Calculation of BSM Kaon B-parameter using Staggered Quarks

Yong-Chull Jang, Hwancheol Jeong , Chulwoo Jung , Jangho Kim, Seonghee Kim, Hyung-Jin Kim, Weonjong Lee, Jaehoon Leem, Jeonghwan Pak, Sungwoo Park, Stephen Sharpe, Boram Yoon

SWME Collaboration

Lattice 2014, New York

BSM Four Fermion Operators

- New $\Delta S = 2$ four-fermion operators that contribute to Kaon Mixing

$$
\begin{aligned}
Q_1 &= [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] \quad \rightarrow \quad B_K \\
Q_2 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1-\gamma_5)d^b] \\
Q_3 &= [\bar{s}^a\sigma_{\mu\nu}(1-\gamma_5)d^a][\bar{s}^b\sigma_{\mu\nu}(1-\gamma_5)d^b] \\
Q_4 &= [\bar{s}^a(1-\gamma_5)d^a][\bar{s}^b(1+\gamma_5)d^b] \\
Q_5 &= [\bar{s}^a\gamma_\mu(1-\gamma_5)d^a][\bar{s}^b\gamma_\mu(1+\gamma_5)d^b]
\end{aligned}
$$

- $\mathcal{H}_{eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu)Q_i(\mu)$
- With the constraint from experiment, calculating corresponding hadronic matrix elements

$$\langle \bar{K}_0 | Q_i | K_0 \rangle$$

can impose strong constraints on BSM models.

Lattice Calculation : BSM B-parameters

- **B-parameters**

$$B_K = \frac{\langle \bar{K}_0 | Q_1 | K_0 \rangle}{8/3 \langle \bar{K}_0 | \bar{s}\gamma_0\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_0\gamma_5 d | K_0 \rangle} \qquad \text{SM, BSM}$$

$$B_i = \frac{\langle \bar{K}_0 | Q_i | K_0 \rangle}{N_i \langle \bar{K}_0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K_0 \rangle} \qquad \text{BSM}$$

