3)(5)% of the proton spin carried by quarks

- i.e. with decomposition $$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$ [X.Ji (1996)]

$$\Rightarrow \Delta\Sigma = \Delta u + \Delta d + \Delta s \approx 33\%$$ [COMPASS (2007)]

- Quark model $\Rightarrow$ $\Delta\Sigma = 1$

$$\Delta q = \int_0^1 [\Delta q(x) + \Delta\bar{q}(x)]dx$$

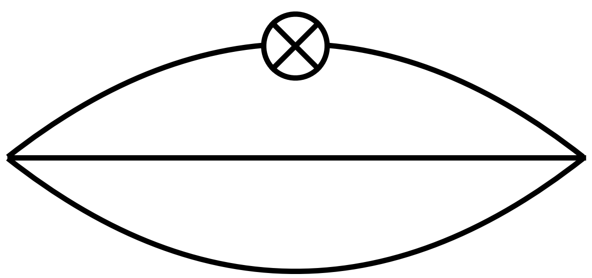
$$\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$

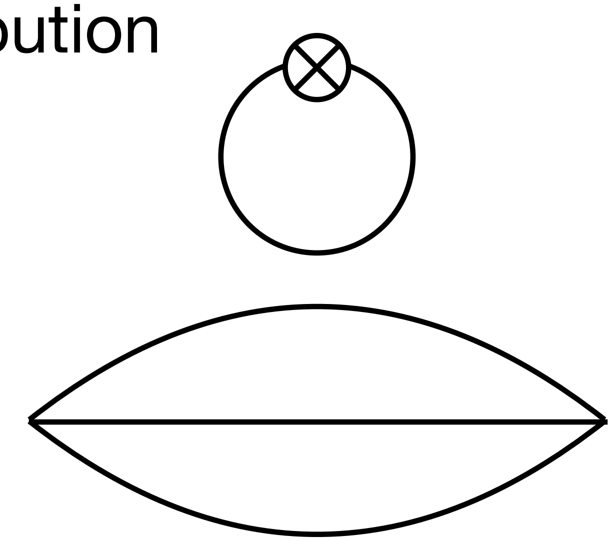
- Can we reproduce this feature directly from QCD?

- Is this suppression a property of the nucleon, or a universal feature?

Motivation for Investigation of Hadron Spin

- Δs is a purely quark-line disconnected contribution





- Much effort to determine Δs experimentally $\int_0^1 g_1^p(x)\,dx = \frac{1}{36}(4a_0 + 3a_3 + a_8)$
  - e.g. COMPASS, HERMES
    $x \geq 0.004$ $\quad$ $x \geq 0.02$

Also g_A and semileptonic hyperon decays assuming SU(3) symmetry

$$-0.15 \leq \Delta s \leq -0.03$$

- A challenge on the lattice
  - Usually tackled through stochastic estimation of nucleon 3pt function
    - e.g. PRL108, 222001 (arXiv:1112.3354)

Many new results, Tuesday late parallel session

$m_\pi = 285\,\mathrm{MeV}$

$\overline{\mathrm{MS}}$ $\quad \mu = \sqrt{7.4}\,\mathrm{GeV}$

$$\Delta s = -0.020(10)(4)$$

Feynman-Hellmann Theorem

- Provides a method for determining hadronic matrix elements from energy shifts
- Suppose we want

$$\langle H|\mathcal{O}|H\rangle$$

- Proceed by

$$S \to S + \lambda \int \mathrm{d}^4x\, \mathcal{O}(x)$$

real parameter

local operator, e.g. $\bar{q}(x)\gamma_5\gamma_3 q(x)$

- FH tells us

$$\frac{\partial E_H(\lambda)}{\partial\lambda} = \frac{1}{2E_H(\lambda)}\left\langle H\left|\frac{\partial S(\lambda)}{\partial\lambda}\right|H\right\rangle$$

$$\Rightarrow \quad \frac{\partial E_H(\lambda)}{\partial\lambda} = \frac{1}{2E_H(\lambda)}\langle H|\mathcal{O}|H\rangle$$

- Calculation of matrix element $\equiv$ hadron spectroscopy

Feynman-Hellmann Theorem

- Most commonly used to determine σ terms since

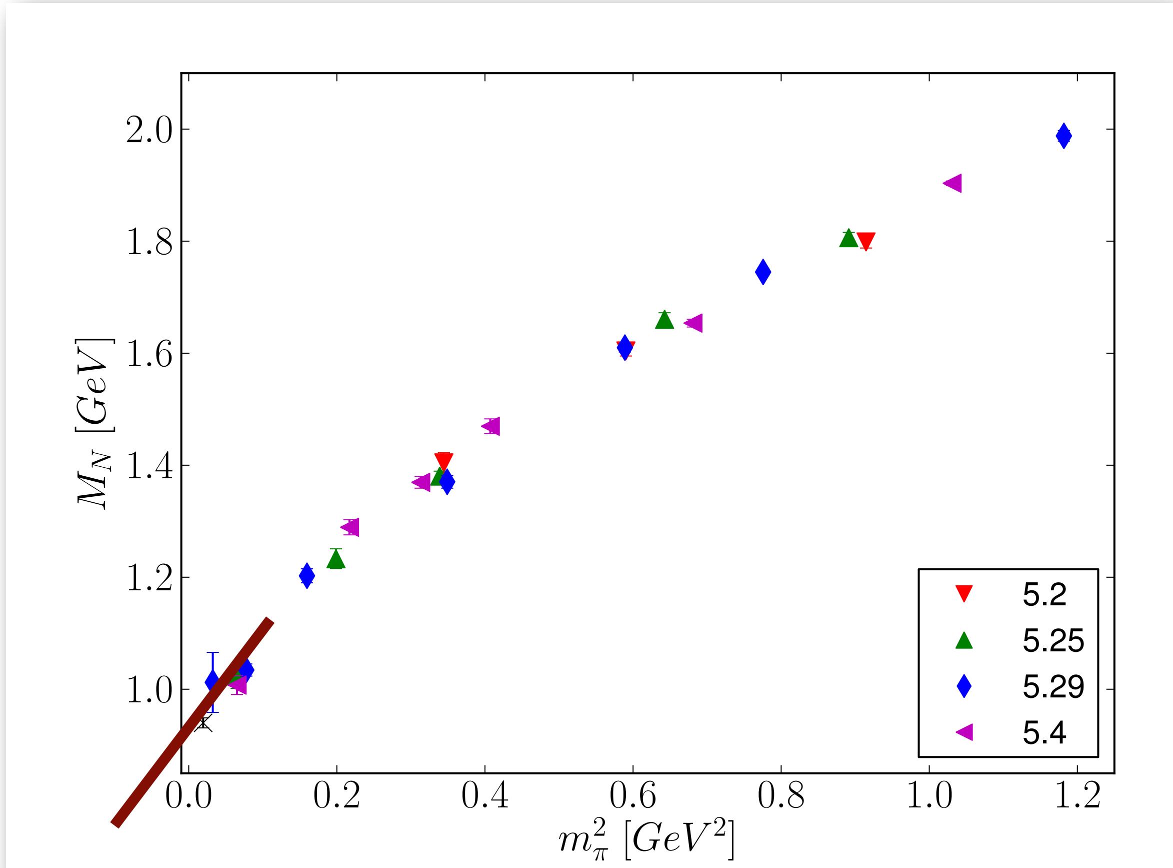
$$\sigma_l^H = m_l \langle H | (\bar{u}u + \bar{d}d) | H \rangle \qquad \sigma_s^H = m_s \langle H | \bar{s}s | H \rangle$$

- and

$$S = \sum_q \left[m_q \bar{q}q + \bar{q}Dq \right]$$

plays the role of λ

$$\sigma_{\pi N} \approx m_\pi^2 \frac{dm_N}{dm_\pi^2}\bigg|_{m_\pi = m_\pi^{\rm phys}}$$



Feynman-Hellmann Theorem

- To access hadron spin fractions, we modify the action to include the axial current

$$S \to S + \lambda \sum_x \bar{q}(x) i\gamma_5\gamma_3 q(x)$$

- FH Theorem then gives

$$\left.\frac{\partial E_H(\lambda)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{2M_H}\langle H|\bar{q}i\gamma_5\gamma_3 q|H\rangle$$

- but for a spin-J hadron with polarisation m in the z-direction

$$\langle H, Jm|\bar{q}i\gamma_5\gamma_3 q|H, Jm\rangle = 2M_H \Delta q^{Jm}$$

$$\Rightarrow \quad \Delta q = \left.\frac{\partial E_H(\lambda)}{\partial\lambda}\right|_{\lambda=0}$$

