Pion masses in 2-flavor QCD with eta condensation

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base on

S.A and M. Creutz, PRL 112(2014) 141603 (arXiv:1402.1837[hep-lat])

1. Introduction

θ term in QCD $\qquad i\theta\frac{1}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}(x)F_{\alpha\beta}(x)\equiv i\theta q(x)$ $\qquad$ CP odd

Neutron Electric Dipole Moment(NEDM)

Experimental bound

$$|\vec{d_n}|\le 6.3\times 10^{-26}e\cdot cm$$

Model estimate

$$|\vec{d_n}|/\theta\simeq 10^{-15}\sim 10^{-17}e\cdot cm$$

$$\theta=\theta_{\rm QCD}+\theta_{\rm EW}\le O(10^{-8})$$

Strong CP problem !

One possible "solution" $\qquad m_u=0 \qquad$ massless up quark

(Lattice QCD already ruled out this ?)

chiral rotation

$$u\longrightarrow e^{i\alpha\gamma_5}u,\quad \bar u\longrightarrow \bar u e^{i\alpha\gamma_5},$$

$$m_u\,\bar u u\rightarrow m_u\,\bar u e^{i2\alpha\gamma_5}u$$

$\theta\longrightarrow\theta'=\theta+2\alpha N_f$ $\quad$ chiral anomaly

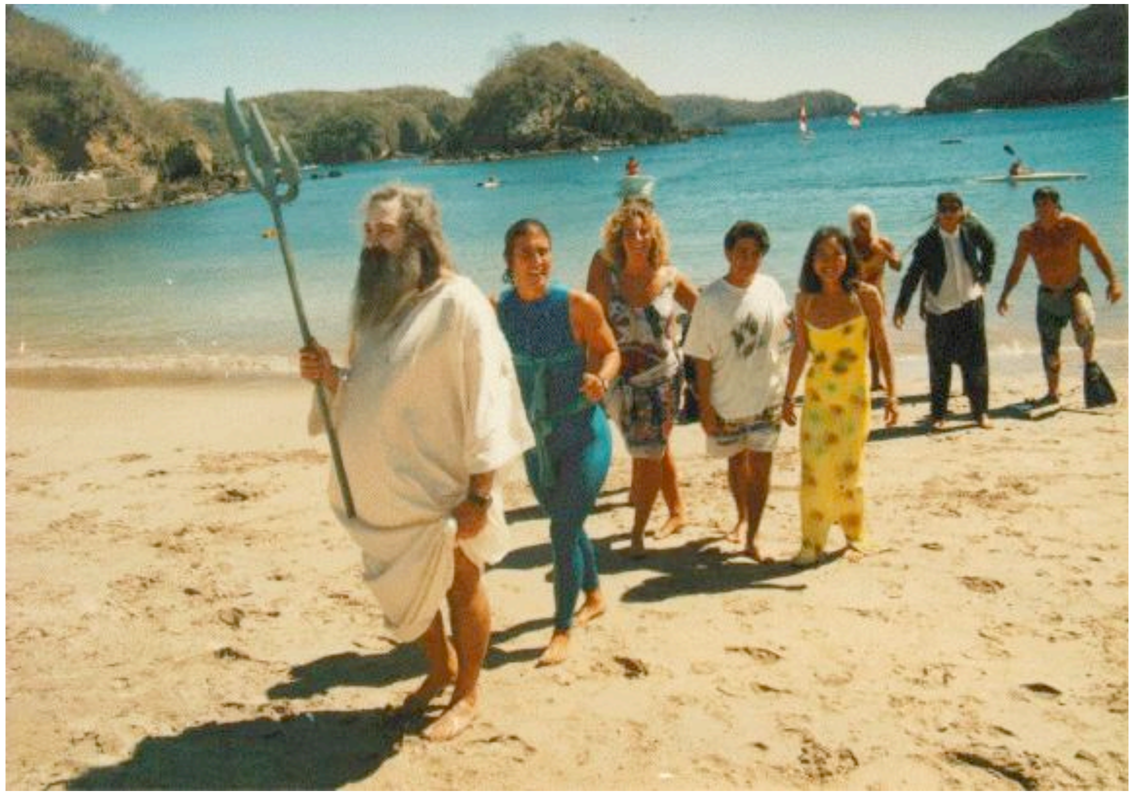
if $m_u=0$, we can make

$$\theta'=0$$

by $\alpha=-\frac{\theta}{2N_f}$

Mike Creutz, “Quark masses, the Dashen phase, and gauge field topology”
arXiv:1306.1245[hep-lat]

Mike’s Oracles



$m_d > 0$ fixed, then

1. Nothing special happens at $m_u = 0$.

2. Massless neutral pion: $m_{\pi^0} = 0$ at $m_u =^{\exists} m_c < 0$.

critical quark mass

3. Pion condensation (Dashen phase): $\langle \pi^0 \rangle \neq 0$ at $m_u < m_c < 0$.

4. $\chi = \infty$ at $m_u = m_c$.

$$\chi = \frac{1}{V}\langle Q^2 \rangle$$ topological susceptibility

5. $\chi = 0$ at $m_u = 0$. ?

In this talk, I show the above properties by ChPT including the anomaly effect.
In addition, we discuss an interesting prediction related to these in 2-flavor QCD.

ChPT with "anomaly"

$$\mathcal{L} = \frac{f^2}{2}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{2}\mathrm{tr}\left(M^\dagger U + U^\dagger M\right) - \underbrace{\frac{\Delta}{2}\left(\det U + \det U^\dagger\right)}_{\text{effect of anomaly}}$$

Note: large N argument by Witten (fundamental rep. for quarks)

$$\frac{\Delta}{2}(\det U + \det U^\dagger) \quad \Rightarrow \quad \frac{c}{N}(\log\det U)^2$$

N=3 quark $\Rightarrow$ fundamental ?