Where, $i = 2, 3, 4, 5$ and $(N_2, N_3, N_4, N_5) = (5/3, 4, -2, 4/3)$

- **Golden Combinations :** G_i

$$G_{23} \equiv \frac{B_2}{B_3} \qquad G_{45} \equiv \frac{B_4}{B_5}$$

$$G_{24} \equiv B_2 \cdot B_4 \qquad G_{21} \equiv \frac{B_2}{B_K}$$

1. Advantage: no SU(2) chiral logs at NLO order in G_i (Golden Combinations)

$N_f = 2 + 1$ QCD: MILC fine lattices - Staggered Quarks

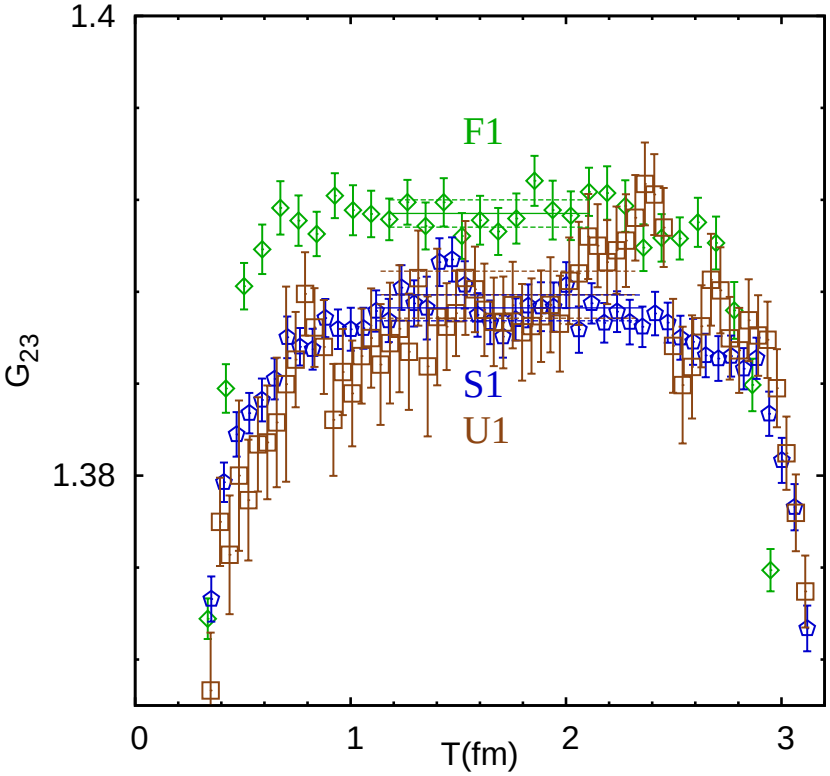
a (fm)	am_l/am_s	geometry	ens×meas	ID	Status
0.09	0.0062/0.0310	$28^3 \times 96$	995×9	F1	
0.09	0.0031/0.0310	$40^3 \times 96$	959×9	F2	
0.09	0.0093/0.0310	$28^3 \times 96$	949×9	F3	
0.09	0.0124/0.0310	$28^3 \times 96$	1995×9	F4	
0.09	0.00465/0.0310	$32^3 \times 96$	651×9	F5	
0.09	0.0062/0.0186	$28^3 \times 96$	950×9	F6	New
0.09	0.0031/0.0186	$40^3 \times 96$	701×9	F7	New
0.09	0.00155/0.0310	$64^3 \times 96$	790×9	F9	New
0.06	0.0036/0.018	$48^3 \times 144$	749×9	S1	
0.06	0.0025/0.018	$56^3 \times 144$	799×9	S2	
0.06	0.0072/0.018	$48^3 \times 144$	593×9	S3	
0.06	0.0054/0.018	$48^3 \times 144$	582×9	S4	
0.06	0.0018/0.018	$64^3 \times 144$	572×9	S5	New
0.045	0.0030/0.015	$64^3 \times 192$	747×1	U1	

Data Analysis

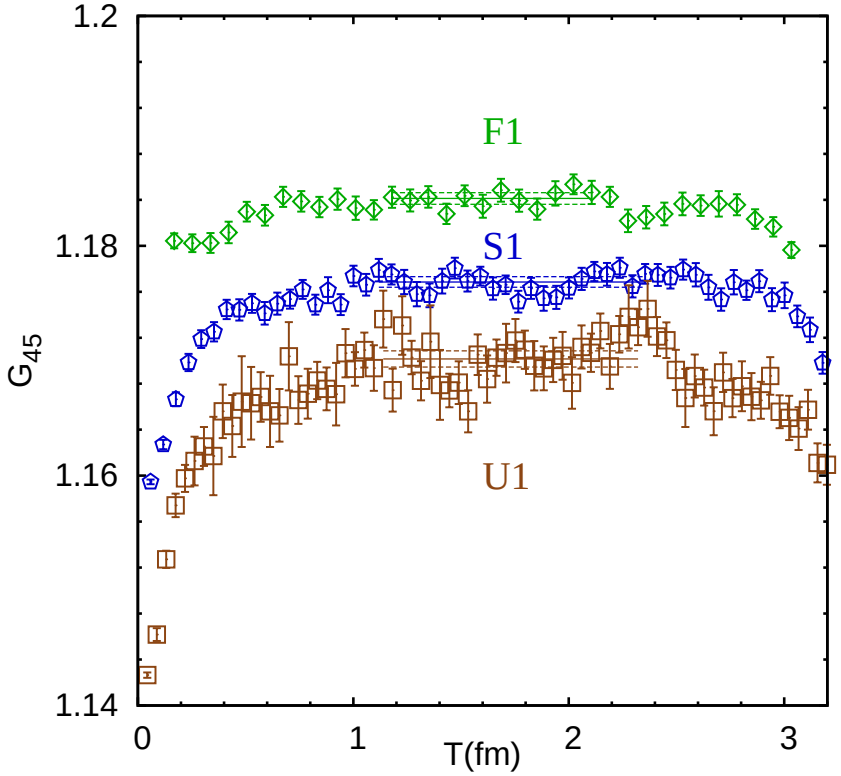
- **Calculate raw data**
  Calculate B_K and G_i for different valence quark mass combinations for each gauge ensemble. ($\overline{\text{MS}}$ scheme with NDR.)
- **Chiral fitting**
  X-fit: Fix valence strange quark mass and extrapolate the light quark mass m_x to physical down quark mass.
  Y-fit: Extrapolate m_y to physical strange quark mass.
- **RG Evolution**
  Obtain results at μ_f =2GeV or 3GeV by running from $\mu_i = 1/a$.
- **Continuum extrapolation**
  Perform [1–3] for different lattices and extrapolate to $a = 0$ and to physical sea quark masses.

