Weak interactions of kaons and pions

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Outline

1 Decay constants $f_K$, $f_\pi$ and $f_K/f_\pi$

2 $K_{I3}$

3 Neutral kaon mixing ($B_K$ and BSM contributions)

4 $K \rightarrow \pi\pi$

Rare kaon decays not covered in this talk (see plenary by Chris Sachrajda)
Situation before lattice 2014

FLAG’13

\[
\begin{align*}
\frac{f_K}{f_\pi} &= 1.194(5) \quad n_f = 2 + 1 + 1 \\
\frac{f_K}{f_\pi} &= 1.192(5) \quad n_f = 2 + 1 \\
\frac{f_K}{f_\pi} &= 1.205(6)(17) \quad n_f = 2
\end{align*}
\]
### Situation before lattice 2014

**FLAG’13**

\[
\begin{align*}
    f_\pi &= 130.2(1.4) \text{ MeV} & n_f &= 2 + 1 \\
    f_K &= 156.3(0.9) \text{ MeV} & n_f &= 2 + 1 \\
    f_K &= 158.1(2.5) \text{ MeV} & n_f &= 2
\end{align*}
\]

2013 \( f_{K^+}/f_{\pi^+} = 1.1947 \) (26)_{stat} (33)_{a^2 extrap} (17)_{FV (2)EM}

2014 \( f_{K^+}/f_{\pi^+} = 1.1956 \) (10)_{stat} ^{+23}_{-14} a^2 extrap (10)_{FV (5)EM}

- \( n_f = 2 + 1 + 1 \) Highly-Improved Staggered Quark (HISQ)
- \( a \sim 0.06, 0.09, 0.12, 0.15 \) fm
- \( m_\pi \sim 135, 200 \) MeV and \( m_\pi L > 3.3 \)

See talk by Javad Komijani, Wednesday@12:10
$f_\pi = 0.1298(9)_{\text{stat}}(4)_{\chi}(2)_{\text{FV}} \text{ GeV}$

$f_K = 0.1556(8)_{\text{stat}}(2)_{\chi}(1)_{\text{FV}} \text{ GeV}$

$f_K/f_\pi = 1.199(5)_{\text{stat}}(6)_{\chi}(1)_{\text{FV}}$

$n_f = 2 + 1$ Domain-Wall fermions

- New Möbius ensembles [Brower, Neff, Orginos '12] combined with existing Shamir ensembles.
- $a \sim 0.084, 1.144 \text{ fm}$, $48^3 \times 96 \times 12$ and $64^3 \times 128 \times 12$
- Physical pion masses $m_\pi \sim 130 \text{ MeV}$ and $m_\pi L > 3.5$
- Finer ensemble $a \sim 0.06$, $32^3 \times 64 \times 12$ with $m_\pi \sim 360 \text{ MeV} \Rightarrow m_\pi L \sim 3.8$
$K_{l3}$ semileptonic form factor I.

Obtain $|V_{us} f_+(0)|$ from the experimental rate

$$\Gamma_{K \to \pi l \nu} = C_K^2 \frac{G_F^2 m_K^5}{192 \pi^2} I \ S_E W \ [1 + 2 \Delta_{SU(2)} + 2 \Delta_{EM}] \ |V_{us} f_+(0)|^2$$

where:

$I$ is the phase space integral evaluated from the shape of the experimental form factor

$\Delta_{SU(2)}$ is the isospin breaking correction

$S_E W$ is the short distance electroweak correction

$\Delta_{EM}$ is the long distance electromagnetic correction

and $f_+(0)$ is the form factor defined from $(q = p - p')$

$$\langle \pi(p')|V_\mu|K(p)\rangle = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2) \quad \text{with} \quad V_\mu = \bar{s} \gamma_\mu u$$

$\Rightarrow$ determine $f_+(0)$ from the lattice to constraint $V_{us}$
Use the the scalar form factor $f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$

$$\langle \pi(p')|V_\mu|K(p)\rangle_{q^2=0} = \frac{m_K^2 - m_\pi^2}{m_s - m_u} f_+(0)$$

- Compute $f_0(q^2)$ for several negative values of $q^2$
- Interpolate to $q^2 = 0$ (or use twisted boundary conditions) \[RBC-UKQCD\]

Or compute $f_+(0)$ from \[Fermilab Lattice and MILC Collaborations Bazavov, et al. '13\]

$$f_+(0) = f_0(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2} \langle \pi(p')|S|K(p)\rangle$$

Form factor can be obtained from $\langle \pi(p')|S|K(p)\rangle$ and from $\langle \pi(p')|V_\mu|K(p)\rangle$
FLAG’13

\[ f_+(0) = 0.9661(32) \quad n_f = 2 + 1 \]
\[ f_+(0) = 0.9560(57)(62) \quad n_f = 2 \]
lattice 2014 update for $K_{l3}$

**RBC-UKQCD**

- New ensembles $48^3$ and $64^3$ at the physical point
- Results obtained from the vector current

See talk by David Murphy, Monday@6:10

<table>
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<tr>
<th>Lattice</th>
<th>$m_\pi$ (MeV)</th>
<th>$f_{+K\pi}^0$</th>
<th>Stat. error</th>
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<td>334</td>
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<tr>
<td><strong>48I (PRELIMINARY)</strong></td>
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<tr>
<td><strong>64I (PRELIMINARY)</strong></td>
<td><strong>139</strong></td>
<td><strong>0.9701(22)</strong></td>
<td><strong>0.2%</strong></td>
</tr>
</tbody>
</table>
ETMc

- 2 + 1 + 1 Twisted Mass / Osterwalder-Seiler fermions
- Results obtained from the vector current
- Preliminary result:

\[ f_+(0) = 0.9683(50)_{\text{stat}} + 42_{\text{chiral}} \]

See talk by Lorenzo Riggio, Friday@5:10
$K^0 - K^0$ mixing, $K \rightarrow \pi\pi$ and CP violation
First discovery of CP violation was made in kaon system in 1964 (Christenson, Cronin, Fitch and Turlay)

