Lattice QCD at non-zero temperature

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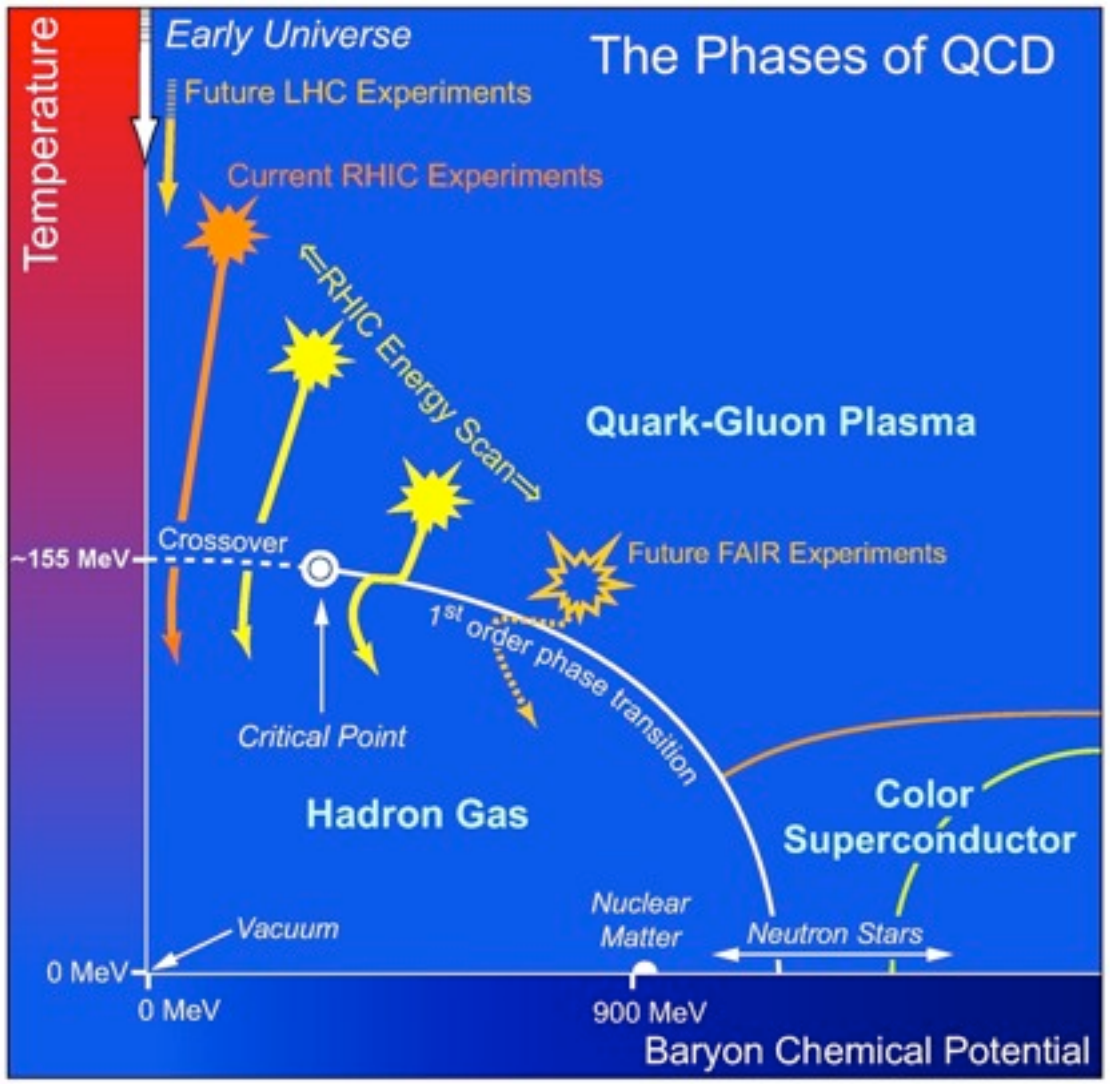
Lattice 2014

Outline

- QCD phase diagram
- Criticality
- Chiral crossover
- Deconfinement and fluctuations
- Equation of state
- Hadrons in the plasma
- Beyond QCD

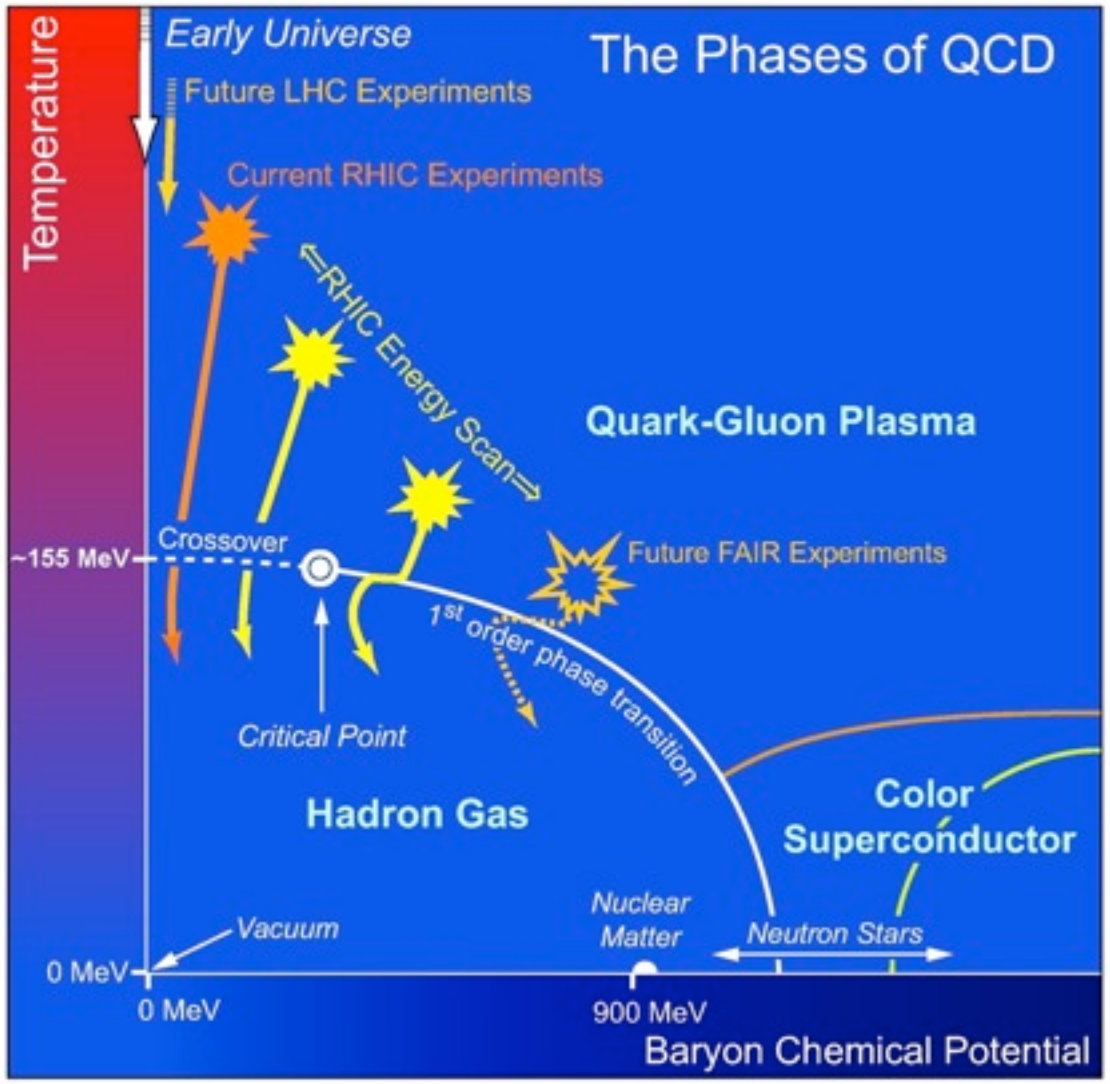
Finite-temperature QCD on the lattice

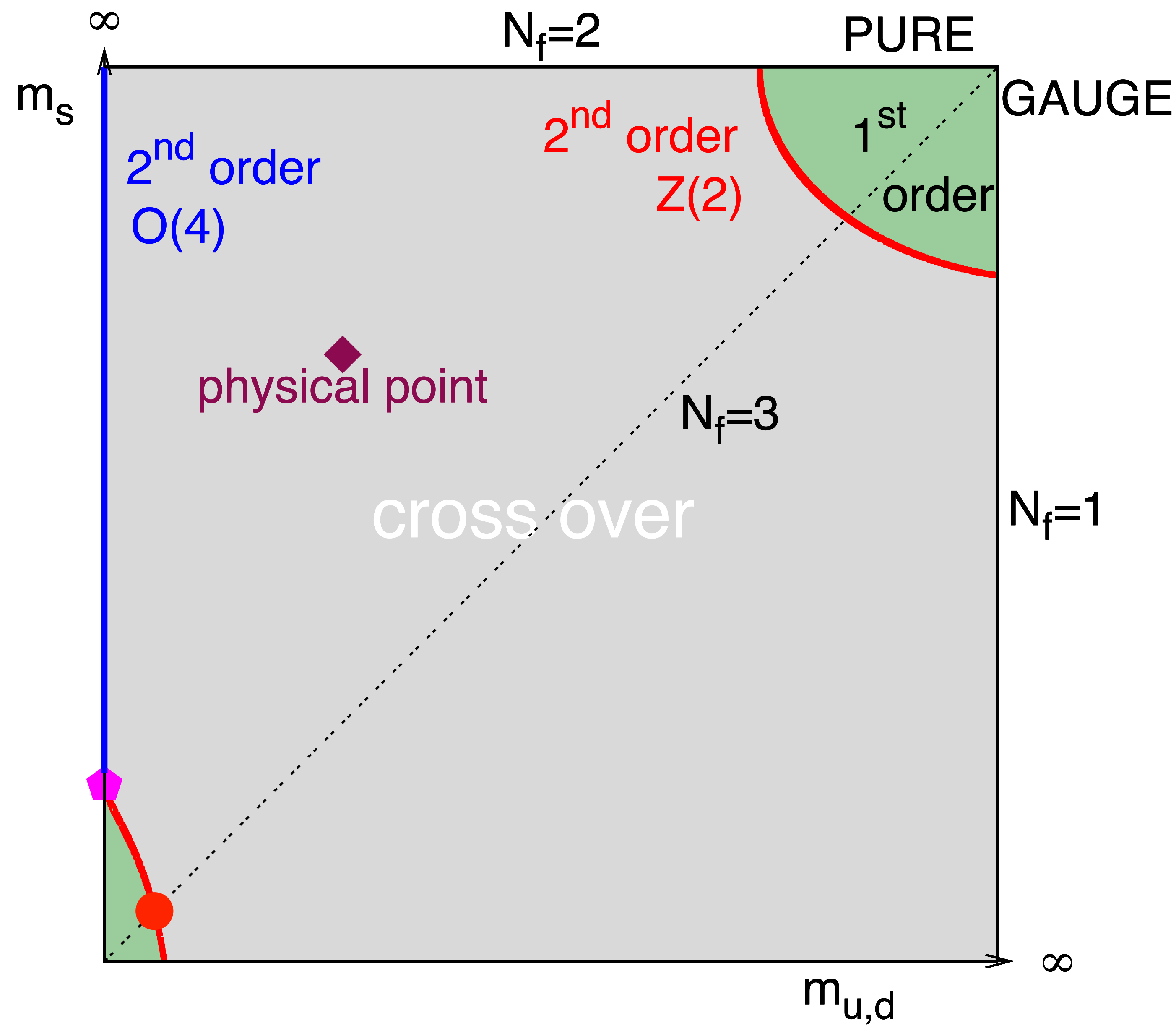


- the phase structure of QCD (and QCD-like theories)
- implications for the physics of the early universe and neutron stars
- connection to heavy-ion experimental program
- (maybe) black hole physics

Collins, Perry, PRL 34 (1975)
Cabbibo, Parisi PLB 59 (1975)

Possible criticality: Columbia plot





Collins, Perry, PRL 34 (1975)

Cabbibo, Parisi PLB 59 (1975)

Pisarski, Wilczek, PRD 29 (1984)

Brown et al., PRL 65 (1990)

Possible criticality: order parameters

- Polyakov loop:

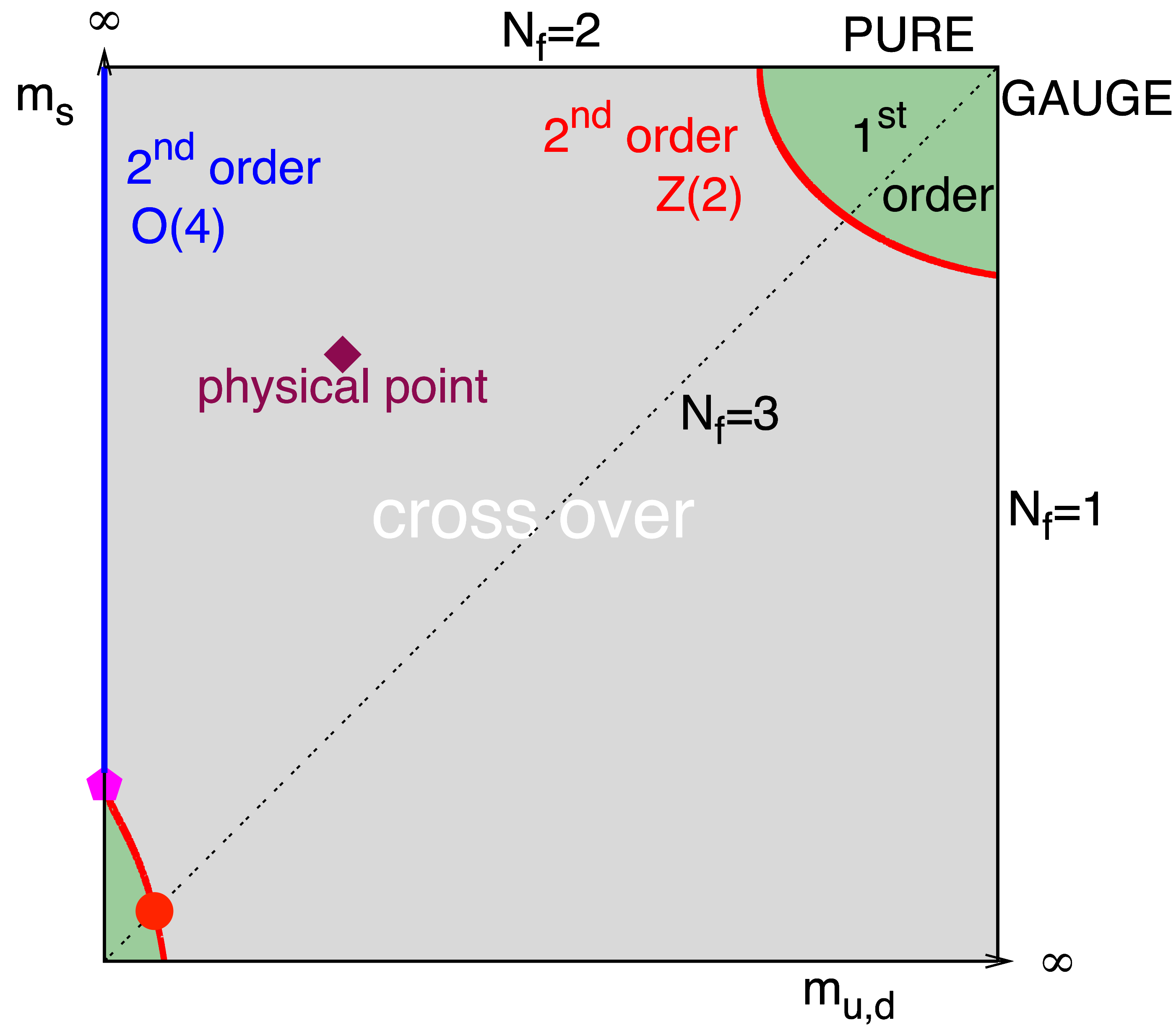
$$L = \left\langle \frac{1}{3} \mathrm{Tr} \prod_{x_0=0}^{N_\tau - 1} U_0(x_0, \vec{x}) \right\rangle$$