Raw Data of G_{23} and G_{45}

- We compare three ensembles which have the same ratio of sea quark mass $m_\ell/m_s = 1/5$.



(a)



(b)

SchPT X-fit and Y-fit of G-parameter

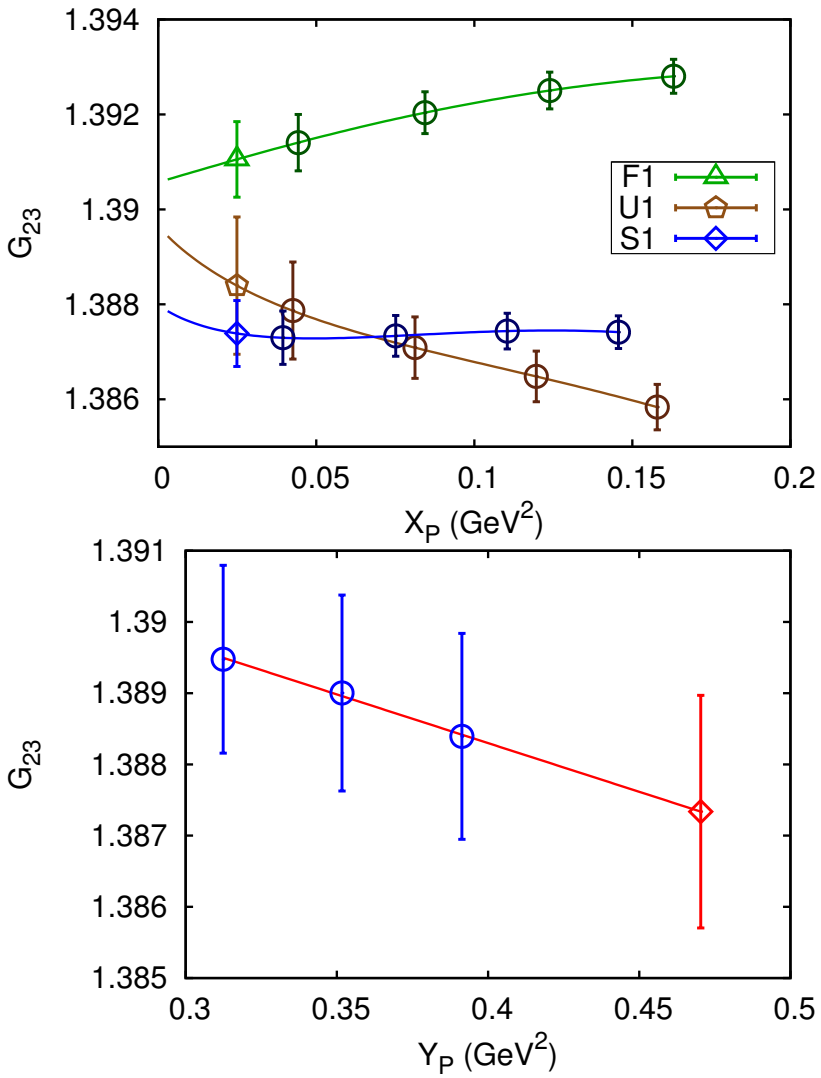
- NNNLO X-fit

$$
\begin{aligned}
G_i \quad & (\text{NNNLO}) \\
= \quad & c_1 + c_2 X + c_3 X^2 \\
+ \quad & c_4 X^2 (\ln(X))^2 \\
+ \quad & c_5 X^2 \ln(X) + c_6 X^3
\end{aligned}
$$