Noble prize in 1980 (Cronin and Fitch)

Very nice measurements of both direct and indirect CP violation

\[
\begin{align*}
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) &= (1.65 \pm 0.26) \times 10^{-3} \\
|\varepsilon| &= (2.228 \pm 0.011) \times 10^{-3}
\end{align*}
\]

Theoretically:
Relate indirect CP violation parameter ($\varepsilon$) to neutral kaon mixing ($B_K$)
Still lacking a quantitative description of direct CP violation ($\varepsilon'$)

Sensitivity to new physics
Flavour eigenstates \( \left( \begin{array}{c}
K^0 = \bar{s} \gamma_5 d \\
\bar{K}^0 = \bar{d} \gamma_5 s
\end{array} \right) \) \( \neq \) CP eigenstates \( |K^0_\pm\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle \mp |\bar{K}^0\rangle \} \). They are mixed in the physical eigenstates

\[
\begin{align*}
|K_L\rangle & \sim |K_0^\pm\rangle + \bar{\epsilon} |K_0^0\rangle \\
|K_S\rangle & \sim |K_0^0\rangle + \bar{\epsilon} |K_0^-\rangle
\end{align*}
\]

Direct and indirect CP violation in \( K \to \pi\pi \)

\[
|K_L\rangle \propto |K_0^-\rangle + \epsilon |K_0^+\rangle
\]

\[
\epsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = |\epsilon| e^{i\phi} \sim \bar{\epsilon}
\]
$K \rightarrow \pi\pi$ amplitudes

Two isospin channels: $\Delta I = 1/2$ and $\Delta I = 3/2$

$K \rightarrow (\pi\pi)_{I=0,2}$

Corresponding amplitudes defined as

$$A[K \rightarrow (\pi\pi)_{I}] = A_I \exp(i\delta_I) \quad /w \ I = 0, 2 \quad \delta = \text{strong phases}$$

$\Delta I = 1/2$ rule

$$\omega = \frac{\Re A_2}{\Re A_0} \sim 1/22 \quad (\text{experimental number})$$

Amplitudes are related to the parameters of CP violation $\varepsilon, \varepsilon'$ via

$$\varepsilon' = \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}} \left[ \frac{\Im(A_2)}{\Re A_2} - \frac{\Im A_0}{\Re A_0} \right]$$

$$\varepsilon = e^{i\phi_\varepsilon} \left[ \frac{\Im(\bar{K}^0|H_{\text{eff}}^{\Delta S=2}|K^0)}{\Delta m_K} + \frac{\Im A_0}{\Re A_0} \right]$$

$\Rightarrow$ Related to $K^0 - \bar{K}^0$ mixing

See poster by Yong-Chull Jang and Weonjong Lee
Neutral kaon mixing in the SM

In the Standard Model, $K^0 - \bar{K}^0$ mixing dominated by box diagrams with $W$ exchange, e.g.

\[
\begin{align*}
\text{Operator product expansion} & \\
H_{\text{eff}}^{\Delta S=2} & = \frac{G_F^2 m_W^2}{16\pi^2} \times F(\text{SM free parameters}) \times C(\mu) \mathcal{O}_{LL}^{\Delta S=2}(\mu)
\end{align*}
\]

Factorise the non-perturbative contribution

\[
\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} F_K^2 M_K^2 B_K(\mu) \quad \text{w/ } \mathcal{O}_{LL}^{\Delta S=2} = (\bar{s}\gamma_\mu(1 - \gamma_5)d)(\bar{s}\gamma^\mu(1 - \gamma_5)d)
\]

$B_K$ is the SM kaon bag parameter

\[
B_K(\mu) = \frac{\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle_{VS}}{\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle_{\text{OPE}}} \]

\[
\mu \ll M_W
\]

$\Delta S = 2$ + long distance $(\Delta S = 1)^2$
Neutral kaon mixing in the SM

In the Standard Model, $K^0 - \bar{K}^0$ mixing dominated by box diagrams with $W$ exchange, e.g.

$$W_{sd} \rightarrow \mu \ll M_W$$

$\Delta S = 2$ (long distance $\Delta S = 1^2$)

$K_L - K_S$ mass difference, long distance contributions:

See plenary talk by Chris Sachrajda, Saturday@10:30
In the SM, only one four-quark operator

\[ O^{\Delta S=2}_{(V-A) \times (V-A)} = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha)(\bar{s}_\beta \gamma^\mu (1 - \gamma_5) d_\beta) \]

Usually parametrised by its bag parameter (renormalization scheme and scale dependent)

\[ B_K = \frac{\langle \bar{K}^0 | O^{\Delta S=2}_{LL} (\mu) | K^0 \rangle}{\langle K^0 | O^{\Delta S=2}_{LL} | K^0 \rangle_{VS}} = \frac{\langle \bar{K}^0 | O^{\Delta S=2}_{LL} (\mu) | K^0 \rangle}{\frac{8}{3} m^2_K f^2_K} \]

Define the Renormalisation-Group-Invariant \( \hat{B}_K \) by

\[ \hat{B}_K = \left( \frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \exp \left\{ \int_0^{\bar{g}(\mu)} dg \left( \frac{\gamma(g)}{\beta(g)} + \frac{\gamma_0}{\beta_0} \right) \right\} B_K(\mu) . \]

Traditionally: give \( B_K^{\overline{\text{MS}}}(2 \text{ GeV}) \) or \( \hat{B}_K \) or \( B_K^{\overline{\text{MS}}}(2 \text{ GeV}) \).