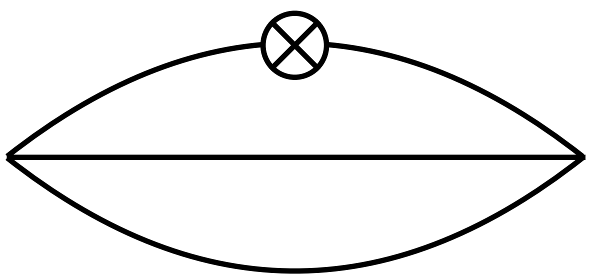
- Also note: reversing hadron polarisation $\equiv$ changing sign of λ

Lattice Set-Up

- N_f =2+1 O(a)-improved Clover fermions ("SLiNC" action)
- Tree-level Symanzik gluon action (plaq + rect)
- Results from a single lattice spacing ($a \sim 0.074$fm), and volume ($32^3 \times 64$)
- Novel method for tuning the quark masses
- Most results are at the SU(3)-symmetric point
- Connected results also at 360MeV
- ~350 measurements from 1500 trajectories

Connected Spin Contributions

- Use existing $N_f=2+1$ configurations

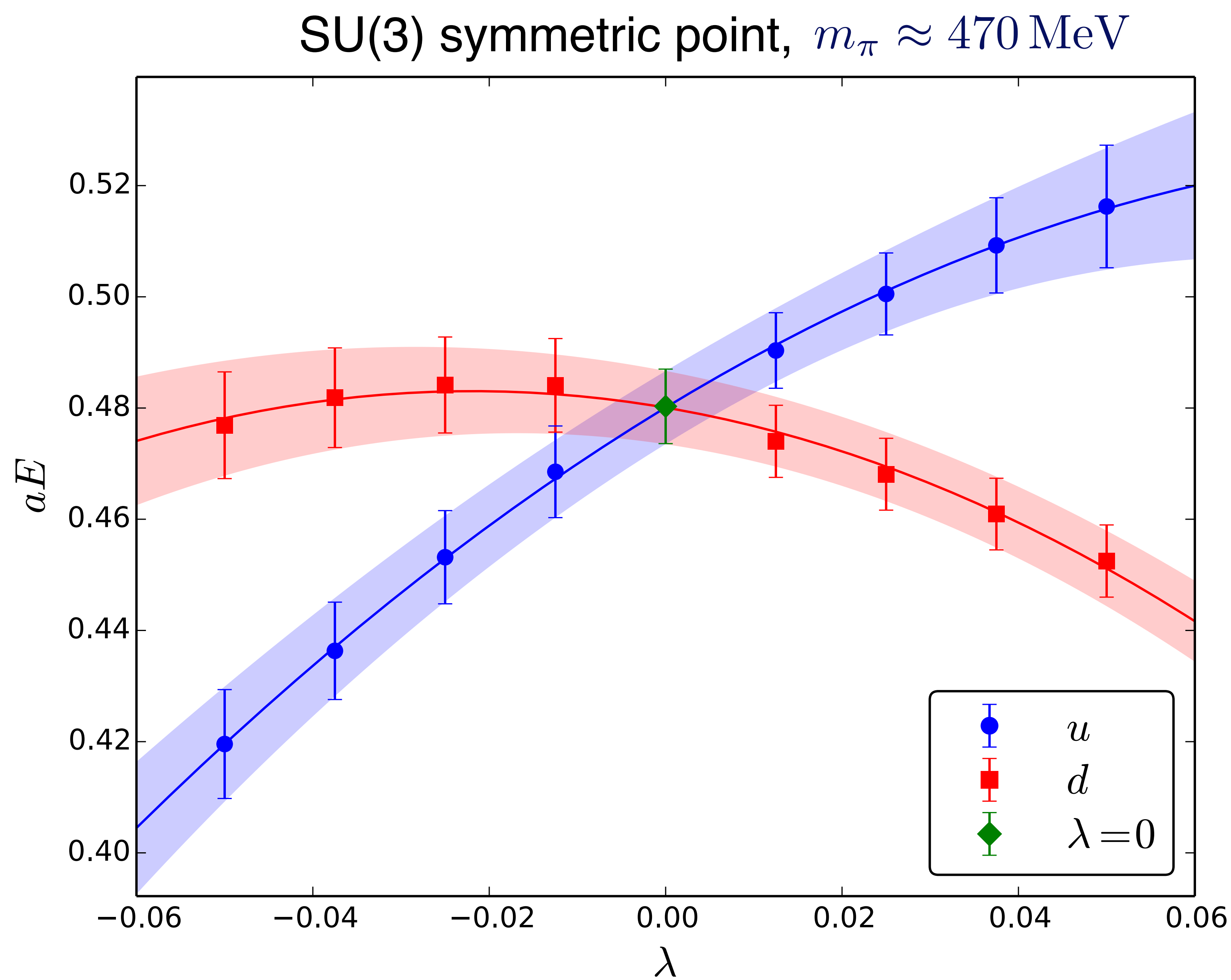


- Modify the action of the valence quarks only
- Allows for comparison with results using standard 3-point function methods
- For more details see

A. Chambers *et al.* (QCDSF), arXiv:1405.3019

Connected Spin Contributions

- Start with nucleon mass v λ



Fit: quadratic in λ → linear terms give Δu and Δd

Connected Spin Contributions

SU(3) symmetric point, $m_\pi \approx 470\,\mathrm{MeV}$

- Linear terms give (unrenormalised):

$$\begin{aligned} \Delta u^{latt}_{\mathrm{conn}} &= 0.97(13) \\ \Delta d^{latt}_{\mathrm{conn}} &= -0.27(11) \end{aligned}$$

- Compare with the 3-point method using a similar size ensemble

$$\begin{aligned} \Delta u^{latt}_{\mathrm{conn}} &= 0.911(29) \\ \Delta d^{latt}_{\mathrm{conn}} &= -0.290(16) \end{aligned}$$

- Good agreement, but statistical error a concern

Connected Spin Contributions

- We can make use of the correlations between the results at different λ obtained on the same set of configurations

- Observe:

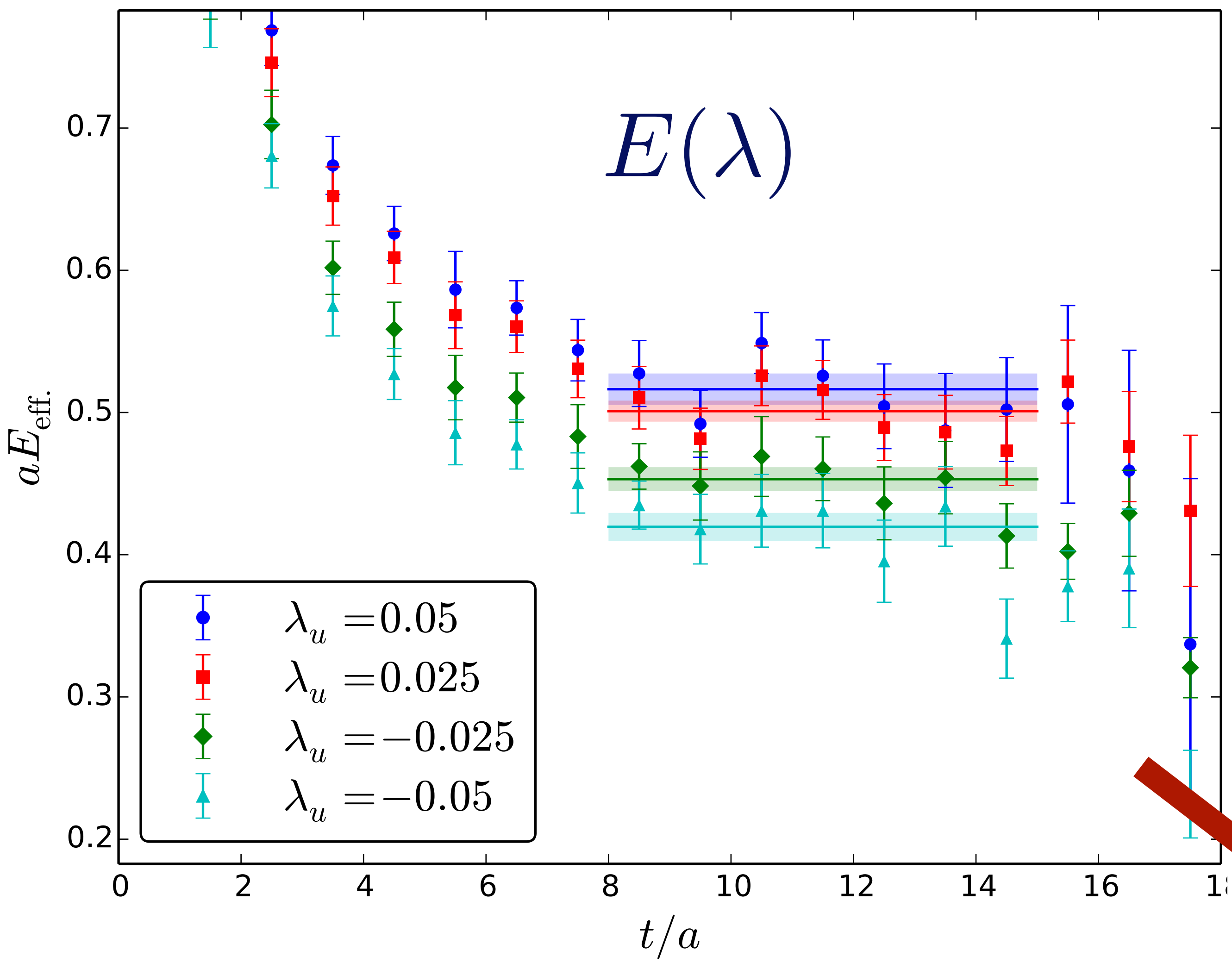
$$E(\lambda) = E(\lambda = 0) + \Delta E(\lambda)$$