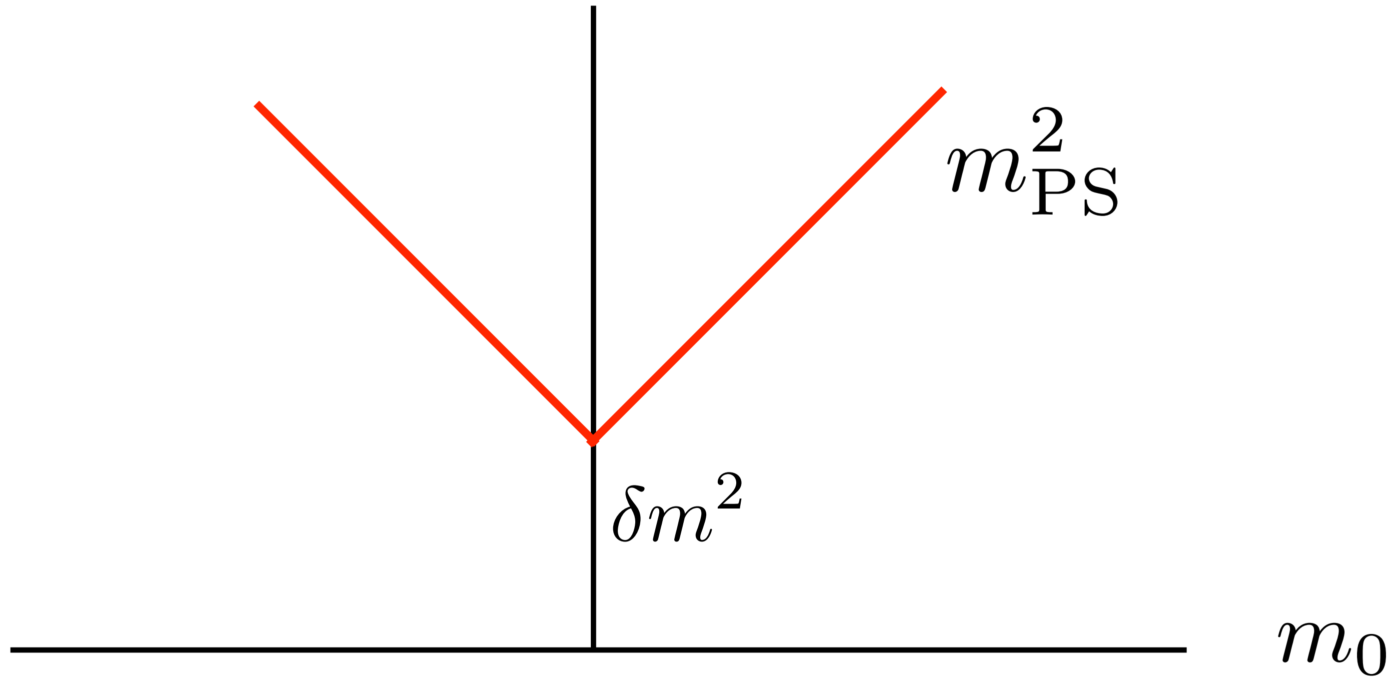
$\Rightarrow$ 2-index anti-symmetric ?

in the large N limit

For simplicity, we use $\frac{\Delta}{2}(\det U + \det U^\dagger)$ but check results with $\frac{c}{N}(\log\det U)^2$

Warm-up: $N_f = 1$ case

naive guess $$m_{\rm PS}^2 = \frac{2B}{f^2}|m_0| + \delta m^2$$ No massless "pion"(eta)



correct behavior $U = U_0 = e^{i\varphi_0}$ vacuum ansatz $m = 2Bm_0$

$$V(\varphi_0) = -(m+\Delta)\cos\varphi_0$$

potential

$$\Rightarrow \quad \varphi_0 = \begin{cases} 0 & m+\Delta > 0 \\ \pi & m+\Delta < 0 \end{cases}$$

minimum

$U(x) = U_0 e^{i\pi(x)/f}$

PS meson field

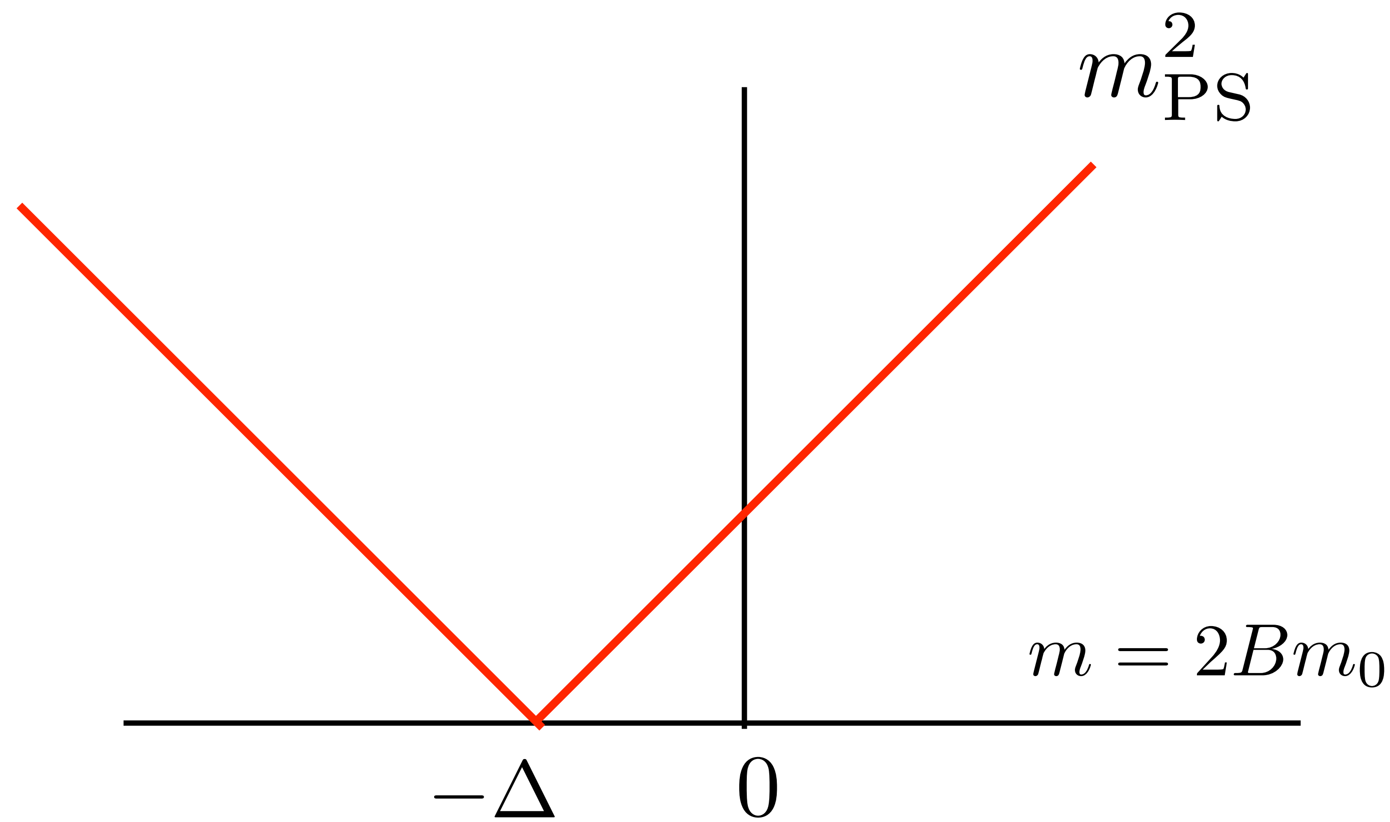
$$\Rightarrow \quad \begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu \pi(x)\partial^\mu \pi(x) - (m+\Delta)U_0 \cos(\pi(x)/f) \\ &= \frac{1}{2}\left[(\partial_\mu \pi(x))^2 + \frac{|m+\Delta|}{f^2}\pi(x)^2\right] + O(\pi^4) \end{aligned}$$