Recently, lattice community starts giving results at a higher scale.
FLAG [Aoki et al., '13-14]
Status before lattice 2014

**BMW '11**

[Dürr, Fodor, Hoelbling, Katz, Krieg, Kurth, Lellouch, Lippert, McNeil, Portelli, Szabó, PLB ’11]

\[ \hat{B}_K = 0.7727(81)_{\text{stat}}(34)_{\text{sys}}(77)_{\text{PT}} \]

- 2 + 1 HEX-smeared clover-improved Wilson fermions,
- Four lattice spacings \( a \sim 0.054 - 0.093 \) fm
- Pion masses down to the physical point
- Non-perturbative-renormalization (NPR) through RI-MOM scheme

**RBC-UKQCD ’12**

[Arthur, Blum, Boyle, Christ, N.G., Hudspith, Izubuchi, Jung, Kelly, Lytle, Mawhinney, Murphy, Ohta, Sachrajda, Soni, Yu, Zanotti, PRD’12]

\[ \hat{B}_K = 0.758(11)_{\text{stat}}(10)_{\chi}(4)_{\text{FV}}(16)_{\text{PT}} \]

- 2 + 1 Domain-Wall fermions
- \( a \sim 0.14 \) fm, IDSDR, \( m_\pi \sim 170 \) MeV (partially quenched 140 MeV)
- \( a \sim 0.85, 0.11 \) fm IW \( m_\pi \) down to \( \sim 290 \) MeV
- NPR with 2 RI-SMOM schemes
\[ \hat{B}_K = 0.7379(47)_{\text{stat}}(365)_{\text{sys}} \]

- 2 + 1 HYP-smeared staggered on aqstqd (MILC) ensembles
- Four lattice spacings \( a \sim 0.045 - 0.12 \) fm
- Pion masses down to 200 MeV
- Renormalisation: 1-loop matching to \( \overline{\text{MS}} \)

\[ \hat{B}_K = 0.729(25)(17) \]

- 2 flavours twisted mass (2 + 1 + 1 in progress)
- Four lattice spacings \( a \sim 0.045 - 0.12 \) fm
- Pion masses down to 200 MeV
- Non-perturbative-renormalization (NPR) through RI-MOM scheme
Non-perturbative scale-evolution

Running between two energy scales $\mu_1$ and $\mu_2$

$$Z(\mu_1) = U(\mu_1, \mu_2)Z(\mu_2)$$

Comparison of the non-perturbative running in RI-MOM with perturbation theory (NLO)
\( n_f = 2 + 1 \) Domain-Wall fermions
- New Möbius ensembles combined with existing Shamir ensembles.
- \( a \sim 0.084, 0.144 \text{ fm}, 48^3 \times 96 \times 12 \) and \( 64^3 \times 128 \times 12 \)
- Physical quark masses \( m_\pi \sim 130 \text{ MeV} \) and \( m_\pi L > 3.5 \)
- Finer ensemble \( a \sim 0.06, 32^3 \times 64 \times 12 \) with \( m_\pi \sim 360 \text{ MeV} \Rightarrow m_\pi L \sim 3.8 \)

\( n_f = 2 + 1 + 1 \) Domain-Wall fermions (Möbius) in progress
RBC-UKQCD PRELIMINARY [Work in progress]

Running to 5 GeV with $2 + 1 + 1$ flavours

$N_f=2+1+1$ B$_K$ step-scaling

RI-SMOM$_{qq}$ scheme

see talk by Julien Frison, Tuesday @5:10
Note about matching to $\overline{\text{MS}}$

- The matching to $\overline{\text{MS}}$ is done at Next-to-leading order
- Difficult to estimate the corresponding systematic error
- NNLO matching factors (between MOM and $\overline{\text{MS}}$) on the wishlist
- RBC-UKQCD uses several intermediate (SMOM) scheme and take the difference for the estimate of the syst. error
- At 2 or 3 GeV this is significantly larger than the naive estimate
- At 5 GeV this error is 1% (see talk by Julien Frison)
See [F. Gabbiani et al '96]

In the SM, neutral kaon mixing occurs through $W$-exchanges → $(V - A) \times (V - A)$

$$O_{1}^{\Delta S=2} = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha) (\bar{s}_\beta \gamma_\mu (1 - \gamma_5) d_\beta),$$

Invariant under Fierz arrangement ⇒ only one color structure

Beyond the SM, other Dirac structure appear at high energy

Low energy description: generic $\Delta S = 2$ effective Hamiltonian $H^{\Delta S= 2} = \sum_{i=1}^{5} C_i(\mu) O_{i}^{\Delta S=2}(\mu)$.

$$O_{2}^{\Delta S=2} = (\bar{s}_\alpha (1 - \gamma_5) d_\alpha) (\bar{s}_\beta (1 - \gamma_5) d_\beta)$$

$$O_{3}^{\Delta S=2} = (\bar{s}_\alpha (1 - \gamma_5) d_\beta) (\bar{s}_\beta (1 - \gamma_5) d_\alpha)$$

$$O_{4}^{\Delta S=2} = (\bar{s}_\alpha (1 - \gamma_5) d_\alpha) (\bar{s}_\beta (1 + \gamma_5) d_\beta)$$

$$O_{5}^{\Delta S=2} = (\bar{s}_\alpha (1 - \gamma_5) d_\beta) (\bar{s}_\beta (1 + \gamma_5) d_\alpha)$$

SUSY basis

Parity partners are redundant if Parity is conserved

On the lattice: compute $\langle \bar{K}^{0} | O_{i}^{\Delta S=2} | K^{0} \rangle$
Mixing pattern given by $SU(3)_L \times SU(3)_R$ decomposition

\[
\begin{align*}
3 \times 3 &= 6 + \bar{3} \\
\bar{3} \times \bar{3} &= \bar{6} + 3 \\
\bar{3} \times 3 &= 1 + 8
\end{align*}
\]

BSM operators

\[
\begin{align*}
O_{2}^{A_{S}=2} &= (\bar{s}_\alpha (1 - \gamma_5) d_\alpha) (\bar{s}_\beta (1 - \gamma_5) d_\beta) \\
O_{3}^{A_{S}=2} &= (\bar{s}_\alpha (1 - \gamma_5) d_\beta) (\bar{s}_\beta (1 - \gamma_5) d_\alpha)
\end{align*}
\]

Under $SU_L(3) \rightarrow \bar{s}_R d_L \bar{s}_R d_L$ Symmetric $\Rightarrow 6_L$
Mixing