Bayesian constraints on $c_{4-6} = 0 \pm 1$.

- Y-fit(U1 ensemble)

$$G_i(\text{Y-fit}) = b_1 + b_2 Y_P$$



Chiral-Continuum Fit

- We use 14 data points from 14 MILC ensembles in the fitting. We extrapolate the results to physical point $a = 0$, $L_P = m_{\pi_0}^2$, and $S_P = m_{s\bar{s}}^2$.
- Fitting functional forms come from the SU(2) SChPT theory.

fit type	fitting functional form	Bayesian Constraints
F_B^1	$d_1 + d_2 \frac{L_P}{\Lambda_\chi^2} + d_3 \frac{S_P}{\Lambda_\chi^2} + d_4 (a\Lambda_Q)^2$	$d_2 \cdots d_4 = 0 \pm 2$
F_B^2	$F_B^1 + d_5 (a\Lambda_Q)^2 \frac{L_P}{\Lambda_\chi^2} + d_6 (a\Lambda_Q)^2 \frac{S_P}{\Lambda_\chi^2}$	$d_2 \cdots d_6 = 0 \pm 2$
F_B^3	$F_B^1 + d_7 (a\Lambda_Q)^2 \alpha_s + d_8 \alpha_s^2 + d_9 (a\Lambda_Q)^4$	$d_2 \cdots d_9 = 0 \pm 2$
F_B^4	$F_B^2 + d_7 (a\Lambda_Q)^2 \alpha_s + d_8 \alpha_s^2 + d_9 (a\Lambda_Q)^4$	$d_2 \cdots d_9 = 0 \pm 2$
F_B^5	$F_B^4 + d_{10} \alpha_s^3 + d_{11} (a\Lambda_Q)^2 \alpha_s^2 + d_{12} (a\Lambda_Q)^4 \alpha_s + d_{13} (a\Lambda_Q)^6$	$d_2 \cdots d_{13} = 0 \pm 2$
F_B^6	$F_B^5 + d_{14} (a\Lambda_Q)^4 \frac{L_P}{\Lambda_\chi^2} + d_{15} (a\Lambda_Q)^2 \alpha_s \frac{L_P}{\Lambda_\chi^2} + d_{16} \alpha_s^2 \frac{L_P}{\Lambda_\chi^2}$	
	$+ d_{17} (a\Lambda_Q)^4 \frac{S_P}{\Lambda_\chi^2} + d_{18} (a\Lambda_Q)^2 \alpha_s \frac{S_P}{\Lambda_\chi^2} + d_{19} \alpha_s^2 \frac{S_P}{\Lambda_\chi^2}$	$d_2 \cdots d_{19} = 0 \pm 2$