$$\Rightarrow \quad m_{\mathrm{PS}}^2 = \frac{|m+\Delta|}{f^2}$$

$m = 0$ is note special

non-symmetric under $m \to -m$

massless PS meson at $m = -\Delta$



2. Phase structure and pion masses at N_f=2

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \equiv 2B \begin{pmatrix} m_{0u} & 0 \\ 0 & m_{0d} \end{pmatrix}$$

mass term

$$U = U_0 = e^{i\varphi_0} \begin{pmatrix} e^{i\varphi_3} & 0 \\ 0 & e^{-i\varphi_3} \end{pmatrix}$$

vacuum

VEV

$$\begin{aligned} \langle \bar{\psi}\psi \rangle &\equiv \frac{1}{2}\mathrm{tr}\,(U_0 + U_0^\dagger) = 2\cos(\varphi_0)\cos(\varphi_3), \\ \langle \bar{\psi}\tau^3\psi \rangle &\equiv \frac{1}{2}\mathrm{tr}\,\tau^3(U_0 + U_0^\dagger) = -2\sin(\varphi_0)\sin(\varphi_3), \end{aligned}$$

$$\begin{aligned} \langle \bar{\psi} i\gamma_5 \psi \rangle &\equiv \frac{1}{2i}\mathrm{tr}\,(U_0 - U_0^\dagger) = 2\sin(\varphi_0)\cos(\varphi_3), \\ \langle \bar{\psi} i\gamma_5 \tau^3 \psi \rangle &\equiv \frac{1}{2i}\mathrm{tr}\,\tau^3(U_0 - U_0^\dagger) = 2\cos(\varphi_0)\sin(\varphi_3). \end{aligned}$$

potential

$$V(\varphi_0, \varphi_3) = -m_u \cos(\varphi_0 + \varphi_3) - m_d \cos(\varphi_0 - \varphi_3) - \Delta \cos(2\varphi_0),$$

$$\begin{aligned} \frac{\partial V}{\partial \varphi_0} &= m_u \sin(\varphi_0 + \varphi_3) + m_d \sin(\varphi_0 - \varphi_3) + 2\Delta \sin(2\varphi_0) = 0 \\ \frac{\partial V}{\partial \varphi_3} &= m_u \sin(\varphi_0 + \varphi_3) - m_d \sin(\varphi_0 - \varphi_3) = 0. \end{aligned}$$

gap equations

$\sin\varphi_0 = \sin\varphi_3 = 0$ is a trivial solution

Non-trivial Solutions

$0 < m_d < \Delta$

$$\sin^2(\varphi_3) = \frac{(m_d - m_u)^2\{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2\}}{4m_u^3 m_d^3}$$
$$\sin^2(\varphi_0) = \frac{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2}{4m_u m_d \Delta^2},$$

$$m_c^- < m_u < m_c^+$$

Dashen phase

$\Delta < m_d$

$$\sin^2(\varphi_3) = \frac{(m_d - m_u)^2\{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2\}}{4m_u^3 m_d^3}$$
$$\sin^2(\varphi_0) = \frac{(m_u + m_d)^2\Delta^2 - m_u^2 m_d^2}{4m_u m_d \Delta^2},$$

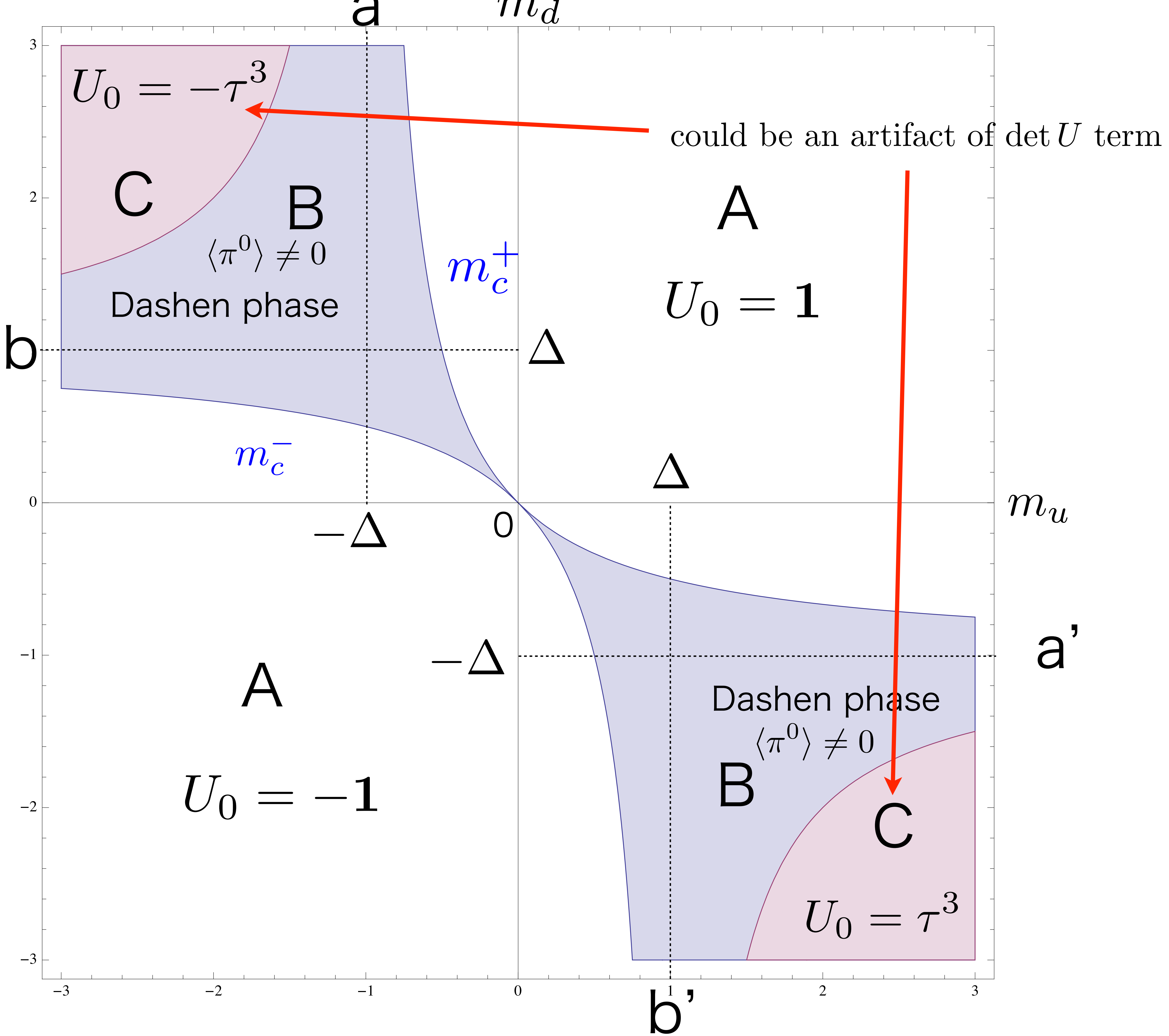
$$-m_c^- < m_u < m_c^+$$

Dashen phase

$$U_0 = \pm \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (\quad \sin^2(\varphi_3) = \sin^2(\varphi_0) = 1 \quad)$$

$$m_u < -m_c^-$$

$$m_c^{\pm} = -\frac{m_d \Delta}{\Delta \pm m_d} < 0,$$



m_d
m_u
a
b
a'
b'
$U_0 = -\tau^3$
could be an artifact of $\det U$ term
C
B
$\langle \pi^0 \rangle \neq 0$
Dashen phase
A
$U_0 = \mathbf{1}$
m_c^+
m_c^-
Δ
$-\Delta$
0
A
$U_0 = -\mathbf{1}$
Dashen phase
$\langle \pi^0 \rangle \neq 0$
B
C
$U_0 = \tau^3$
3
2
0
-1
-2
-3
-3
-2
-1
0
1
2
3