- Mixing pattern given by $SU(3)_L \times SU(3)_R$ decomposition
  
  \begin{align*}
  3 \times 3 & = 6 + \bar{3} \\
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O_{2}^{\Delta S=2} = (\bar{s}_\alpha (1 - \gamma_5)d_\alpha)(\bar{s}_\beta (1 - \gamma_5)d_\beta) \\
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Under $SU_R(3) \rightarrow \bar{s}_R d_L \bar{s}_R d_L$  Symmetric $\Rightarrow \bar{6}_R$
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\[
\begin{align*}
O_{4}^{\Delta S=2} &= (\bar{s}_\alpha (1 - \gamma_5)d_\alpha) (\bar{s}_\beta (1 + \gamma_5)d_\beta) \\
O_{5}^{\Delta S=2} &= (\bar{s}_\alpha (1 - \gamma_5)d_\beta) (\bar{s}_\beta (1 + \gamma_5)d_\alpha)
\end{align*}
\]

Under $SU_L(3) \longrightarrow \bar{s}_R d_L \bar{s}_L d_R$  \hspace{1cm} Non-flavour singlet $\Rightarrow 8_L$
Mixing pattern given by $SU(3)_L \times SU(3)_R$ decomposition

\[
\begin{align*}
3 \times 3 &= 6 + \bar{3} \\
\bar{3} \times \bar{3} &= \bar{6} + 3 \\
\bar{3} \times 3 &= 1 + 8
\end{align*}
\]

BSM operators

\[
\begin{align*}
O_{4}^{\Delta S=2} &= (\bar{s}_{\alpha} (1 - \gamma_5) d_{\alpha}) (\bar{s}_{\beta} (1 + \gamma_5) d_{\beta}) \\
O_{5}^{\Delta S=2} &= (\bar{s}_{\alpha} (1 - \gamma_5) d_{\beta}) (\bar{s}_{\beta} (1 + \gamma_5) d_{\alpha})
\end{align*}
\]

Under $SU_R(3) \longrightarrow \bar{s}_R d_L \bar{s}_L d_R$ Non-flavour singlet $\Rightarrow 8_R$
Mixing pattern and $SU(3)_\chi PT$

- $O_1 \in (27, 1)$ renormalises multiplicatively
- $O_2, O_3 \in (6, \bar{6})$ mix together
- $O_4, O_5 \in (8, 8)$ mix together
- Renormalization matrix is block diagonal $1_{(27, 1)} + (2 \times 2)_{(6, \bar{6})} + (2 \times 2)_{(8, 8)}$
- In the chiral limit $O_1 \to m_P^2$ and $O_{i \geq 2} \to \text{Cst}$

$$\Rightarrow \text{Expect } \frac{\langle \bar{K}^0 | O_{BSM} | K^0 \rangle}{\langle \bar{K}^0 | O_{SM} | K^0 \rangle} \to \frac{1}{m_P^2}$$
Normalisation

\( \langle \bar{K}^0 | O | K^0 \rangle \) are dimension-four quantities

Different normalisations exit

- Bag parameters \( B' \)'s, like \( B = \frac{\langle \bar{K}^0 | O_1 | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle_{VS}} \)

\[
B_1 = B_K = \frac{\langle \bar{K}^0 | O_1 | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}
\]

\[
B_{i \geq 2} = \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle}
\]

- Ratios \( R' \)'s [Babich, N.G., Hoelbling, Howard, Lellouch, Rebbi '06]

\[
R_i^{BSM}(m_P) = \left[ \frac{f_K^2}{m_K^2} \right]_{\text{expt}} \left[ \frac{m_K^2}{f_K^2} \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle} \right]_{\text{lat}}
\]

- Golden combinations \( G_s \) [Bailey, Kim, Lee, Sharpe '12, Bećirević, Villadoro '04]

Ratios or products of \( B \) parameters free of chiral logs at NLO
Situation before Lattice’14

- $n_f = 2 + 1$ Domain-Wall [RBC-UKQCD ’12]
- $n_f = 2$ [ETMc ’12] and preliminary $n_f = 2 + 1 + 1$ Twisted Mass [ETMc @ lat’13]
- $n_f = 2 + 1$ staggered [SWME ’13]

RBC-UKQCD and ETMc found compatible results, but tension observed by SWME
Update from SWME

See poster by Jaehoon Leem

Different (but equivalent) choice of basis

\[ O_{2}^{\Delta S=2} = (\bar{s}_\alpha (1 - \gamma_5) d_\alpha) (\bar{s}_\beta (1 - \gamma_5) d_\beta) \]
\[ O_{3}^{\Delta S=2} = (\bar{s}_\alpha \sigma_{\mu \nu} (1 - \gamma_5) d_\alpha) (\bar{s}_\beta \sigma_{\mu \nu} (1 - \gamma_5) d_\beta) \]
\[ O_{4}^{\Delta S=2} = (\bar{s}_\alpha (1 - \gamma_5) d_\alpha) (\bar{s}_\beta (1 + \gamma_5) d_\beta) \]
\[ O_{5}^{\Delta S=2} = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha) (\bar{s}_\beta \gamma_\mu (1 + \gamma_5) d_\beta) \]

BSM bag parameters defined by

\[ B_i = \frac{\langle \bar{K}^0 | O_i^{\Delta S=2} | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle} \]

where \( N_{2\ldots.5} = 5/3, 4, -2, 4/3 \)

Golden combinations \( G_i \)

\[ G_{23} = \frac{B_2}{B_3} \]
\[ G_{24} = B_2 \times B_4 \]
\[ G_{45} = \frac{B_4}{B_5} \]
\[ G_{21} = \frac{B_2}{B_K} \]

No \( \chi^{a1} \) logs at NLO
SWME results and comparison

slide from Jaehoon Leem

Preliminary Result

- We obtain BSM B-parameters $B_i$ from the results of golden combination $G_i$ and $B_K$.
- The dominant systematic error comes from the perturbative matching (4.4%).

