Weak interactions of kaons and pions

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Lattice 2014, Columbia University, June 24, 2014

Outline

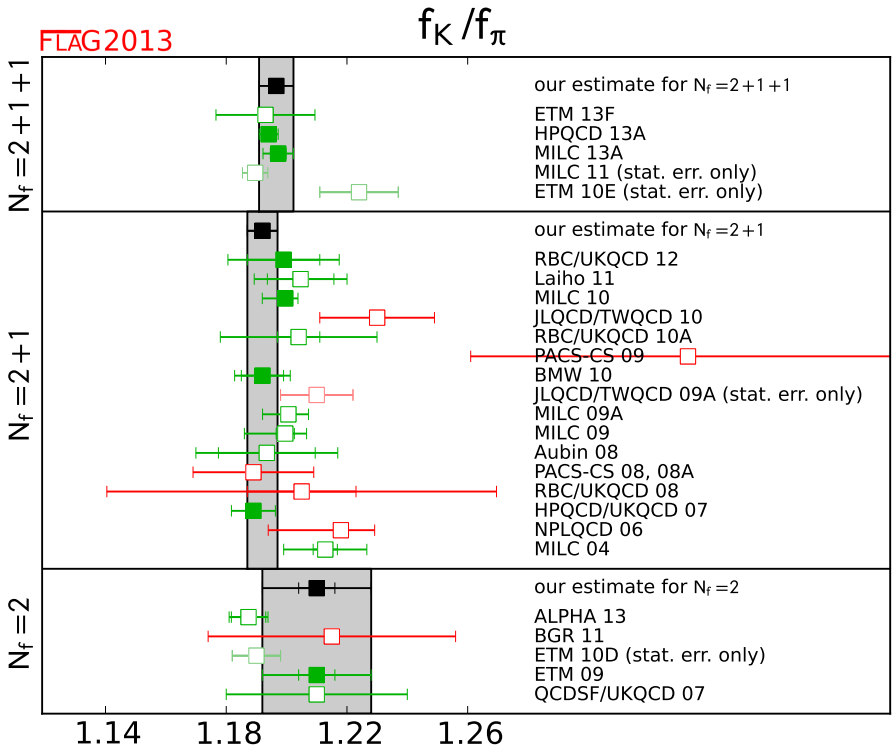
1. Decay constants f_K, f_π and f_K/f_π
2. K_{l3}
3. Neutral kaon mixing (B_K and BSM contributions)
4. $K \to \pi\pi$

Rare kaon decays not covered in this talk (see plenary by Chris Sachrajda)

Situation before lattice 2014

FLAG'13

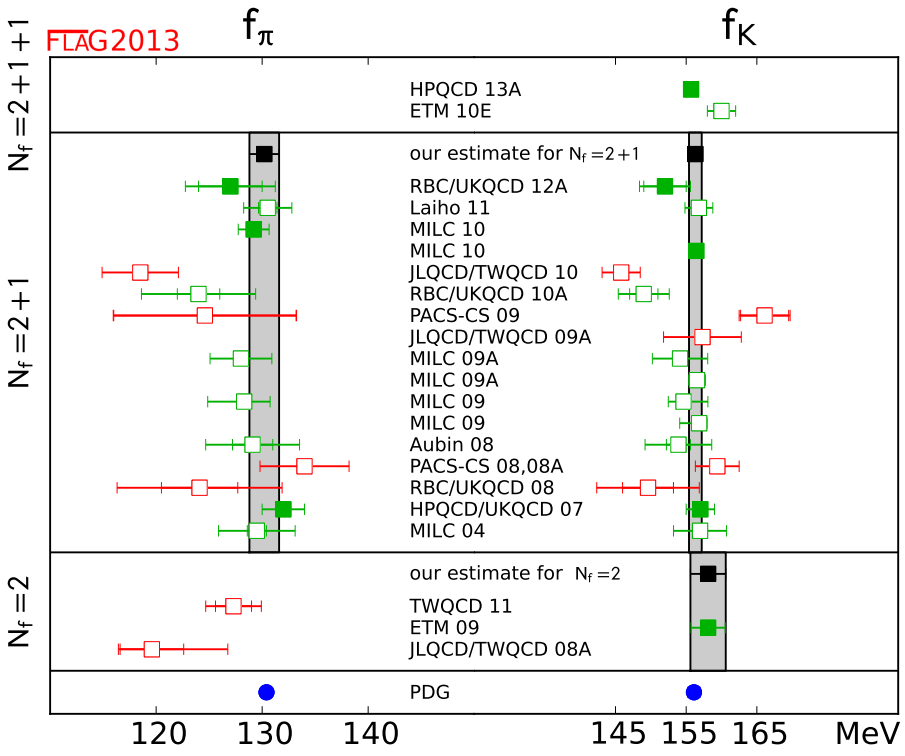
$$f_K/f_\pi = 1.194(5) \qquad n_f = 2+1+1$$
$$f_K/f_\pi = 1.192(5) \qquad n_f = 2+1$$
$$f_K/f_\pi = 1.205(6)(17) \qquad n_f = 2$$



Situation before lattice 2014

FLAG'13

$$f_\pi = 130.2(1.4)\ \mathrm{MeV} \qquad n_f = 2+1$$
$$f_K = 156.3(0.9)\ \mathrm{MeV} \qquad n_f = 2+1$$
$$f_K = 158.1(2.5)\ \mathrm{MeV} \qquad n_f = 2$$



Fermilab/MILC [*A. Bazavov et al.*, Phys.Rev.Lett. 110 (2013) 172003 & PoS LATTICE 2013]

2013	$f_{K^+}/f_{\pi^+} = 1.1947$	$(26)_{\rm stat}\ (33)_{a^2\ {\rm extrap}}(17)_{\rm FV}(2)_{\rm EM}$
2014	$f_{K^+}/f_{\pi^+} = 1.1956$	$(10)_{\rm stat}\ {}^{+23}_{-14}\vert_{a^2\ {\rm extrap}}(10)_{\rm FV}(5)_{\rm EM}$

- $n_f = 2+1+1$ Highly-Improved Staggered Quark (HISQ)
- $a \sim 0.06, 0.09, 0.12, 0.15\ \mathrm{fm}$
- $m_\pi \sim 135, 200\ \mathrm{MeV}$ and $m_\pi L > 3.3$

See talk by Javad Komijani, Wednesday@12:10

RBC-UKQCD PRELIMINARY (draft in final stage)

$$
\begin{aligned}
f_\pi &= 0.1298(9)_{\rm stat}(4)_\chi(2)_{\rm FV}\ \ {\rm GeV} \\
f_K &= 0.1556(8)_{\rm stat}(2)_\chi(1)_{\rm FV}\ \ {\rm GeV} \\
f_K/f_\pi &= 1.199(5)_{\rm stat}(6)_\chi(1)_{\rm FV}
\end{aligned}
$$

$n_f = 2+1$ Domain-Wall fermions

- New Möbius ensembles [Brower, Neff, Orginos '12] combined with existing Shamir ensembles.
- $a \sim 0.084, 1.144\ {\rm fm}$, $48^3 \times 96 \times 12$ and $64^3 \times 128 \times 12$
- Physical pion masses $m_\pi \sim 130\ {\rm MeV}$ and $m_\pi L > 3.5$
- Finer ensemble $a \sim 0.06$, $32^3 \times 64 \times 12$ with $m_\pi \sim 360\ {\rm MeV} \Rightarrow m_\pi L \sim 3.8$)

K_{l3}

K_{l3} semileptonic form factor I.

Obtain $|V_{us} f_+(0)|$ from the experimental rate

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2}\, I\, S_{EW}\ \left[1 + 2\Delta_{SU(2)} + 2\Delta_{EM}\right]\ |V_{us}\, f_+(0)|^2$$

where:

I is the phase space integral evaluated from the shape of the experimental form factor

$\Delta_{SU(2)}$ is the ispospin breaking correction

S_{EW} is the short distance electroweak correction

Δ_{EM} is the long distance electromagnetic correction

and $f_+(0)$ is the form factor defined from ($q = p - p'$)

$$\langle\pi(p')|V_\mu|K(p)\rangle \quad = \quad (p_\mu + p'_\mu)\, f_+(q^2) \,+\, (p_\mu - p'_\mu)\, f_-(q^2) \qquad \text{with } V_\mu = \bar{s}\gamma_\mu u$$

$\Rightarrow$ determine $f_+(0)$ from the lattice to constraint V_{us}

K_{l3} semileptonic form factor II.

Use the the scalar form factor $f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$

$$\langle \pi(p')|V_\mu|K(p)\rangle_{q^2=0} = \frac{m_K^2 - m_\pi^2}{m_s - m_u} f_+(0)$$

- Compute $f_0(q^2)$ for several negative values of q^2
- Interpolate to $q^2 = 0$ (or use twisted boundary conditions) [RBC-UKQCD]

Or compute $f_+(0)$ from [Fermilab Lattice and MILC Collaborations *Bazavov, et al. '13*]

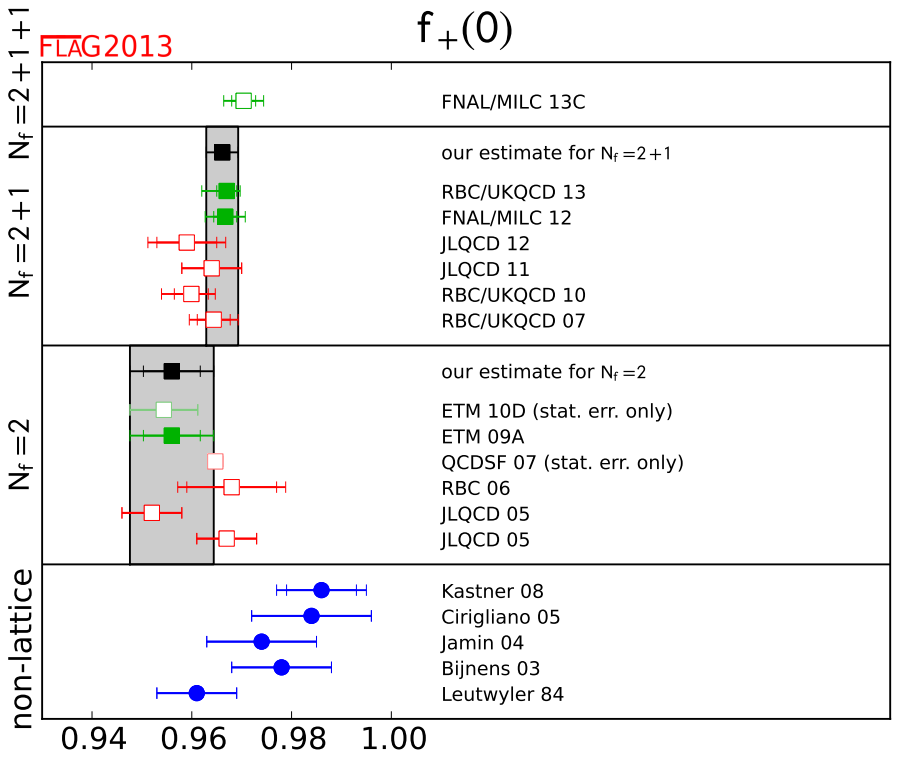
$$f_+(0) = f_0(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2} \langle \pi(p')|S|K(p)\rangle$$

