Simulating $\mathcal{N} = 4$ Yang-Mills

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Why (lattice) $\mathcal{N} = 4$ Yang-Mills?

- Finite QFT - true at 1 loop even on lattice!
- Conformally invariant in continuum. How does this get restored on lattice as $V \to \infty$ and $a \to 0$?
- Cornerstone of AdSCFT correspondence.
- Only known example of 4D theory which admits a SUSY preserving discretization. Lattice formulation defines theory outside of perturbation theory.
- Gravity as ($\mathcal{N} = 4$) Yang-Mills squared ...

People

Many people contributed to development of lattice formulation eg. Unsal, Kaplan, Sugino, Kawamoto, Hanada, Joseph,...
Here, report on recent results from (somewhat) large scale simulations with:

- Tom DeGrand, CU Boulder
- Poul Damgaard, NBI
- Joel Giedt, RPI
- David Schaich, Syracuse U.
- Aarti Veernala, Syracuse U.
- S.C

Topics

- Introduction.
- Key ingredients in lattice formulation.
- Continuum limit. Restoration of full SUSY (Joel Giedt)
- Practical issues:
  - Regulating flat directions (S.C)
  - Suppressing U(1) monopoles (S.C)
  - Sign problems (or lack of them) (David Schaich)
- Static potential (David Schaich)

Key ingredients

Continuum $\mathcal{N} = 4$ YM obtained by dimensional reduction of 5D theory:

$$S = \mathcal{Q} \int d^5x \left(\chi_{ab} \mathcal{F}_{ab} + \eta \left[\overline{\mathcal{D}}_a, \mathcal{D}_a \right] + \frac{1}{2} \eta d \right) + \int d^5x \, \epsilon_{abcde} \chi_{ab} \overline{\mathcal{D}}_c \chi_{de}$$

Usual fields	Twisted fields
$A_\mu, \mu = 1 \ldots 4 \quad \phi_i, i = 1 \ldots 6$	$\mathcal{A}_a, a = 1 \ldots 5$
$\Psi^f, f = 1 \ldots 4$	$\eta, \psi_a, \chi_{ab}, a, b = 1 \ldots 5$

Complex bosons: $\mathcal{A}_a = A_a + i\phi_a$, $\mathcal{D}_a = \partial_a + \mathcal{A}_a$, $\mathcal{F}_{ab} = [\mathcal{D}_a, \mathcal{D}_b]$

$\mathcal{Q}$ is scalar supersymmetry

Scalar supersymmetry

$$\boxed{\text{Where did } \mathcal{Q} \text{ come from ?}}$$

Appearance of scalar fermion η implies scalar SUSY.
Action:

$$\mathcal{Q}A_a = \psi_a \quad \mathcal{Q}\psi_a = 0 \quad + \ldots \quad \text{similar on other fields}$$

$$\text{Notice } \boxed{\mathcal{Q}^2 = 0} \text{ !}$$

- Any action of form $S = \mathcal{Q}\,(\text{something})$ will be trivially invariant under $\mathcal{Q}$.
- This is how theory evades usual problems of lattice susy

Some lattice details

- Place all fields on links (η degenerate case - site field). Gauge transform like endpoints.
- Prescription exists for replacing derivatives by gauge covariant finite difference operators.
- But what lattice to use ? Natural to look for 4D lattice with a basis of 5 equivalent basis vectors – $\boxed{A_4^* \text{ lattice}}$