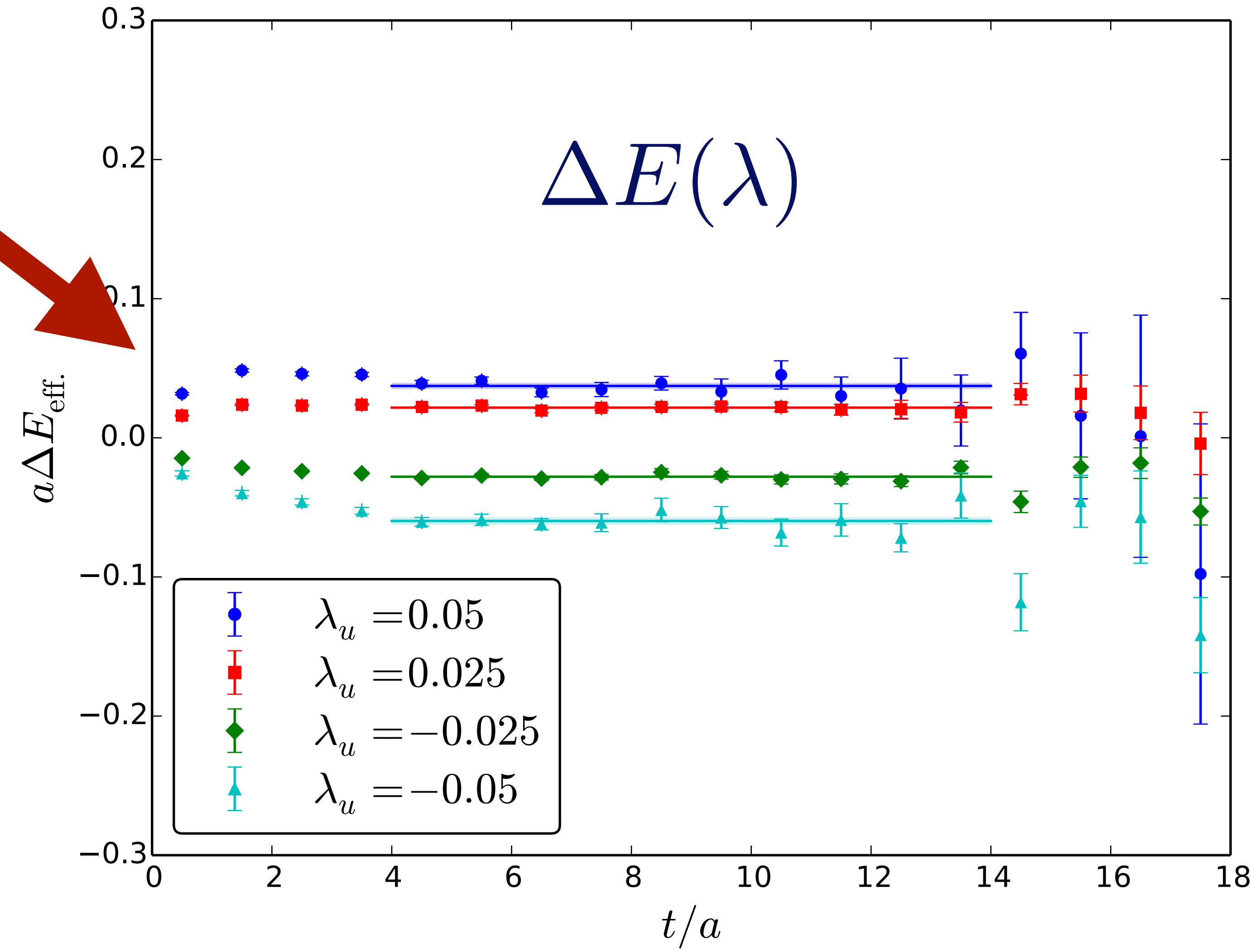
$$\Rightarrow \qquad \Delta q = \left.\frac{\partial \Delta E_H(\lambda)}{\partial \lambda}\right|_{\lambda=0}$$

- And $\Delta E_H(\lambda)$ can be computed from a ratio of 2-point functions

$$\frac{C(\lambda, t)}{C(\lambda = 0, t)} \xrightarrow{\text{large } t} \propto e^{-\Delta E_H(\lambda) t}$$

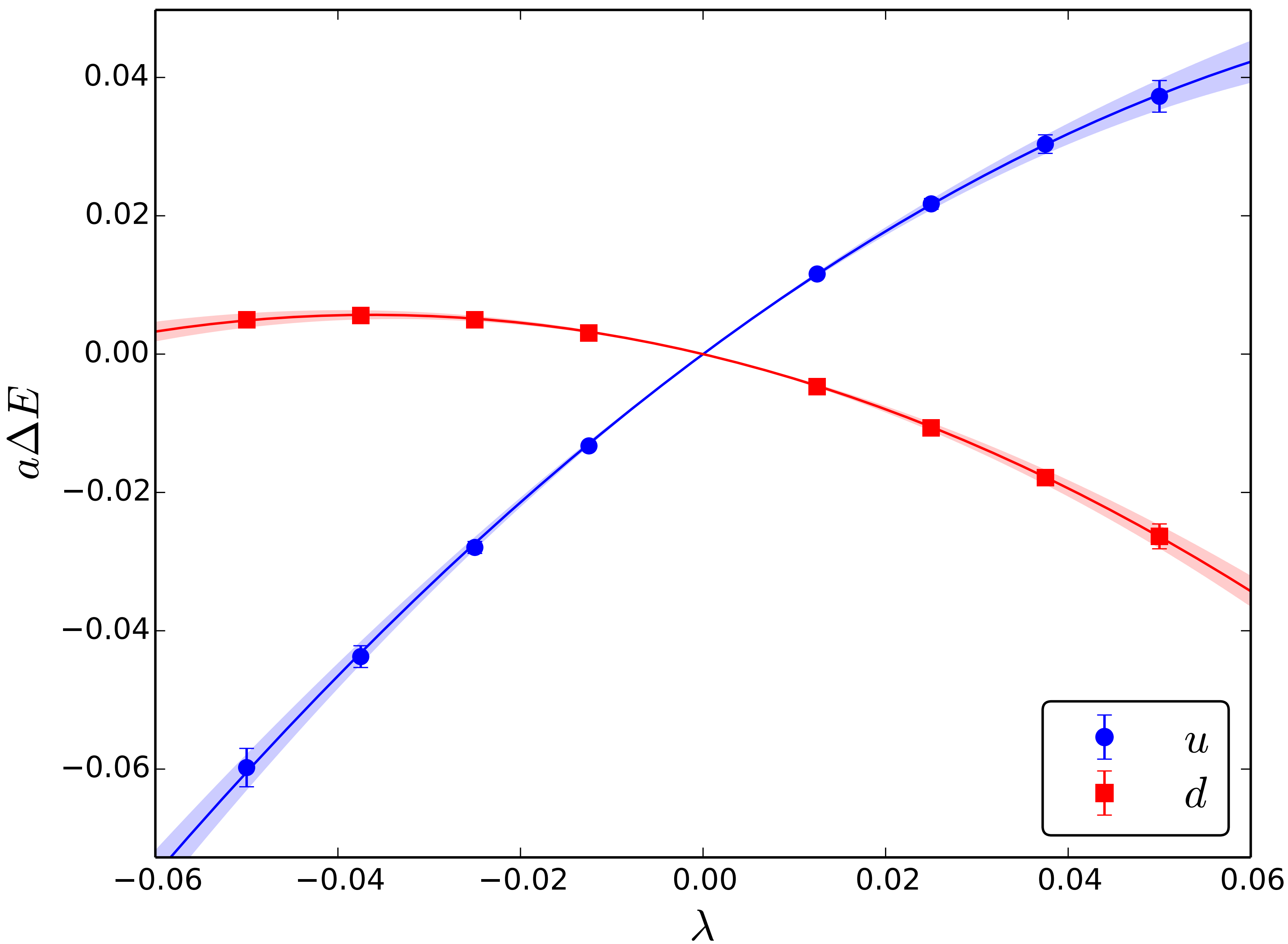
Connected Spin Contributions





Connected Spin Contributions

- Energy shift v λ



Linear terms give:

$$\Delta u^{latt}_{\mathrm{conn}} = \quad 0.990(20)$$
$$\Delta d^{latt}_{\mathrm{conn}} = -0.313(14)$$