Form factor can be obtained from $\langle \pi(p')|S|K(p)\rangle$ and from $\langle \pi(p')|V_\mu|K(p)\rangle$

Situation before lattice 2014

FLAG'13

$$f_+(0) = 0.9661(32) \qquad n_f = 2+1$$
$$f_+(0) = 0.9560(57)(62) \qquad n_f = 2$$



RBC-UKQCD

- New ensembles 48^3 and 64^3 at the physical point
- Results obtained from the vector current

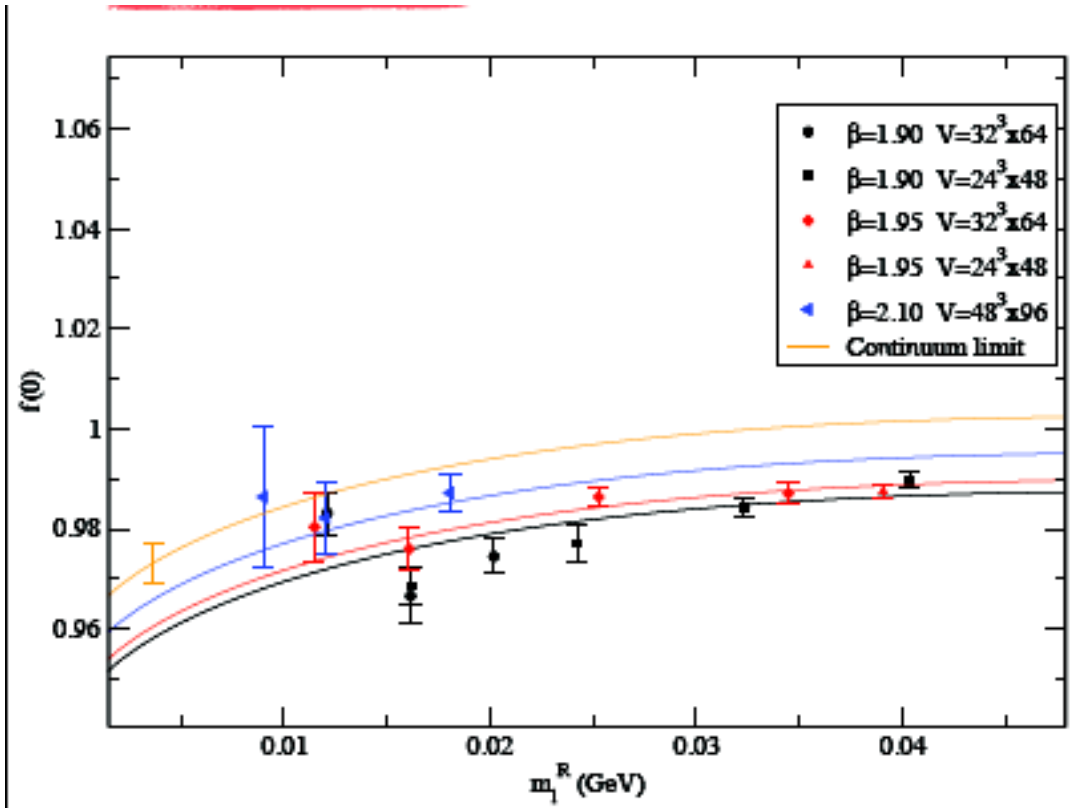
See talk by David Murphy, Monday@6:10

Lattice	m_π (MeV)	$f_+^{K\pi}(0)$	Stat. error
24I	678	0.9992(1)	0.01%
24I	563	0.9956(4)	0.04%
24I	422	0.9870(9)	0.09%
24I	334	0.9760(43)	0.4%
24I	334	0.9858(28)	0.3%
48I (PRELIMINARY)	**139**	**0.9727(25)**	**0.3%**
32ID	248	0.9771(21)	0.2%
32ID	171	0.9710(45)	0.5%
32I	398	0.9904(17)	0.2%
32I	349	0.9845(23)	0.2%
32I	295	0.9826(35)	0.4%
64I (PRELIMINARY)	**139**	**0.9701(22)**	**0.2%**

ETMc

- $2+1+1$ Twisted Mass / Osterwalder-Seiler fermions
- Results obtained from the vector current
- Preliminary result:

$$f_+(0) = 0.9683(50)_{\text{stat+fit}}(42)_{\text{chiral}}$$



See talk by Lorenzo Riggio, Friday@5:10

$K^0 - K^0$ mixing, $K \to \pi\pi$ and CP violation

Background: Kaon decays and CP violation

- First discovery of CP violation was made in kaon system in 1964 (Christenson, Cronin, Fitch and Turlay)
- Noble prize in 1980 (Cronin and Fitch)
- Very nice measurements of both direct and indirect CP violation

$$\begin{cases} Re\left(\frac{\varepsilon'}{\varepsilon}\right) & = (1.65 \pm 0.26) \times 10^{-3} \\ |\varepsilon| & = (2.228 \pm 0.011) \times 10^{-3} \end{cases}$$

- Theoretically:

  Relate indirect CP violation parameter (ϵ) to neutral kaon mixing (B_K)

  Still lacking a quantitative description of direct CP violation (ε')

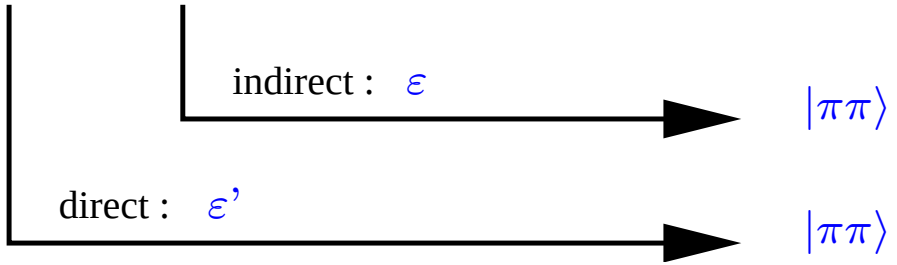
- Sensitivity to new physics

Background: Kaon decays and CP violation

Flavour eigenstates $\begin{pmatrix} K^0 = \bar{s}\gamma_5 d \\ \overline{K}^0 = \bar{d}\gamma_5 s \end{pmatrix} \neq$ CP eigenstates $|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle \mp |\overline{K}^0\rangle\}$ They are mixed in the physical eigenstates $\begin{cases} |K_L\rangle & \sim & |K^0_-\rangle + \bar{\varepsilon}|K^0_+\rangle \\ |K_S\rangle & \sim & |K^0_+\rangle + \bar{\varepsilon}|K^0_-\rangle \end{cases}$

Direct and indirect CP violation in $K \to \pi\pi$

$$|K_L\rangle \propto |K_-\rangle + \varepsilon|K_+\rangle$$



$$\epsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = |\epsilon|e^{i\phi_\epsilon} \sim \bar{\epsilon}$$

$K \to \pi\pi$ amplitudes

Two isospin channels: $\Delta I = 1/2$ and $\Delta I = 3/2$

$$K \to (\pi\pi)_{I=0,2}$$

Corresponding amplitudes defined as

$$A[K \to (\pi\pi)_{\mathrm{I}}] = A_{\mathrm{I}} \exp(i\delta_{\mathrm{I}}) \qquad /\mathrm{w}\ \mathrm{I} = 0, 2 \qquad \delta = \text{strong phases}$$

$\Delta I = 1/2$ rule

$$\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_o} \sim 1/22 \qquad \text{(experimental number)}$$

Amplitudes are related to the parameters of CP violation $\varepsilon, \varepsilon'$ via

$$\epsilon' = \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

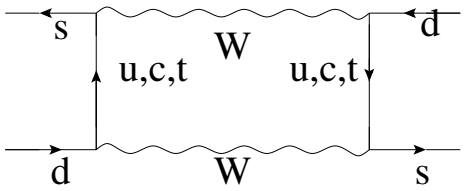
$$\epsilon = e^{i\phi_\epsilon} \left[\frac{\mathrm{Im}\langle \bar{K}^0 | H_{\mathrm{eff}}^{\Delta S=2} | K^0 \rangle}{\Delta m_K} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

$\Rightarrow$ Related to $K^0 - \bar{K}^0$ mixing

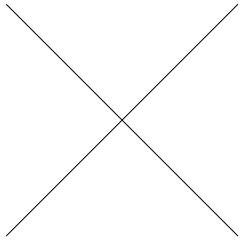
See poster by Yong-Chull Jang and Weonjong Lee

Neutral kaon mixing in the SM

In the Standard Model, $K^0 - \bar{K}^0$ mixing dominated by box diagrams with W exchange, e.g.



$\mu \ll M_W$
OPE



$\Delta S = 2$

$+$ long distance
$(\Delta S = 1)^2$

Operator product expansion

$$H_{\rm eff}^{\Delta S=2} = \frac{G_F^2 m_W^2}{16\pi^2} \times F(\text{SM free parameters}) \times C(\mu)\mathcal{O}_{LL}^{\Delta S=2}(\mu)$$

Factorise the non-perturbative contribution

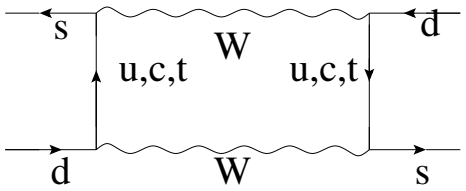
$$\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} F_K^2 M_K^2 B_K(\mu) \qquad \text{w/ } \mathcal{O}_{LL}^{\Delta S=2} = (\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma^\mu(1-\gamma_5)d)$$

B_K is the SM kaon bag parameter

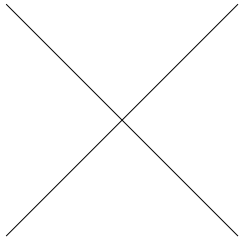
$$B_K(\mu) = \frac{\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle}{\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle_{\rm VS}}$$

Neutral kaon mixing in the SM

In the Standard Model, $K^0 - \bar{K}^0$ mixing dominated by box diagrams with W exchange, e.g.