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<th>SWME</th>
<th>RBC&amp;UKQCD</th>
<th>ETM</th>
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<td>$\mu = 2\text{GeV}$</td>
<td>$\mu = 3\text{GeV}$</td>
<td>$\mu = 3\text{GeV}$</td>
</tr>
<tr>
<td>$B_K$</td>
<td>0.537(04)(24)</td>
<td>0.518(04)(23)</td>
<td>0.53(2)</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0.576(05)(25)</td>
<td>0.532(05)(23)</td>
<td>0.43(5)</td>
</tr>
<tr>
<td>$B_{3\text{Buras}}$</td>
<td>0.385(05)(17)</td>
<td>0.363(05)(16)</td>
<td>N.A.</td>
</tr>
<tr>
<td>$B_{3\text{SU3Y}}$</td>
<td>0.862(07)(38)</td>
<td>0.785(07)(34)</td>
<td>0.75(9)</td>
</tr>
<tr>
<td>$B_4$</td>
<td>0.914(29)(40)</td>
<td>0.913(32)(40)</td>
<td>0.69(7)</td>
</tr>
<tr>
<td>$B_5$</td>
<td>0.661(20)(29)</td>
<td>0.660(22)(29)</td>
<td>0.47(6)</td>
</tr>
</tbody>
</table>

$\sim 3\sigma$ discrepancy/tension for $B_{4,5}$
SWME results and comparison

- Is the tension due to the matching to $\overline{\text{MS}}$?
- Systematic errors dominated by the perturbative renormalization procedure
- NPR implementation is on the way

See talk by Jangho Kim, Tuesday@5:10
Lattice 2014 update

RBC-UKQCD  [Boyle, N.G., Hudspith, Lytle, Sachrajda]

- $R_i$ from 2 + 1 Domain-Wall fermions
- Main limitation of previous work: single lattice spacing and only RI-MOM scheme
- New lattice spacing and NPR with RI-SMOM schemes

Non-perturbative renormalisation matrix can be obtained with great precision

- Volume source  [Göckeler et al, QCDSF ’98] ⇒ tiny statistical errors
- Keep the momenta orientation fixed and use twisted boundary condition ⇒ control discretisation effects
- Non-Exceptional kinematic (RI-SMOM) to avoid unwanted IR effects (chiral symmetry breaking, pole subtraction)

Unfortunately, the 1 − loop matching coefficient RI-SMOM $\rightarrow \overline{\text{MS}}$ are not known for the $(6, \bar{6})$ operators (for the $(8, 8)$ we can use  [Lehner & Sturm ’11])

In RI-MOM (exceptional kinematic), the pole subtractions seem to be mandatory

⇒ hard to estimate the associated systematic error
Non-perturbative running of the $(8,8)$ operators

- $n_f = 2$ non-perturbatively improved clover fermions
- Schrödinger functional, massless limit
- Non-perturbative evolution between $0.438 \text{ GeV}$ and $56 \text{ GeV}$

see talk by Mauro Papinutto Tuesday @ 2:55
Non-perturbative running of the $(8,8)$ operators

- $n_f = 2$ non-perturbatively improved clover fermions
- Schrödinger functional, massless limit
- Non-perturbative evolution between $0.438 \text{ GeV}$ and $56 \text{ GeV}$

see talk by Mauro Papinutto
Tuesday @ 2:55
2 + 1 Domain-Wall on asqtad (MILC configurations)

- Same setup as used for $B_K$  
  [Laiho & Van de Water’11]

- 3 lattice spacings

see talk by Maxwell Hansen
\[ K \rightarrow \pi\pi \]
Overview of the computation

Some references: [Bernard @ TASI’89, RBC PRD’01, Lellouch @ Les houches ’09]

Operator Product expansion

\[ \bar{d} \bar{s} \bar{d} \bar{u} \rightarrow \bar{s} \bar{d} \bar{u} \]

Describe \( K \rightarrow (\pi \pi)_{I=0,2} \) with an effective Hamiltonian

\[ H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} \left( V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \right) Q_i(\mu) \right\} \]

Short distance effects factorized in the Wilson coefficients \( y_i, z_i \)

Long distance effects factorized in the matrix elements

\[ \langle \pi \pi | Q_i | K \rangle \rightarrow \text{Lattice} \]
4-quark operators

\[ Q_1 = (\bar{s}d)_{V-A}(\bar{u}u)_{V-A} \quad Q_2 = \text{color mixed} \]
4-quark operators

Electroweak penguins

\[ Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V+A} \quad Q_8 = \text{color mixed} \]

\[ Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V-A} \quad Q_{10} = \text{color mixed} \]
4-quark operators

QCD penguins

\[ Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V-A} \]
\[ Q_4 = \text{color mixed} \]

\[ Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V+A} \]
\[ Q_6 = \text{color mixed} \]
$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Irrep of $SU(3)_L \otimes SU(3)_R$

\[
\bar{3} \otimes 3 = 8 + 1
\]
\[
\bar{8} \otimes 8 = 27 + 10 + 10 + 8 + 8 + 1
\]

Decomposition of the 4-quark operators gives

\[
Q_{1,2} = Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2}
\]
\[
Q_{3,4} = Q_{3,4}^{(8,1),\Delta I=1/2}
\]
\[
Q_{5,6} = Q_{5,6}^{(8,1),\Delta I=1/2}
\]
\[
Q_{7,8} = Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2}
\]
\[
Q_{9,10} = Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2}
\]
Only 7 are independent: one $(27, 1)$ four $(8, 1)$, and two $(8, 8)$, $\Rightarrow$ we called them $Q'$