Recall 3-point results:

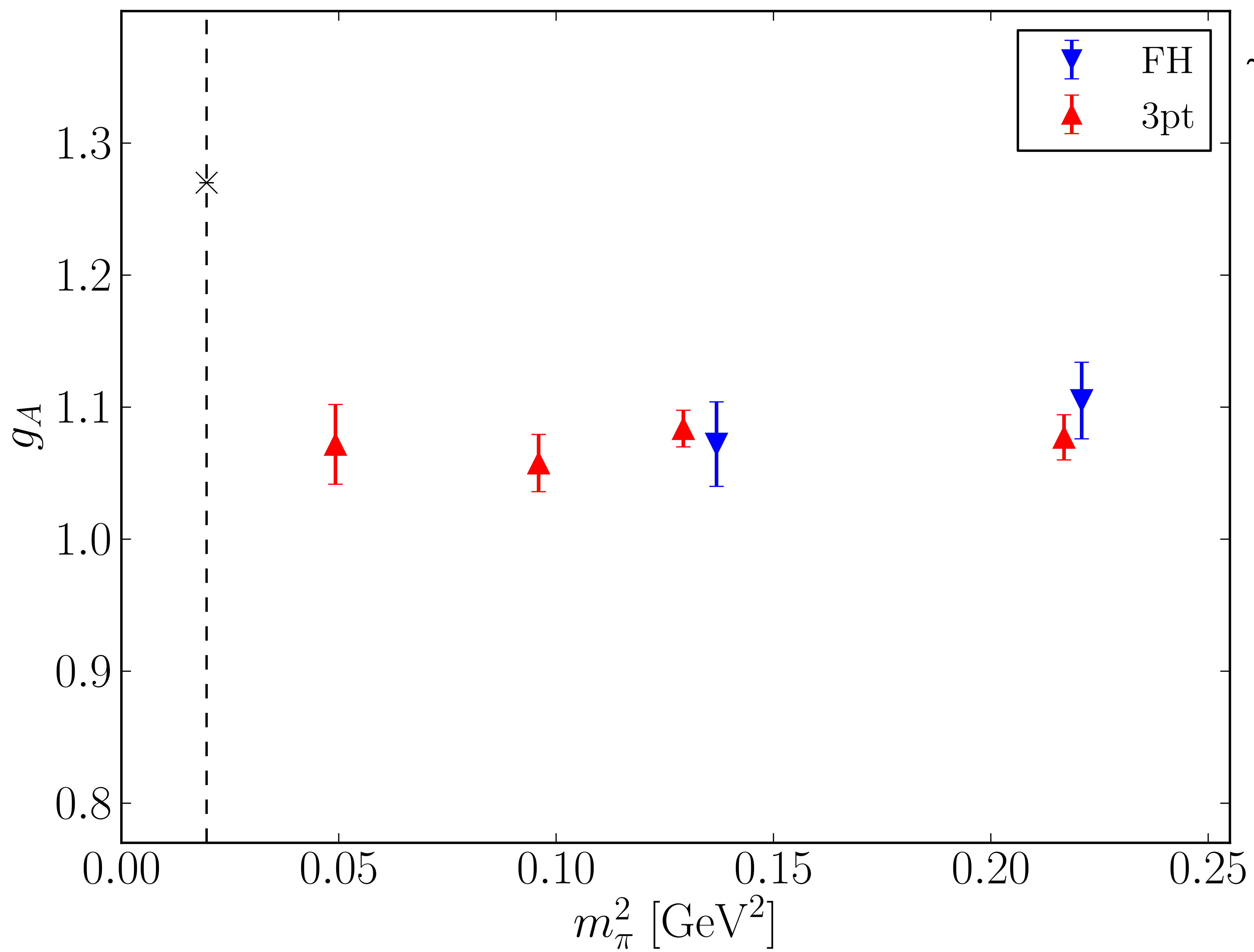
$$\Delta u^{latt}_{\mathrm{conn}} = \quad 0.911(29)$$
$$\Delta d^{latt}_{\mathrm{conn}} = -0.290(16)$$

- Rough agreement
- Possible excited state contamination in 3-point function results?

g_A

$Z_A = 0.867(4)$ [M. Constantinou *et al.* (in preparation)]