$\mu \ll M_W$
OPE



$\Delta S = 2$

$+$ long distance
$(\Delta S = 1)^2$

$K_L - K_S$ mass difference,
long distance contributions:

See plenary talk by Chris Sachrajda, Saturday@10:30

Neutral kaon mixing in the SM: B_K

In the SM, only one four-quark operator

$$\mathcal{O}^{\Delta S=2}_{(V-A)\times(V-A)} = (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{s}_\beta \gamma^\mu (1-\gamma_5) d_\beta)$$

Usually parametrised by its bag parameter (renormalization scheme and scale dependent)

$$B_K = \frac{\langle \bar{K}^0 | \mathcal{O}^{\Delta S=2}_{LL}(\mu) | K^0 \rangle}{\langle \bar{K}^0 | \mathcal{O}^{\Delta S=2}_{LL} | K^0 \rangle_{\rm VS}} = \frac{\langle \bar{K}^0 | \mathcal{O}^{\Delta S=2}_{LL}(\mu) | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

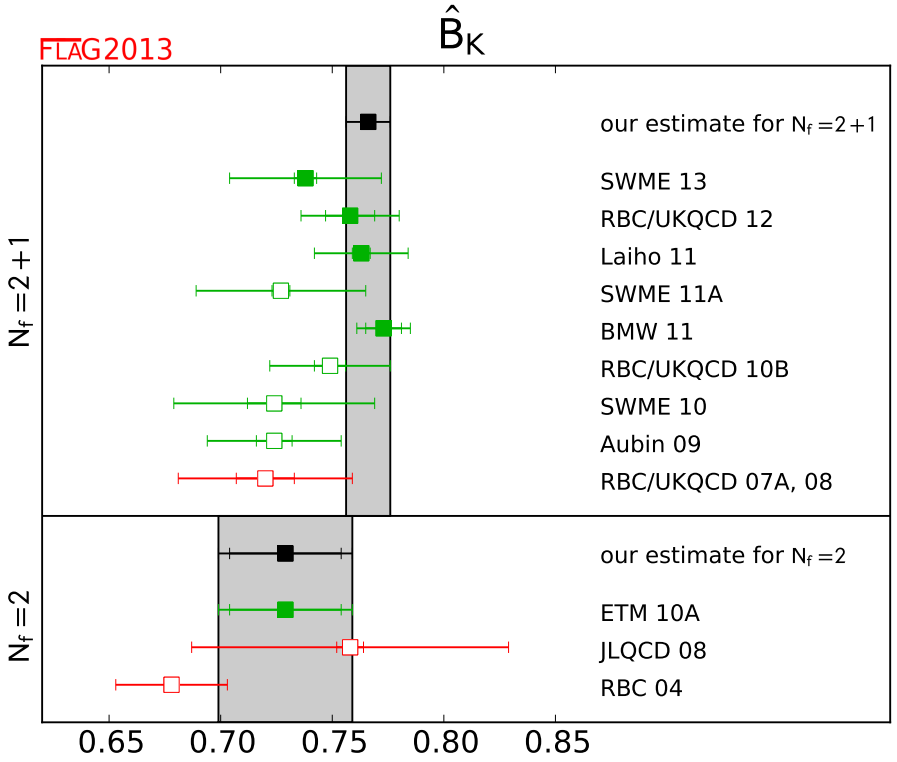
Define the Renormalisation-Group-Invariant $\hat{B}_K$ by

$$\hat{B}_K = \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \exp\left\{ \int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma(g)}{\beta(g)} + \frac{\gamma_0}{\beta_0 g} \right) \right\} B_K(\mu) \ .$$

Traditionally: give $B_K^{\overline{\rm MS}}(2\ \mathrm{GeV})$ or $\hat{B}_K$ or $B_K^{\overline{\rm MS}}(2\ \mathrm{GeV})$.

Recently, lattice community starts giving results at a higher scale.

FLAG [Aoki et al., '13-14]



Status before lattice 2014

BMW '11 [*Dürr, Fodor, Hoelbling, Katz, Krieg, Kurth, Lellouch, Lippert, McNeil, Portelli, Szabó*, PLB '11]

$$\hat{B}_K = 0.7727(81)_{\text{stat}}(34)_{\text{sys}}(77)_{\text{PT}}$$

- $2+1$ HEX-smeared clover-improved Wilson fermions,
- Four lattice spacings $a \sim 0.054 - 0.093\ \mathrm{fm}$
- Pion masses down to the physical point
- Non-perturbative-renormalization (NPR) through RI-MOM scheme

RBC-UKQCD '12 [*Arthur, Blum, Boyle, Christ,N.G., Hudspith, Izubuchi, Jung, Kelly, Lytle, Mawhinney, Murphy, Ohta, Sachrajda, Soni, Yu, Zanotti], PRD'12*

$$\hat{B}_K = 0.758(11)_{\text{stat}}(10)_{\chi}(4)_{\text{FV}}(16)_{\text{PT}}$$

- $2+1$ Domain-Wall fermions
- $a \sim 0.14\ \mathrm{fm}$, IDSDR, $m_\pi \sim 170\ \mathrm{MeV}$ (partially quenched 140 MeV)
- $a \sim 0.85, 0.11\ \mathrm{fm}$ IW m_π down to $\sim 290\ \mathrm{MeV}$
- NPR with 2 RI-SMOM schemes

SWME '14 [Bae, Jang, Jeong, Jung, H.J.Kim, Ja.Kim, Jo.Kim, K.Kim, S.Kim, Lee, Jaehoon Leem, Pak, Park, Sharpe, Yoon]

$$\hat{B}_K = 0.7379(47)_{\text{stat}}(365)_{\text{sys}}$$

- $2+1$ HYP-smeared staggered on aqstqd (MILC) ensembles
- Four lattice spacings $a \sim 0.045 - 0.12\ \mathrm{fm}$
- Pion masses down to 200 MeV
- Renormalisation: 1-loop matching to $\overline{\mathrm{MS}}$

ETMc

$$\hat{B}_K = 0.729(25)(17)$$

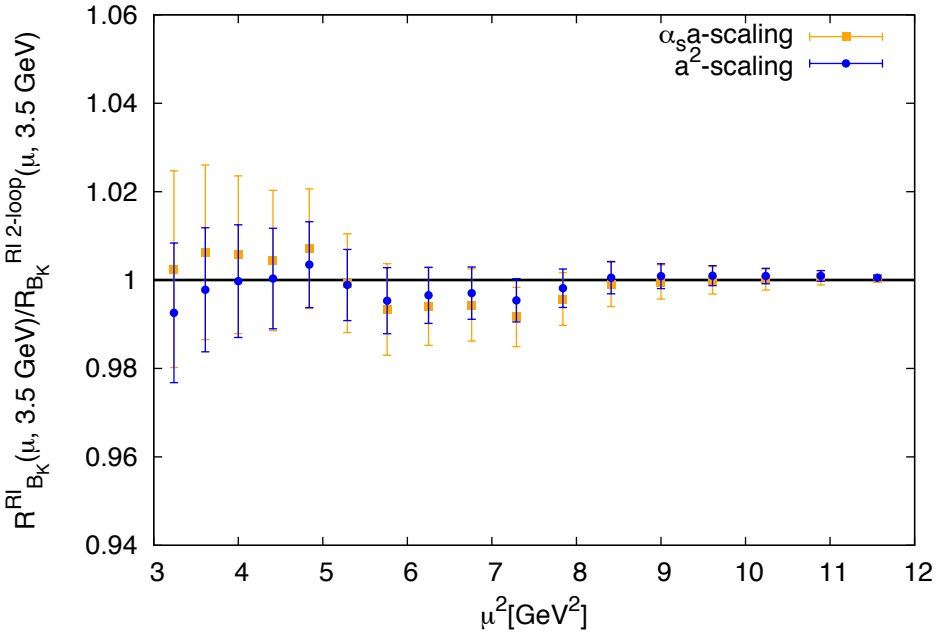
- 2 flavours twisted mass ($2+1+1$ in progress)
- Four lattice spacings $a \sim 0.045 - 0.12\ \mathrm{fm}$
- Pion masses down to 200 MeV
- Non-perturbative-renormalization (NPR) through RI-MOM scheme

Non-perturbative scale-evolution

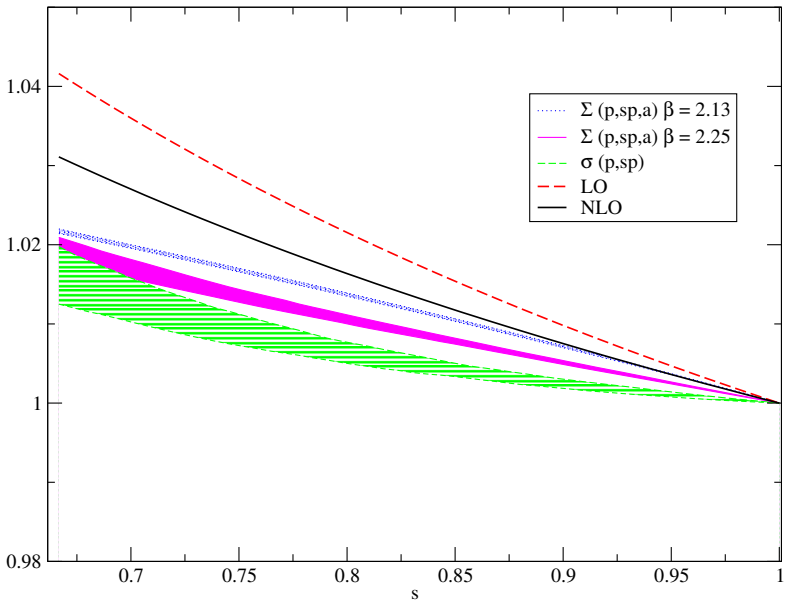
Running between two energy scales μ_1 and μ_2

$$Z(\mu_1) = U(\mu_1, \mu_2) Z(\mu_2)$$

Comparison of the non-perturbative running in RI-MOM with perturbation theory (NLO)