\[
\begin{align*}
(27, 1) & \quad Q_1' = Q_{1}^{(27,1), \Delta I=3/2} + Q_{1}^{(27,1), \Delta I=1/2} \\
(8, 1) & \quad Q_2' = Q_{2}^{(8,1), \Delta I=1/2} \\
 & \quad Q_3' = Q_{3}^{(8,1), \Delta I=1/2} \\
 & \quad Q_5' = Q_{5}^{(8,1), \Delta I=1/2} \\
 & \quad Q_6' = Q_{6}^{(8,1), \Delta I=1/2} \\
(8, 8) & \quad Q_7' = Q_{7}^{(8,8), \Delta I=3/2} + Q_{7}^{(8,8), \Delta I=1/2} \\
 & \quad Q_8' = Q_{8}^{(8,8), \Delta I=3/2} + Q_{8}^{(8,8), \Delta I=1/2}
\end{align*}
\]
$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Only 7 are independent: one $(27, 1)$ four $(8, 1)$, and two $(8, 8)$, $\Rightarrow$ we called them $Q'$

\[
(27, 1) \quad Q'_1 = Q'_{(27,1), \Delta I=3/2} + Q'_{(27,1), \Delta I=1/2}
\]

\[
(8, 1) \quad Q'_2 = Q'_{(8,1), \Delta I=1/2} \\
Q'_3 = Q'_{(8,1), \Delta I=1/2} \\
Q'_5 = Q'_{(8,1), \Delta I=1/2} \\
Q'_6 = Q'_{(8,1), \Delta I=1/2}
\]

\[
(8, 8) \quad Q'_7 = Q'_{(8,8), \Delta I=3/2} + Q'_{(8,8), \Delta I=1/2} \\
Q'_8 = Q'_{(8,8), \Delta I=3/2} + Q'_{(8,8), \Delta I=1/2}
\]
$K \rightarrow (\pi \pi)_{I=2}$ by the RBC-UKQCD collaborations
Overview of the computation

- Lellouch-Lüscher method \cite{Lellouch Lüscher '00} to obtain the physical matrix element from the finite-volume Euclidean amplitude and the derivative of the phase shift

Combine

- Wigner-Eckart theorem (Exact up to isospin symmetry breaking )

\[
\langle \pi^+(p_1)\pi^0(p_2)|O^{\Delta I=3/2}_{\Delta I^Z=1/2}|K^+\rangle = 3/2\langle \pi^+(p_1)\pi^+(p_2)|O^{\Delta I=3/2}_{\Delta I^Z=3/2}|K^+\rangle
\]

and then compute the unphysical process \( K^+ \to \pi^+\pi^+ \)

- Use Anti-periodic B.C. to eliminate the unwanted (wrong-kinematic) state \cite{Sachrajda & Villadoro '05}

- Renormalise at low energy \( \mu_0 \sim 1.1 \text{ GeV} \) on the IDSDR and run non-perturbatively using finer lattices to \( \mu = 3 \text{ GeV} \) and match to \( \overline{\text{MS}} \) \cite{Arthur, Boyle '10, Arthur, Boyle, N.G., Kelly, Lytle '11}

\[
\lim_{a_1 \to 0} \left[ Z(\mu_1, a_1) Z^{-1}(\mu_0, a_1) \right] \times Z(\mu_0, a_0) = Z(\mu_1, a_0)
\]

\begin{align*}
\text{fine lattice} & \quad & \text{coarse lattice}
\end{align*}
“Pilot” computation of the full process


Unphysical:

- “Heavy” pions (lightest $\sim m_\pi \sim 300$ MeV), small volume
- Non-physical kinematics: pions at rest
\( A_0 \) from RBC-UKQCD

“Pilot” computation of the full process


Unphysical:

- “Heavy” pions (lightest \( \sim m_\pi \sim 300 \text{ MeV} \)), small volume

- Non-physical kinematics: pions at rest

But “complete”:

- Two-pion state

- All the contractions of the 7 fourk-operators are computed

- Renormalisation done non-perturbatively
“Pilot” computation of the full process


Unphysical:

- “Heavy” pions (lightest $\sim m_\pi \sim 300$ MeV), small volume
- Non-physical kinematics: pions at rest

But “complete”:

- Two-pion state
- All the contractions of the 7 fourk-operators are computed
- Renormalisation done non-perturbatively

obtain

\[
\begin{align*}
\text{Re } A_0 &= 3.80(82) \times 10^{-7} \text{GeV} \\
\text{Im } A_0 &= -2.5(2.2) \times 10^{-11} \text{GeV}
\end{align*}
\]
Toward an quantitative understanding of the $\Delta I = 1/2$ rule

We combine our physical computation of $\Delta I = 3/2$ part is our non-physical computation of the $\Delta I = 1/2$

<table>
<thead>
<tr>
<th></th>
<th>$1/\alpha$</th>
<th>$m_\pi$ [MeV]</th>
<th>$m_K$ [MeV]</th>
<th>Re$A_2$ [$10^{-8}$GeV]</th>
<th>Re$A_0$ [$10^{-8}$ GeV]</th>
<th>$\frac{\text{Re} A_0}{\text{Re} A_2}$</th>
<th>kinematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$16^3$ IW</td>
<td>$1.73(3)$</td>
<td>$422(7)$</td>
<td>$878(15)$</td>
<td>$4.911(31)$</td>
<td>$45(10)$</td>
<td>$9.1(2.1)$</td>
<td>threshold</td>
</tr>
<tr>
<td>$24^3$ IW</td>
<td>$1.73(3)$</td>
<td>$329(6)$</td>
<td>$662(11)$</td>
<td>$2.668(14)$</td>
<td>$32.1(4.6)$</td>
<td>$12.0(1.7)$</td>
<td>threshold</td>
</tr>
<tr>
<td>$32^3$ ID</td>
<td>$1.36(1)$</td>
<td>$142.9(1.1)$</td>
<td>$511.3(3.9)$</td>
<td>$1.38(5)(26)$</td>
<td>-</td>
<td>-</td>
<td>physical</td>
</tr>
<tr>
<td>Exp</td>
<td>–</td>
<td>$135 - 140$</td>
<td>$494 - 498$</td>
<td>$1.479(4)$</td>
<td>$33.2(2)$</td>
<td>$22.45(6)$</td>
<td></td>
</tr>
</tbody>
</table>