- Compare with 3-point method

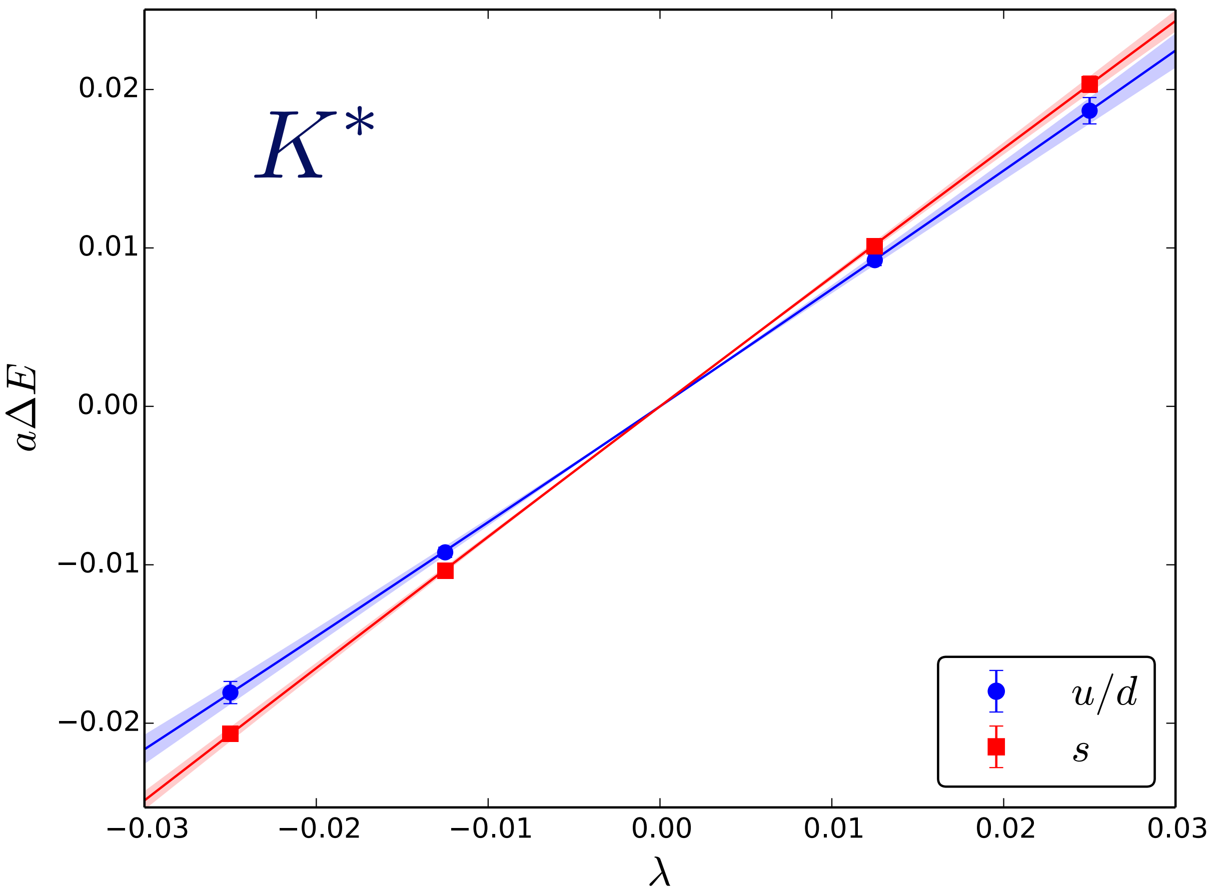


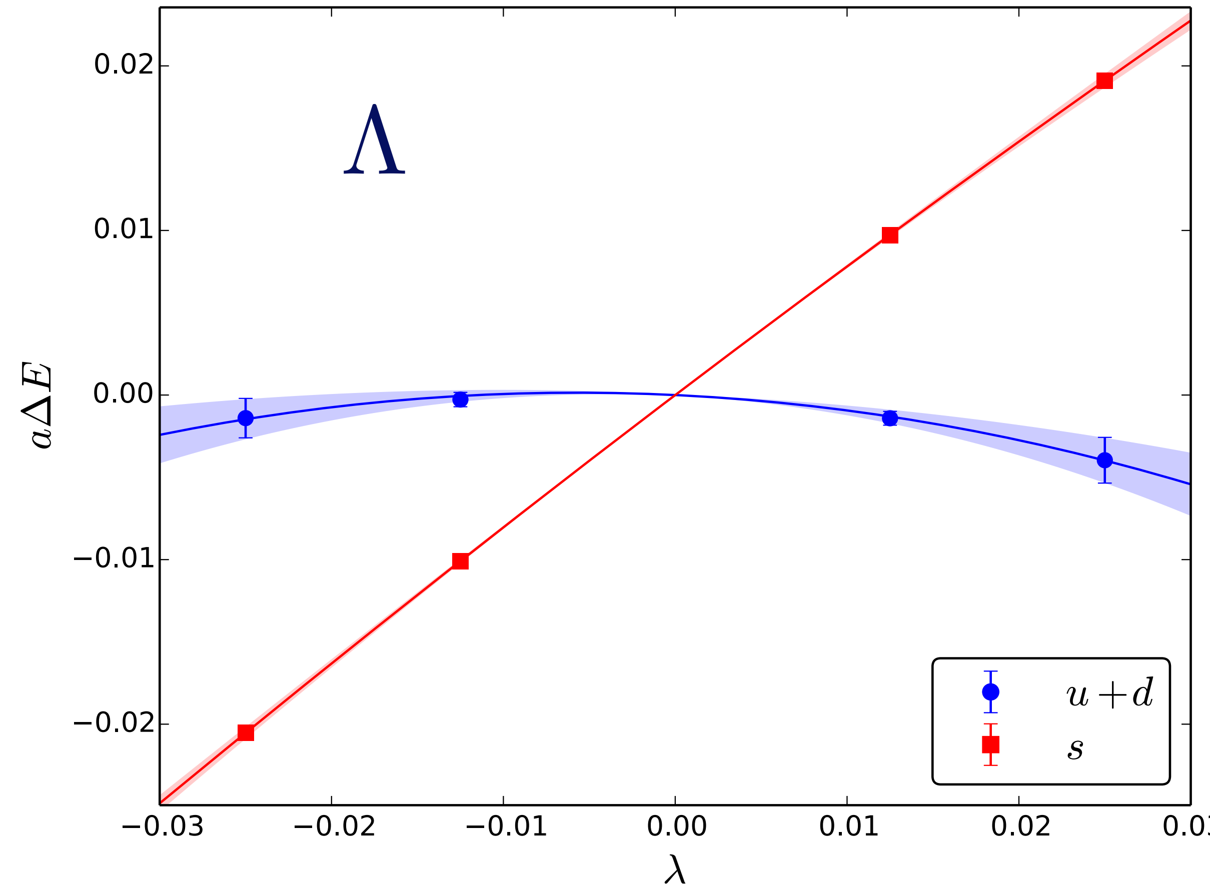
~350 measurements

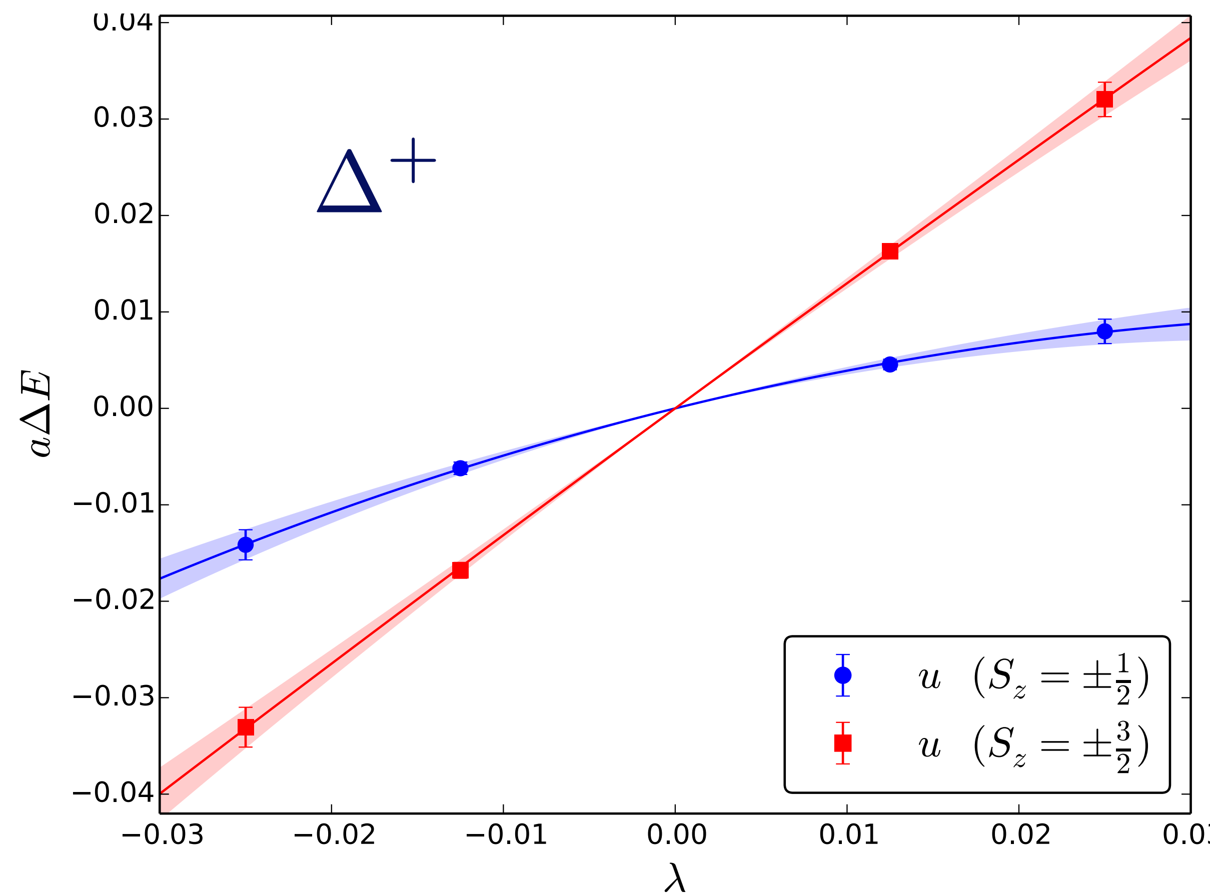
~800-1000 measurements

Connected Spin Contributions

- Repeat for other hadrons away from SU(3)-symmetric point $m_\pi \approx 360\,\mathrm{MeV}$