BMW'11
$U_{NP}(\mu, 3.5)/U_{2\text{-loop}}(\mu, 3.5)$



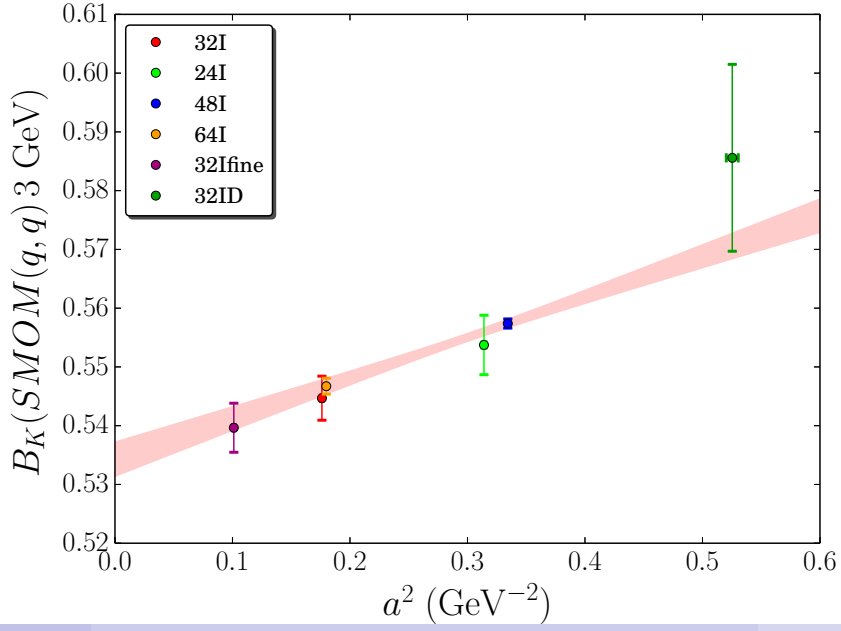
RBC-UKQCD'10
$U_{NP}(2, 3)$ vs $U_{2\text{-loop}}(2, 3)$

RBC-UKQCD PRELIMINARY [Work in progress, draft in final stage]

$n_f = 2 + 1$ Domain-Wall fermions

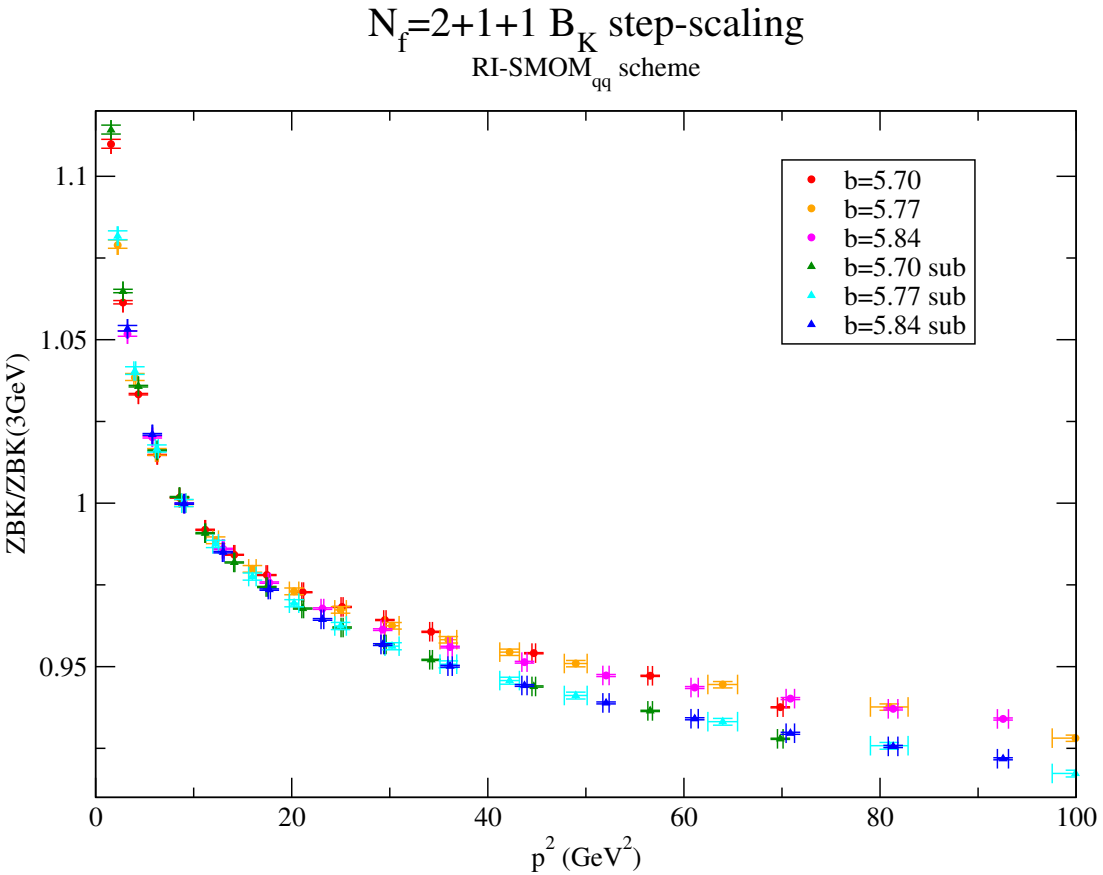
- New Möbius ensembles combined with existing Shamir ensembles.
- $a \sim 0.084, 0.144$ fm, $48^3 \times 96 \times 12$ and $64^3 \times 128 \times 12$
- Physical quark masses $m_\pi \sim 130$ MeV and $m_\pi L > 3.5$
- Finer ensemble $a \sim 0.06$, $32^3 \times 64 \times 12$ with $m_\pi \sim 360$ MeV $\Rightarrow m_\pi L \sim 3.8$)

$n_f = 2 + 1 + 1$ Domain-Wall fermions (Möbius) in progress



RBC-UKQCD PRELIMINARY [Work in progress]

Running to 5 GeV with $2+1+1$ flavours



see talk by Julien Frison, Tuesday @5:10

Note about matching to $\overline{\mathrm{MS}}$

- The matching to $\overline{\mathrm{MS}}$ is done at Next-to-leading order
- Difficult to estimate the corresponding systematic error
- NNLO matching factors (between MOM and $\overline{\mathrm{MS}}$) on the wishlist
- RBC-UKQCD uses several intermediate (SMOM) scheme and take the difference for the estimate of the syst. error
- At 2 or 3 GeV this is significantly larger than the naive estimate
- At 5 *GeV* this error is 1% (see talk by Julien Frison)

Standard Model and Beyond

See [F. Gabbiani et al '96]

In the SM, neutral kaon mixing occurs through W-exchanges $\rightarrow$ $(V-A)\times(V-A)$

$$O_1^{\Delta s=2} = (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta \gamma_\mu (1-\gamma_5) d_\beta)\,,$$

Invariant under Fierz arrangement $\Rightarrow$ only one color structure

Beyond the SM, other Dirac structure appear at high energy

Low energy description: generic $\Delta S = 2$ effective Hamiltonian $H^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu)\, O_i^{\Delta S=2}(\mu)\,.$

SUSY basis

$$\begin{aligned}
O_2^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1-\gamma_5) d_\beta) \\
O_3^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1-\gamma_5) d_\alpha) \\
O_4^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1+\gamma_5) d_\beta) \\
O_5^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1+\gamma_5) d_\alpha)
\end{aligned}$$

Parity partners are redundant if Parity is conserved

On the lattice: compute $\langle \bar{K}^0 | O_i^{\Delta S=2} | K^0 \rangle$

Mixing

- Mixing pattern given by $SU(3)_L \times SU(3)_R$ decomposition

$$\begin{aligned} 3 \times 3 &= 6 + \bar{3} \\ \bar{3} \times \bar{3} &= \bar{6} + 3 \\ \bar{3} \times 3 &= 1 + 8 \end{aligned}$$

BSM operators

$$\begin{aligned} O_2^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)(\bar{s}_\beta (1-\gamma_5) d_\beta) \\ O_3^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)(\bar{s}_\beta (1-\gamma_5) d_\alpha) \end{aligned}$$

Under $SU_L(3) \longrightarrow \bar{s}_R d_L \bar{s}_R d_L$ Symmetric $\Rightarrow 6_L$

Mixing

- Mixing pattern given by $SU(3)_L \times SU(3)_R$ decomposition

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Under $SU_R(3) \longrightarrow \bar{s}_R d_L \bar{s}_R d_L$ Symmetric $\Rightarrow \bar{6}_R$

Mixing

- Mixing pattern given by $SU(3)_L \times SU(3)_R$ decomposition

$$\begin{aligned} 3 \times 3 &= 6 + \bar{3} \\ \bar{3} \times \bar{3} &= \bar{6} + 3 \\ \bar{3} \times 3 &= 1 + 8 \end{aligned}$$

BSM operators

$$\begin{aligned} O_4^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)(\bar{s}_\beta (1+\gamma_5) d_\beta) \\ O_5^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)(\bar{s}_\beta (1+\gamma_5) d_\alpha) \end{aligned}$$

Under $SU_L(3) \longrightarrow \bar{s}_R d_L \bar{s}_L d_R$ Non-flavour singlet $\Rightarrow 8_L$

Mixing

- Mixing pattern given by $SU(3)_L \times SU(3)_R$ decomposition

$$\begin{aligned} 3 \times 3 &= 6 + \bar{3} \\ \bar{3} \times \bar{3} &= \bar{6} + 3 \\ \bar{3} \times 3 &= 1 + 8 \end{aligned}$$

BSM operators

$$\begin{aligned} O_4^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)(\bar{s}_\beta (1+\gamma_5) d_\beta) \\ O_5^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)(\bar{s}_\beta (1+\gamma_5) d_\alpha) \end{aligned}$$

Under $SU_R(3) \longrightarrow \bar{s}_R d_L \bar{s}_L d_R$ Non-flavour singlet $\Rightarrow 8_R$

Mixing pattern and $SU(3)\chi PT$

- $O_1 \in (27,1)$ renormalises multiplicatively
- $O_2, O_3 \in (6,\bar{6})$ mix together
- $O_4, O_5 \in (8,8)$ mix together
- Renormalization matrix is block diagonal $1_{(27,1)} \; + \; (2\times 2)_{(6,\bar{6})} \; + \; (2\times 2)_{(8,8)}$
- In the chiral limit $O_1 \to m_P^2$ and $O_{i\geq 2} \to \mathrm{Cst}$

$$\Rightarrow \text{Expect } \frac{\langle \bar{K}^0 | O_{BSM} | K^0 \rangle}{\langle \bar{K}^0 | O_{SM} | K^0 \rangle} \to \frac{1}{m_P^2}$$

Normalisation

$\langle \bar{K}^0 | O | K^0 \rangle$ are dimension-four quantities

Different normalisations exit

- Bag parameters $B's$, like $B = \frac{\langle \bar{K}^0 | O_1 | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle_{VS}}$

$$B_1 = B_K = \frac{\langle \bar{K}^0 | O_1 | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

$$B_{i \geq 2} = \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle}$$

- Ratios $R's$ [Babich, N.G., Hoelbling, Howard, Lellouch, Rebbi '06]

$$R_i^{\mathrm{BSM}}(m_P) = \left[\frac{f_K^2}{m_K^2} \right]_{\mathrm{expt}} \left[\frac{m_K^2}{f_K^2} \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle} \right]_{\mathrm{latt}}$$