Pattern which could explain the $\Delta I = 1/2$ enhancement

[Boyle, Christ, N.G., Goode, Izubuchi, Janowski, Lehner, Liu, Lytle, Sachrajda, Soni, Zhang, PRL’13]
Two kinds of contraction for each $\Delta I = 3/2$ operator

Contraction 1

Contraction 2

$Re A_2$ is dominated by the tree level operator ($EWP \sim 1\%$):

Naive factorisation approach: $2 \sim 1/3$ ①

Our computation: $2 \sim -0.7 ①$

$\Rightarrow$ large cancellation in $Re A_2$
Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator

- Re$A_2$ is dominated by the tree level operator
  (EWP $\sim 1\%$):
  
- Naive factorisation approach: $\odot \sim 1/3\odot$

- Our computation: $\odot \sim -0.7\odot$

$\Rightarrow$ large cancellation in Re$A_2$
Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator

- $\text{Re}A_2$ is dominated by the tree level operator ($\text{EWP} \sim 1\%$):
- Naive factorisation approach: $2 \sim 1/3 1$
- Our computation: $2 \sim -0.7 1$

$\Rightarrow$ large cancellation in $\text{Re}A_2$
Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Re$A_0$ is also dominated by the tree level operators

<table>
<thead>
<tr>
<th>i</th>
<th>$Q_{i}^{\text{lat}}$ [GeV]</th>
<th>$Q_{i}^{\text{MS-NDR}}$ [GeV]</th>
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<tbody>
<tr>
<td>1</td>
<td>$8.1(4.6) \times 10^{-8}$</td>
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</tr>
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<tr>
<td>4</td>
<td>$-\infty$</td>
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<tr>
<td>10</td>
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Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Re$A_0$ is also dominated by the tree level operators

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Re$A_0 = 3.2(0.5) \times 10^{-7}$

Dominant contribution to $Q_2^{\text{lat}}$ is $\propto (\mathcal{2} \mathcal{2} - \mathcal{1}) \Rightarrow$ Enhancement in Re$A_0$
Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Re$A_0$ is also dominated by the tree level operators

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Dominant contribution to $Q_2^{\text{lat}}$ is $\propto \left( \frac{\Delta}{2} - \frac{\Gamma}{1} \right) \Rightarrow$ Enhancement in Re$A_0$

\[
\frac{\text{Re}A_0}{\text{Re}A_2} \sim \frac{2\Delta - \Gamma}{\Gamma + 2\Delta}
\]

With this unphysical kinematics, we find

\[
\frac{\text{Re}A_0}{\text{Re}A_2} = \begin{cases} 
9.1(2.1) & \text{for } m_K = 878 \text{ MeV } m_\pi = 422 \text{ MeV} \\
12.0(1.7) & \text{for } m_K = 662 \text{ MeV } m_\pi = 329 \text{ MeV}
\end{cases}
\]
Main limitation on the previous computation: only one coarse lattice spacing

IDSDR $32^3 \times 64$, with $a^{-1} \sim 1.37$ GeV $\Rightarrow a \sim 0.14$ fm, $L \sim 4.6$ fm

Current computation:

two lattice spacing, $n_f = 2 + 1$, large volume at the physical point

New discretisation of the Domain-Wall fermion formulation: Möbius [Brower, Neff, Orginos '12]

- $48^3 \times 96$, with $a^{-1} \sim 1.729$ GeV $\Rightarrow a \sim 0.11$ fm, $L \sim 5.5$ fm
- $64^3 \times 128$ with $a^{-1} \sim 2.358$ GeV $\Rightarrow a \sim 0.84$ fm, $L \sim 5.4$ fm
- $a m_{res} \sim 10^{-4}$

Status: Computation finished, draft in final stage
### Lattice 2014 update

- **$\Delta l = 3/2$**

  Main limitation on the previous computation: only one coarse lattice spacing

  IDSDR $32^3 \times 64$, with $a^{-1} \sim 1.37$ GeV $\Rightarrow a \sim 0.14$ fm, $L \sim 4.6$ fm

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  - $am_{res} \sim 10^{-4}$

  Status: Computation finished, draft in final stage

- **$\Delta l = 1/2$**

  Main limitation on the previous computation: non-physical kinematic

  New formulation: G-parity boundary conditions

  Status: First computation almost finished

See talks by Chris Kelly and by Daiqian Zhang, Monday
K \to (\pi \pi)_{I=2} \text{ Lattice 2014 update}

2012 \quad \text{[Blum, Boyle, Christ, N.G., Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, PRL’12, PRD’12]}
Re A_2 = 1.381(46)_{\text{stat}}(258)_{\text{syst}} 10^{-8} \text{ GeV}
Im A_2 = -6.54(46)_{\text{stat}}(120)_{\text{syst}} 10^{-13} \text{ GeV}

2014 \quad \text{[RBC-UKQCD Work in progress, draft in final stage]}

Preliminary: systematic budget not complete

see also talk by T. Janowski @ lat’13 \quad \text{[Janowski, Sachrajda, Boyle, Christ, Mawhinney, Yin, Zhang, N.G., Lytle]}
Other computations of $K \rightarrow (\pi \pi)$
\[ K \rightarrow \pi \pi \text{ with improved Wilson fermions} \]

[N. Ishizuka, K.I. Ishikawa, A. Ukawa, T. Yoshie]

- Direct computation with 2-pion at rest
- both \( \Delta I = 1/2 \) and \( \Delta I = 3/2 \)
- \( 2 + 1 \) improved Wilson fermions on Iwasaki gauge config
- \( 32^3 \times 64, \sim 0.091 \text{ fm}, L \sim 2.91 \text{ fm} \)

- Perturbative operator renormalization (1 loop)
  after non-perturbative subtraction of the lower dimensional operator \( P \).
  \[
  Q_i^{\text{MS}}(\mu) = \sum_j Z_{ij}(\mu) \cdot \left[ Q_j^{\text{lat}} - \alpha_j P \right]
  \]
  \[ P = \bar{s}\gamma_5 d, \quad \alpha_j = \frac{\langle 0 | Q_j | K \rangle}{\langle 0 | P | K \rangle}, \quad Z_{ij}(\mu): 1 \text{ loop} \]