- Chiral condensate:

$$\langle \bar{\psi}\psi \rangle_q = \frac{T}{V} \frac{\partial \ln Z}{\partial m_q}$$

- Chiral susceptibility:

$$\chi_q = \frac{\partial \langle \bar{\psi}\psi \rangle_q}{\partial m_q}$$



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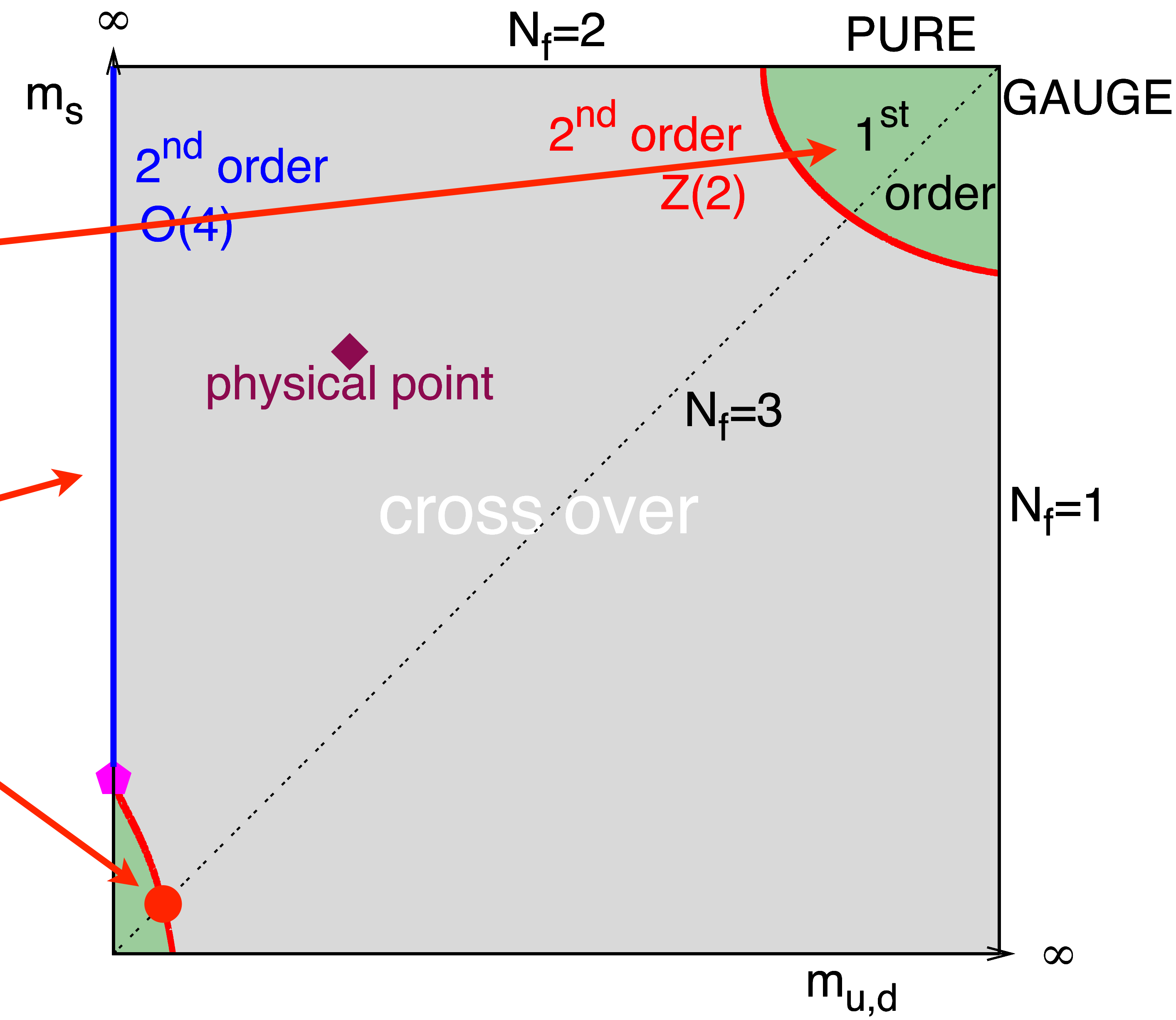


Real world

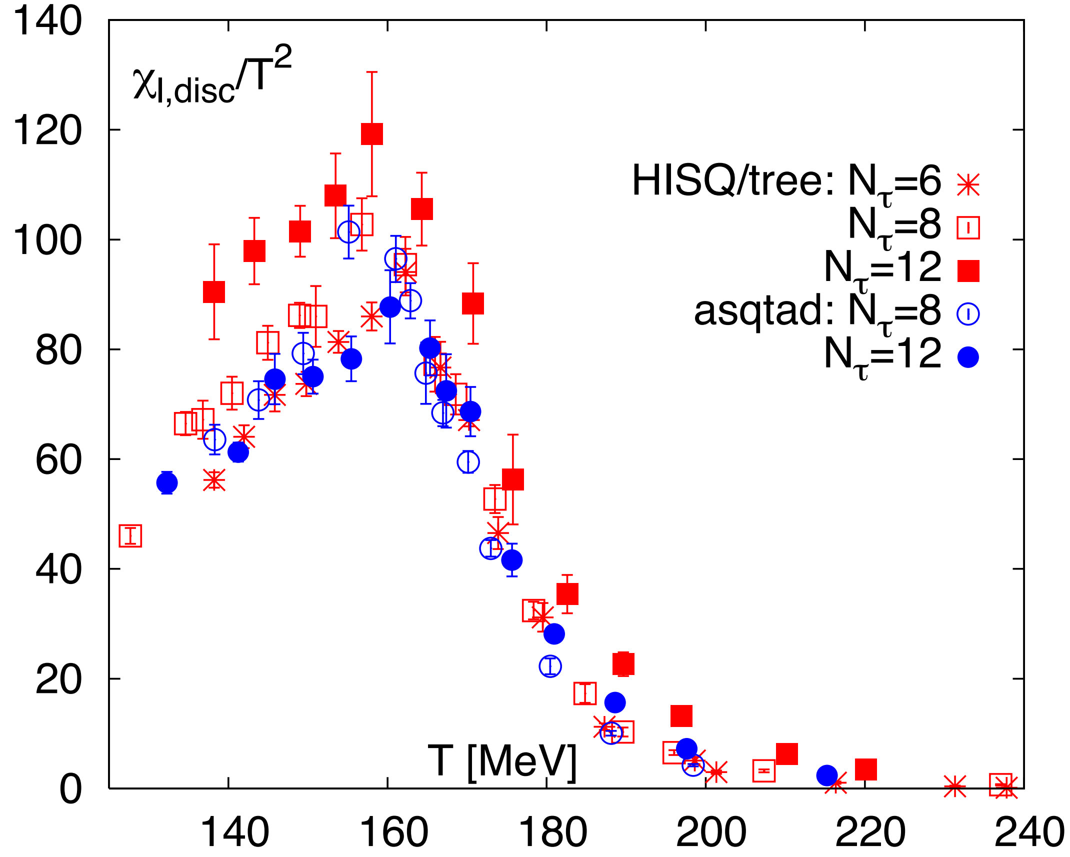
- No order parameter at the physical point
- Can define chiral crossover temperature from the peak of the disconnected chiral susceptibility (light quark masses are small)
- Deconfinement – susceptibilities of conserved charges:

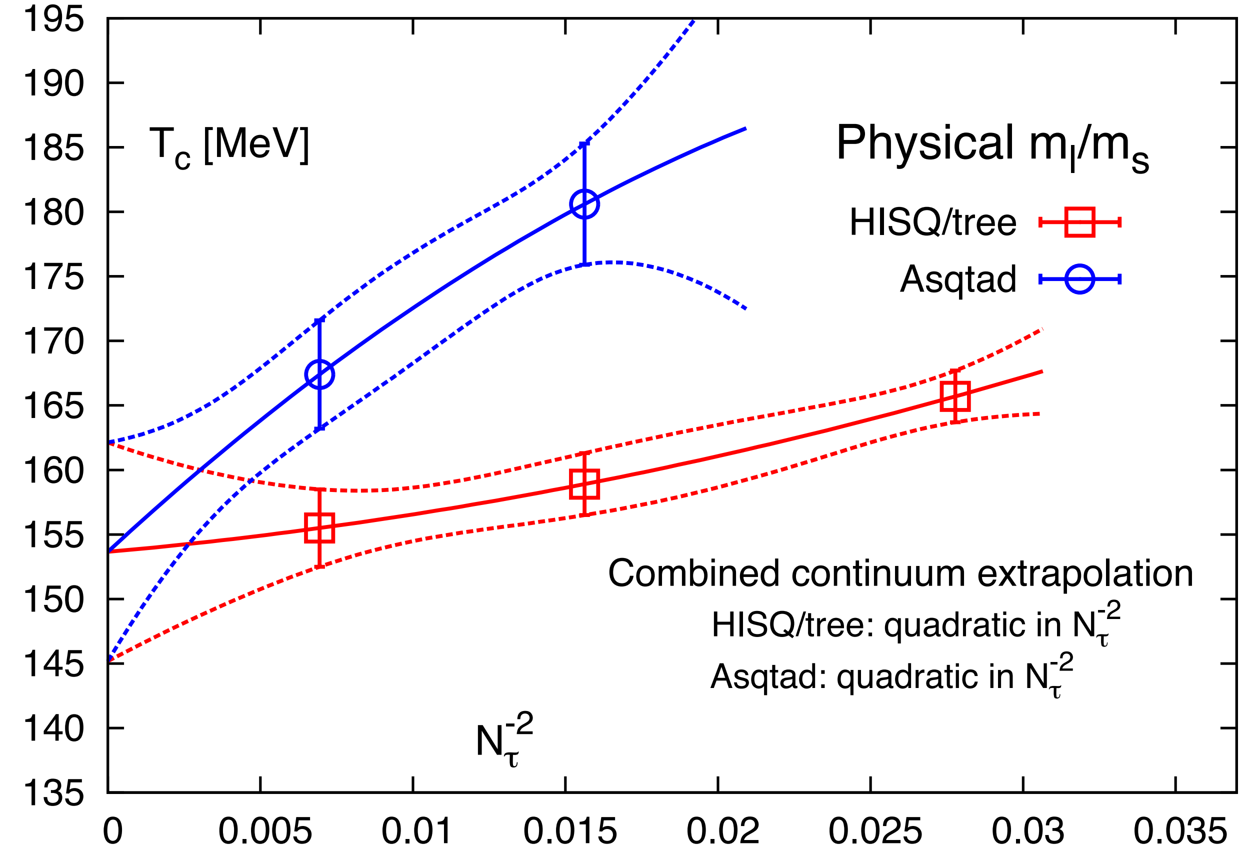
$$\chi_{klm}^{BQS} = \left. \frac{\partial^{(k+l+m)}[P(\mu_B,\mu_Q,\mu_S)/T^4]}{\partial(\mu_B/T)^k \partial(\mu_Q/T)^l \partial(\mu_S/T)^m} \right|_{\vec{\mu}=0}$$

- Deconfinement – equation of state $P(T), \varepsilon(T)$

Chiral crossover temperature

- well established with staggered





$$T_c = 147(2)(3)\ \text{MeV}$$

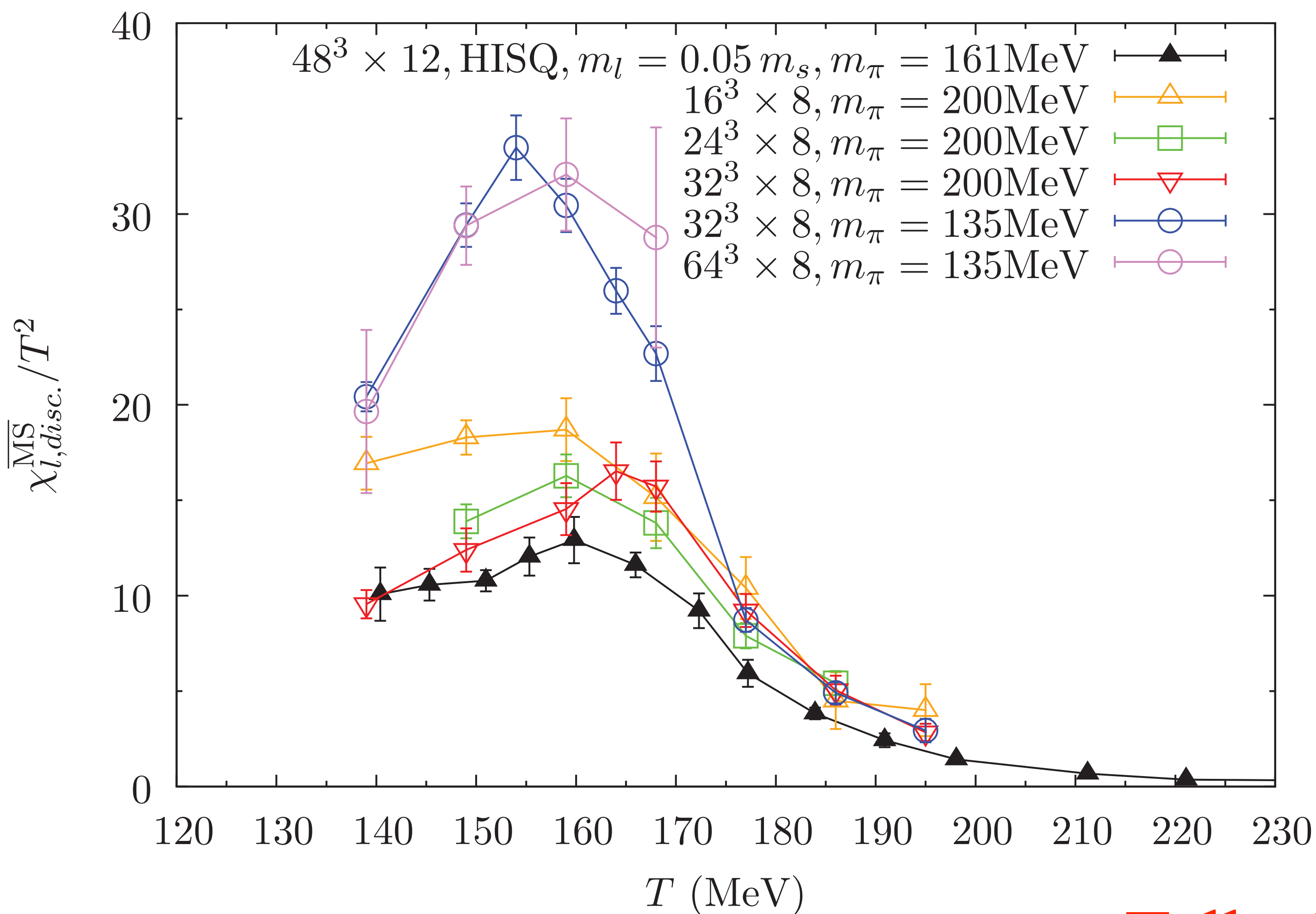
Borsanyi et al. [WB] (2010)

$$T_c = 154(9)\ \text{MeV}$$

Bazavov et al. [HotQCD] (2012)

Chiral crossover temperature

- new: 2+1 domain-wall at the physical pion mass(!)