A_4: set of points in 5D hypercubic lattice Z^5 which satisfy

$$n_1 + n_2 + n_3 + n_4 + n_5 = 0$$

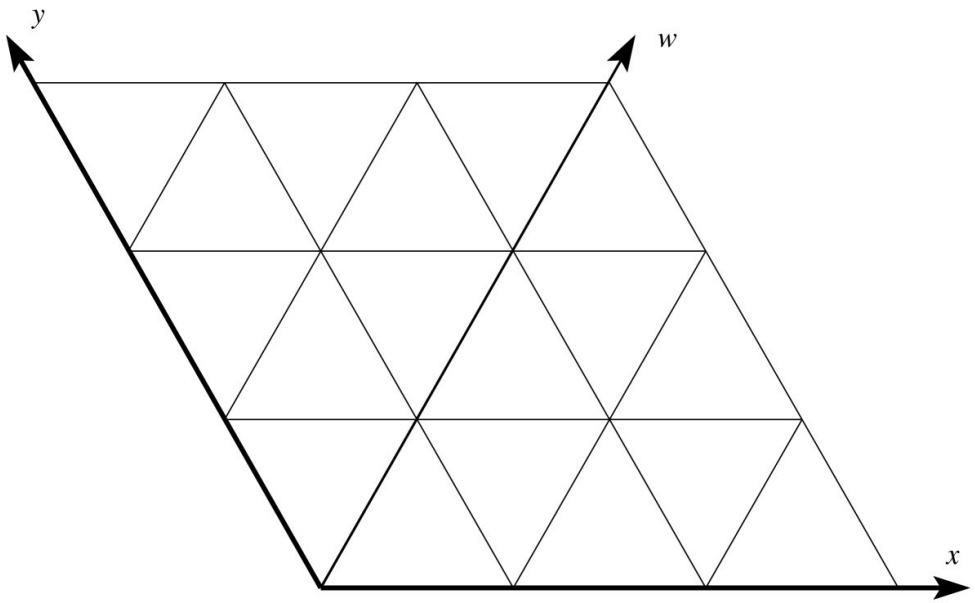
A_4^* is just dual lattice to A_4.

(Also: weight lattice of $SU(5)$, basis vectors for 4-simplex, ...)

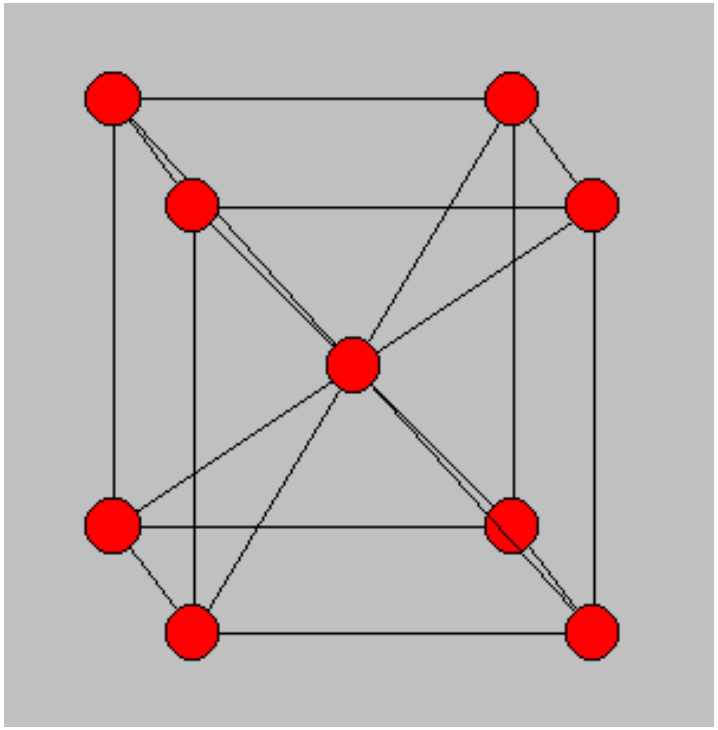
Examples of A_d^*

Symmetry group: S_{d+1}. Low lying irreps match $SO(d)$

$d = 2$



$d = 3$



Advantages of formulation

Single exact SUSY is enough to:

- Pair boson/fermion states
- Classical moduli space survives in quantum theory: no scalar potential developed to all orders in lattice perturbation theory
- Fine tuning is reduced to single log tuning (Joel)
- beta function of lattice theory vanishes at 1loop.
- Certain quantities eg partition function can be computed exactly at 1-loop.

Novel Features

- Exact SUSY requires complexified links in algebra of $U(N)$!