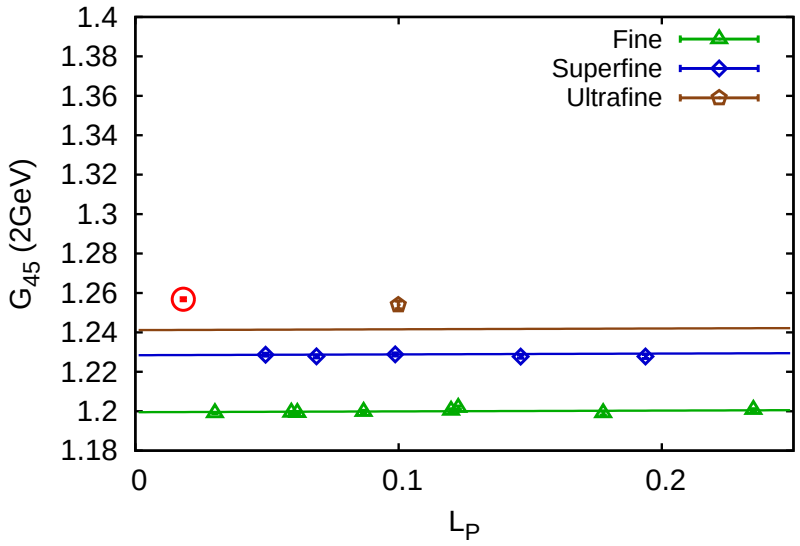
Chiral-Continuum Fit : Fitting quality

- We can see that the χ^2 values for fitting functional forms get saturated as we add higher order terms in the fitting functional forms. We choose F_B^1-fit results as central values for B_K, G_{24}, and G_{21}. For G_{23} and G_{45}, we choose those of F_B^4 as the central values.

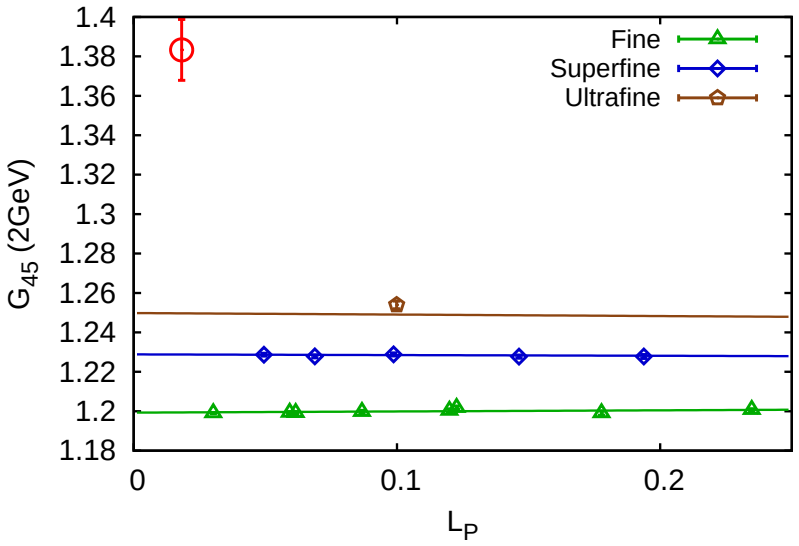
fit type	B_K	G_{23}	G_{45}	G_{24}	G_{21}
F_B^1	1.49	2.01	4.06	1.08	1.25
F_B^2	1.49	1.86	3.75	1.02	1.22
F_B^3	1.48	1.42	1.53	0.93	1.19
F_B^4	1.48	1.32	1.38	0.91	1.18
F_B^5	1.48	1.30	1.33	0.90	1.17
F_B^6	1.48	1.22	1.15	0.88	1.13

Chiral-Continuum Fit of G_{45}

- The result of Chiral-Continuum fit. The straight line in the plots represents the value of fitting function at fixed S_P and a^2 for fine($a \approx 0.09$fm), superfine ($a \approx 0.06$fm), and ultrafine($a \approx 0.045$fm) gauge ensembles.