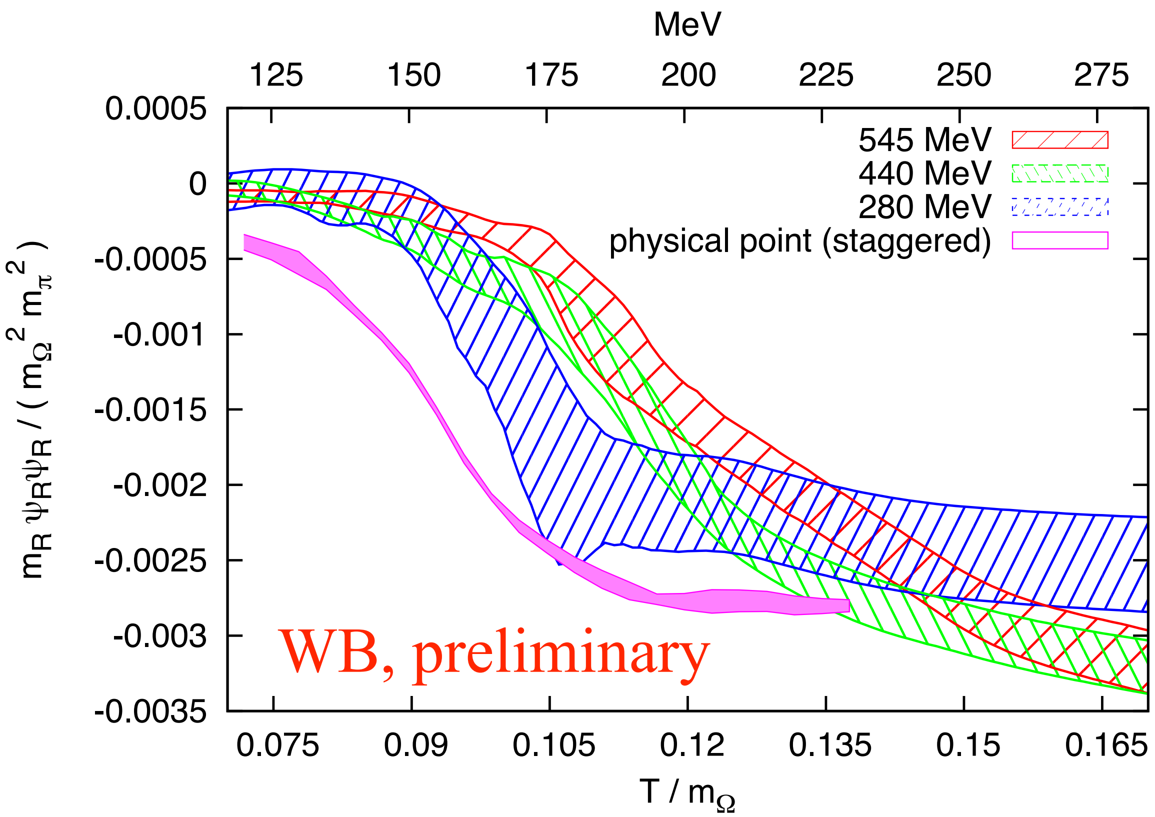


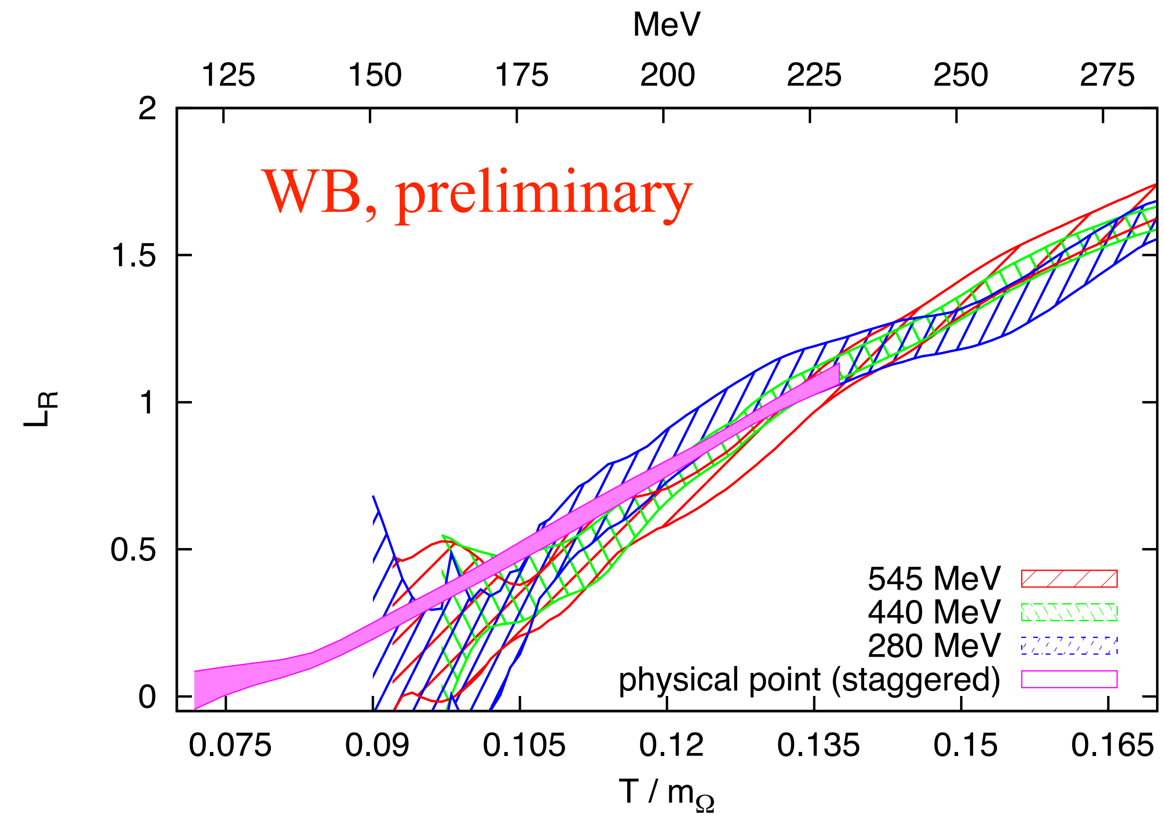
Talk: Schroeder, Thu

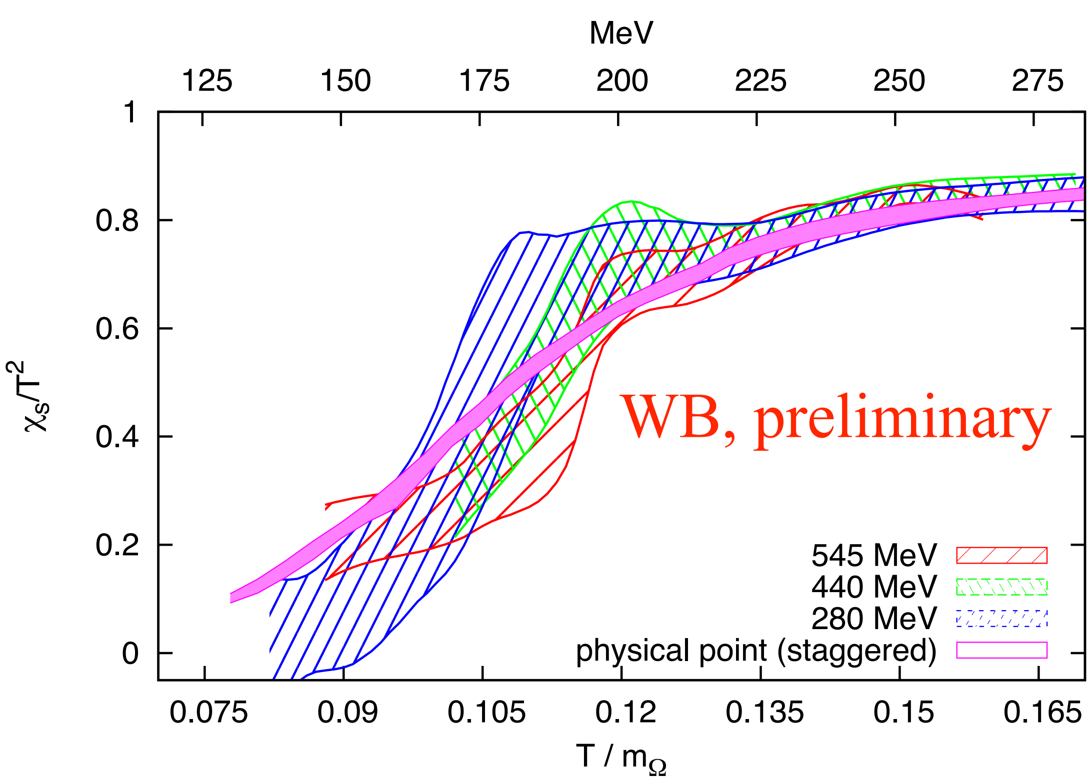
$$T_c = 155(9)\ \mathrm{MeV}$$

Bhattacharya et al. [HotQCD], 1402.5175

Crossover region, 2+1 Wilson







- Stout improved
- Pion down to 280 MeV
- Condensate decreasing in the right direction

Talk: Trombitas, Thu

Hadron Resonance Gas model

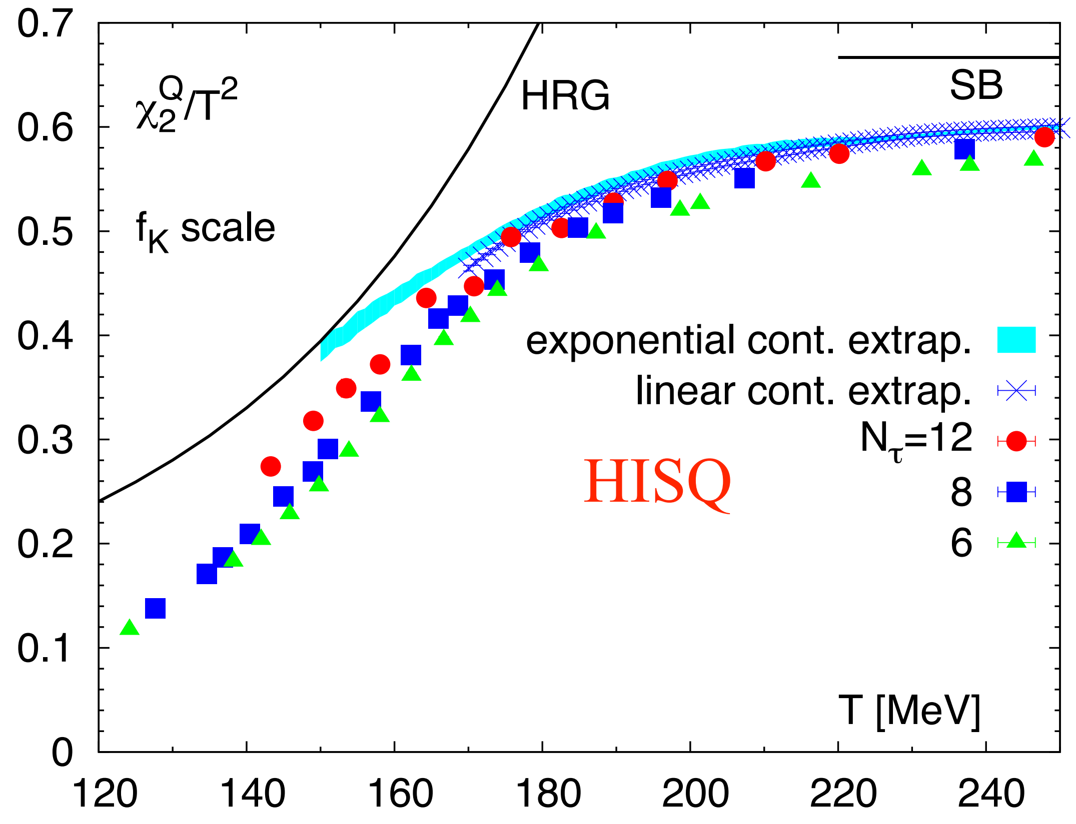
$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3} \sum_{i \in\ mesons} \ln \mathcal{Z}^M_{m_i}(T, V, \mu_{X^a}) + \frac{1}{VT^3} \sum_{i \in\ baryons} \ln \mathcal{Z}^B_{m_i}(T, V, \mu_{X^a})$$

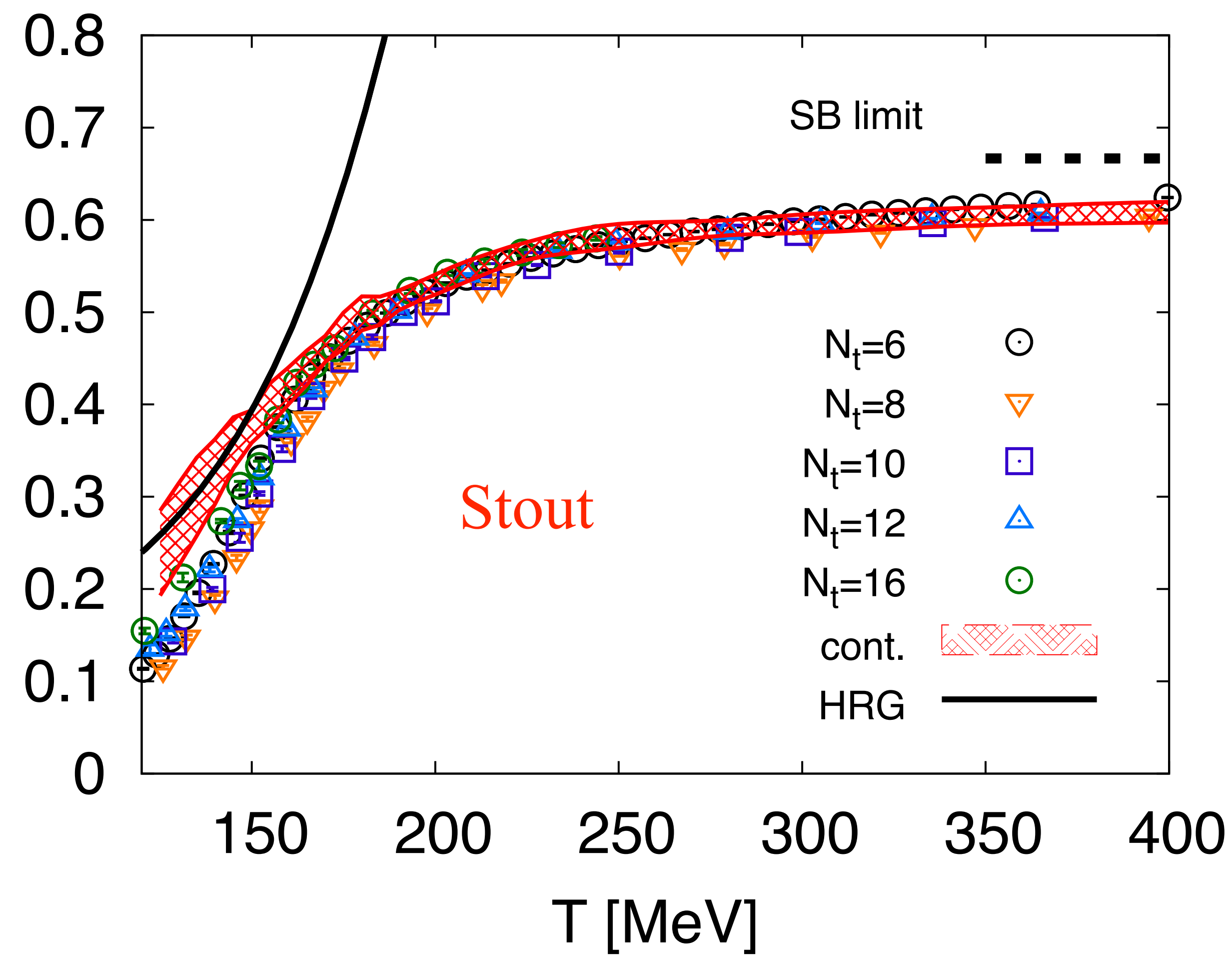
$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T})$$

$$\varepsilon_i = \sqrt{k^2 + m_i^2} \qquad \ln z_i = \sum_a X_i^a \mu_{X^a}/T$$

Hagedorn, Nuovo Cim. 35 (1965)