PS meson masses

$$U(x) = U_0 e^{i\Pi(x)/f}, \qquad \Pi(x) = \begin{pmatrix} \frac{\eta(x)+\pi_0(x)}{\sqrt{2}} & \pi_-(x) \\ \pi_+(x) & \frac{\eta(x)-\pi_0(x)}{\sqrt{2}} \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}^{(2)} &= \frac{1}{2}\left\{(\partial_\mu \pi_0(x))^2 + (\partial_\mu \eta(x))^2 + 2\partial_\mu \pi_+(x)\partial^\mu \pi_-(x)\right\} + \frac{\delta m}{2f^2}\eta^2(x) \\ &+ \frac{m_+(\vec{\varphi})}{4f^2}\left\{\eta^2(x) + \pi_0^2(x) + 2\pi_+(x)\pi_-(x)\right\} - \frac{m_-(\vec{\varphi})}{2f^2}\underline{\eta(x)\pi_0(x)}, (26) \end{aligned}$$

$\pi_0 - \eta$ mixing

$$m_{\pi_\pm}^2 = \frac{m_+(\vec{\varphi})}{2f^2}$$ charged pion

$$m_{\tilde{\pi}_0}^2 = \frac{1}{2f^2}\left[m_+(\vec{\varphi}) + \delta m - X\right]$$ neutral pion

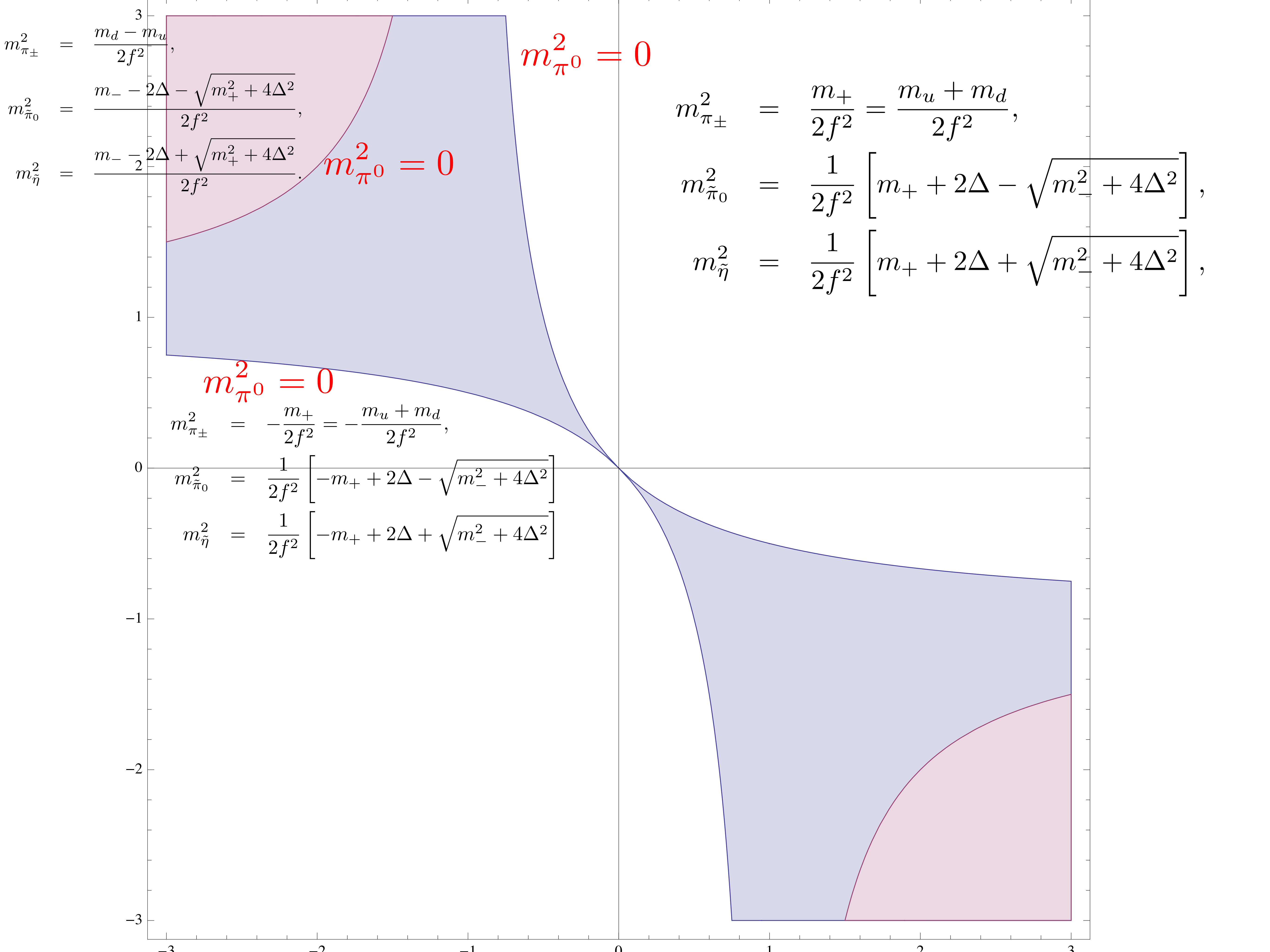
$$m_{\tilde{\eta}}^2 = \frac{1}{2f^2}\left[m_+(\vec{\varphi}) + \delta m + X\right]$$ eta meson