- Golden combinations G_s [Bailey, Kim, Lee, Sharpe '12 , Bećirević, Villadoro '04]

  Ratios or products of B parameters free of chiral logs at NLO

Situation before Lattice'14

- $n_f = 2 + 1$ Domain-Wall [RBC-UKQCD '12]
- $n_f = 2$ [ETMc '12] and preliminary $n_f = 2 + 1 + 1$ Twisted Mass [ETMc @ lat'13]
- $n_f = 2 + 1$ staggered [SWME '13]

RBC-UKQCD and ETMc found compatible results, but tension observed by SWME

See poster by Jaehoon Leem

- Different (but equivalent) choice of basis

$$
\begin{aligned}
O_2^{\Delta S=2} &= (\bar{s}_\alpha(1-\gamma_5)d_\alpha)\,(\bar{s}_\beta(1-\gamma_5)d_\beta) \\
O_3^{\Delta S=2} &= (\bar{s}_\alpha\sigma_{\mu\nu}(1-\gamma_5)d_\alpha)\,(\bar{s}_\beta\sigma_{\mu\nu}(1-\gamma_5)d_\beta) \\
O_4^{\Delta S=2} &= (\bar{s}_\alpha(1-\gamma_5)d_\alpha)\,(\bar{s}_\beta(1+\gamma_5)d_\beta) \\
O_5^{\Delta S=2} &= (\bar{s}_\alpha\gamma_\mu(1-\gamma_5)d_\alpha)\,(\bar{s}_\beta\gamma_\mu(1+\gamma_5)d_\beta)
\end{aligned}
$$

- BSM bag parameters defined by

$$
B_i = \frac{\langle \bar{K}^0 | O_i^{\Delta S=2} | K^0 \rangle}{N_i \langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}
$$

  where $N_{2\ldots5} = 5/3, 4, -2, 4/3$

- Golden combinations G_i

$$
\begin{aligned}
G_{23} &= \frac{B_2}{B_3} & G_{45} &= \frac{B_4}{B_5} \\
G_{24} &= B_2 \times B_4 & G_{21} &= \frac{B_2}{B_K}
\end{aligned}
$$

- No χ^{al} logs at NLO

slide from Jaehoon Leem

Preliminary Result

- We obtain BSM B-parameters B_i from the results of golden combination G_i and B_K.
- The dominant systematic error comes from the perturbative matching.(4.4%)

	SWME		RBC&UKQCD	ETM
	$\mu = 2$GeV	$\mu = 3$GeV	$\mu = 3$ GeV	$\mu = 3$ GeV
B_K	0.537(04)(24)	0.518(04)(23)	0.53(2)	0.51(2)
B_2	0.576(05)(25)	0.532(05)(23)	0.43(5)	0.47(2)
B_3^{Buras}	0.385(05)(17)	0.363(05)(16)	N.A.	N.A.
B_3^{SUSY}	0.862(07)(38)	0.785(07)(34)	0.75(9)	0.78(4)
B_4	0.914(29)(40)	0.913(32)(40)	0.69(7)	0.75(3)
B_5	0.661(20)(29)	0.660(22)(29)	0.47(6)	0.60(3)

$\sim 3\sigma$ discrepancy/tension for $B_{4,5}$

SWME results and comparison

- Is the tension due to the matching to $\overline{\mathrm{MS}}$?
- Systematic errors dominated by the perturbative renormalization procedure
- NPR implementation is on the way

See talk by Jangho Kim,
Tuesday@5:10

RBC-UKQCD [Boyle, N.G.,Hudspith, Lytle, Sachrajda]

- R_i from $2+1$ Domain-Wall fermions
- Main limitation of previous work: single lattice spacing and only RI-MOM scheme
- New lattice spacing and NPR with RI-SMOM schemes

Non-perturbative renormalisation matrix can be obtained with great precision

- Volume source [Göckeler et al, QCDSF '98] $\Rightarrow$ tiny statistical errors
- Keep the momenta orientation fixed and use twisted boundary condition $\Rightarrow$ control disctretisation effects
- Non-Exceptional kinematic (RI-SMOM) to avoid unwanted IR effects (chiral symmetry breaking, pole subtraction)

Unfortunately, the $1-$ *loop* matching coefficient RI-SMOM $\rightarrow \overline{\mathrm{MS}}$ are not known for the $(6,\bar{6})$ operators (for the $(8,8)$ we can use [Lehner & Sturm '11])

In RI-MOM (exceptional kinematic), the pole subtractions seem to be mandatory

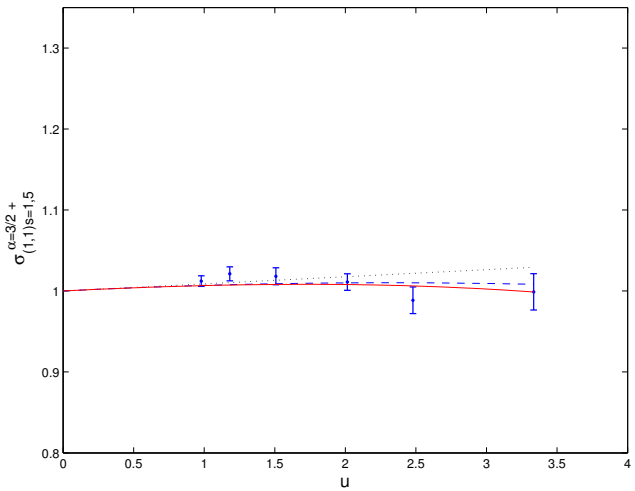
$\Rightarrow$ hard to estimate the associated systematic error

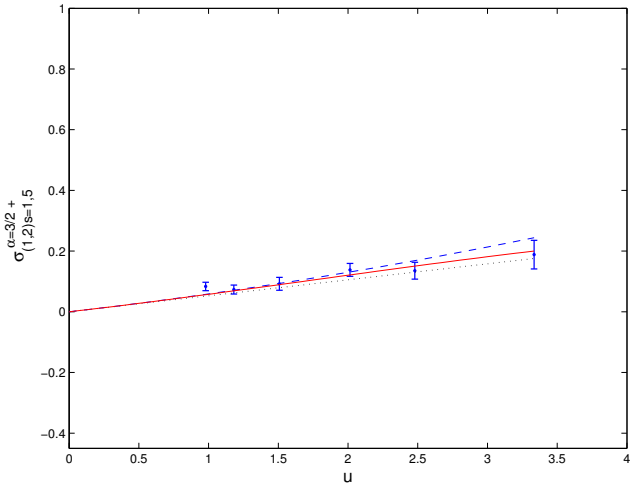
ALPHA'14 [*Papinutto, Pena, Preti*]

Non-perturbative running of the $(8, 8)$ operators

- $n_f = 2$ non-perturbatively improved clover fermions
- Schrödinger functional, massless limit
- Non-perturbative evolution between 0.438 GeV and 56 GeV

see talk by
Mauro Papinutto
Tuesday @ 2:55





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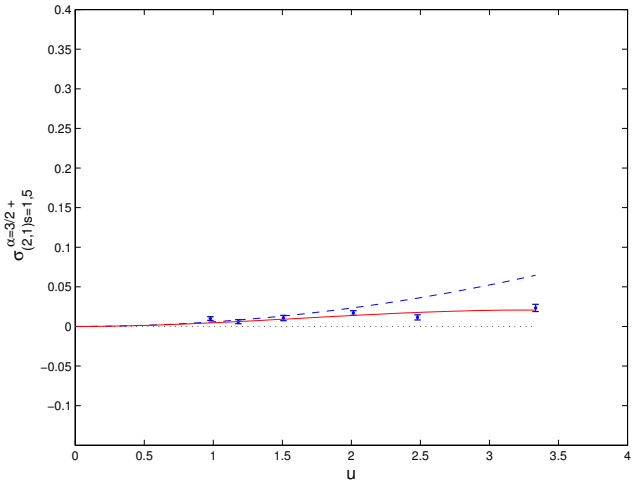
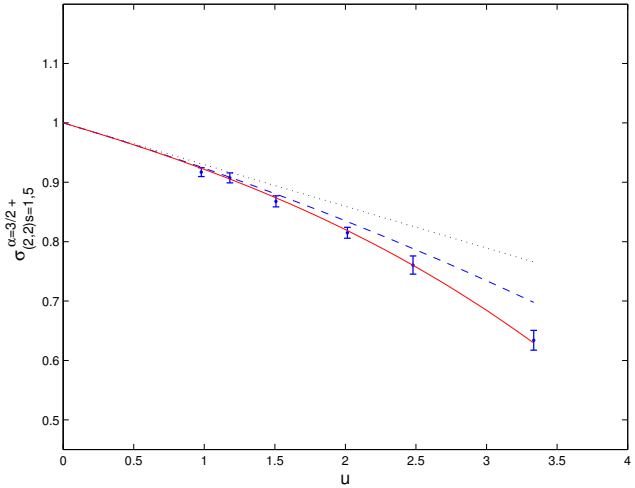




Lattice 2014 update

[*Hansen, Laiho,Van de Water*]

- $2+1$ Domain-Wall on asqtad (MILC configurations)
- Same setup as used for B_K [Laiho & Van de Water'11]
- 3 lattice spacings

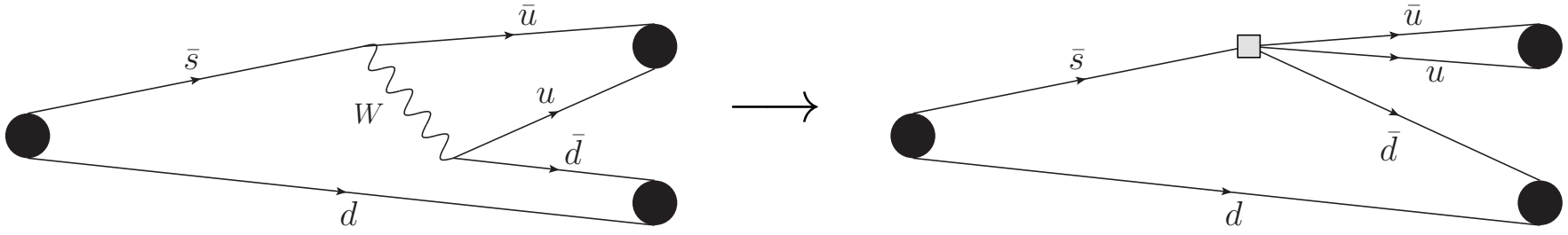
see talk by Maxwell Hansen

$$K \to \pi\pi$$

Overview of the computation

Some references: [Bernard @ TASI'89, RBC PRD'01, Lellouch @ Les houches '09]