Electric charge fluctuations



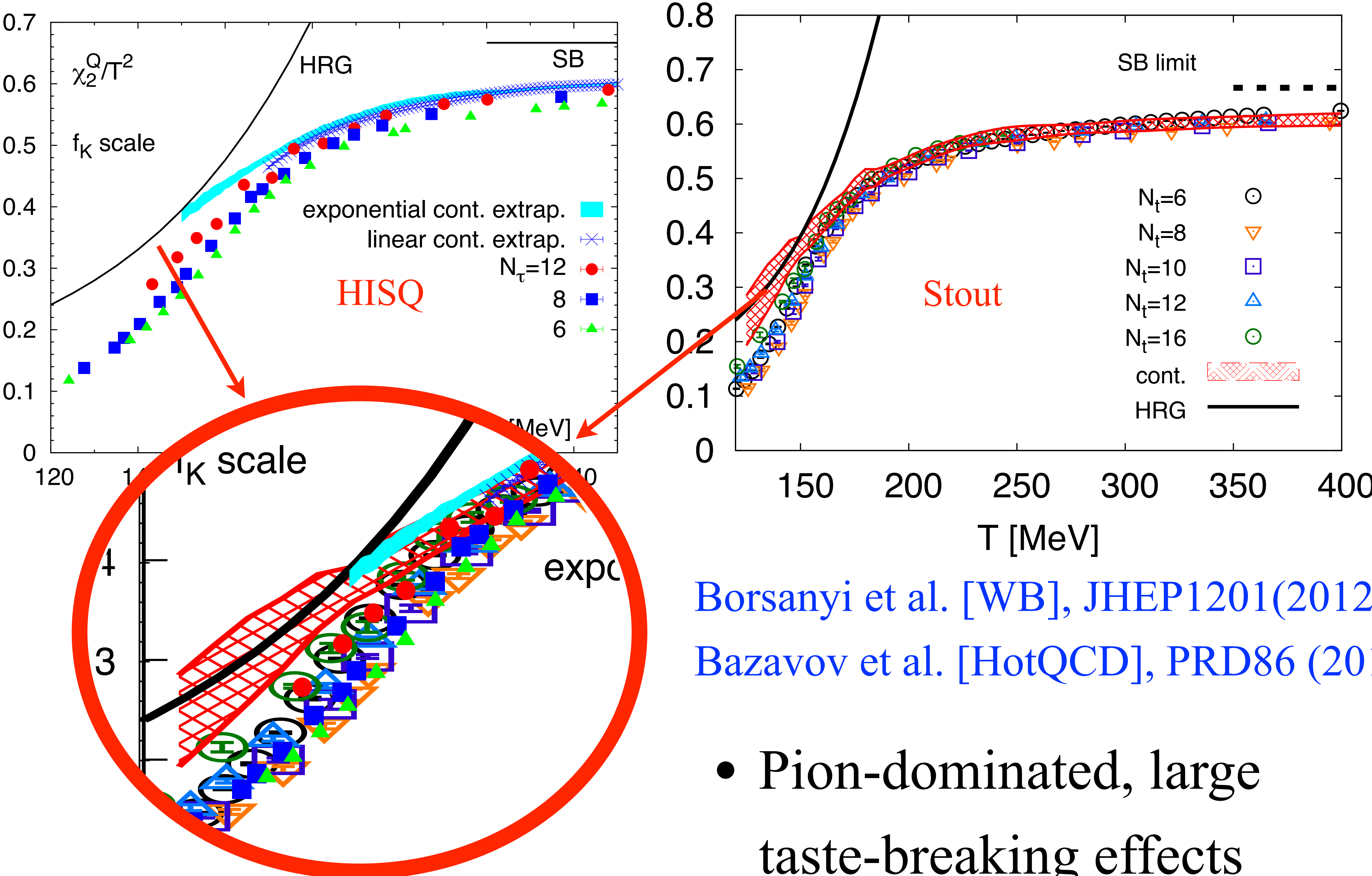


Borsanyi et al. [WB], JHEP1201(2012)

Bazavov et al. [HotQCD], PRD86 (2012)

- Relevant for heavy-ion experiments

Electric charge fluctuations



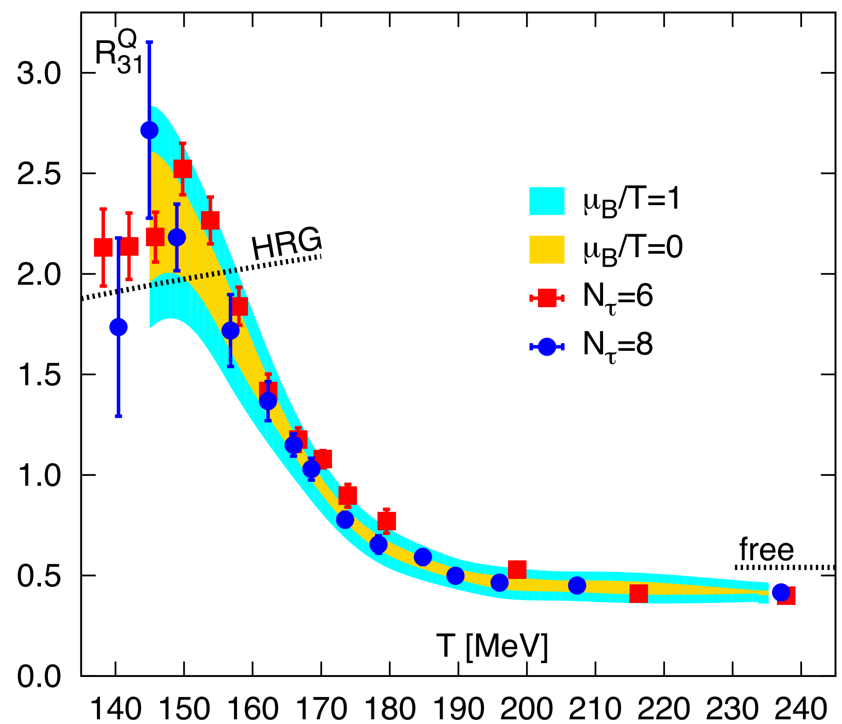
Borsanyi et al. [WB], JHEP1201(2012)

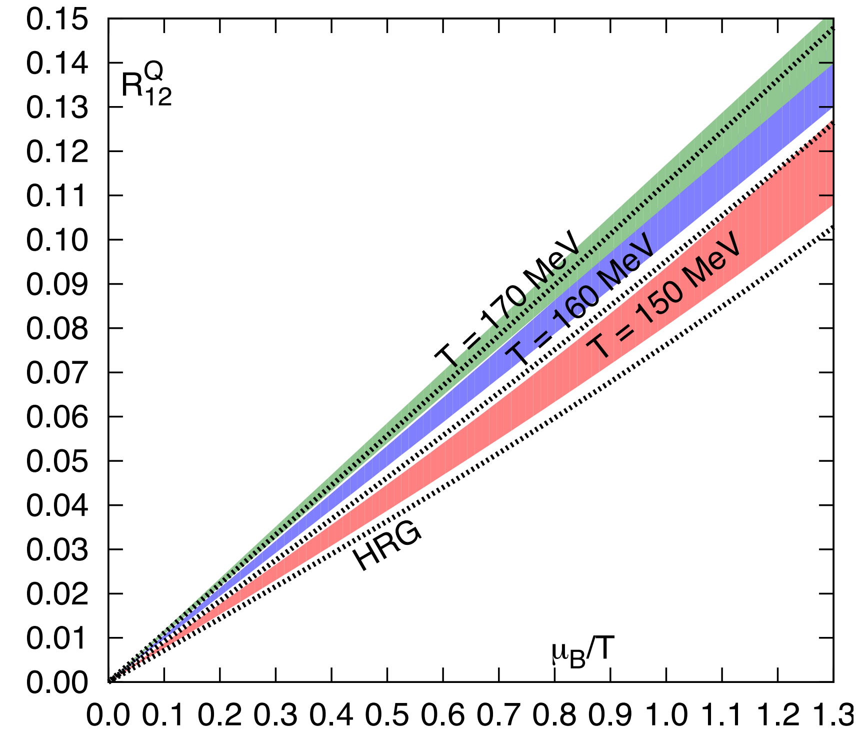
Bazavov et al. [HotQCD], PRD86 (2012)

- Pion-dominated, large taste-breaking effects

Freeze-out parameters

- Ratios of electric charge cumulants can be measured experimentally, can define thermometer and baryometer (discussed at Lattice2013)





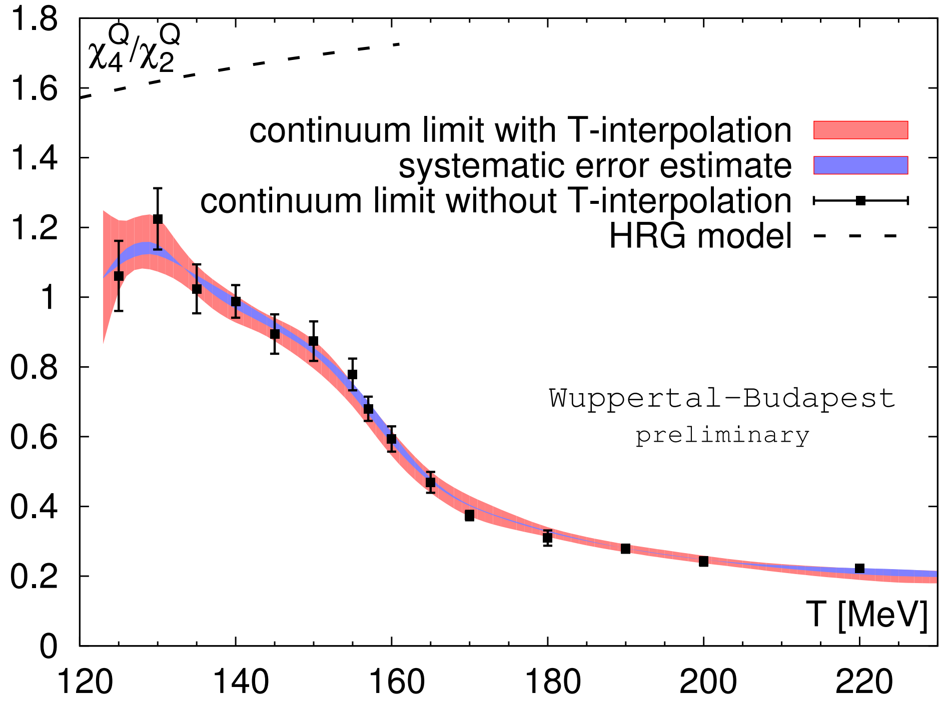
Bazavov et al. [BNL-Bielefeld], PRL109 (2012)

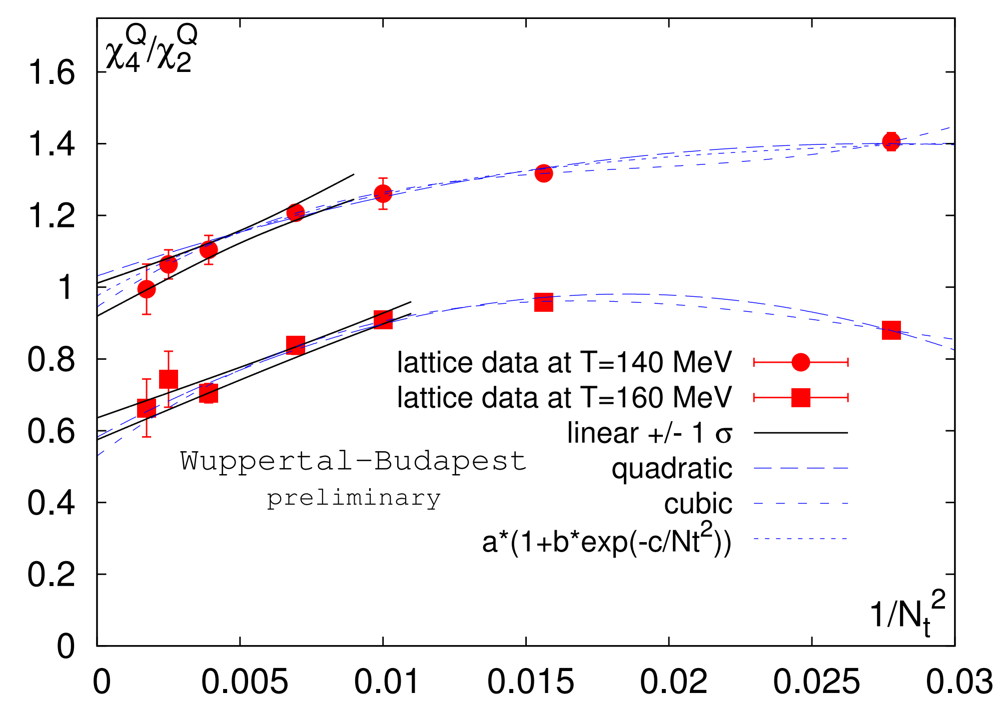
Borsanyi et al. [WB], PRL111 (2013)

Talk: Schmidt, Mon – *B*, *Q*, *S* cumulants up to 6th order

Freeze-out parameters

- 4stout, 2+1+1, electric charge kurtosis
- Physical pion mass, $N_t = 6, ..., 24$