$$m_{\tilde{\pi}_0} < m_{\pi_\pm}$$

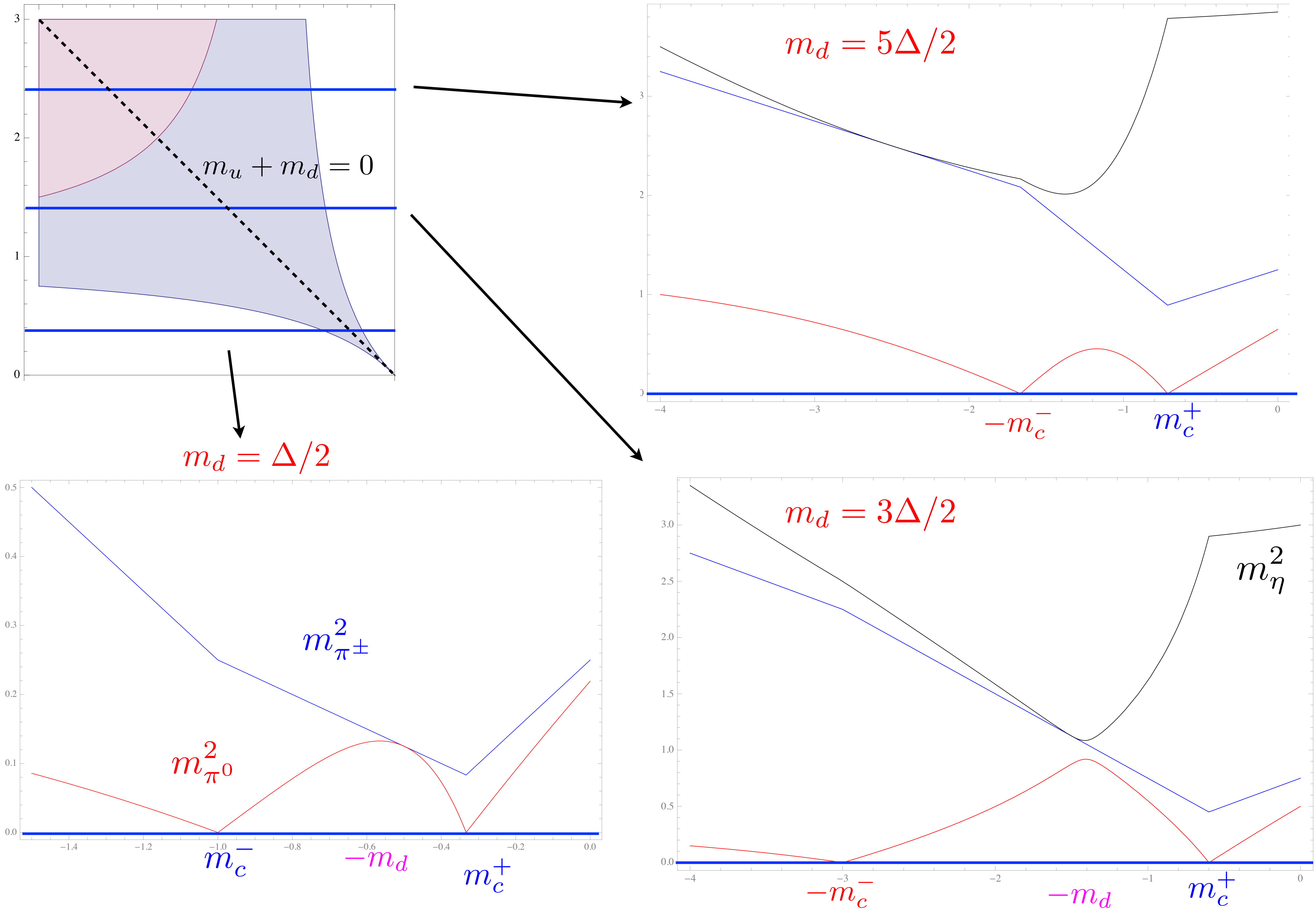
$$X = \sqrt{m_-(\vec{\varphi})^2 + \delta m^2},$$

$$\begin{aligned} m_\pm(\vec{\varphi}) &= m_d \cos(\varphi_0 - \varphi_3) \pm m_u \cos(\varphi_0 + \varphi_3) \\ &= m_\pm \cos(\varphi_0)\cos(\varphi_3) + m_\mp \sin(\varphi_0)\sin(\varphi_3), \\ \delta m &= 2\Delta \cos(2\varphi_0). \end{aligned}$$

$$m_\pm = m_d \pm m_u.$$



$m^2_{\pi^\pm} = \frac{m_d - m_u}{2f^2}$,
$m^2_{\tilde{\pi}_0} = \frac{m_- - 2\Delta - \sqrt{m_+^2 + 4\Delta^2}}{2f^2}$,
$m^2_{\tilde{\eta}} = \frac{m_- - 2\Delta + \sqrt{m_+^2 + 4\Delta^2}}{2f^2}$.
$m^2_{\pi^0} = 0$
$m^2_{\pi^\pm} = \frac{m_+}{2f^2} = \frac{m_u + m_d}{2f^2}$,
$m^2_{\tilde{\pi}_0} = \frac{1}{2f^2}\left[m_+ + 2\Delta - \sqrt{m_-^2 + 4\Delta^2}\right]$,
$m^2_{\tilde{\eta}} = \frac{1}{2f^2}\left[m_+ + 2\Delta + \sqrt{m_-^2 + 4\Delta^2}\right]$,
$m^2_{\pi^0} = 0$
$m^2_{\pi^0} = 0$
$m^2_{\pi^\pm} = -\frac{m_+}{2f^2} = -\frac{m_u + m_d}{2f^2}$,
$m^2_{\tilde{\pi}_0} = \frac{1}{2f^2}\left[-m_+ + 2\Delta - \sqrt{m_-^2 + 4\Delta^2}\right]$
$m^2_{\tilde{\eta}} = \frac{1}{2f^2}\left[-m_+ + 2\Delta + \sqrt{m_-^2 + 4\Delta^2}\right]$
3
2
1
0
−1
−2
−3


$m_u + m_d = 0$
$m_d = 5\Delta/2$
$-m_c^-$
m_c^+
$m_d = \Delta/2$
$m^2_{\pi^\pm}$
$m^2_{\pi^0}$
m_c^-
$-m_d$
m_c^+
$m_d = 3\Delta/2$
m^2_η
$-m_c^-$
$-m_d$
m_c^+

3. Topological susceptibility and massless up quark

(anomalous) WT identities

$$\langle \left[\partial^\mu A^a_\mu(x) + \bar\psi(x)\{M, T^a\}\gamma_5\psi(x) - 2N_f\delta^{a0}q(x)\right]\mathcal{O}(y)\rangle \quad = \quad \delta^{(4)}(x-y)\langle\delta^a\mathcal{O}(y)\rangle$$

$$q(x) \quad = \quad \frac{g^2}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}G_{\mu\nu}(x)G_{\alpha\beta}(x),$$ topological charge density

$\mathcal{O}(y) = q(y)$ with $a = 0, 3$

Integrating over x.

$$\chi \quad \equiv \int d^4x\, \langle q(x)q(y)\rangle = \frac{2m_u}{N_f}\int d^4x\ \langle \bar u\gamma_5 u(x)q(y)\rangle\,.$$

?