Operator Product expansion



Describe $K \to (\pi\pi)_{\mathrm{I}=0,2}$ with an effective Hamiltonian

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \Big\{ \sum_{i=1}^{10} \big(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \big) Q_i(\mu) \Big\}$$

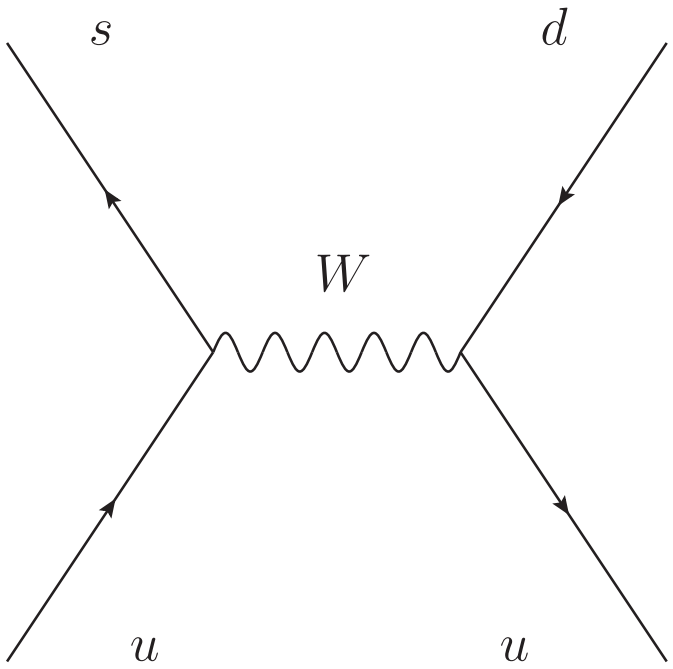
Short distance effects factorized in the Wilson coefficients y_i, z_i

Long distance effects factorized in the matrix elements

$$\langle \pi\pi | Q_i | K \rangle \longrightarrow \text{ Lattice}$$

4-quark operators

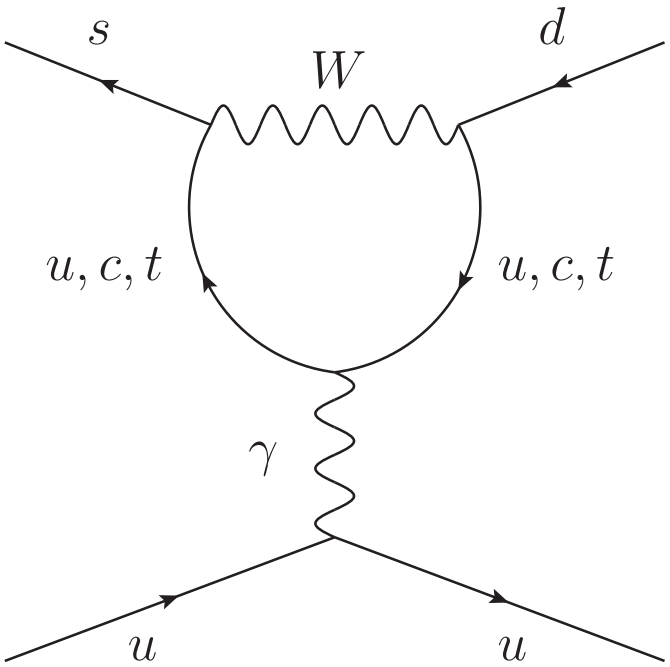
Current diagrams



$Q_1 = (\bar{s}d)_{\mathrm{V-A}}(\bar{u}u)_{\mathrm{V-A}}$ $\qquad$ $Q_2 =$ color mixed

4-quark operators

Electroweak penguins

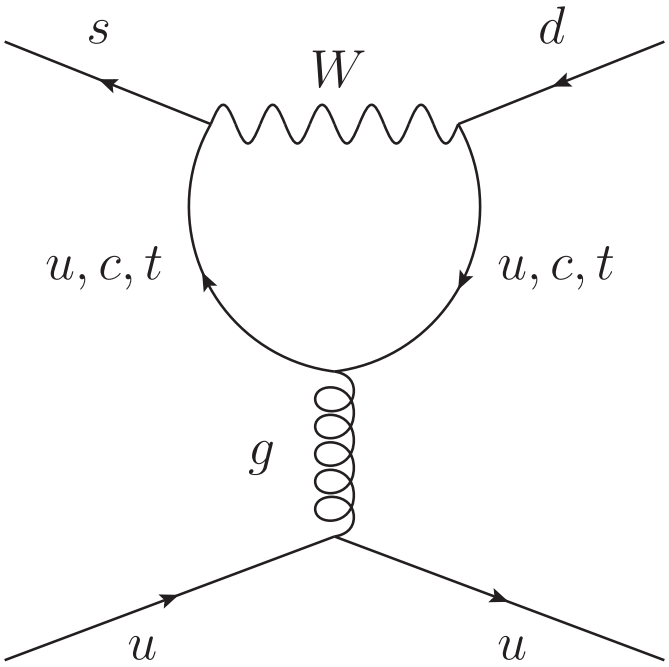


$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V+A}} \qquad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V-A}} \qquad Q_{10} = \text{color mixed}$$

4-quark operators

QCD penguins



$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V-A}} \qquad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V+A}} \qquad Q_6 = \text{color mixed}$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Irrep of $SU(3)_L \otimes SU(3)_R$

$$\begin{aligned} \overline{3} \otimes 3 &= 8 + 1 \\ \overline{8} \otimes 8 &= 27 + \overline{10} + 10 + 8 + 8 + 1 \end{aligned}$$

Decomposition of the 4-quark operators gives

$$\begin{aligned} Q_{1,2} &= Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2} \\ Q_{3,4} &= Q_{3,4}^{(8,1),\Delta I=1/2} \\ Q_{5,6} &= Q_{5,6}^{(8,1),\Delta I=1/2} \\ Q_{7,8} &= Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2} \\ Q_{9,10} &= Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2} \end{aligned}$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Only 7 are independent: one $(27,1)$ four $(8,1)$, and two $(8,8)$, $\Rightarrow$ we called them Q'

$$
\begin{array}{lll}
(27,1) & Q'_1 & = \quad Q_1'^{(27,1),\Delta I=3/2} + Q_1'^{(27,1),\Delta I=1/2} \\
(8,1) & Q'_2 & = \quad Q_2'^{(8,1),\Delta I=1/2} \\
 & Q'_3 & = \quad Q_3'^{(8,1),\Delta I=1/2} \\
 & Q'_5 & = \quad Q_5'^{(8,1),\Delta I=1/2} \\
 & Q'_6 & = \quad Q_6'^{(8,1),\Delta I=1/2} \\
(8,8) & Q'_7 & = \quad Q_7'^{(8,8),\Delta I=3/2} + Q_7'^{(8,8),\Delta I=1/2} \\
 & Q'_8 & = \quad Q_8'^{(8,8),\Delta I=3/2} + Q_8'^{(8,8),\Delta I=1/2}
\end{array}
$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Only 7 are independent: one $(27,1)$ four $(8,1)$, and two $(8,8)$, $\Rightarrow$ we called them Q'

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 & Q'_6 & = \quad Q_6'^{(8,1),\Delta I=1/2} \\
(8,8) & Q'_7 & = \quad Q_7'^{(8,8),\Delta I=3/2} + Q_7'^{(8,8),\Delta I=1/2} \\
 & Q'_8 & = \quad Q_8'^{(8,8),\Delta I=3/2} + Q_8'^{(8,8),\Delta I=1/2}
\end{array}
$$

$K \to (\pi\pi)_{I=2}$ by the RBC-UKQCD collaborations

A_2 from RBC-UKQCD

[Blum, Boyle, Christ, N.G.,Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, *PRL'12, PRD'12*]

- $2+1$ Domain-Wall on IDSDR $a \sim 0.14\ \mathrm{fm}$
- lightest unitary pion mass $\sim 170\ \mathrm{MeV}$ (partially quenched 140 MeV)
- NPR thourgh RI-SMOM schemes

Overview of the computation

- Lellouch-Lüscher method [Lellouch Lüscher '00] to obtain the physical matrix element from the finite-volume Euclidiean amplitude and the derivative of the phase shift
- Combine
  - Wigner-Eckart theorem (Exact up to isospin symmetry breaking)

$$\langle \pi^+(p_1)\pi^0(p_2)|O^{\Delta I=3/2}_{\Delta I_Z=1/2}|K^+\rangle = 3/2\langle \pi^+(p_1)\pi^+(p_2)|O^{\Delta I=3/2}_{\Delta I_Z=3/2}|K^+\rangle$$

  and then compute the unphysical process $K^+ \to \pi^+\pi^+$

  - Use Anti-periodic B.C. to eliminate the unwanted (wrong-kinematic) state [Sachrajda & Villadoro '05]
- Renormalise at low energy $\mu_0 \sim 1.1\ \mathrm{GeV}$ on the IDSDR and run non-perturbatively using finer lattices to $\mu = 3\ \mathrm{GeV}$ and match to $\overline{\mathrm{MS}}$ [Arthur, Boyle '10, Arthur, Boyle, N.G. , Kelly, Lytle '11]

$$\lim_{a_1\to 0}\underbrace{\left[Z(\mu_1,a_1)Z^{-1}(\mu_0,a_1)\right]}_{\text{fine lattice}} \times \underbrace{Z(\mu_0,a_0)}_{\text{coarse lattice}} = Z(\mu_1,a_0)$$

A_0 from RBC-UKQCD

"Pilot" computation of the full process

[T. Blum, Boyle, Christ, N.G., Goode, Izubuchi, Lehner, Liu, Mawhinney, Sachrajda, Soni, Sturm, Yin, Zhou, PRD'11].