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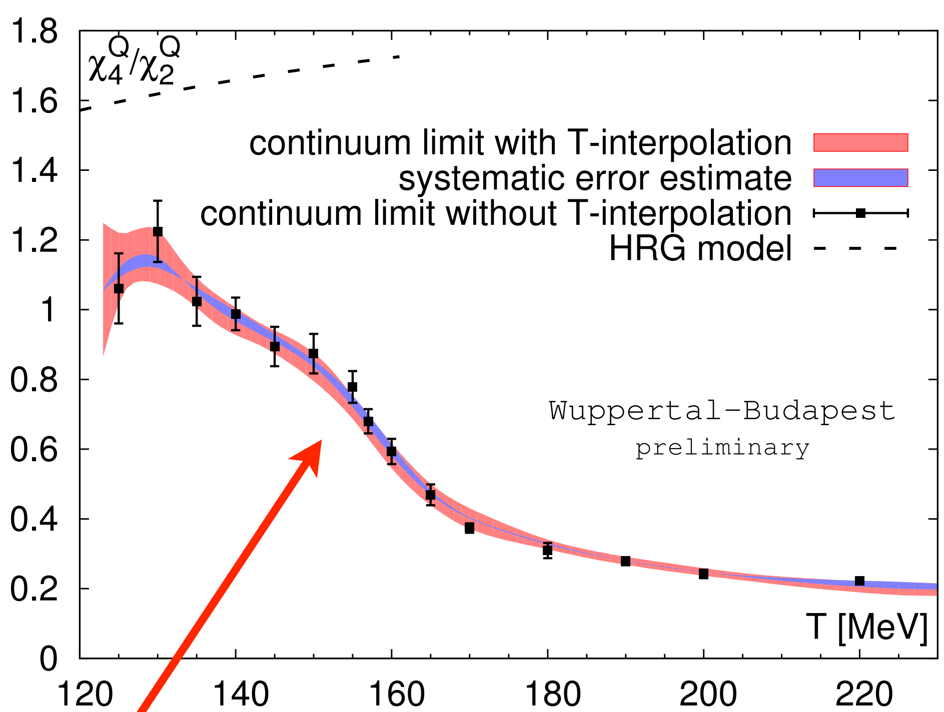
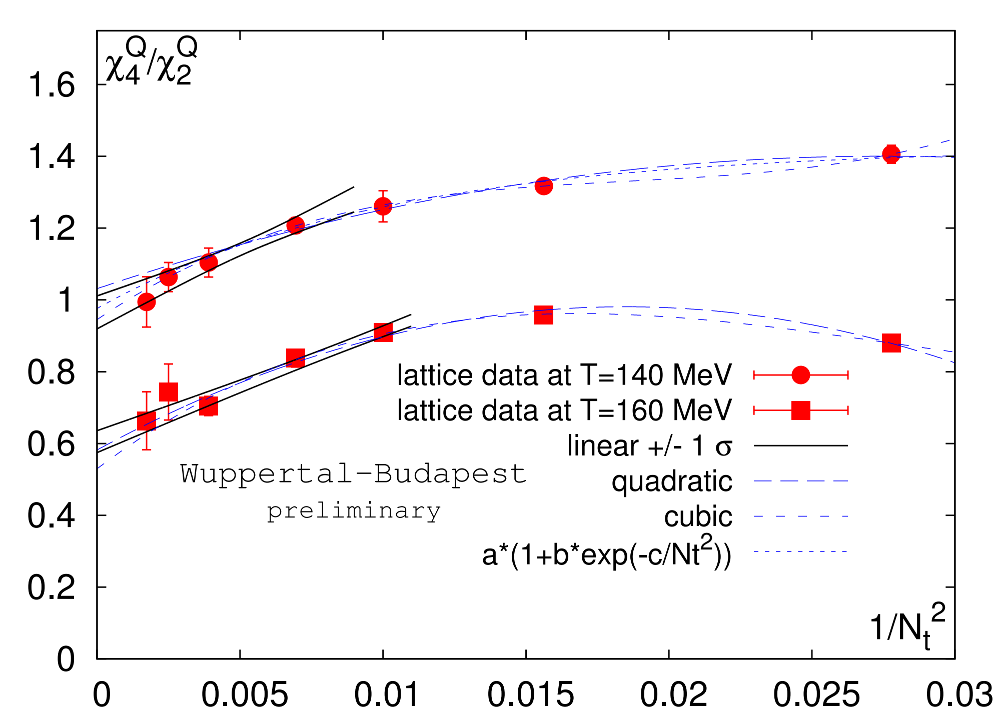
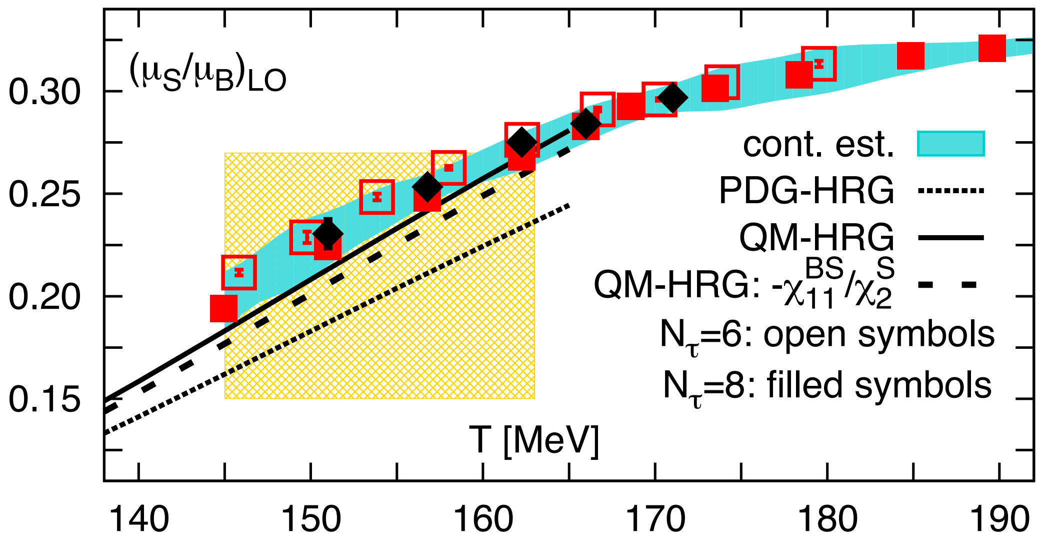




Thermometer for LHC

Talk: Borsanyi, Mon

Thermodynamics and missing states



- Strangeness neutrality:

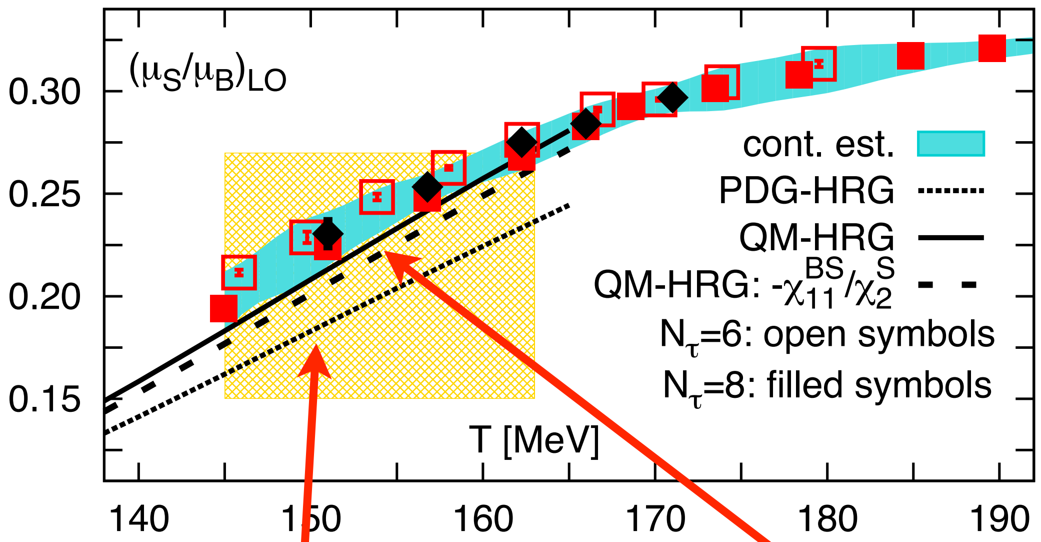
$$\mu_S = \mu_S(\mu_B, T)$$

- At leading order:

$$\left(\frac{\mu_S}{\mu_B}\right)_{LO} = -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\mu_Q}{\mu_B}$$

- What spectrum to use in HRG?

Thermodynamics and missing states



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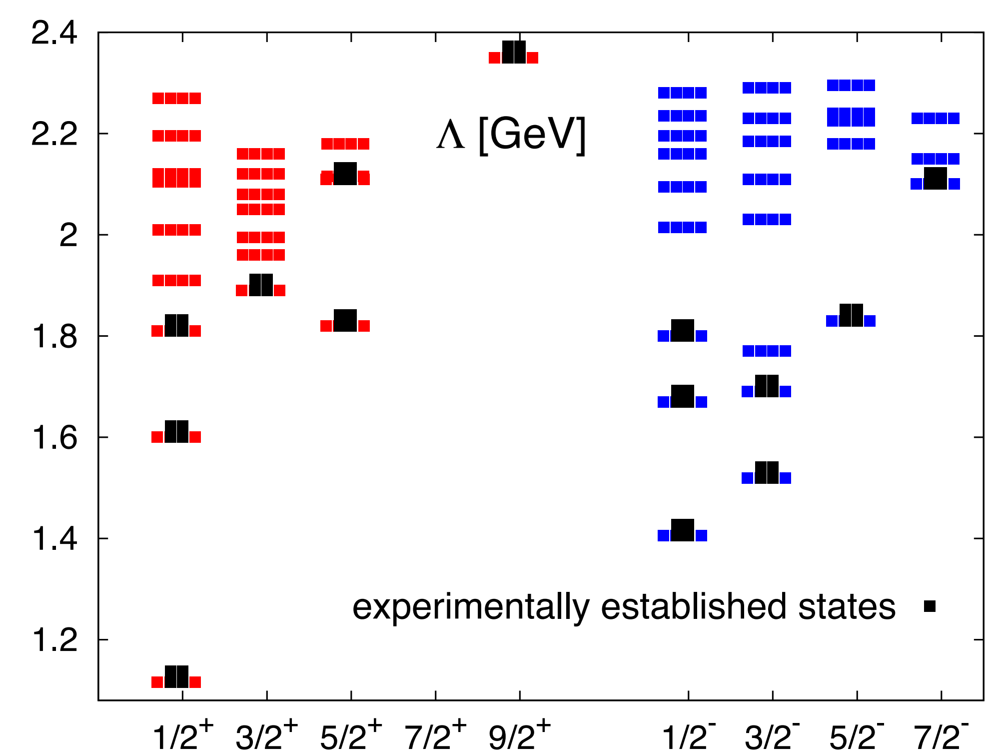
PDG

Quark Models



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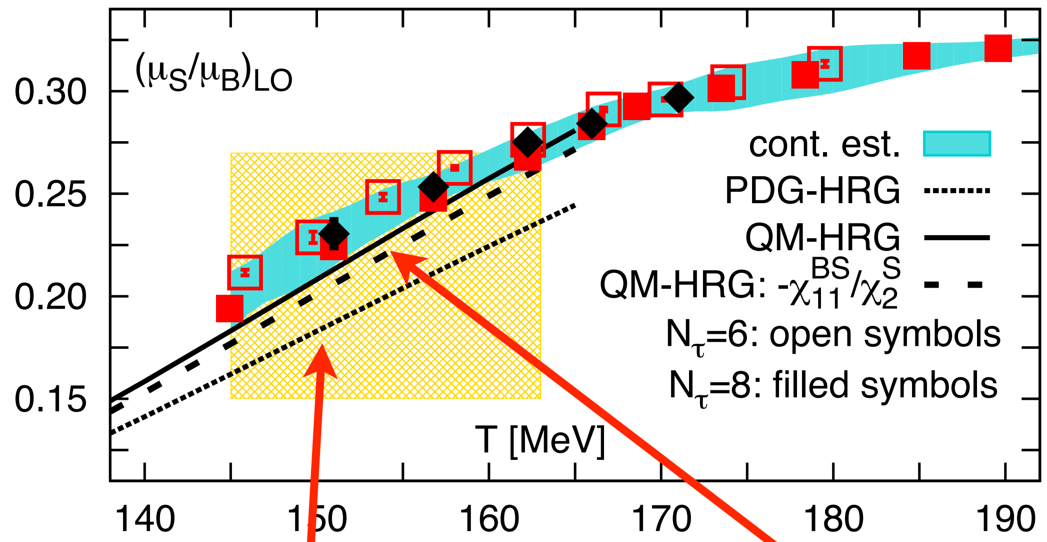

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- What spectrum to use in HRG?

PDG

Quark Models

Talk: Ding, Mon

- Lattice favors QM-HRG with larger than currently observed set of strange mesons and baryons
- Similarly in the charm sector

Bazavov et al. [BNL-Bielefeld], 1404.6511, 1404.4043

Equation of state

- Trace anomaly:

$$\varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a} \quad \Rightarrow \quad \frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}$$

- Requires UV subtractions, very computationally extensive

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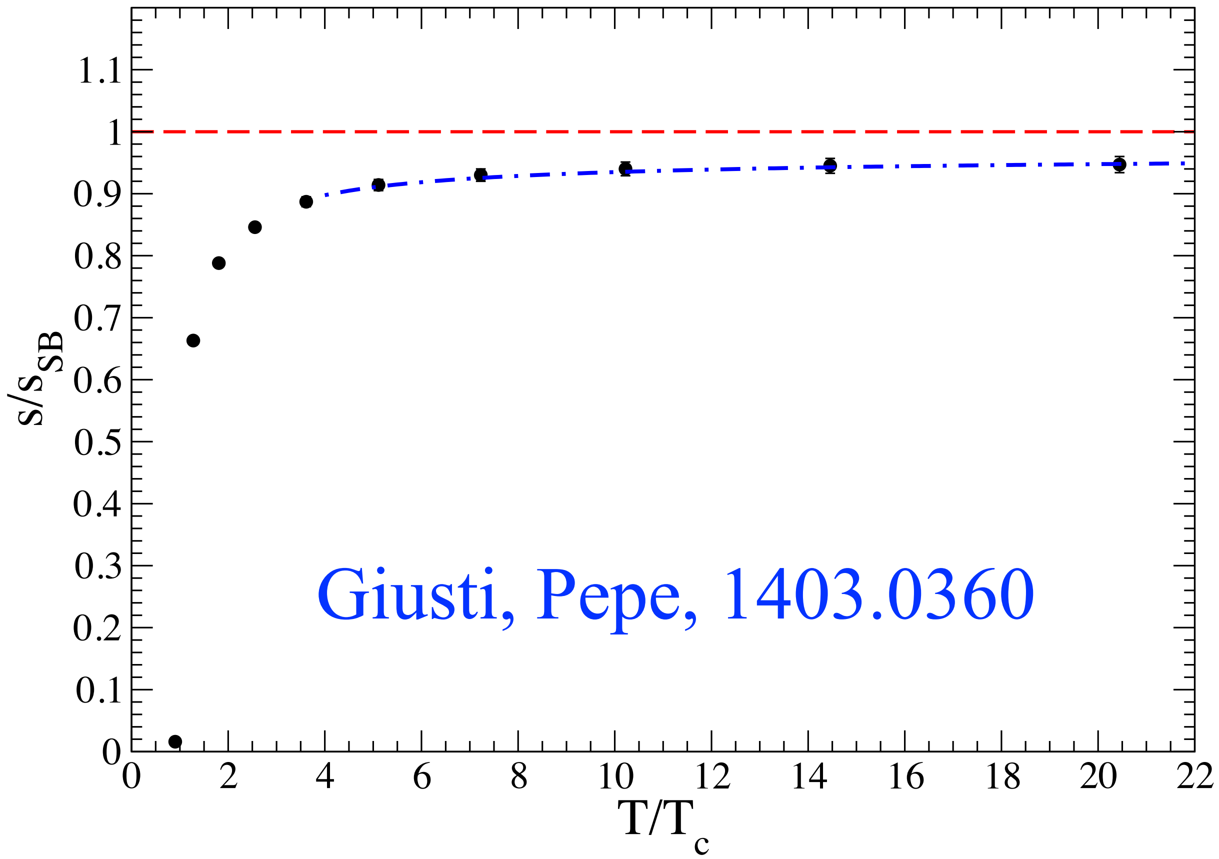
- Requires UV subtractions, very computationally extensive
- New scheme: calculate entropy in the moving frame, shifted boundary conditions

$$\frac{s(T)}{T^3} = -\frac{(1+\boldsymbol{\xi}^2)}{\xi_k}\frac{\langle T_{0k}\rangle_{\boldsymbol{\xi}}}{T^4}$$