$$\chi = \infty \text{ at } m_{\pi^0} = 0 \text{ and } m_u \neq 0$$

$$\chi = 0 \text{ at } m_{\pi^0} \neq 0 \text{ and } m_u = 0$$

Anomalous WT identities in N_f=2 ChPT

WT identities

$$\langle \delta_x S \mathcal{O}(y)\rangle = \delta^{(4)}(x-y)\,\langle \delta\mathcal{O}(y)\rangle$$

$$\delta_x S = i\theta(x)\left[\partial^\mu A_\mu(x) + \mathrm{tr}\,\{MU^\dagger(x) - M^\dagger U(x)\} - \underbrace{\Delta\{\det U(x) - \det U^\dagger(x)\}}_{2N_f q(x):\ \text{topological charge density}}\right],$$

$$A_\mu(x) = f^2\mathrm{tr}\,\{U^\dagger(x)\partial_\mu U(x) - U\partial_\mu U^\dagger(x)\}$$

$$2N_f\chi = \underbrace{\frac{\Delta^2}{4}\int d^4x\,\langle\{\det U(x) - \det U^\dagger(x)\}\{\det U(y) - \det U^\dagger(y)\}\rangle}_{-\frac{2\Delta^2}{f^2}\int d^4x\,\langle\eta(x)\eta(y)\rangle} + \frac{\Delta}{2}\underbrace{\langle\det U(y) + \det U^\dagger(y)\rangle}_{\text{effect of contact term}\ =\Delta},$$

$$2N_f\chi = -\frac{4\Delta^2 m_+(\vec\varphi)}{m_+^2(\vec\varphi) - m_-^2(\vec\varphi) + 2m_+(\vec\varphi)\delta m} + \Delta.$$

$m_u = 0 \Rightarrow m_+(\vec{\varphi}) = m_-(\vec{\varphi}) = m_d$ and $\delta m = 2\Delta$

$$\Rightarrow \boxed{2N_f\chi \;=\; -\frac{4\Delta^2 m_d}{4m_d\Delta} + \Delta = 0,}$$

$$m^2_{\pi^0} = 0 \Rightarrow \int d^4x\,\langle \eta(x)\eta(y)\rangle \;=\; \frac{1}{2X}\left(\frac{X_-}{m^2_{\tilde{\pi}_0}} + \frac{X_+}{m^2_{\tilde{\eta}}}\right) \longrightarrow \infty$$

$$\Rightarrow \boxed{2N_f\chi \;\to\; -\infty, \quad m_{\tilde{\pi}_0} \to 0,}$$

4. An interesting application

$$m_u = -m_d = -m \qquad \theta = 0$$

chiral rotations

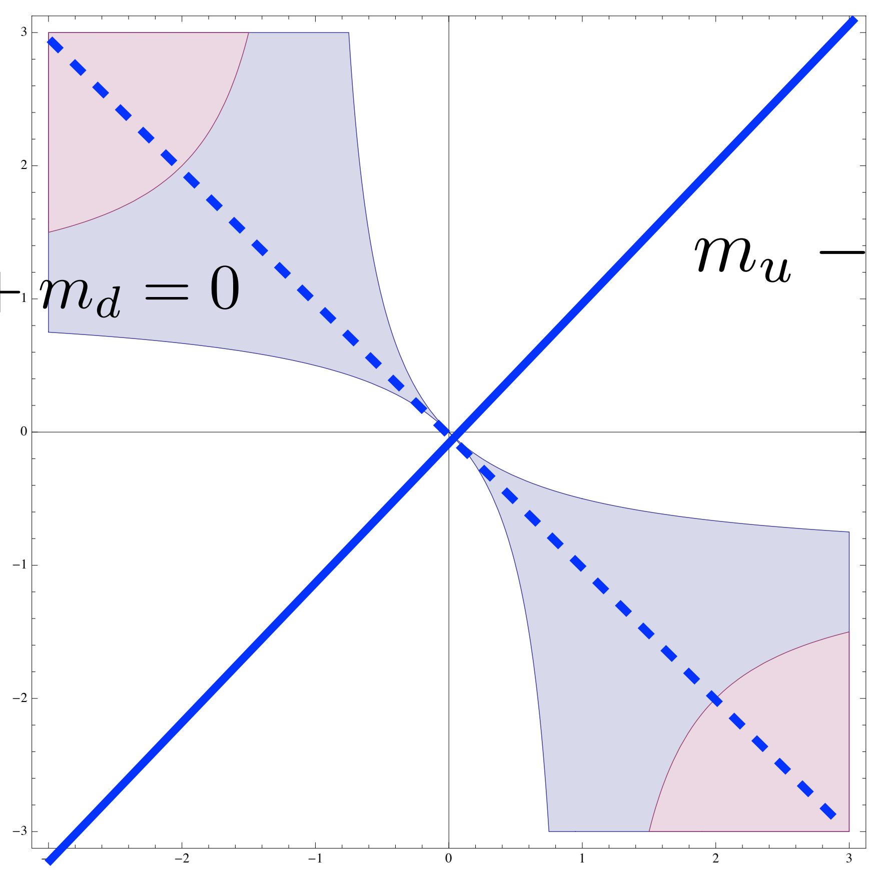
$$m_u = m_d = m \qquad \theta = \pi$$

$$M = \begin{pmatrix} -m & 0 \\ 0 & m \end{pmatrix}$$

$$V_R = e^{i(\theta_0 + \theta_3 \tau^3)} = V_L^\dagger, \qquad M' = V_L^\dagger M V_R = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$\theta_0 = \theta_3 = \pi/4,$$