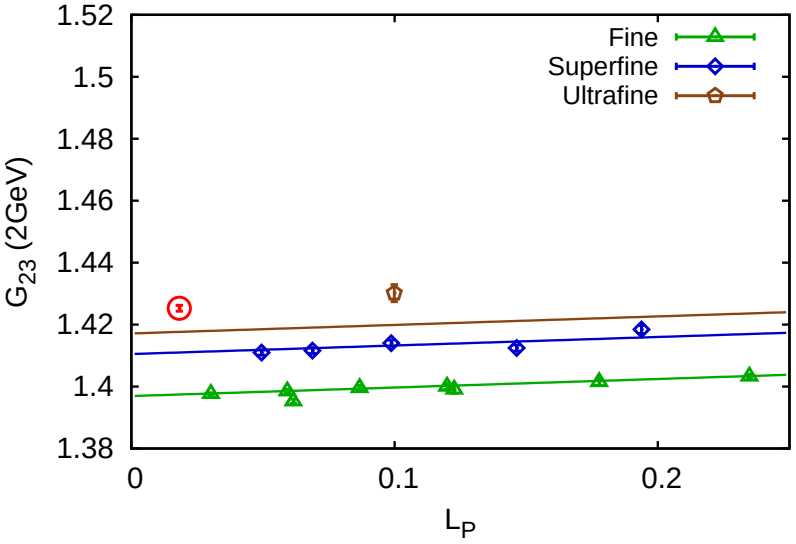


(c) F_B^1 , $\chi^2/\text{dof} = 4.06$

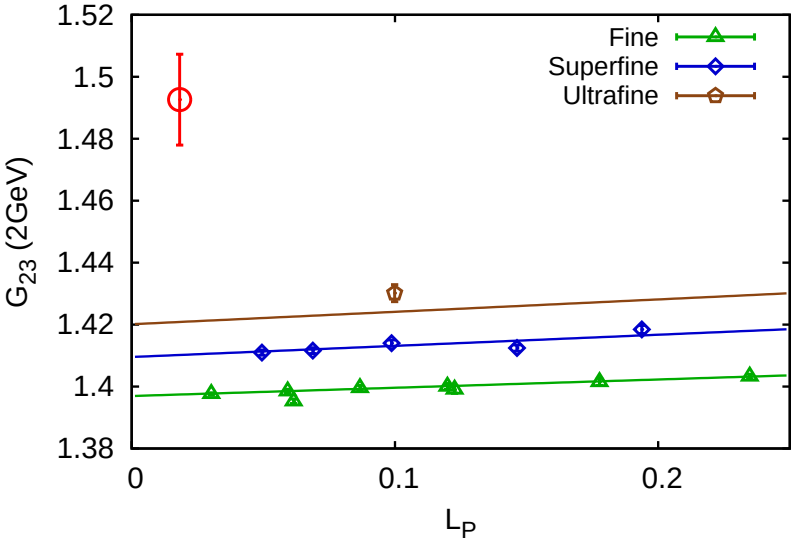


(d) F_B^4 , $\chi^2/\text{dof} = 1.38$

Chiral-Continuum Fit of G_{23}

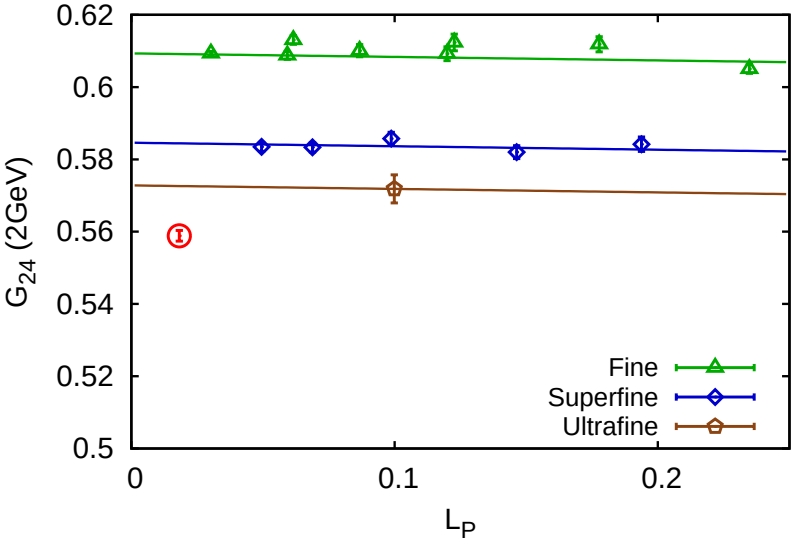


(e) F_B^1 , $\chi^2/\text{dof} = 2.01$

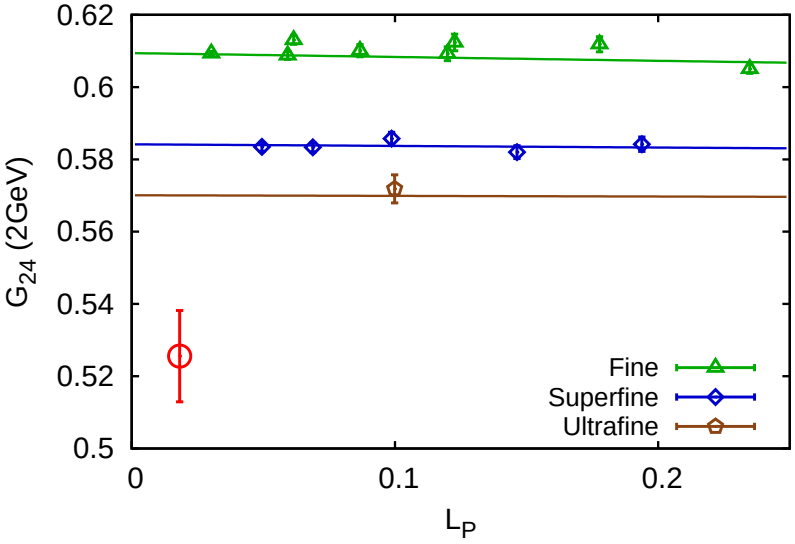


(f) F_B^4 , $\chi^2/\text{dof} = 1.32$

Chiral-Continuum Fit of G_{24}

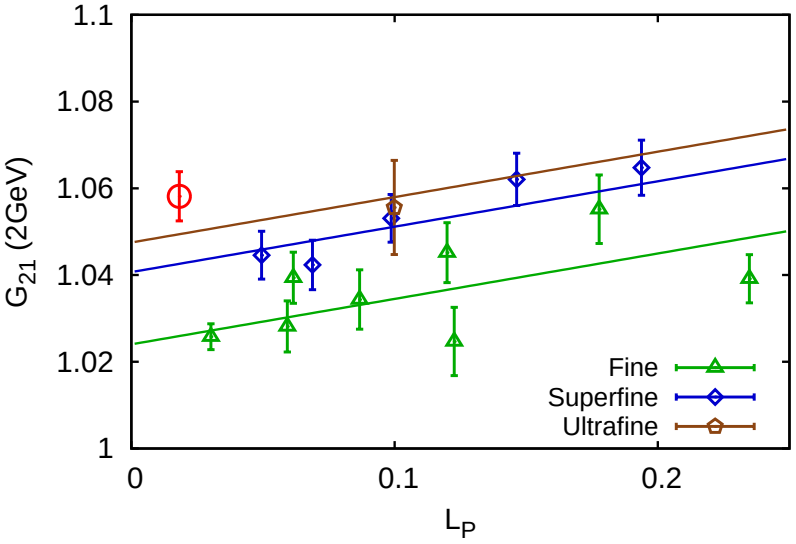


(g) F_B^1 , $\chi^2/\text{dof} = 1.08$

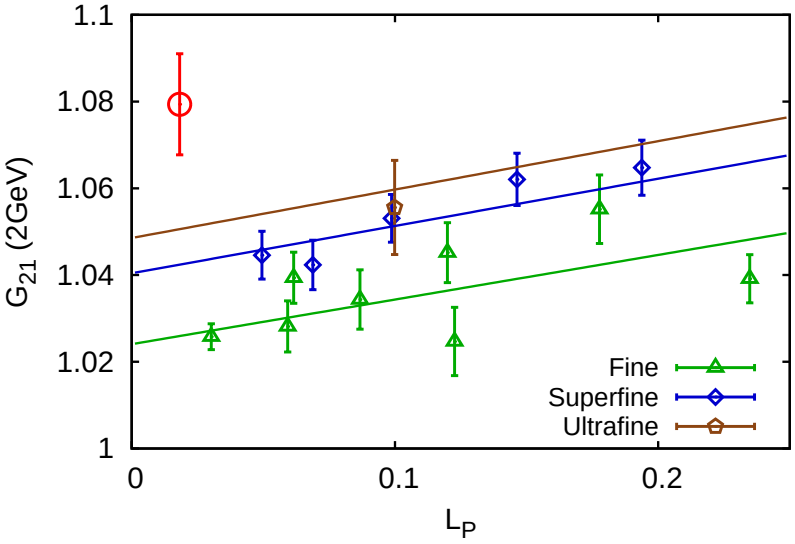


(h) F_B^4 , $\chi^2/\text{dof} = 0.91$

Chiral-Continuum Fit of G_{21}



(i) F_B^1 , $\chi^2/\text{dof} = 1.25$



(j) F_B^4 , $\chi^2/\text{dof} = 1.18$

Historical Progress

- 2013(PRD)[1] $\rightarrow$ 2014(ens) : Add more gauge ensembles.
- 2014(ens) $\rightarrow$ 2014(A.D.) : Correct two-loop contribution to pseudoscalar anomalous dimension.