Plenary: Kitazawa, Sat – EoS with gradient flow

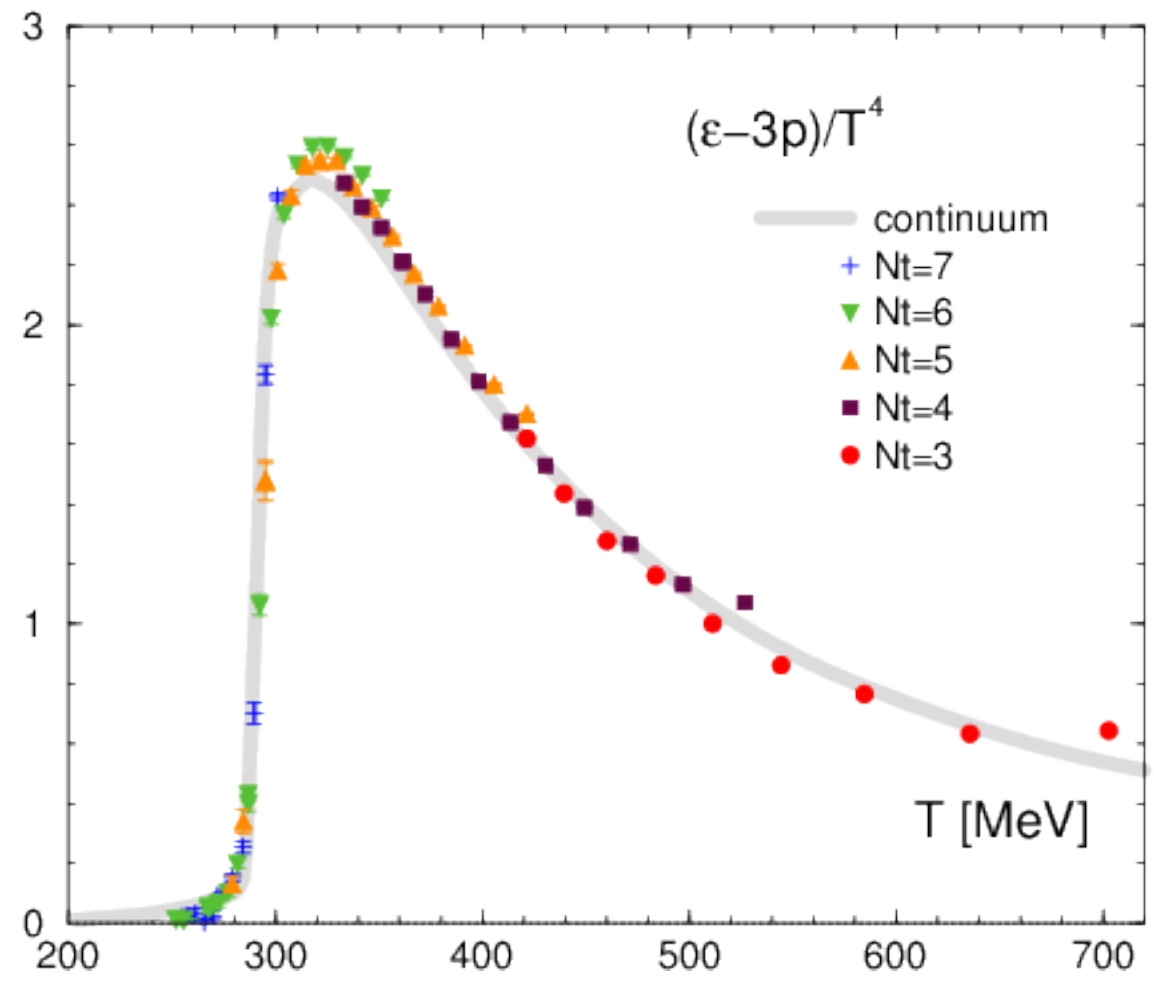
Equation of state

- Shifted boundary conditions in the fixed scale approach
- Enable to reach various temperatures at the same lattice spacing

Continuum from:

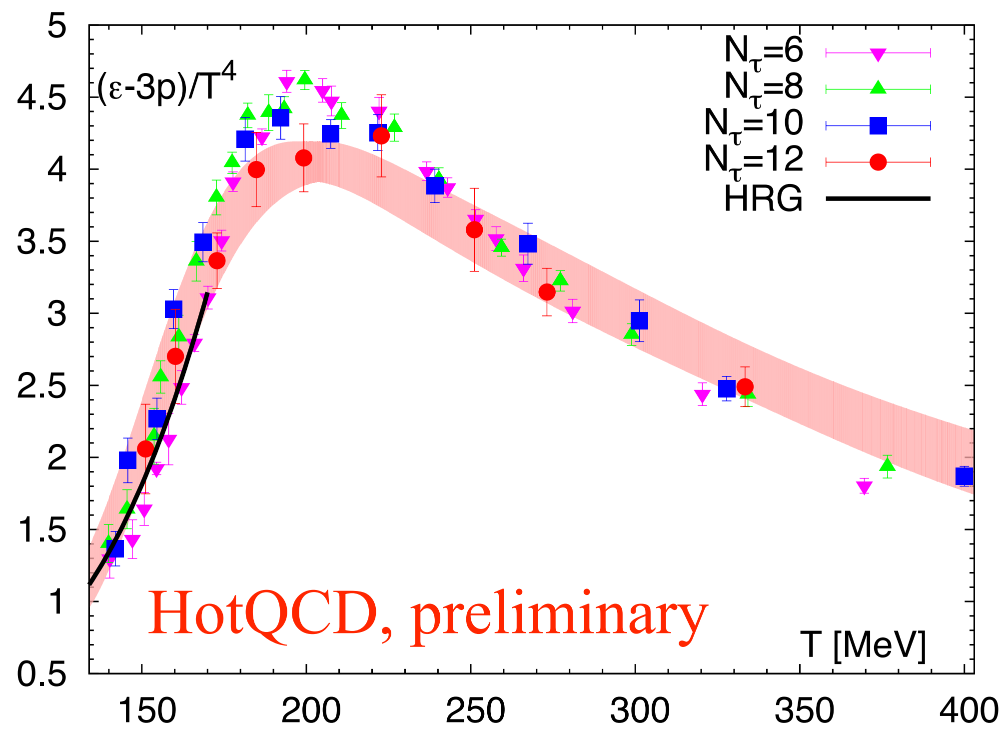
Borsanyi et al. [WB], JHEP07(2012)

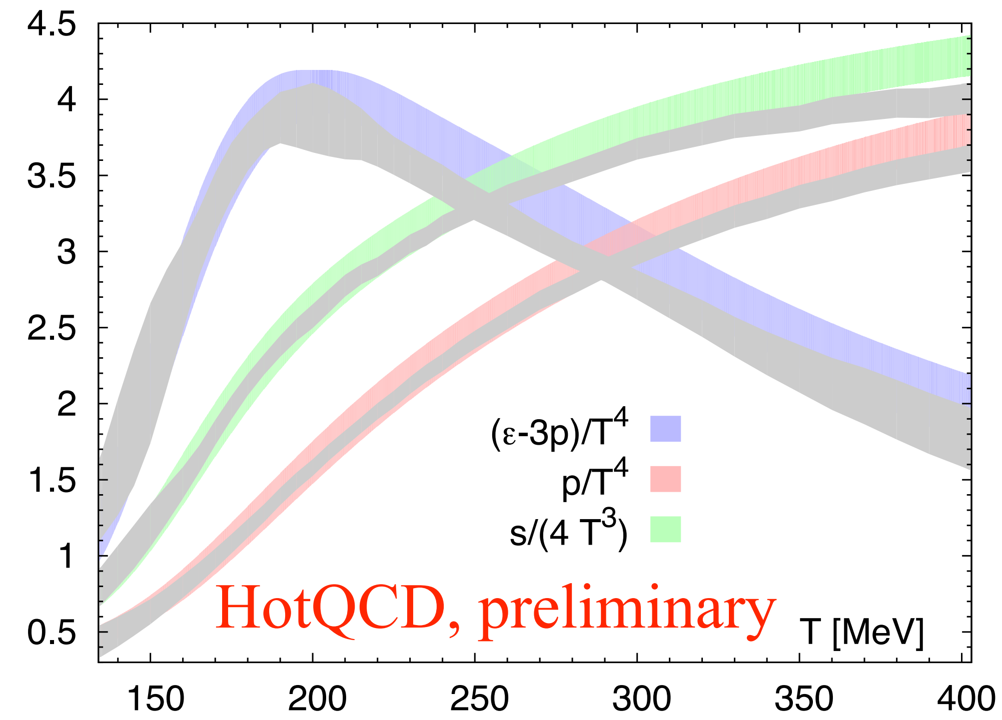
Talk: Umeda, Thu



Equation of state

- Continuum extrapolated equation of state with HISQ





- Agreement with stout Borsanyi et al. [WB], PLB 370 (2014)

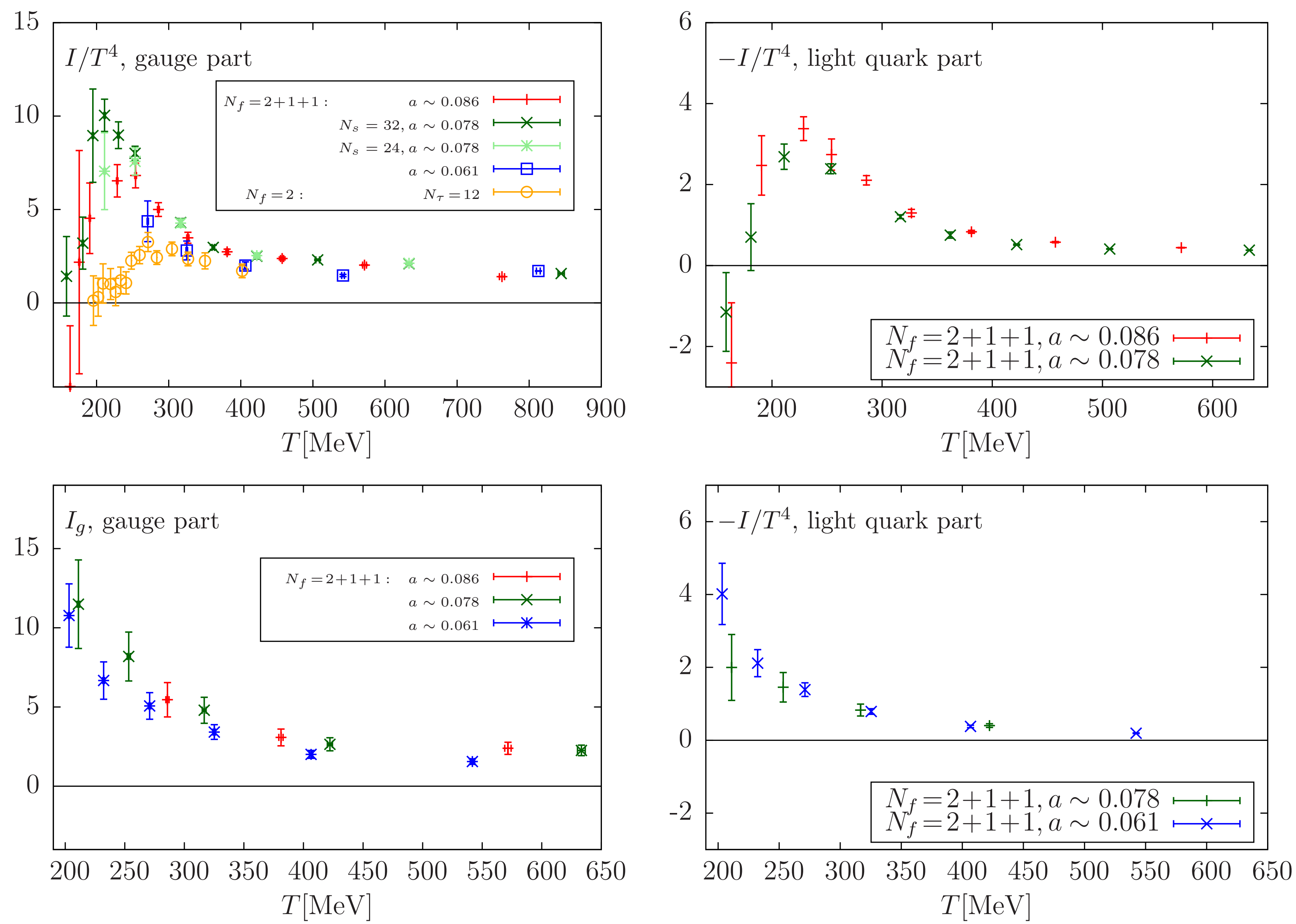
Talk: Bhattacharya, Thu

- Stout equation of state with the dynamical charm

Talk: Krieg, Thu

Equation of state

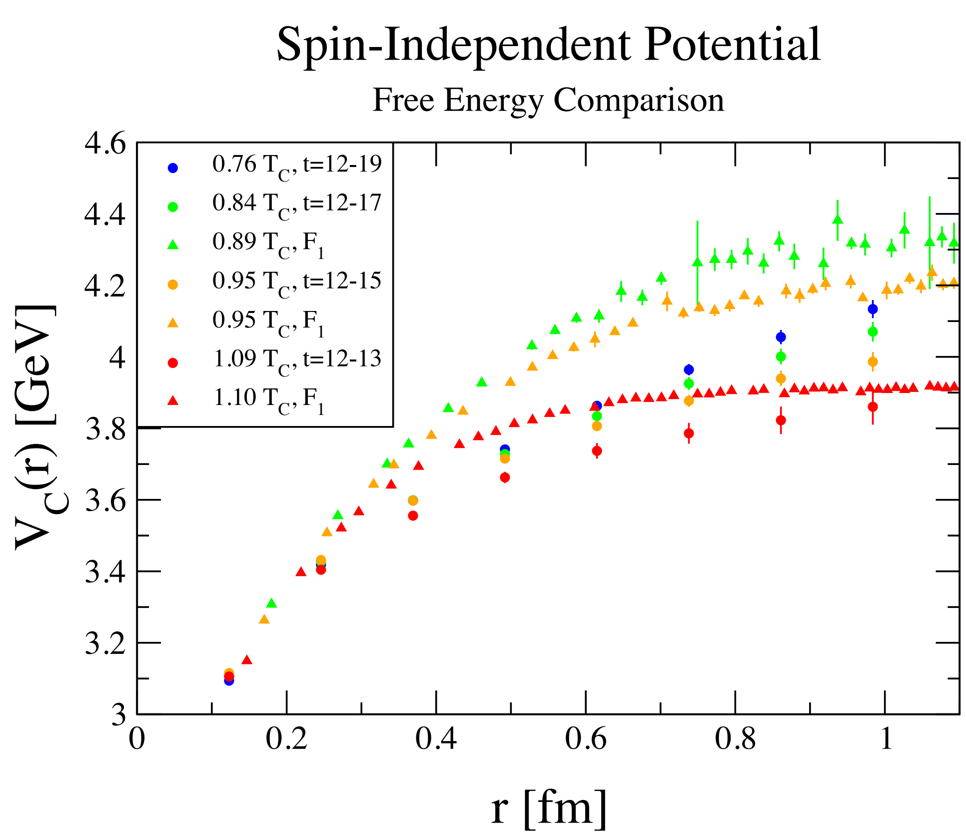
- tmfT collaboration, 2+1+1 TM, pion 220 and 400 MeV