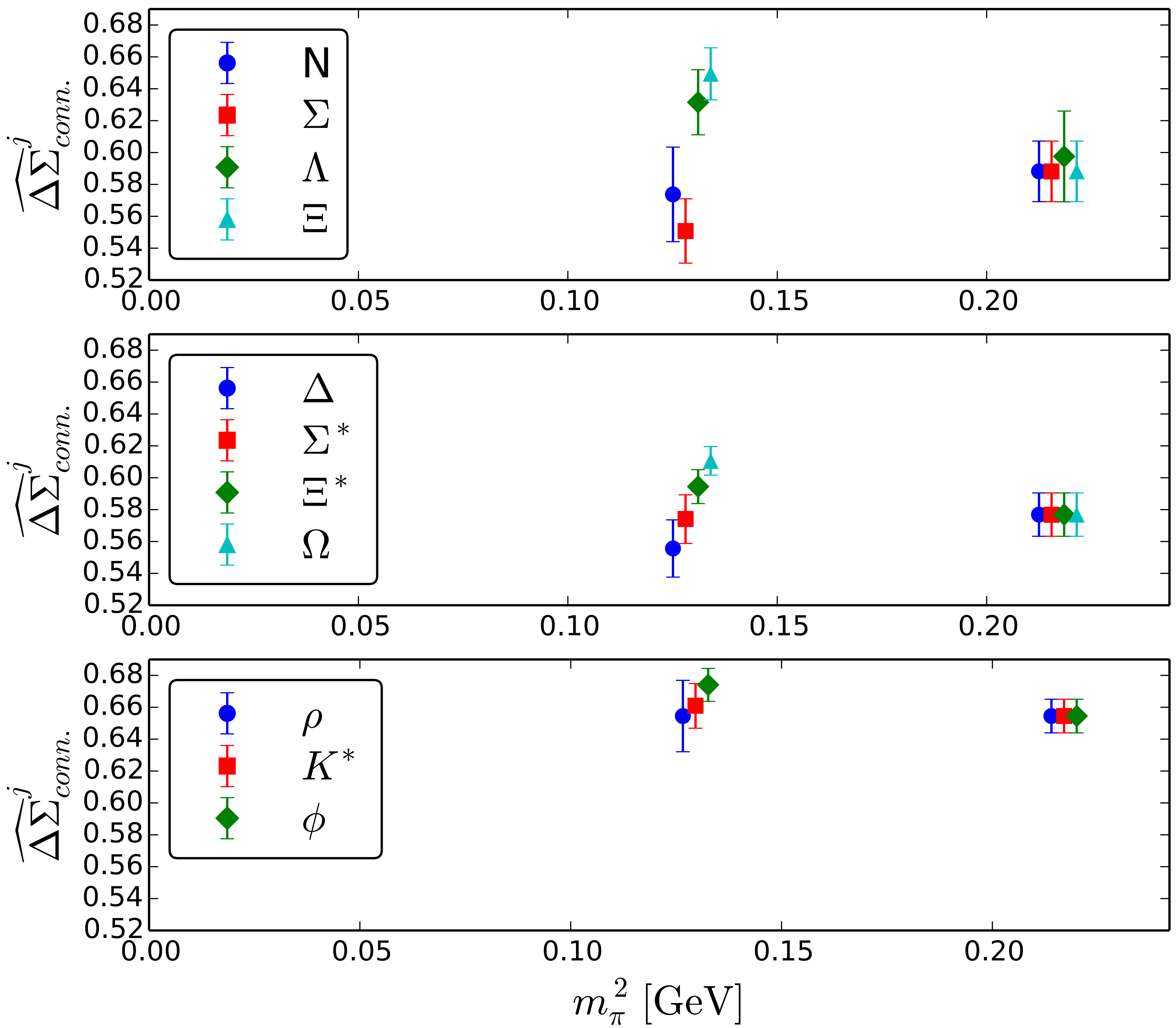


Connected Spin Contributions - Summary

- Convert quark spin contributions to spin fractions

$$\widehat{\Delta\Sigma}^J = \frac{\Delta\Sigma^J}{2J}$$

- using $Z_A = 0.867(4)$



Disconnected Spin Contributions

- To compute the disconnected contributions to Δq

➡ Include operator in action for HMC

- Problem: the term we have added to the fermion matrix

$$M \to M(\lambda) = M_0 + \lambda\, i\gamma_5\gamma_3$$

- does not satisfy γ_5 hermiticity

$$M^\dagger(\lambda) = \gamma_5 M(-\lambda)\gamma_5$$

$$\Rightarrow \quad \det[M(\lambda)]^* = \det[M(-\lambda)]$$

- and we have a sign problem

Disconnected Spin Contributions

- Solution: instead add the γ_5 hermitian operator to M

$$M \to M(\lambda) = M_0 + \lambda\, \gamma_5 \gamma_3$$

- Which is fine since we are interested in small perturbations around $\lambda = 0$

$$\left.\frac{\partial S(\lambda)}{\partial \lambda}\right|_{\lambda=0}$$

- Except that now the correlation functions will pick up a phase

$$C(\lambda, t) \xrightarrow{\text{large } t} A(\lambda) e^{-E(\lambda)t} e^{i\phi(\lambda)t}$$

- where

$$E(\lambda) = E(\lambda = 0) + \mathcal{O}(\lambda^2)$$

$$\phi(\lambda) = \lambda \Delta q + \mathcal{O}(\lambda^3)$$

Disconnected Spin Contributions

- Imaginary part of the correlator now ≠ 0 $\qquad \propto e^{-Mt}\sin(\phi t)$

- Real part $\qquad \propto e^{-Mt}\cos(\phi t)$

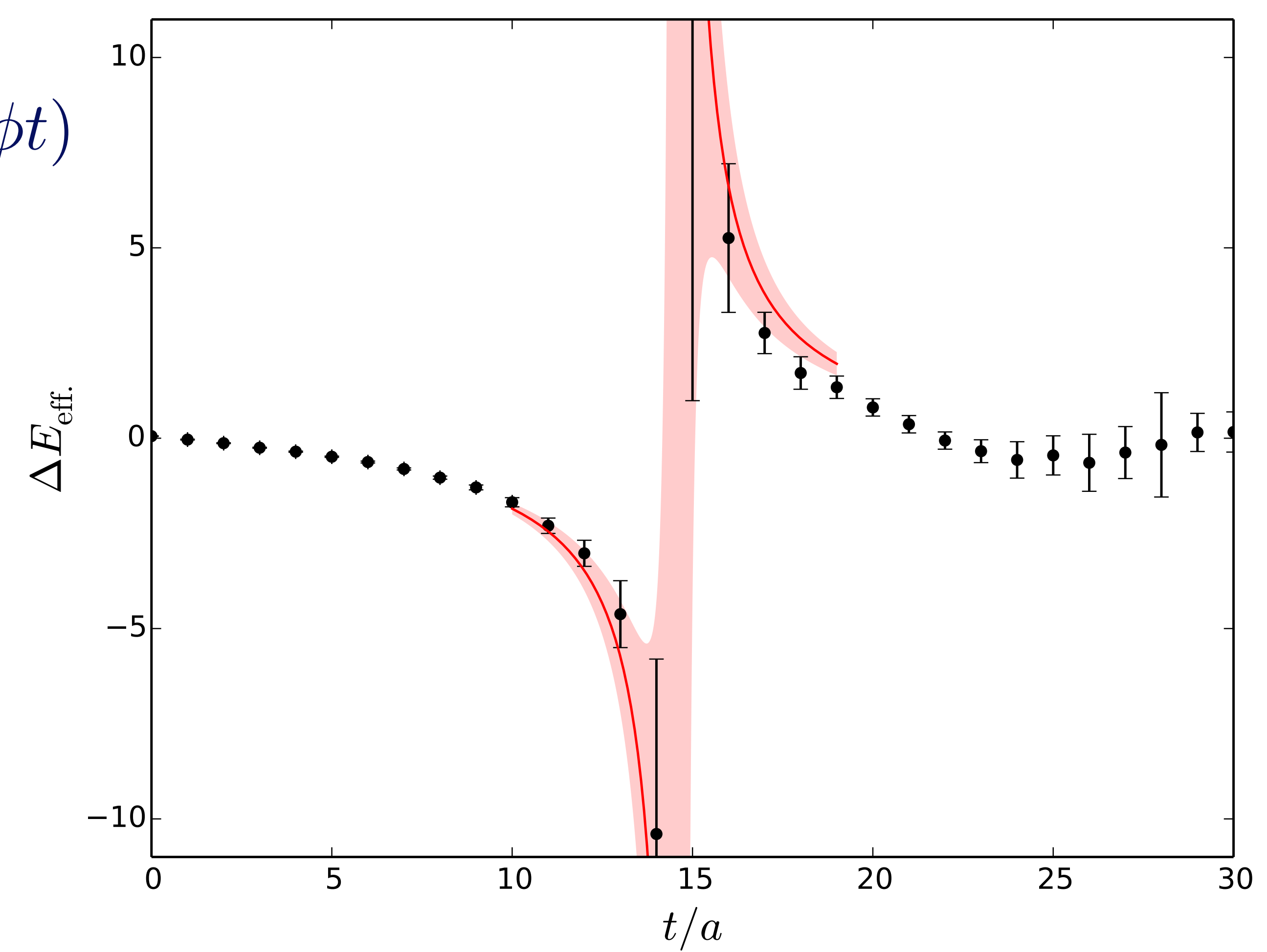
- Using the spin up/down projectors $\Gamma_{\pm} = \frac{1}{4}(1+\gamma_4)(1\pm i\gamma_5\gamma_3)$

$$\mathcal{R}\Big[C_{\pm}(\lambda,t)\Big] \xrightarrow{\text{large } t} A(\lambda)e^{-E(\lambda)t}\cos(\phi t)$$

$$\mathcal{I}\Big[C_{\pm}(\lambda,t)\Big] \xrightarrow{\text{large } t} \pm A(\lambda)e^{-E(\lambda)t}\sin(\phi t)$$