$$\mathcal{U}_a(x) = \sum_{i=1}^{N^2} T^i \mathcal{U}_a^i(x)$$

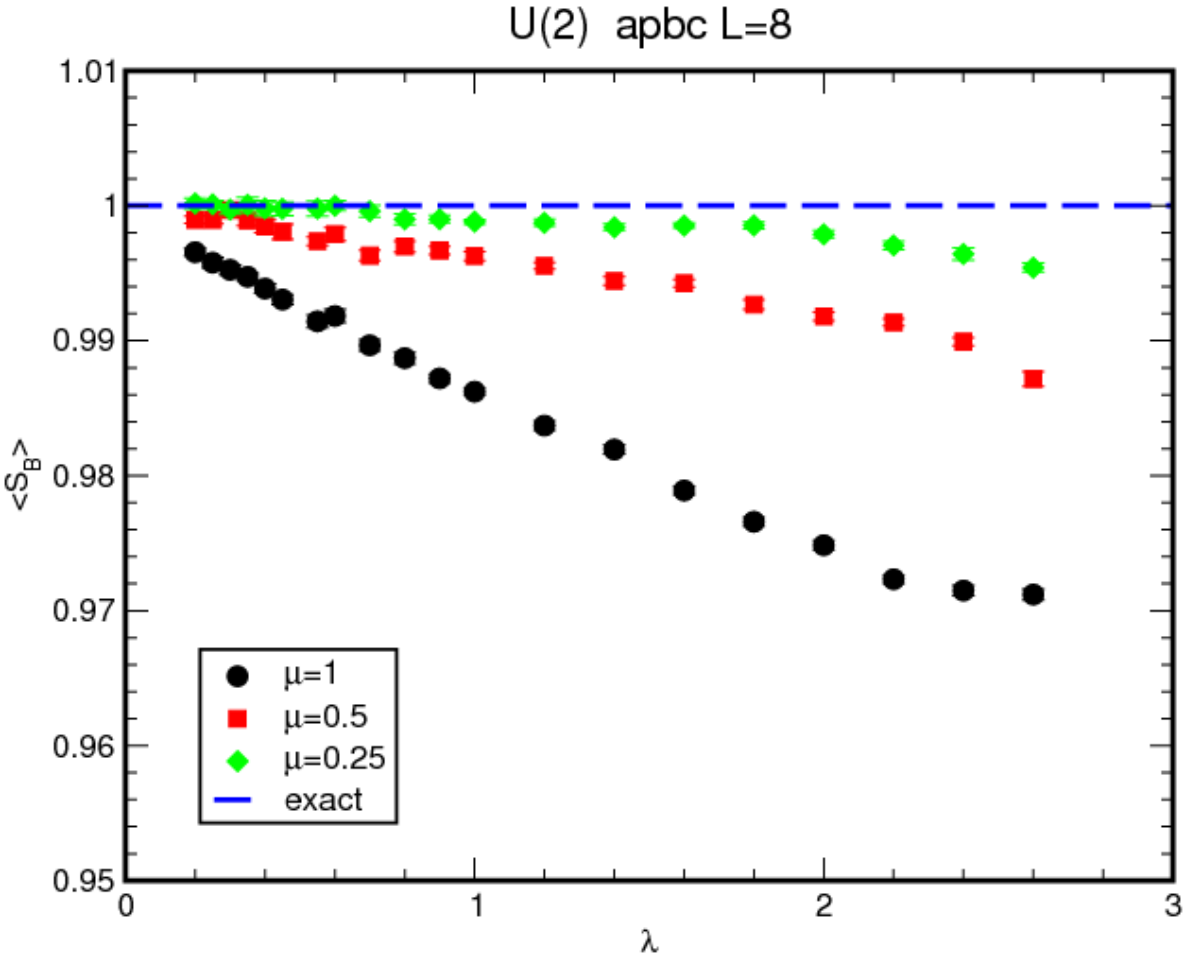
- Naive continuum limit requires $\mathcal{U}_a^0 = 1 + \ldots$ ($T^0 \equiv I_N$)
- One of many possible vacua .. stabilize by adding potential term

$$\delta S_1 = \mu^2 \sum_{x,a} \left(\frac{1}{N} \mathrm{Tr}\, \mathcal{U}_a(x) \overline{\mathcal{U}}_a(x) - 1 \right)^2$$

- Selects correct vacuum state. Breaks exact SUSY but all counter terms must vanish as $\mu \to 0$.

Restoration of exact $\mathcal{Q}$ SUSY

$\mathcal{Q}$ Ward identity:

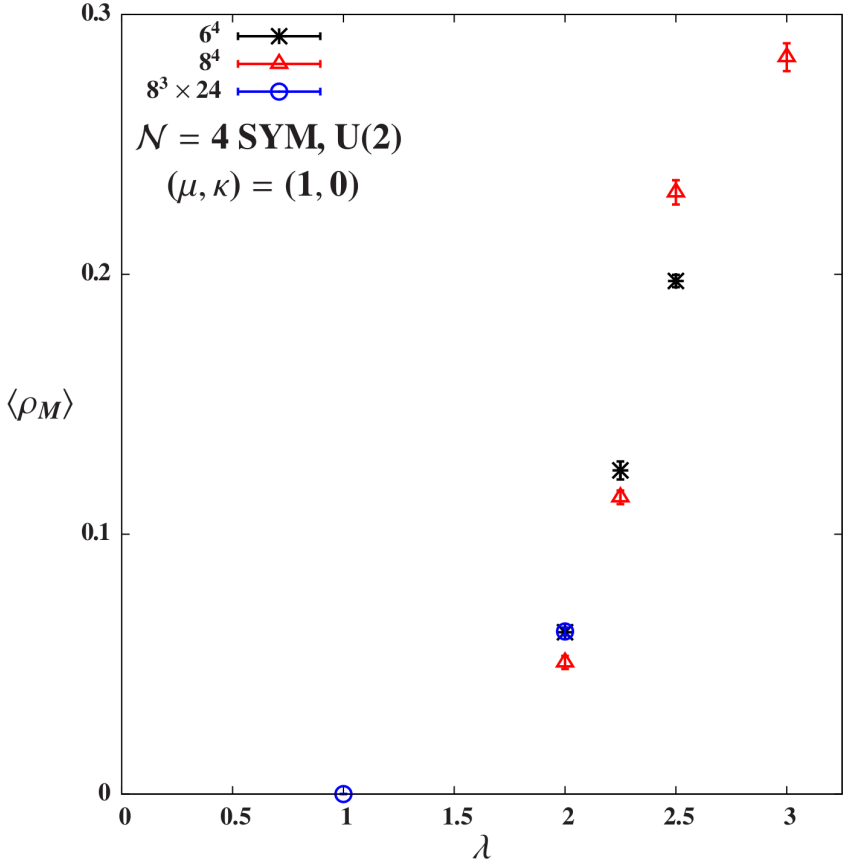


But ..

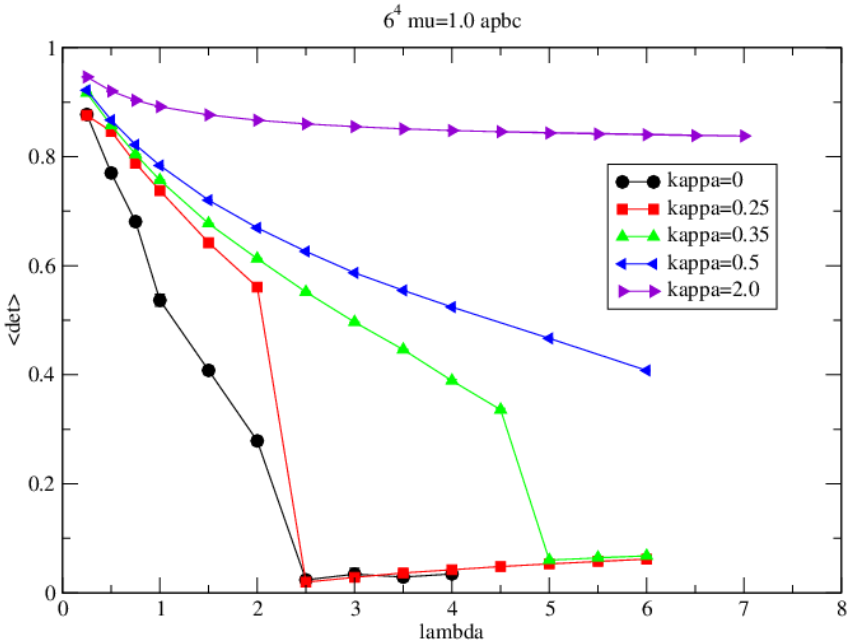
Unfortunately this is not quite enough ...

Confinement of U(1) at strong coupling

U(1) monopole density



det(plaquette)



The fix ..