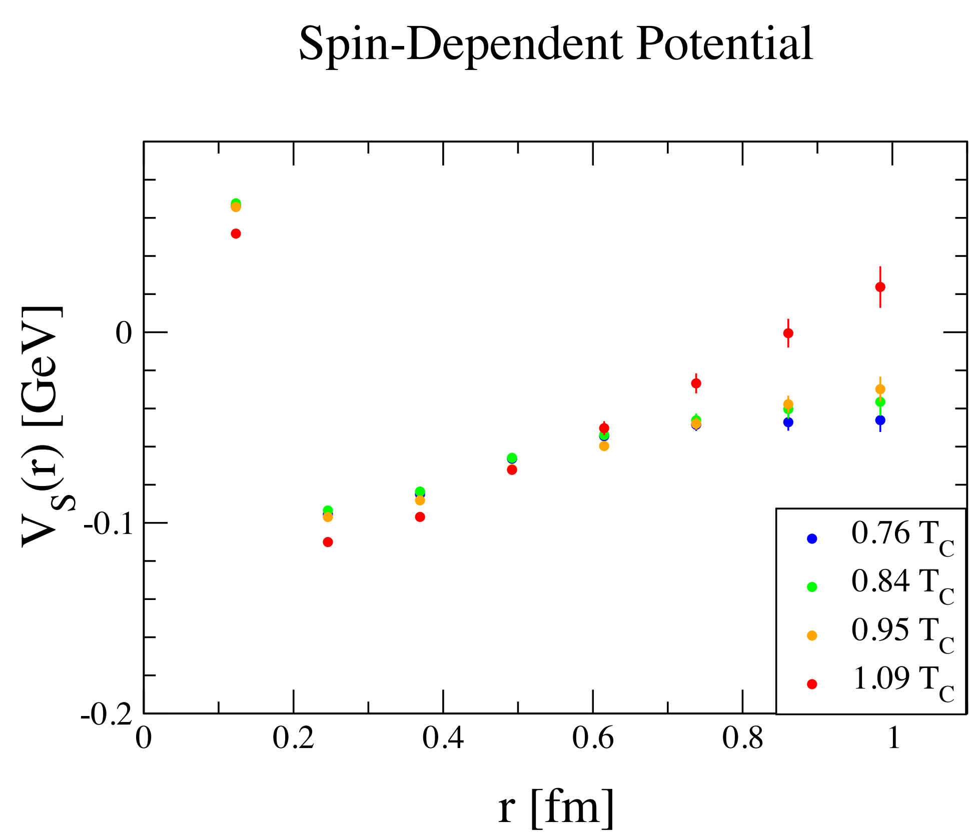


QM2014: Ilgenfritz

Heavy quark potential

- Reconstruction of the spectral function, real and imaginary part of the potential Talk: Rothkopf, Wed
- Singlet and octet potential Rossi, Testa, PRD87 (2013)
- Charmonium S-wave potential Allton et al., publication in preparation





Quarkonium spectral functions

- Challenge: reconstruct spectral functions from Euclidean correlators

Quarkonium spectral functions

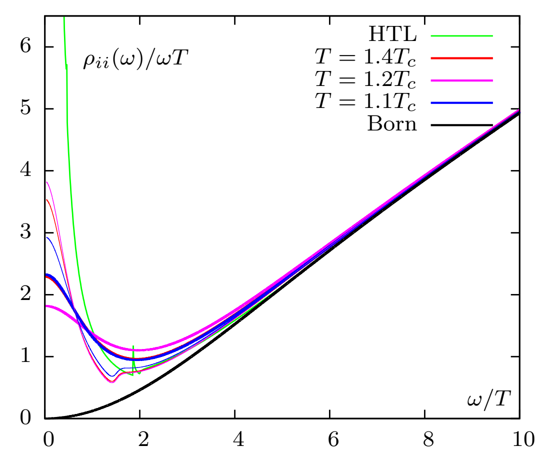
- Challenge: reconstruct spectral functions from Euclidean correlators

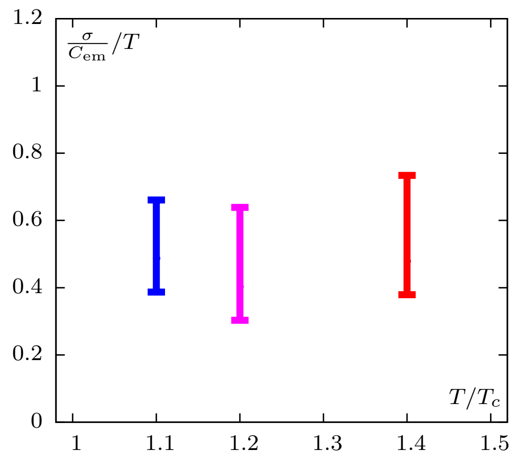
$$
\begin{aligned}
G(\tau,\vec{p}) &= \int_0^\infty d\omega\sigma(\omega,\vec{p})K(\omega,\tau) \\
K(\omega,\tau) &= \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}.
\end{aligned}
$$

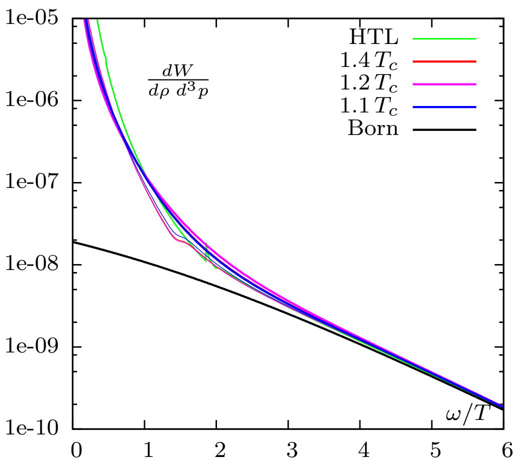
- Needed to understand the fate of various heavy quarkonia states in the plasma, calculate transport properties

Quarkonium spectral functions

- Quenched, vector channel   Talk: Meyer, Wed







Quarkonium spectral functions

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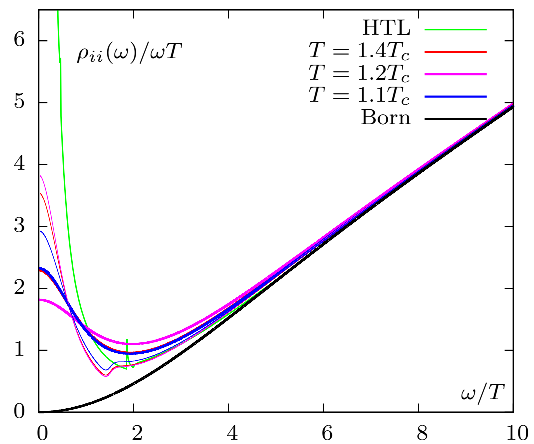
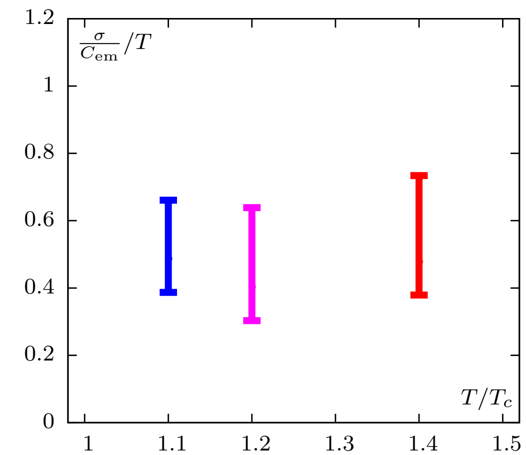
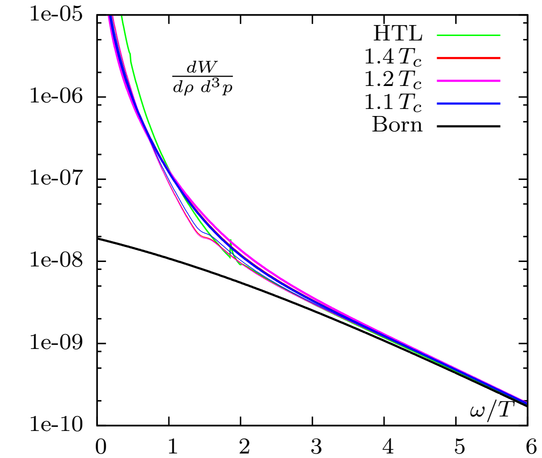
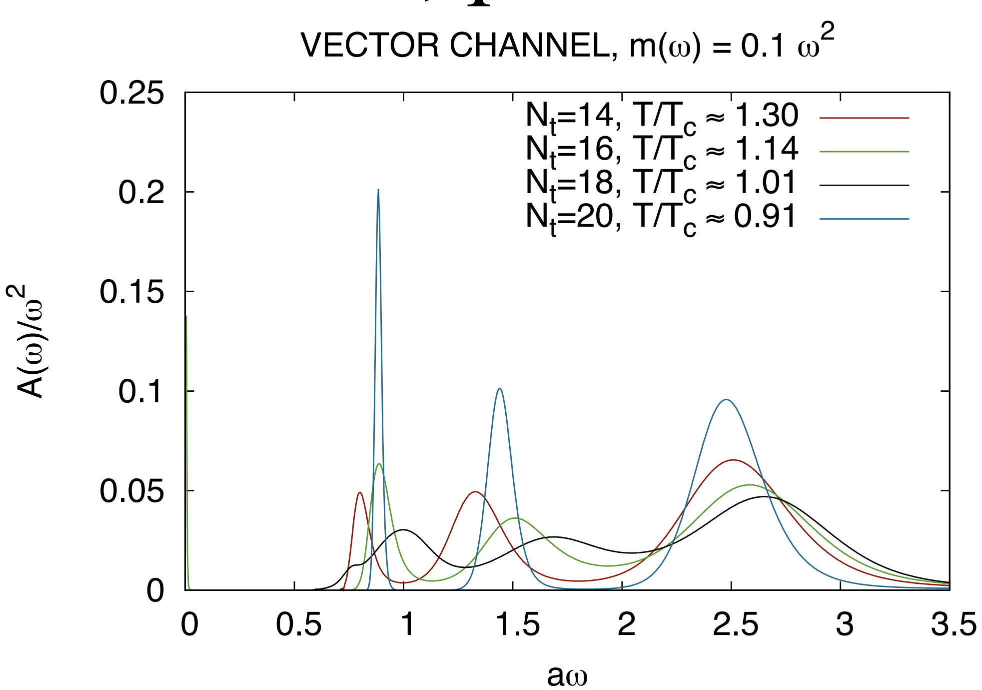
Talk: Meyer, Wed