Unphysical:

- "Heavy" pions (lightest $\sim m_\pi \sim 300\ \mathrm{MeV}$), small volume
- Non-physical kinematics: pions at rest

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But "complete":

- Two-pion state
- All the contractions of the 7 fourk-operators are computed
- Renormalisation done non-perturbatively

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But "complete":

- Two-pion state
- All the contractions of the 7 fourk-operators are computed
- Renormalisation done non-perturbatively

obtain

$$\begin{aligned} \mathrm{Re}\, A_0 &= 3.80(82) \times 10^{-7}\mathrm{GeV} \\ \mathrm{Im}\, A_0 &= -2.5(2.2) \times 10^{-11}\mathrm{GeV} \end{aligned}$$

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

We combine our physical computation of $\Delta I = 3/2$ part is our non-physical computation of the $\Delta I = 1/2$

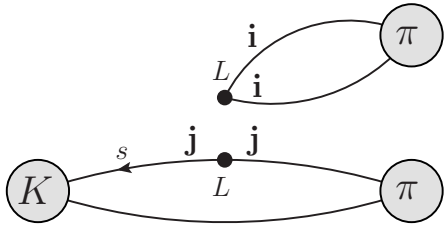
	$1/a$ [GeV]	m_π [MeV]	m_K [MeV]	$\mathrm{Re}A_2$ [10^{-8}GeV]	$\mathrm{Re}A_0$ [10^{-8} GeV]	$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}$	kinematics
16^3 **IW**	1.73(3)	422(7)	878(15)	4.911(31)	45(10)	9.1(2.1)	threshold
24^3 **IW**	1.73(3)	329(6)	662(11)	2.668(14)	32.1(4.6)	12.0(1.7)	threshold
32^3 **ID**	1.36(1)	142.9(1.1)	511.3(3.9)	1.38(5)(26)	-	-	physical
Exp	–	135 - 140	494 - 498	1.479(4)	33.2(2)	22.45(6)	

Pattern which could explain the $\Delta I = 1/2$ enhancement

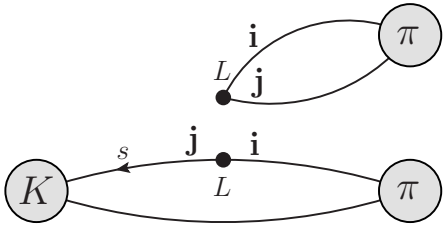
[Boyle, Christ, N.G., Goode, Izubuchi, Janowski, Lehner, Liu, Lytle, Sachrajda, Soni, Zhang, PRL'13]

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator



Contraction ①



Contraction ②

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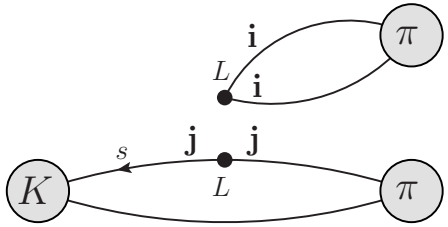

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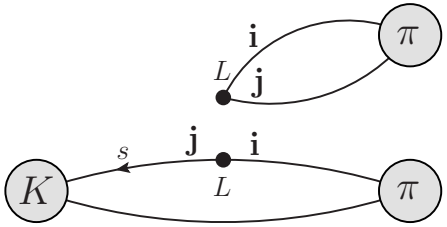


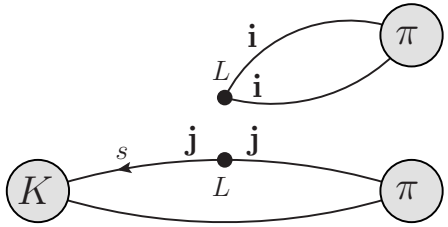
Contraction ②

- ReA_2 is dominated by the tree level operator (EWP $\sim 1\%$):
- Naive factorisation approach: ② $\sim 1/3$①
- Our computation: ② $\sim -0.7$①

$\Rightarrow$ large cancellation in ReA_2

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator



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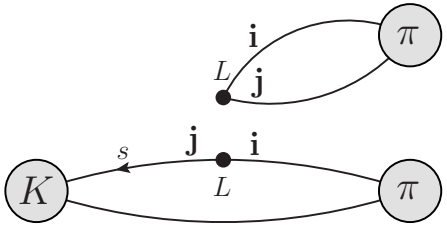

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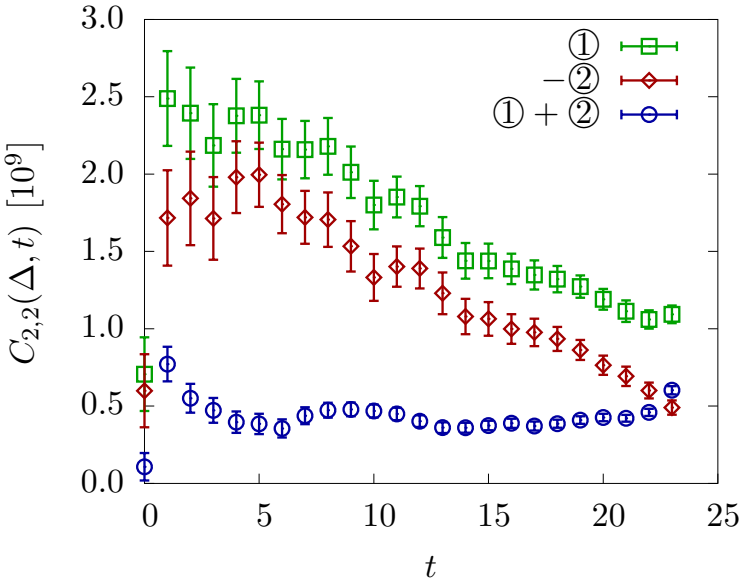

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

ReA_0 is also dominated by the tree level operators

i	$Q_i^{\rm lat}$ [GeV]	$Q_i^{\overline{\rm MS}\text{-NDR}}$ [GeV]
1	8.1(4.6) 10^{-8}	6.6(3.1) 10^{-8}
2	2.5(0.6) 10^{-7}	2.6(0.5) 10^{-7}
3	-0.6(1.0) 10^{-8}	5.4(6.7) 10^{-10}
4	–	2.3(2.1) 10^{-9}
5	-1.2(0.5) 10^{-9}	4.0(2.6) 10^{-10}
6	4.7(1.7) 10^{-9}	-7.0(2.4) 10^{-9}
7	1.5(0.1) 10^{-10}	6.3(0.5) 10^{-11}
8	-4.7(0.2) 10^{-10}	-3.9(0.1) 10^{-10}
9	–	2.0(0.6) 10^{-14}
10	–	1.6(0.5) 10^{-11}
ReA_0	3.2(0.5) 10^{-7}	3.2(0.5) 10^{-7}

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

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7	1.5(0.1) 10^{-10}	6.3(0.5) 10^{-11}
8	-4.7(0.2) 10^{-10}	-3.9(0.1) 10^{-10}
9	–	2.0(0.6) 10^{-14}
10	–	1.6(0.5) 10^{-11}
ReA_0	3.2(0.5) 10^{-7}	3.2(0.5) 10^{-7}

Dominant contribution to $Q_2^{\rm lat}$ is $\propto$ (2② − ①) $\Rightarrow$ Enhancement in ReA_0

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

ReA_0 is also dominated by the tree level operators

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2	2.5(0.6) 10^{-7}	2.6(0.5) 10^{-7}
3	-0.6(1.0) 10^{-8}	5.4(6.7) 10^{-10}
4	–	2.3(2.1) 10^{-9}
5	-1.2(0.5) 10^{-9}	4.0(2.6) 10^{-10}
6	4.7(1.7) 10^{-9}	-7.0(2.4) 10^{-9}
7	1.5(0.1) 10^{-10}	6.3(0.5) 10^{-11}
8	-4.7(0.2) 10^{-10}	-3.9(0.1) 10^{-10}
9	–	2.0(0.6) 10^{-14}
10	–	1.6(0.5) 10^{-11}
ReA_0	3.2(0.5) 10^{-7}	3.2(0.5) 10^{-7}

Dominant contribution to $Q_2^{\rm lat}$ is $\propto$ (2② − ①) $\Rightarrow$ Enhancement in ReA_0

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim \frac{2② - ①}{① + ②}$$

With this unphysical kinematics, we find

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 9.1(2.1) \text{ for } m_K = 878\ \mathrm{MeV}\ m_\pi = 422\ \mathrm{MeV}$$

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 12.0(1.7) \text{ for } m_K = 662\ \mathrm{MeV}\ m_\pi = 329\ \mathrm{MeV}$$

Lattice 2014 update

- $\Delta I = 3/2$

  Main limitation on the previous computation : only one coarse lattice spacing

  IDSDR $32^3 \times 64$, with $a^{-1} \sim 1.37\ \mathrm{GeV} \Rightarrow a \sim 0.14\ \mathrm{fm}$, $L \sim 4.6\ \mathrm{fm}$

  Current computation:

  two lattice spacing, $n_f = 2 + 1$, large volume at the physical point

  New discretisation of the Domain-Wall fermion forumlation: Möbius [Brower, Neff, Orginos '12]

  - $48^3 \times 96$, with $a^{-1} \sim 1.729\ \mathrm{GeV} \Rightarrow a \sim 0.11\ \mathrm{fm}$, $L \sim 5.5\ \mathrm{fm}$
  - $64^3 \times 128$ with $a^{-1} \sim 2.358\ \mathrm{GeV} \Rightarrow a \sim 0.84\ \mathrm{fm}$, $L \sim 5.4\ \mathrm{fm}$
  - $am_{res} \sim 10^{-4}$

  Status: Computation finished, draft in final stage

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  Status: Computation finished, draft in final stage

- $\Delta I = 1/2$

  Main limitation on the previous computation : non-physical kinematic

  New formulation: G-parity boundary conditions

  Status: First computation almost finished

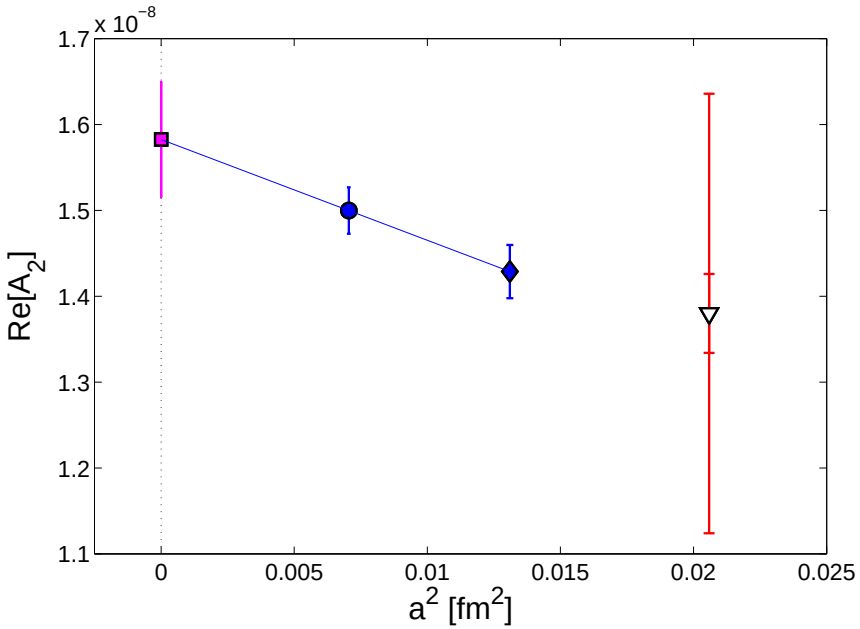
See talks by Chris Kelly and by Daiqian Zhang, Monday