Add to action a term that (approximately) projects $U(N) \to SU(N)$

$$\delta S_2 = \kappa \sum_{x,\mu<\nu} |\det P_{\mu\nu} - 1|^2$$

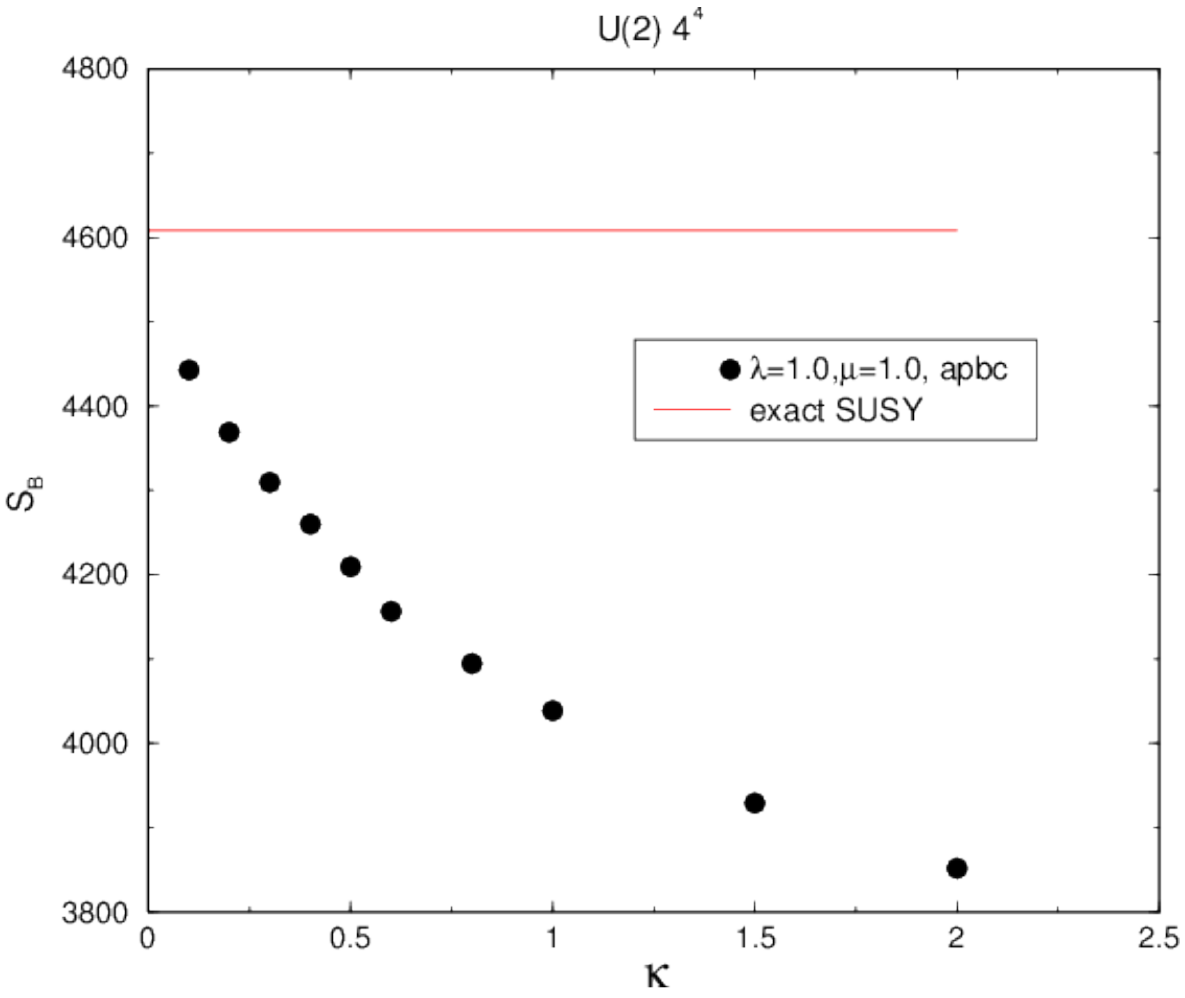
To leading order

$$\delta S_2 = 2\kappa \sum_{x,\mu<\nu} \left(1 - \cos F^0_{ab}\right) + \ldots$$

For $\kappa > 0.5$ U(1) sector weakly coupled and monopole density very small.
Marginal coupling to sector which decouples in continuum limit.
Extrapolate $\kappa \to 0$?

Allows us to push to strong coupling in non-abelian sector

Kappa dependence



Simulations

- RHMC algorithm to handle Pfaffian with multiple time scale Omelyan integrator.
- Code base extension to MILC. Arbitrary numbers of colors. A_4^* lattice communication.
- Lattices stored as hypercubic $\{n_\mu\}$ with additional body-diagonal link. Map to physical space-time needed only for correlators and only at analysis stage. $R = \sum_{\nu=1}^{4} \hat{\mathbf{e}}_\nu n_\nu$
- 6^4, 8^4, $8^3 \times 24$, $16^3 \times 32$ lattices with apbc for fermions in temporal direction.

Summary

- Currently employing large(ish) simulations to study new lattice formulation of $\mathcal{N} = 4$ super Yang-Mills.
- Retains exact SUSY. Reduces dramatically number of couplings needed to tune to supersymmetric continuum limit (Joel's talk).
- "Naive formulation" requires supplementary couplings (μ, κ). Limit $\mu, \kappa \to 0$ under control.
- No sign problem (David's talk)
- No confinement even at strong coupling (David's talk).

Starting to look at physically interesting quantities eg. anomalous dimensions (Konishi) ....