$$\Delta \qquad\qquad -\Delta$$



$$m_u + m_d = 0$$

$$m_u - m_d = 0$$

ChPT analysis

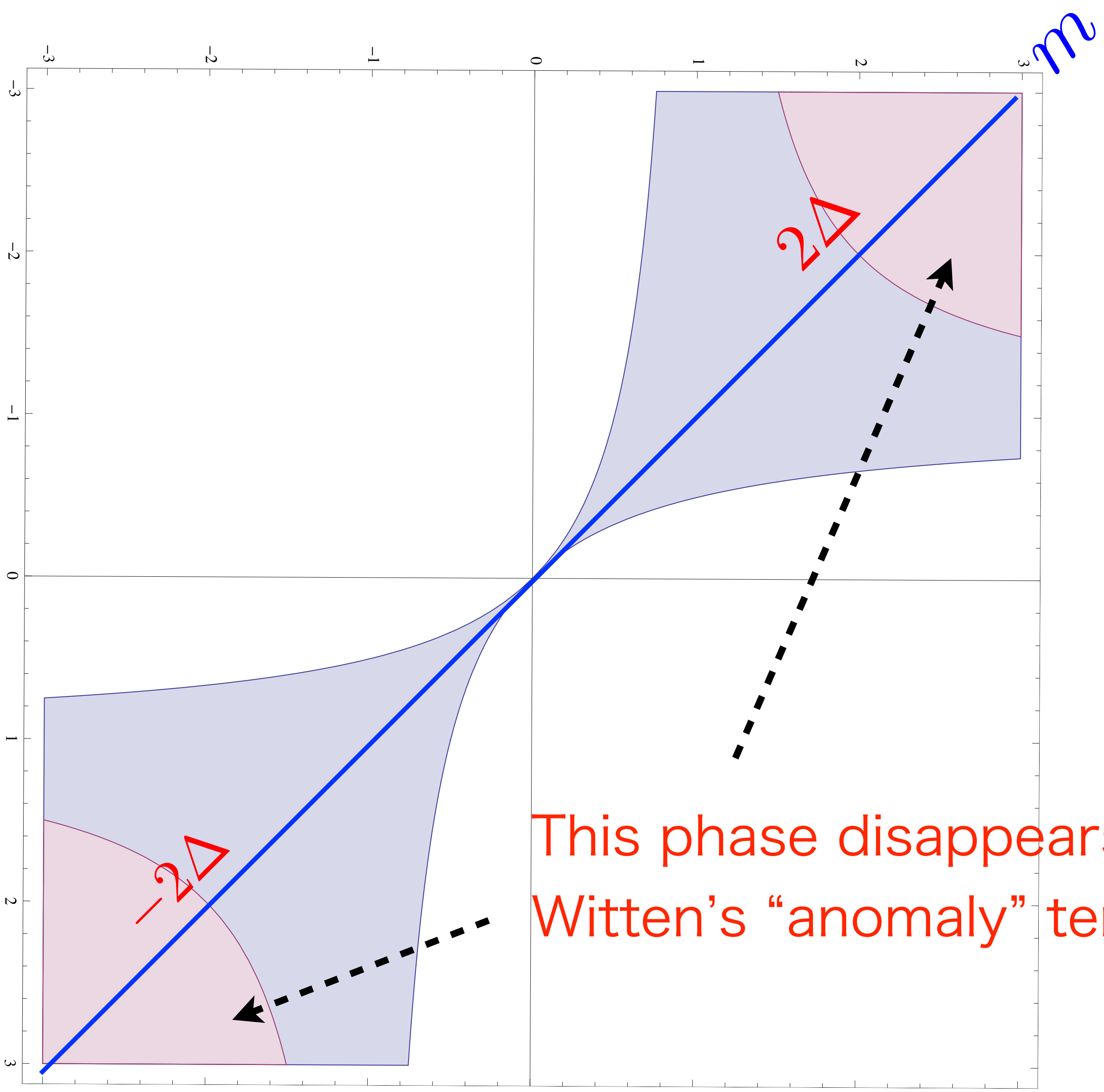
VEV

$$\langle\bar{\psi}\psi\rangle^2 + \langle\bar{\psi}i\gamma_5\psi\rangle^2 = 4$$

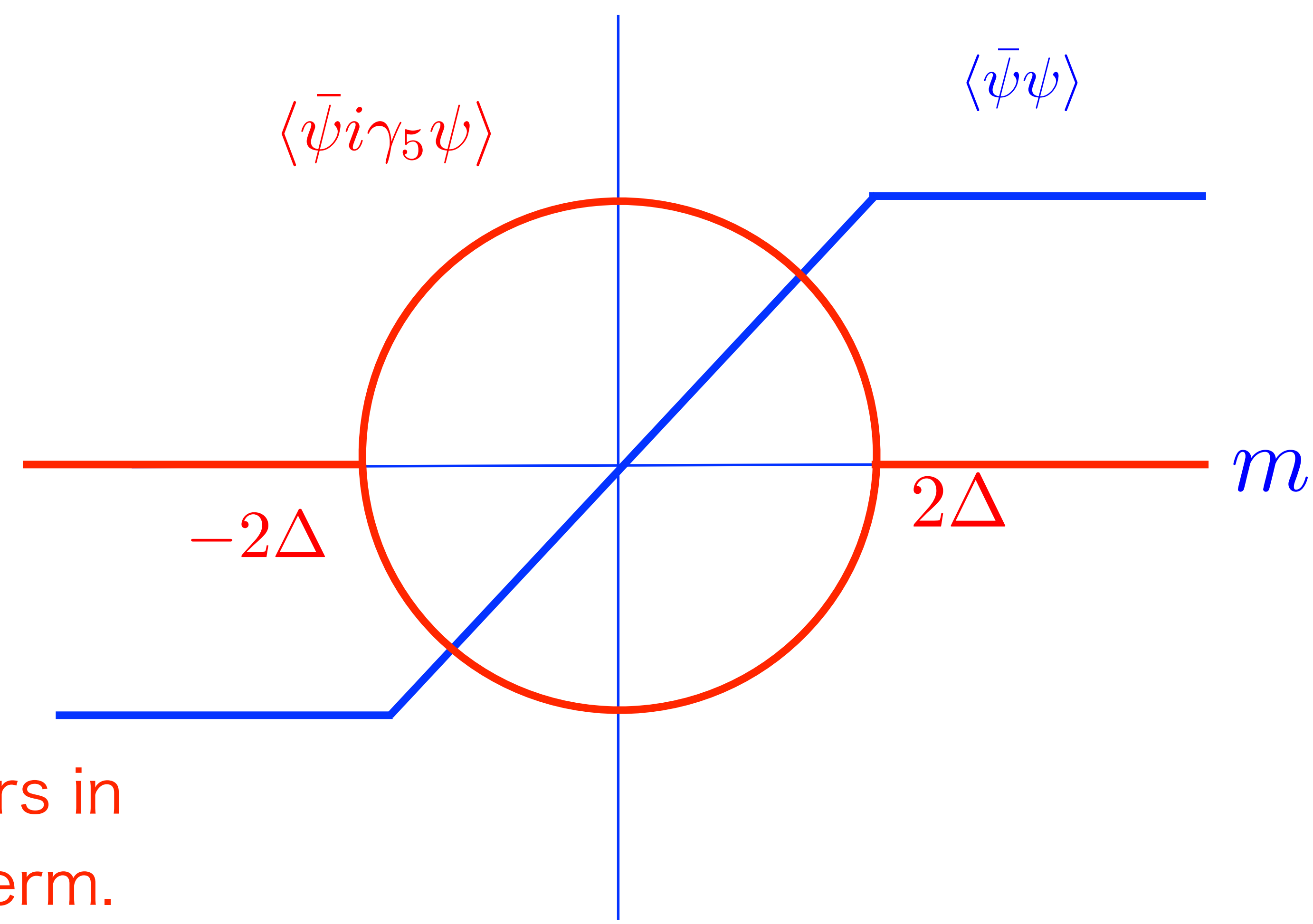
$$\begin{aligned} \cos\varphi_3 &= 1 \\ \cos\varphi_0 &= \begin{cases} 1, & 2\Delta \le m \\ \dfrac{m}{2\Delta}, & -2\Delta < m < 2\Delta \\ -1, & m \le -2\Delta \end{cases} \end{aligned}$$

$$\langle\bar{\psi}i\gamma_5\psi\rangle = 2\sin\varphi_0\cos\varphi_3 = \begin{cases} 0, & m^2 \ge 4\Delta^2 \\ \pm 2\sqrt{1-\dfrac{m^2}{4\Delta^2}}, & m^2 < 4\Delta^2 \end{cases}$$

$$\langle\bar{\psi}\psi\rangle = 2\cos\varphi_0\cos\varphi_3 = \begin{cases} 2, & 2\Delta \le m \\ \dfrac{m}{\Delta}, & -2\Delta < m < 2\Delta \\ -2, & m \le -2\Delta \end{cases},$$



This phase disappears in Witten's "anomaly" term.