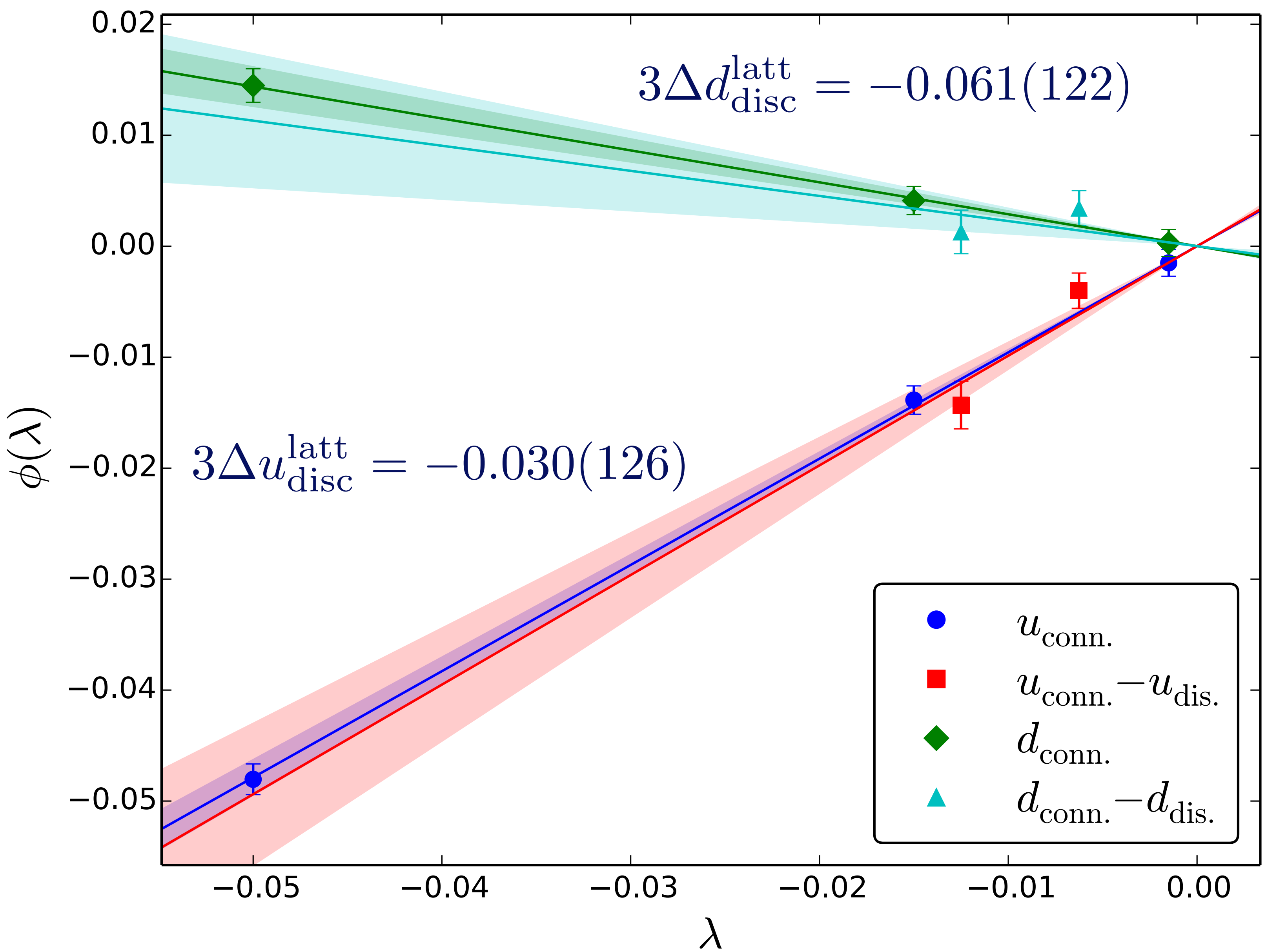
- Motivates the correlated ratio

$$\frac{\mathcal{I}\Big[C_{+}(\lambda,t)\Big]-\mathcal{I}\Big[C_{-}(\lambda,t)\Big]}{\mathcal{R}\Big[C_{+}(\lambda,t)\Big]+\mathcal{R}\Big[C_{-}(\lambda,t)\Big]} \xrightarrow{\text{large } t} \tan(\phi t)$$



Disconnected Spin Contributions

SU(3) symmetric point, $m_\pi \approx 470\,\mathrm{MeV}$



$$\Delta u^{\mathrm{latt}}_{\mathrm{disc}} = \Delta d^{\mathrm{latt}}_{\mathrm{disc}} = \Delta s^{\mathrm{latt}}_{\mathrm{disc}} = -0.015(41)$$

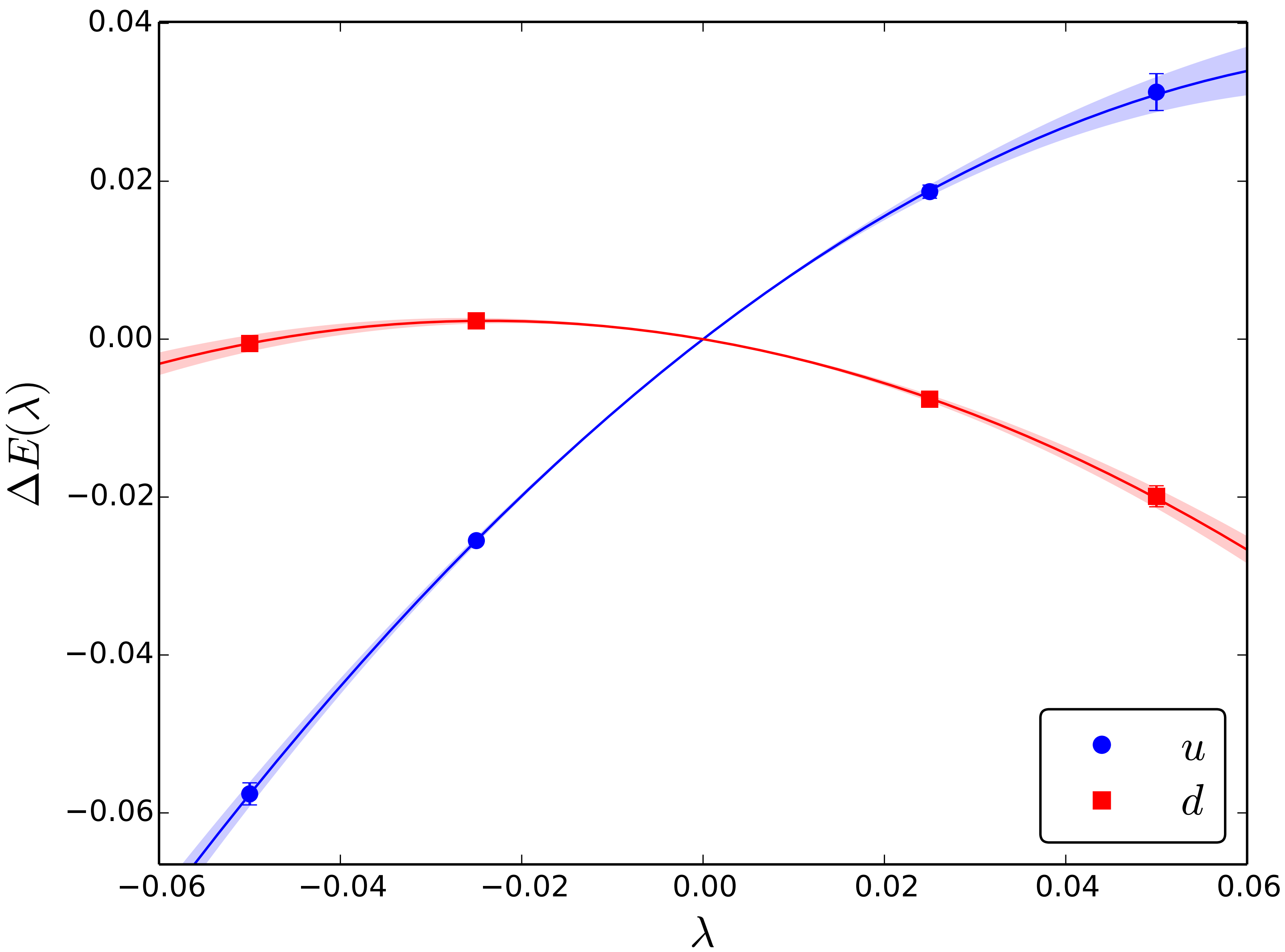
$$Z_A = 0.867(4) \quad \Rightarrow \quad \Delta u_{\mathrm{disc}} = \Delta d_{\mathrm{disc}} = \Delta s_{\mathrm{disc}} = -0.013(36)$$