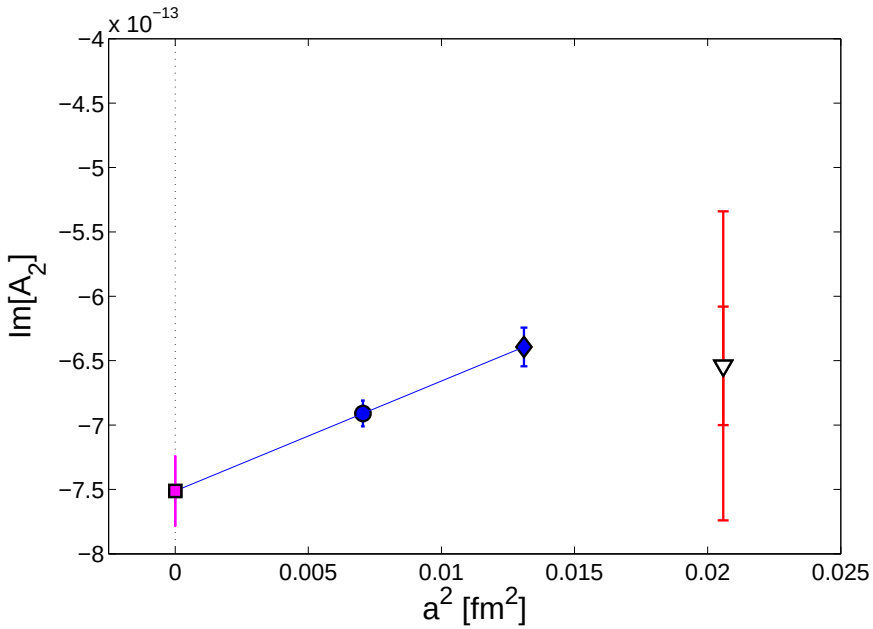
$K \to (\pi\pi)_{I=2}$ Lattice 2014 update

2012 [Blum, Boyle, Christ, N.G.,Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, *PRL'12, PRD'12*]

$\mathrm{Re}\, A_2 = 1.381(46)_{\mathrm{stat}}(258)_{\mathrm{syst}}\, 10^{-8}\ \mathrm{GeV}$ $\mathrm{Im}\, A_2 = -6.54(46)_{\mathrm{stat}}(120)_{\mathrm{syst}} 10^{-13}\ \mathrm{GeV}$

2014 [RBC-UKQCD Work in progress, draft in final stage]





Preliminary: systematic budget not complete

see also talk by T.Janowski @ lat'13 [Janowski, Sachrajda, Boyle, Christ, Mawhinney, Yin, Zhang, N.G., Lytle]

Other computations of $K \to (\pi\pi)$

$K \to \pi\pi$ with improved Wilson fermions

[N. Ishizuka , K.I. Ishikawa , A. Ukawa , T. Yoshie]

- Direct computation with 2-pion at rest
- both $\Delta I = 1/2$ and $\Delta I = 3/2$
- $2+1$ improved Wilson fermions on Iwasaki gauge config
- $32^3 \times 64$, ~ 0.091 fm, $L \sim 2.91$ fm

- Perturbative operator renormalization (1 loop)
  after non-perturbative subtraction of the lower dimensional operator P.

$$Q_i^{\overline{\rm MS}}(\mu) = \sum_j Z_{ij}(\mu) \cdot [\, Q_j^{\rm lat} - \alpha_j P \,]$$

$$P = \bar{s}\gamma_5 d \ , \ \alpha_j = \frac{\langle 0|Q_j|K\rangle}{\langle 0|P|K\rangle} \ , \ Z_{ij}(\mu) : \ 1 \text{ loop}$$

- For calculations of the quark loops in the "eye" and the disconnected diagrams,
  hopping parameter expansion (4th order) and
  truncated solver method (N_T=5) are used.

(proposed by G.S. Bali et al., (CPC 181(2010)1570))

See talk by Naruhito Ishizuka
(Monday 4:30)

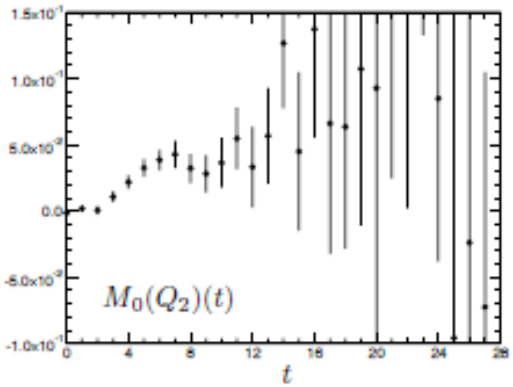
From Naruhito Ishizuka's talk

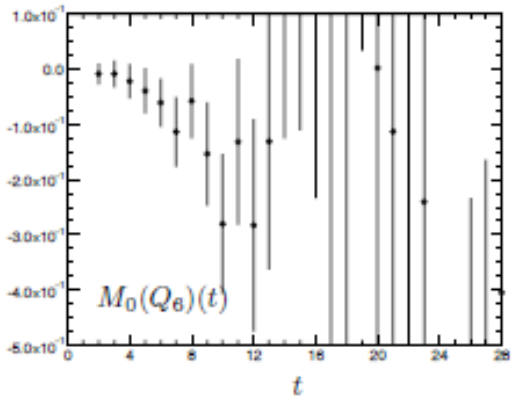
Results

Effective matrix elements :

$$M_I(Q_j)(t) = \langle 0|\, K(t_K)\, Q_j(t)\, (\pi\pi)_I(t_\pi)\, |0\rangle \times e^{m_K(t_K - t) + E^I_{\pi\pi}(t - t_\pi)}$$

$$\propto \langle K|Q_j|\pi\pi; I\rangle \quad \text{for} \quad t_K \gg t \gg t_\pi \qquad (\, t_K = 24\ ,\ t_\pi = 0\ ,\ t : \text{run}\ )$$





: signals are seen in t=[9,12].

Result of decay amplitudes :

	Ours	Experiment
m_π (MeV)	280	140
$\mathrm{Re}A_2$ ($\times 10^{-8}$ GeV)	2.426 ± 0.038	1.479 ± 0.004
$\mathrm{Re}A_0$ ($\times 10^{-8}$ GeV)	60 ± 36	33.2 ± 0.2
$\mathrm{Re}A_0/\mathrm{Re}A_2$	25 ± 15	22.45 ± 0.06
$\mathrm{Re}(\epsilon'/\epsilon)$ ($\times 10^{-3}$)	0.80 ± 2.54	1.66 ± 0.23

- Enhancement of $\Delta I = 1/2$ process is seen.
- Further improvement of statics is necessary for ϵ'/ϵ .

Role of the charm mass in $K \to \pi\pi$ with improved Wilson fermions

Ongoing effort, updated in [Endress & Pena '12, Endress, Pena, Sivalingam '14]

- Charm is kept active in the effective Hamiltonian
- Matching to $SU(3)$ (heavy charm) and $SU(4)$ (unphysical light charm) chiral Lagrangian
- Computation of the LEC as a function of m_c

- Technically demanding, as requires to compute "eye contractions", see [Endress, Pena, Sivalingam '14]
- Implementation with quenched overlap fermions on a single lattice spacing
- First results indicate an enhancement in $Re(A_0)/Re(A_2)$ as m_C increases.
- Hard to know at the moment if the enhancement will be enough to give a factor 20 (charm is still far from its physical value)

ETMc

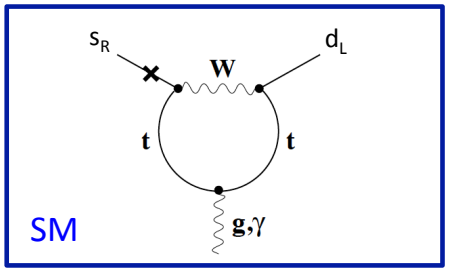
- $2+1+1$ Twisted Mass / Osterwalder-Seiler fermions
- Pion mass down to ~ 210 MeV
- three lattice spacings $a \sim 0.06 - 0.09$ fm

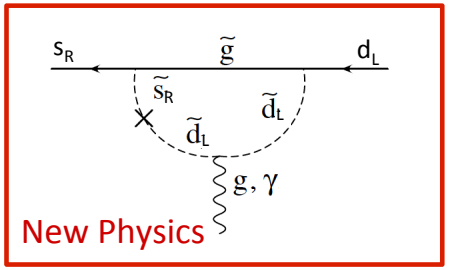
The effective ΔS=1 Hamiltonian of dim=5 contains four magnetic operators:

$$H_{eff}^{\Delta S=1,d=5} = \sum_{i=\pm}\left(C_\gamma^i Q_\gamma^i + C_g^i Q_g^i\right) + h.c.$$

$$Q_\gamma^\pm = \frac{Q_d e}{16\pi^2}\left(\bar{s}_L \sigma^{\mu\nu} F_{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} F_{\mu\nu} d_L\right)$$

$$Q_g^\pm = \frac{g}{16\pi^2}\left(\bar{s}_L \sigma^{\mu\nu} G_{\mu\nu} d_R \pm \bar{s}_R \sigma^{\mu\nu} G_{\mu\nu} d_L\right)$$





	C_{SM}	C_{NP}	For $M_{NP} \sim 1$ TeV :
• Dim = 5 ➡	$\sim 1/M_W$	$\sim 1/M_{NP}$	$\sim 10^{-3} / 10^{-2} \sim 10^{-1}$
• $\Delta F \neq 0$ ➡	$\sim \alpha_w(M_W)$	$\sim \alpha_s(M_{NP})$	$\sim 0.09 / 0.03 \sim 3$
• LR chirality ➡	$\sim m_s / M_W$	$\sim \delta_{LR}$	? / $10^{-3} \sim 10^3 - 1$?

See talk by Vittorio Lubicz Wednesday@10:20 and poster by Marios Costa

Conclusions

Exciting time for kaon/pion physics

- Various collaborations are reaching the physical point
- For the decay constants, or semi-leptonic form factors, we are reaching a precision such that EM corrections become significant (see plenary talk by Antonin Portelli, Thursday@11:30)
- Computation of new quantities (eg: chromomagnetic operator)
- New computations of the neutral kaon mixing matrix elements (B_K and BSM)
- Continuum limit of $K \to (\pi\pi)_{I=2}$ at the physical point
- First realistic results of $K \to (\pi\pi)_{I=0}$ (with physical kinematics) should be available in a few months, thanks to G-parity boundary conditions
- Various collaborations are computing the BSM neutral kaon matrix elements
- NPR is at mature stage with the RI-SMOM schemes, but some matching coefficients are highly needed: NNLO (2-loops matching) for B_K and NLO (1-loop matching) or the $(6, \bar{6})$ BSM operators