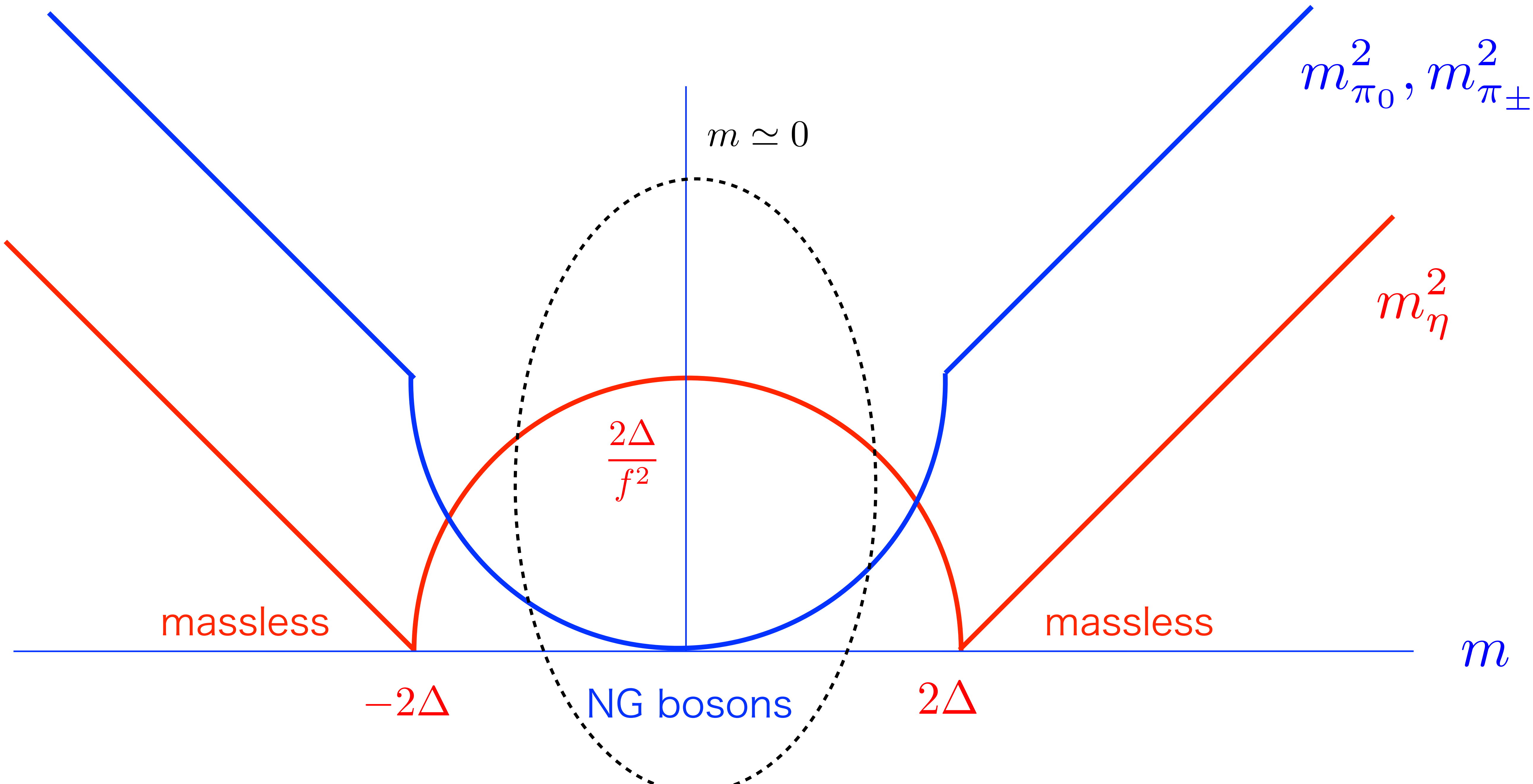
Spontaneous CP violation !

(eta condensation)

PS meson masses

$$m_{\pi_\pm}^2 = m_{\pi_0}^2 = \begin{cases} \frac{1}{2f^2} 2|m|, & m^2 \geq 4\Delta^2 \\ \frac{1}{2f^2}\frac{m^2}{\Delta}, & m^2 < 4\Delta^2 \end{cases} \qquad m_\eta^2 = \begin{cases} \frac{1}{2f^2}\left[2|m| - 4\Delta\right], & m^2 \geq 4\Delta^2 \\ \frac{1}{2f^2}\frac{4\Delta^2 - m^2}{\Delta}, & m^2 < 4\Delta^2 \end{cases},$$

non-standard PCAC relation !



$$\boxed{m_\pi^2 = \frac{1}{2f^2}\frac{m^2}{\Delta}}$$

How can we get this from WT-identities ?

$$\langle\{\partial^\mu A^3_\mu + m\,\mathrm{tr}\tau^3(U^\dagger - U)\}(x)\mathcal{O}(y)\rangle = \langle\delta^x\mathcal{O}(y)\rangle$$

taking $\mathcal{O} = \mathrm{tr}\,\tau^3(U^\dagger - U)$ and integrating over x

$$\Rightarrow\quad m\int d^4x\,\langle \underbrace{\mathrm{tr}\,\tau^3(U^\dagger - U)(x)}_{= -i\frac{2\sqrt{2}}{f}\cos\varphi_0\,\pi_0(x)}\,\mathrm{tr}\,\tau^3(U^\dagger - U)(y)\rangle = -2\langle \underbrace{\mathrm{tr}(U + U^\dagger)(y)}_{= 4\cos\varphi_0}\rangle$$

$$\Rightarrow\quad m\frac{\cos^2\varphi_0}{f^2}\underbrace{\int d^4x\langle\pi_0(x)\pi_0(y)\rangle}_{= \frac{1}{m^2_{\pi_0}}} = \cos\varphi_0 \quad\Rightarrow\quad m^2_{\pi_0} = \frac{m}{f^2}\underbrace{\cos\varphi_0}_{= \frac{m}{2\Delta}}$$

$$\Rightarrow\quad \boxed{m^2_{\pi_0} = \frac{m}{f^2}\frac{m}{2\Delta}}$$

one m form WTI, the other m from VEV.

5. Conclusions

Using ChPT with anomaly effect, we show

1. $m_u = 0$ is nothing special if $m_d \neq 0$. (no symmetry)

2. At $m_u = m_c^{\pm}, -m_c^{-} \neq 0$, $m_{\pi^0} = 0$.

3. $\langle \pi^0 \rangle \neq 0$ at $m_c^{-}(-m_c^{-}) < m_u < m_c^{+}$. Dashen phase

rooted Staggered quark can not reproduce this.

4. $\chi = \infty$ at $m_u = m_c$.

5. $\chi = 0$ at $m_u = 0$. $\Rightarrow$ a solution to strong CP problem

$\boxed{\text{Mike's Oracles are confirmed by ChPT.}}$

New predictions for 2-flavor QCD with $m_u = m_d$ and $\theta = \pi$

1. Spontaneous CP violation : $\langle \eta \rangle \neq 0$

2. Non-standard PCAC relation: $m_\pi^2 \propto m_q^2$