Tensor Charge - Connected

SU(3) symmetric point, $m_\pi \approx 470\,\mathrm{MeV}$

- Energy shift v λ

$$M \to M(\lambda) = M_0 + \lambda\,\gamma_5\sigma_{34}$$



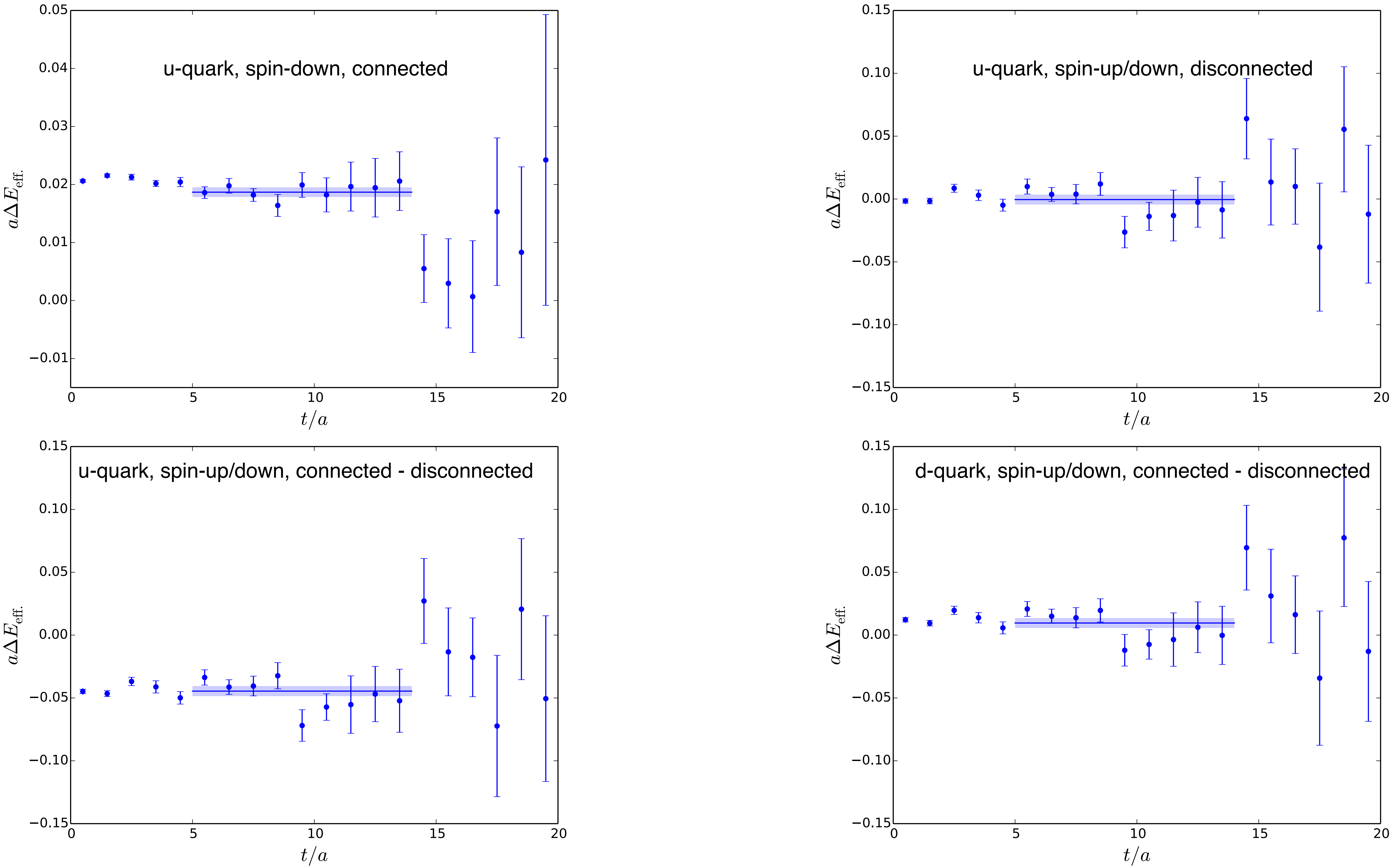
Linear terms give (500 measurements):

$$\delta u^{latt}_{\mathrm{conn}} = 0.886(17)$$
$$\delta d^{latt}_{\mathrm{conn}} = -0.199(12)$$

3-point results (1500 measurements):

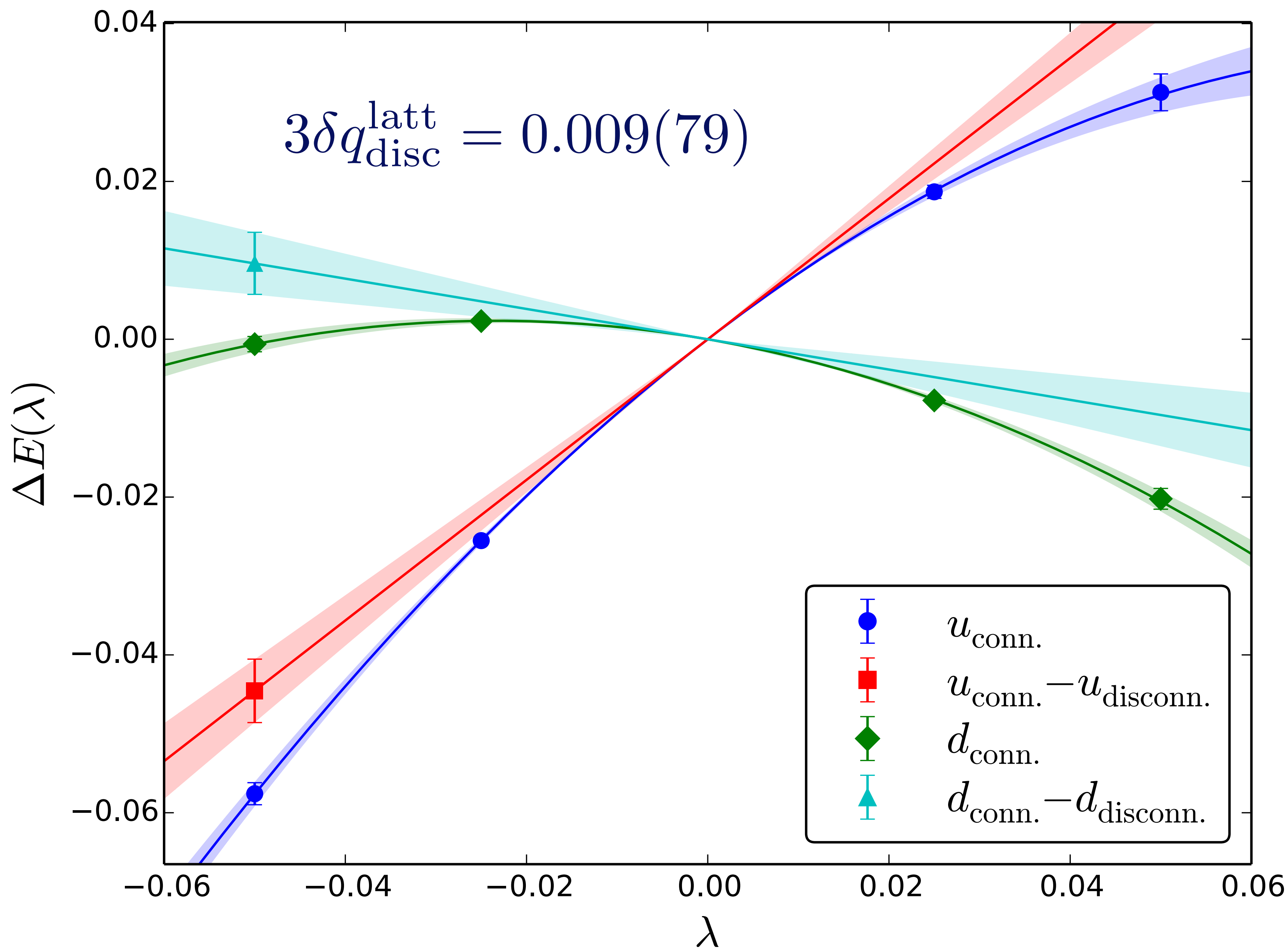
$$\delta u^{latt}_{\mathrm{conn}} = 0.858(14)$$
$$\delta d^{latt}_{\mathrm{conn}} = -0.200(4)$$

Tensor Charge - Disconnected



Tensor Charge - Disconnected

SU(3) symmetric point, $m_\pi \approx 470\,\mathrm{MeV}$



$Z_T^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = 0.995(1)$ → $\delta q_{\mathrm{disc}} = 0.003(26)$

Summary

- Feynman-Hellmann method
  - Alternative to conventional 3-point function methods for computing matrix elements
  - Demonstrated by computing connected and disconnected contributions to

$$\Delta q \quad \text{and} \quad \delta q$$

- Advantages
  - Simple to implement
  - Good control over excited state contamination
  - Excellent for studying a single operator in many hadrons
- Disadvantages
  - Different inversions (gauge configurations) for each operator and λ
- At the SU(3)-symmetric point, disconnected Δq and δq consistent with zero