- 2014(A.D.) $\rightarrow$ 2014(final) : Change fit type from F_B^1 to F_B^4 for G_{23} and G_{45}.

$\mu = 3$GeV	2013(PRD)[1]	2014(ens)	2014(A.D.)	2014(final)
B_K	0.519(7)(23)	0.518(3)	0.518(4)	0.518(4)(24)
B_2	0.549(3)(28)	0.547(1)	0.525(1)	0.525(1)(25)
B_3^{Buras}	0.390(2)(17)	0.390(1)	0.375(1)	0.358(4)(23)
B_3^{SUSY}	0.790(30)	0.783(2)	0.750(2)	0.774(6)(34)
B_4	1.033(6)(46)	1.024(1)	0.981(3)	0.981(3)(71)
B_5	0.855(6)(43)	0.853(3)	0.817(2)	0.748(9)(76)

[1] SWME Collaboration, Phys.ReV. **D88**,071503(2013)

Preliminary Results and Comparison

- We obtain B_i from results of G_i and B_K.

$$\text{Dominant error} \begin{cases} \text{Perturbative matching : 4.4\%} \\ \text{Chiral-continuum extrap}(|F_B^1 - F_B^4|) : 1.3 \sim 10.1\% \end{cases}$$

	SWME		RBC&UKQCD	ETM
	$\mu = 2$GeV	$\mu = 3$GeV	$\mu = 3$ GeV	$\mu = 3$ GeV
B_K	0.537(4)(25)	0.518(4)(24)	0.53(2)	0.51(2)
B_2	0.568(1)(27)	0.525(1)(25)	0.43(5)	0.47(2)
B_3^{Buras}	0.380(4)(25)	0.358(4)(23)	N.A.	N.A.
B_3^{SUSY}	0.849(6)(37)	0.774(6)(34)	0.75(9)	0.78(4)
B_4	0.984(3)(73)	0.981(3)(71)	0.69(7)	0.75(3)
B_5	0.712(9)(78)	0.748(9)(76)	0.47(6)	0.60(3)

Conclusion

- The B_K parameter agrees with the results from RBC & UKQCD and ETM collaboration.
- In the case of BSM B-parameter, there is $2\sigma(B_2)$ to $4\sigma(B_4$ and $B_5)$ discrepancy beween our result and those from RBC & UKQCD and ETM collaboration.
- We guess that the discrepancy comes from the difference in matching. We use perturbative matching, whereas RBC & UKQCD and ETM collaboration use NPR (non-perturbative renormalization).
- To confirm our guess, we will obtain the matching factor using NPR (Jangho Kim) in near future.