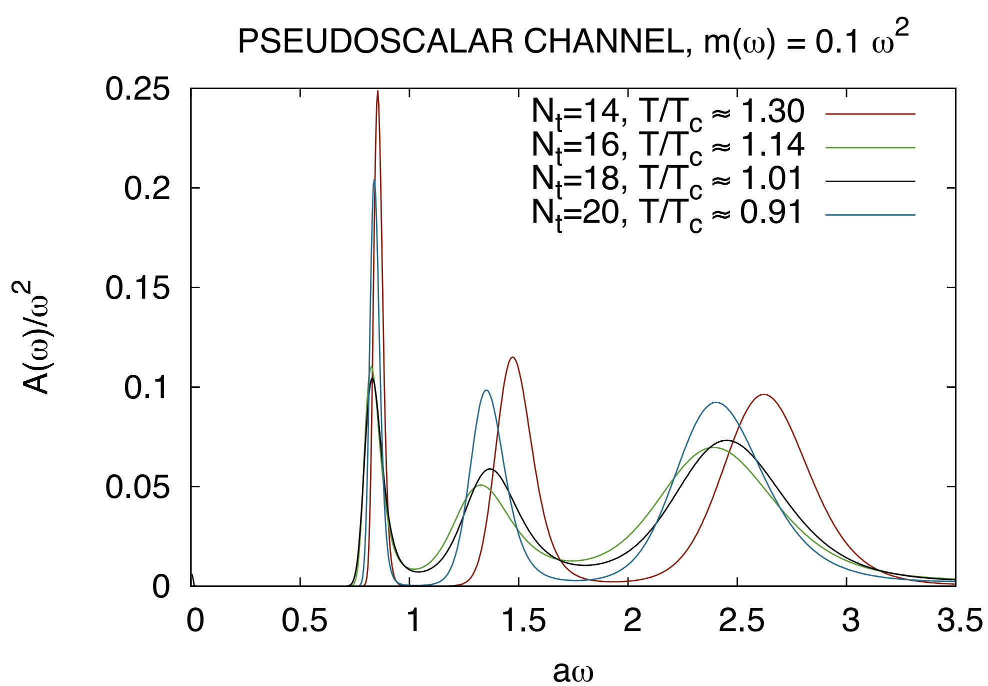






- 2+1 Wilson, pseudoscalar and vector channel



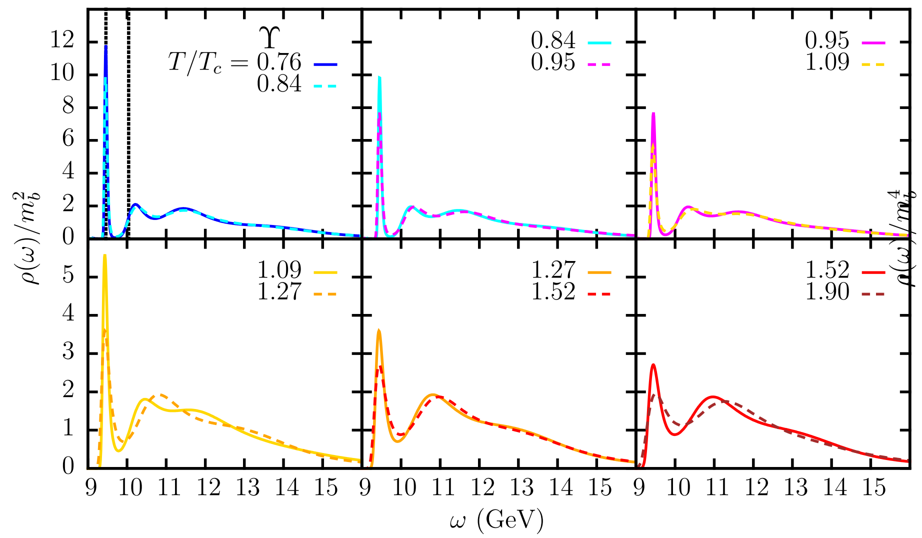


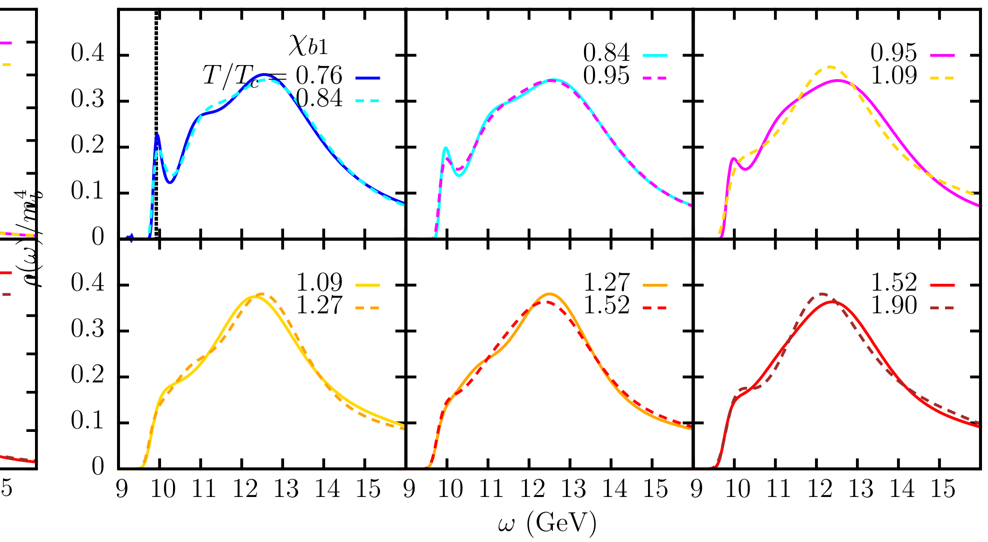
Borsanyi et al. [WB], 1401.5940

Talk: Pasztor, Wed

Quarkonium spectral functions

- S- and P-wave bottomonium, NRQCD on 2+1 Wilson





Aarts et al. [FASTSUM], 1402.6210

Talk: Harris, Wed

Quarkonium spectral functions

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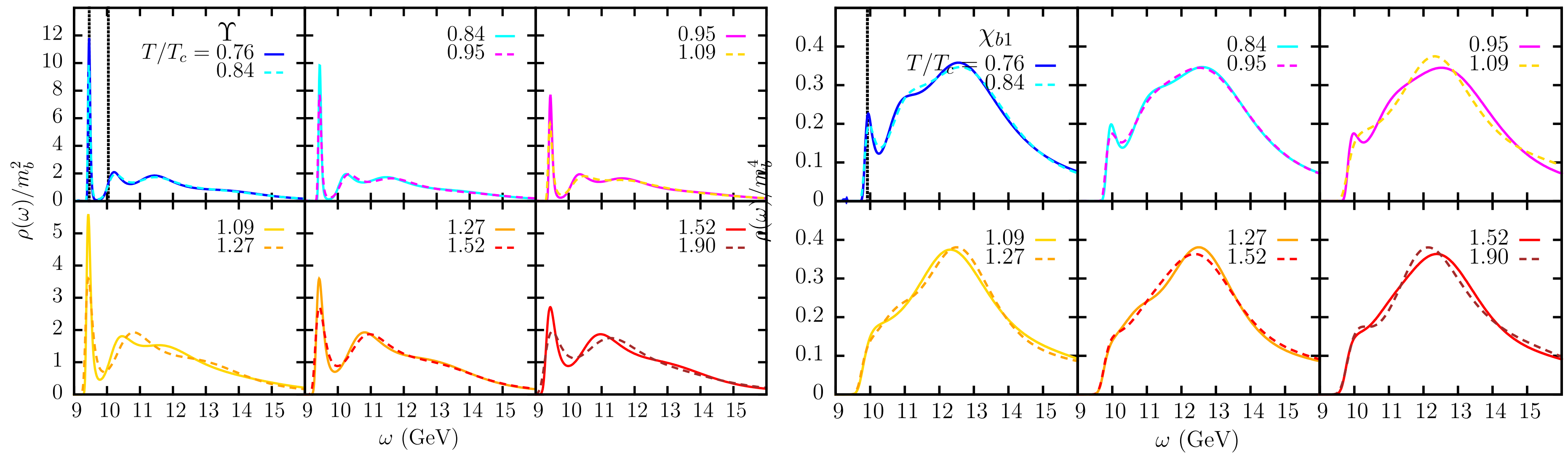
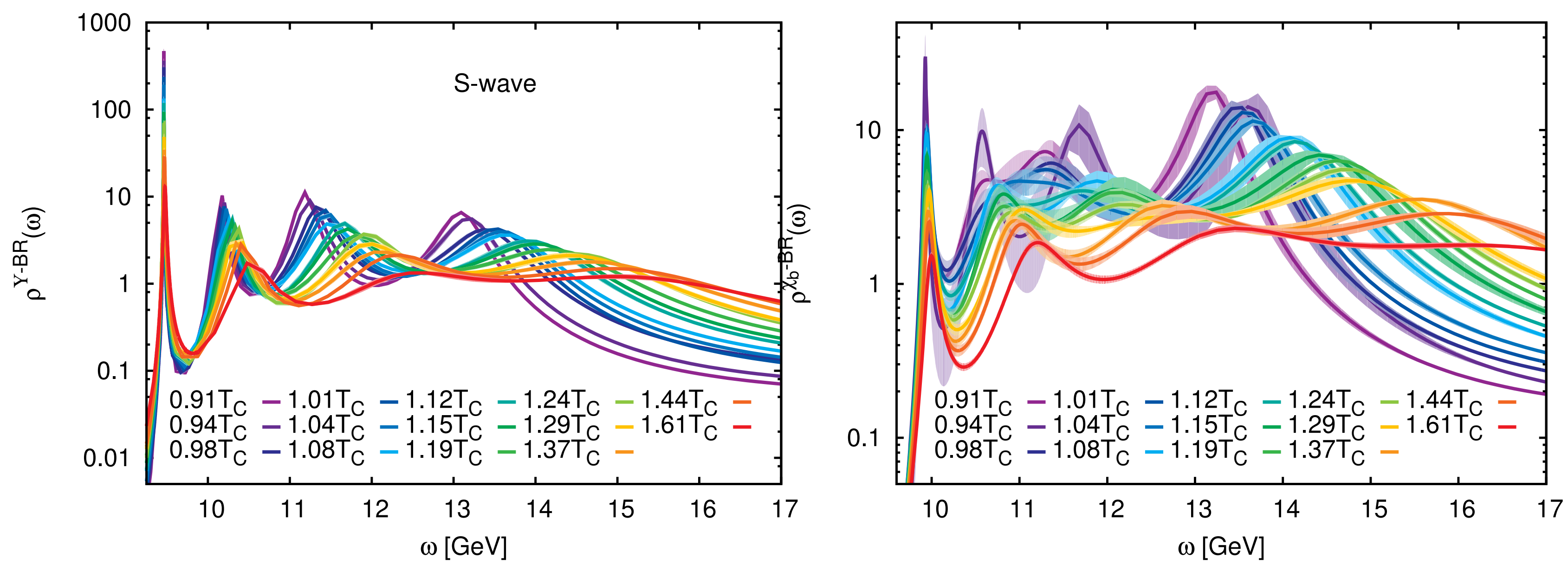


Aarts et al. [FASTSUM], 1402.6210

Talk: Harris, Wed

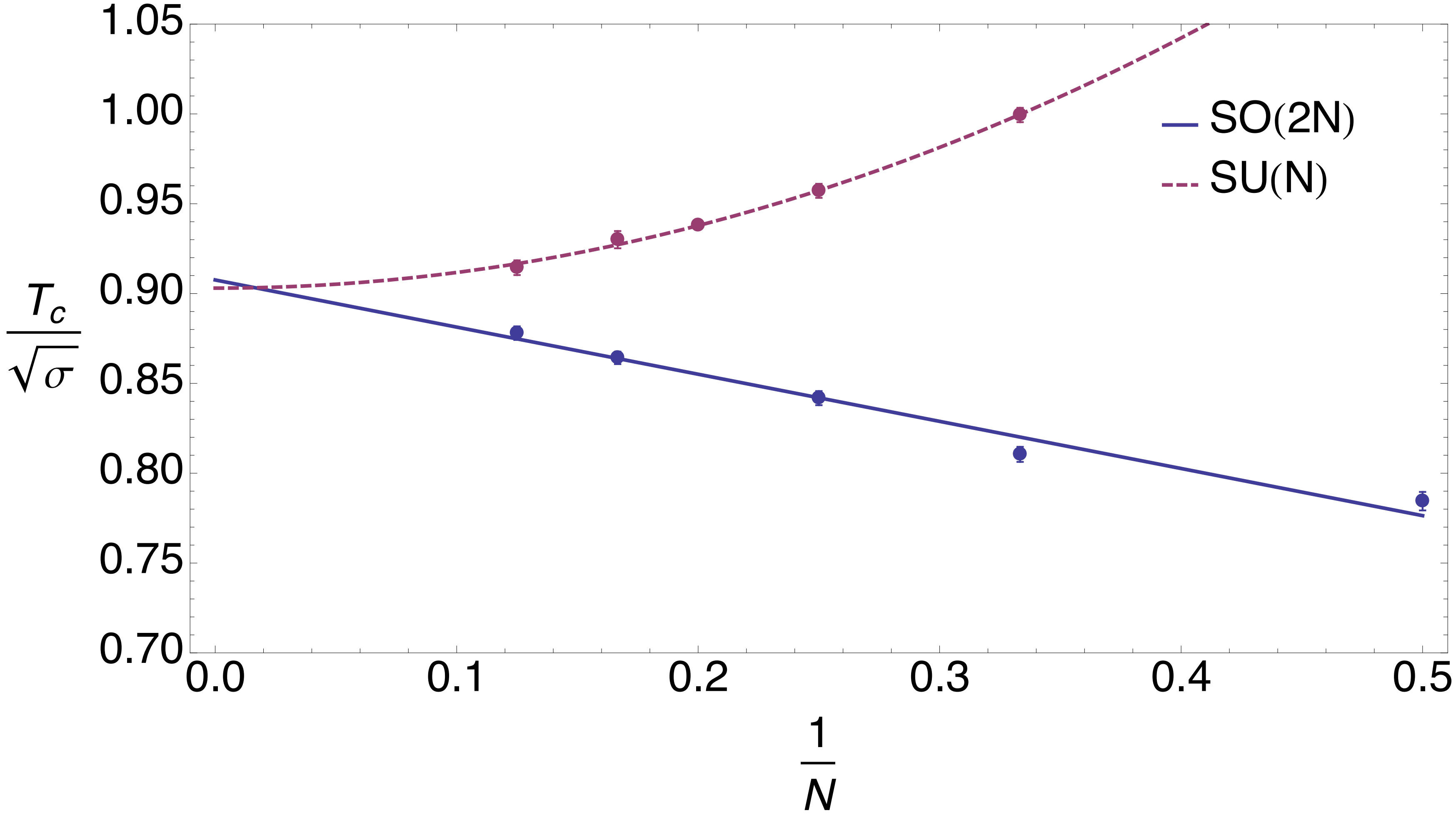
- S- and P-wave bottomonium, NRQCD on HISQ



Poster: Petreczky

Beyond QCD

- Testing orbifold equivalence for large number of colors



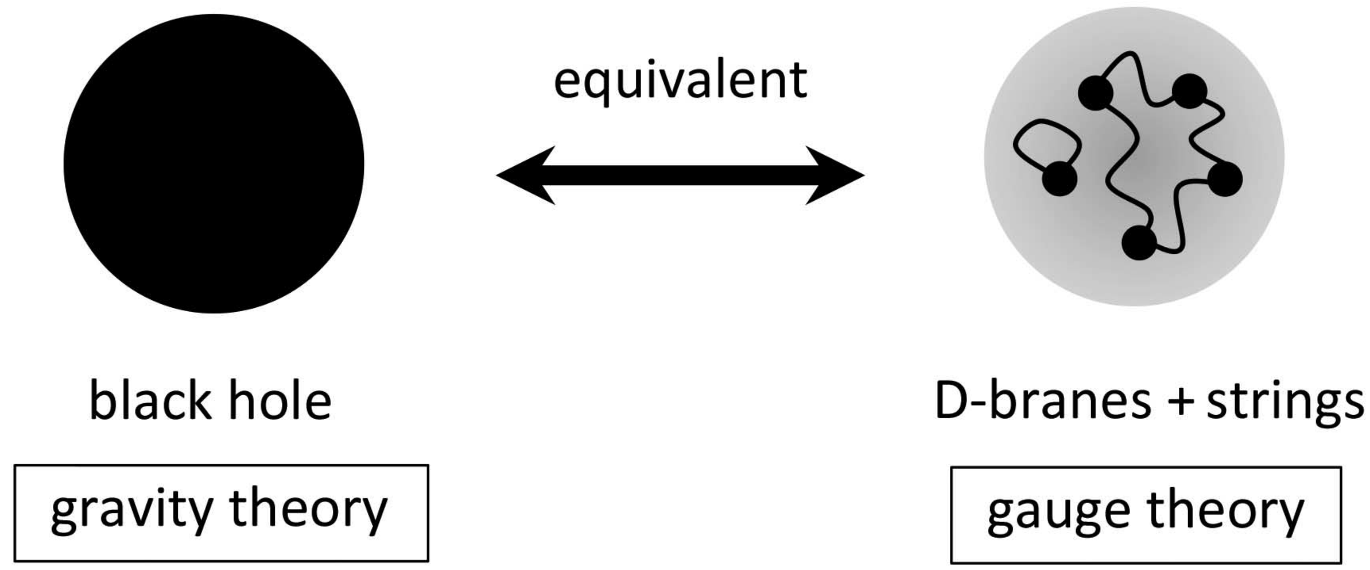
$$\left.\frac{T_c}{\sqrt{\sigma}}\right|_{SO(\infty)} = 0.9076(41)$$

$$\left.\frac{T_c}{\sqrt{\sigma}}\right|_{SU(\infty)} = 0.9030(29)$$

Talk: Lau, Fri

Beyond QCD

- Gauge-gravity duality



Beyond QCD

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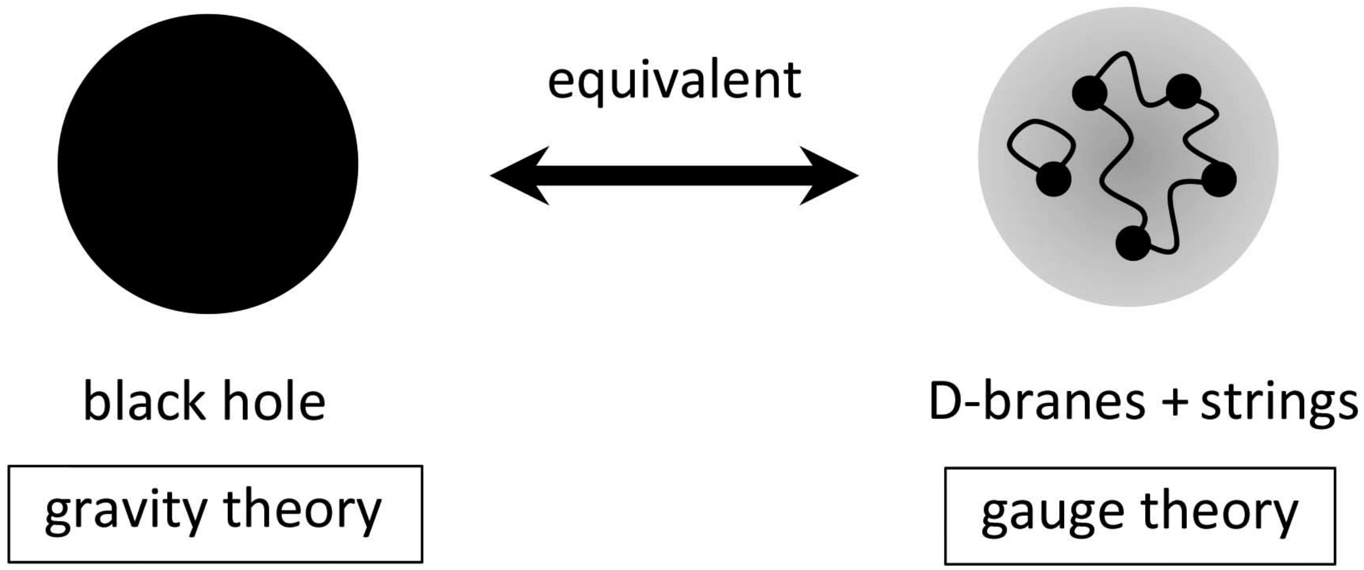

- Solve the gauge side (with lattice techniques) to study formation of a black hole

Hanada et al., Science 344 (2014)
Hanada, Maltz, Susskind, 1405.1732

Talk: Hanada, Mon

Not covered in the talk

- Columbia plot along the three-flavor line

  Talks: Nakamura, Tue; Jin, Tue

- Properties of the spectrum of the Dirac operator at low and high temperature Giordano, Kovacs, Pittler, PRL 112 (2014)

  Talks: Kovacs, Wed; Giordano, Wed; Pittler, Wed

- Jet quenching parameter

  Panero, Rummukainen, Schäfer, PRL 112 (2014)

- ChPT at finite temperature

  Talk: Robaina, Fri

Thanks to:

Chris Allton, Szabolcs Borsanyi, Bastian Brandt, Masanori Hanada, Tim Harris, Tamas Kovacs, Richard Lau, Florian Meyer, Michael Mueller-Preussker, Yoshifumi Nakamura, Daniel Nogradi, Marco Panero, Michele Pepe, Giancarlo Rossi, Alexander Rothkopf, Finn Stokes, Takashi Umeda, Ettore Vicari

Conclusions

- Agreement on the chiral crossover temperature among staggered and now chiral fermions
- Simulations with Wilson fermions are catching up
- Lattice calculations reached the level of precision to test phenomenological models and make connection to heavy-ion experiments
- Lattice provides hints about thermodynamic relevance of the yet-unobserved states in the strange and charm sector
- Agreement between the 2+1 HISQ and stout equation of state
- New techniques for calculating the equation of state
- Good progress in reconstruction of spectral functions, new techniques introduced

Thank you!