- For calculations of the quark loops in the “eye” and the disconnected diagrams,
  hopping parameter expansion (4th order) and
  truncated solver method (\( N_\tau = 5 \)) are used.
  (proposed by G.S. Bali et al., (CPC 181(2010)1570))

---

See talk by Naruhito Ishizuka
(Monday 4:30)
$K \to \pi\pi$ with improved Wilson fermions

From Naruhito Ishizuka’s talk

**Results**

Effective matrix elements:

\[
M_I(Q_j)(t) = \langle 0 | K(t_K) Q_j(t) (\pi\pi)_I(t_\pi) | 0 \rangle \times e^{m_K(t_K-t)+E_{\pi\pi}(t-t_\pi)} \\
\propto \langle K | Q_j \pi\pi; I \rangle \quad \text{for} \quad t_K \gg t \gg t_\pi \quad (t_K = 24, \ t_\pi = 0, \ t : \text{run})
\]

![Graphs](image)

**Result of decay amplitudes:**

<table>
<thead>
<tr>
<th></th>
<th>Ours</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\pi$ (MeV)</td>
<td>280</td>
<td>140</td>
</tr>
<tr>
<td>$\Re A_2 \times 10^{-8}$ GeV</td>
<td>2.426 ± 0.038</td>
<td>1.479 ± 0.004</td>
</tr>
<tr>
<td>$\Re A_0 \times 10^{-8}$ GeV</td>
<td>60 ± 36</td>
<td>33.2 ± 0.2</td>
</tr>
<tr>
<td>$\Re A_0 / \Re A_2$</td>
<td>25 ± 15</td>
<td>22.45 ± 0.06</td>
</tr>
<tr>
<td>$\Re(e'/e) \times 10^{-3}$</td>
<td>0.80 ± 2.54</td>
<td>1.66 ± 0.23</td>
</tr>
</tbody>
</table>

- Enhancement of $\Delta I = 1/2$ process is seen.
- Further improvement of statics is necessary for $e'/e$. 

Nicolas Garron (Trinity College Dublin)  
Weak interactions of kaons and pions  
June 24, 2014  
49 / 52
Role of the charm mass in $K \rightarrow \pi\pi$ with improved Wilson fermions

Ongoing effort, updated in [Endress & Pena '12, Endress, Pena, Sivalingam '14]

- Charm is kept active in the effective Hamiltonian
- Matching to $SU(3)$ (heavy charm) and $SU(4)$ (unphysical light charm) chiral Lagrangian
- Computation of the LEC as a function of $m_c$

- Technically demanding, as requires to compute “eye contractions”, see [Endress, Pena, Sivalingam '14]

- Implementation with quenched overlap fermions on a single lattice spacing

- First results indicate an enhancement in $\text{Re}(A_0)/\text{Re}(A_2)$ as $m_C$ increases.

- Hard to know at the moment if the enhancement will be enough to give a factor 20 (charm is still far from its physical value)
Lattice 2014 update: Chromomagnetic operator in $K \to \pi$

**ETMc**

- $2 + 1 + 1$ Twisted Mass / Osterwalder-Seiler fermions
- Pion mass down to $\sim 210$ MeV
- three lattice spacings $a \sim 0.06 - 0.09$ fm

The effective $\Delta S=1$ Hamiltonian of $\text{dim}=5$ contains four magnetic operators:

$$H_{\text{eff}}^{\Delta S=1, d=5} = \sum_{i=\pm} \left( C_{i}^{\gamma} Q_{i}^{\gamma} + C_{i}^{g} Q_{i}^{g} \right) + \text{h.c.}$$

$$Q_{\gamma}^{\pm} = \frac{g}{16\pi^{2}} \left( \bar{s}_{R} \sigma^{\mu\nu} F_{\mu\nu} d_{L} \pm \bar{s}_{R} \sigma^{\mu\nu} F_{\mu\nu} d_{L} \right)$$

$$Q_{g}^{\pm} = \frac{g}{16\pi^{2}} \left( \bar{s}_{R} \sigma^{\mu\nu} G_{\mu\nu} d_{L} \pm \bar{s}_{R} \sigma^{\mu\nu} G_{\mu\nu} d_{L} \right)$$

**New Physics**

See talk by Vittorio Lubicz Wednesday@10:20 and poster by Marios Costa

<table>
<thead>
<tr>
<th>$C_{\text{SM}}$</th>
<th>$C_{\text{NP}}$</th>
<th>For $M_{\text{NP}} \sim 1$ TeV:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Dim} = 5$</td>
<td>$\sim 1/M_{W}$</td>
<td>$\sim 1/M_{NP}$</td>
</tr>
<tr>
<td>$\Delta F \neq 0$</td>
<td>$\sim \alpha_{w}(M_{W})$</td>
<td>$\sim \alpha_{s}(M_{NP})$</td>
</tr>
<tr>
<td>LR chirality</td>
<td>$\sim m_{s}/M_{W}$</td>
<td>$\sim \delta_{LR}$</td>
</tr>
</tbody>
</table>
Conclusions

Exciting time for kaon/pion physics

- Various collaborations are reaching the physical point
- For the decay constants, or semi-leptonic form factors, we are reaching a precision such that EM corrections become significant (see plenary talk by Antonin Portelli, Thursday@11:30)
- Computation of new quantities (eg: chromomagnetic operator)
- New computations of the neutral kaon mixing matrix elements ($B_K$ and BSM)
- Continuum limit of $K \to (\pi\pi)_{I=2}$ at the physical point
- First realistic results of $K \to (\pi\pi)_{I=0}$ (with physical kinematics) should be available in a few months, thanks to G-parity boundary conditions
- Various collaborations are computing the BSM neutral kaon matrix elements
- NPR is at mature stage with the RI-SMOM schemes, but some matching coefficients are highly needed: NNLO (2-loops matching) for $B_K$ and NLO (1-loop matching) or the $(6, \bar{6})$ BSM operators