Density of States Method for the Effective Center Model of QCD

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Basic idea

- At finite density lattice QCD faces the complex phase problem $\Rightarrow$ alternative methods!
- Approach: We explore the **density of states method** [1]
- Model theory: The effective center model of QCD
- Crosscheck: Exact results from the **dual representation** [2]

Effective center model of QCD

The theory is described by the action:

$$S[P] = -\sum_x \left[\tau \sum_{\nu=1}^{3} (P_x^\star P_{x+\hat{\nu}} + c.c. - 2) + \kappa e^{\mu}(P_x - 1) + \kappa e^{-\mu}(P_x^\star - 1) \right] ,$$

$P_x \in \mathbb{Z}_3 = \{1, e^{i2\pi/3}, e^{-i2\pi/3}\}$, $S[P] = 0$ if $P_x = 1, \forall x$.

Rewriting the action

$N_0, N_\pm$ is the number of spins $= 1$, $e^{\pm i2\pi/3}$ and $\Delta N = N_+ - N_-$.

$$S[P] = -\tau \sum_x \sum_{\nu=1}^{3} [P_x^\star P_{x+\hat{\nu}} + c.c. - 2] - 3\kappa \cosh(\mu)(N_0[P] - V) - i\sqrt{3}\kappa \sinh(\mu)\Delta N[P]$$

Partition sum: $Z = \sum_{\{P\}} e^{-S_R[P]} \cos\left(\sqrt{3}\kappa \sinh(\mu)\Delta N[P]\right)$

Density of states

Definition of a weighted density of states:

$$\rho(d) = \sum_{\{P\}} e^{-S_R[P]} \delta(d - \Delta N[P])$$

Rewriting the partition sum in terms of $\rho(d)$:

$$Z = \sum_{d=-V}^{V} \rho(d) \cos(\sqrt{3}\kappa \sinh(\mu) d)$$

Calculating the density of states

Spline ansatz in the interval $d \in [d_0, d_0 + \delta d), \delta d \in \mathbb{N}$,

$$\rho_{d_0}(d) = \exp\left[\Delta\zeta(d_0)(d - d_0) + \zeta(d_0 - \delta d)\delta d\right] ,$$

with $\Delta\zeta(d_0) = \zeta(d_0) - \zeta(d_0 - \delta d)$.

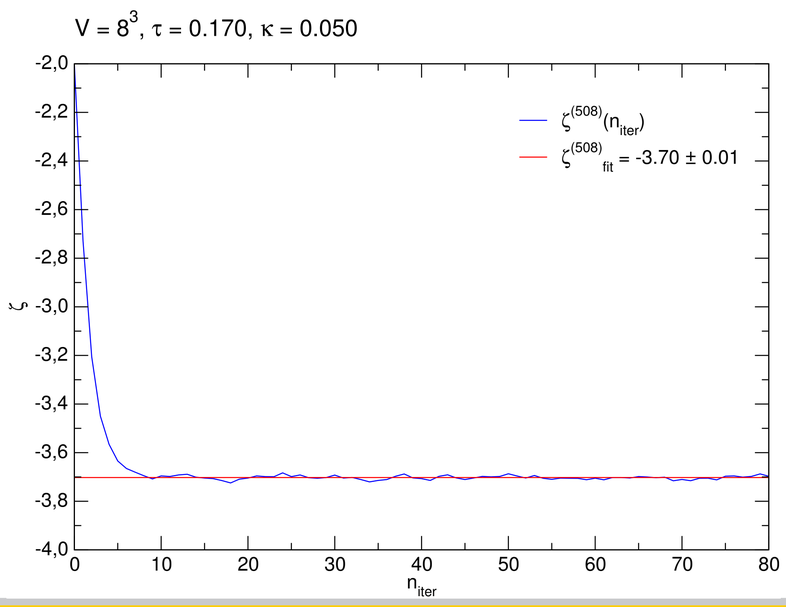
Computing the coefficients using restricted Monte Carlo updates [1]:

$$\langle\langle \Delta d \rangle\rangle(\zeta) = \frac{1}{N} \sum_{d=d_0}^{d_0+\delta d} \rho_{d_0}(d) e^{-\zeta d} \Delta d ,$$

with $\Delta d = d - d_0 - \delta d/2$. Using an iterative approach:

$$\Delta\zeta^{(n+1)}(d_0) = \Delta\zeta^{(n)}(d_0) + \frac{6}{\delta d(1 + \delta d/2)} \langle\langle \Delta d \rangle\rangle \left(\Delta\zeta^{(n)}(d_0)\right)$$

Convergence of the iteration:



Reconstruction of $\rho(d)$

By using the symmetry

$$\rho(d) = \rho(-d) ,$$

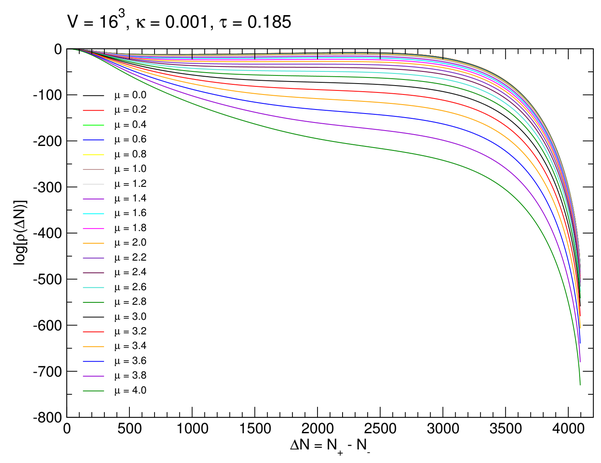
and demanding

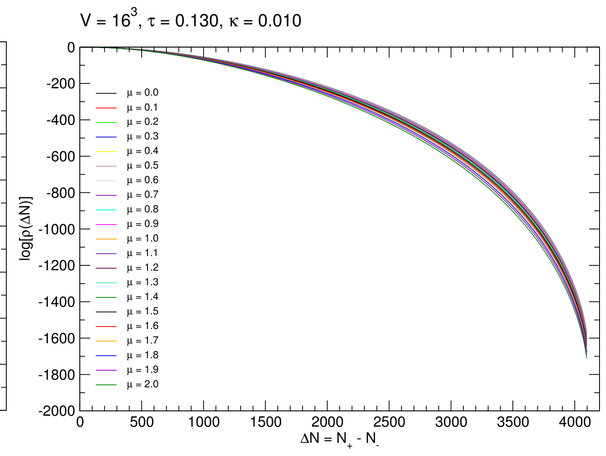
$$\zeta(-\delta d) = 0 \Rightarrow \rho(0) = 1 ,$$

we get the complete density of states:

$$\log[\rho(d)] = \sum_{n=0}^{N-1} \left[(d - n\delta d)\Delta\zeta(n\delta d) + \delta d \sum_{m=0}^{n-1} \Delta\zeta(m\delta d) \right] \times \begin{cases} 1, & d \in [n, n+1)\delta d \\ 0, & \text{otherwise} \end{cases}$$

Examples for $\log(\rho)$:





Observables

The expectation value of the observable $\mathcal{O}$ is given by

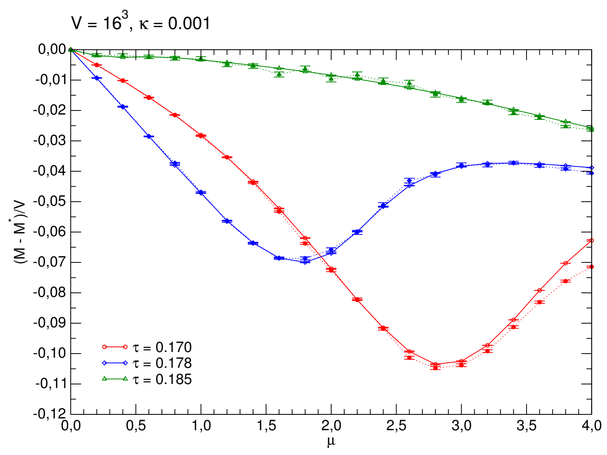
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \sum_{d=-V}^{V} \rho(d) \cos\left(\sqrt{3}\kappa \sinh(\mu) d\right) \mathcal{O}(d) .$$

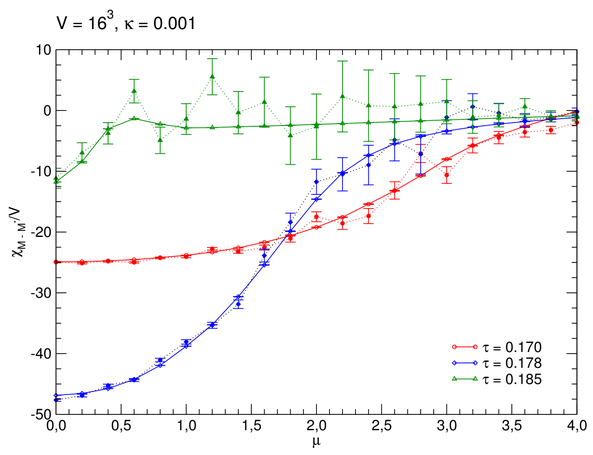
We choose the observable

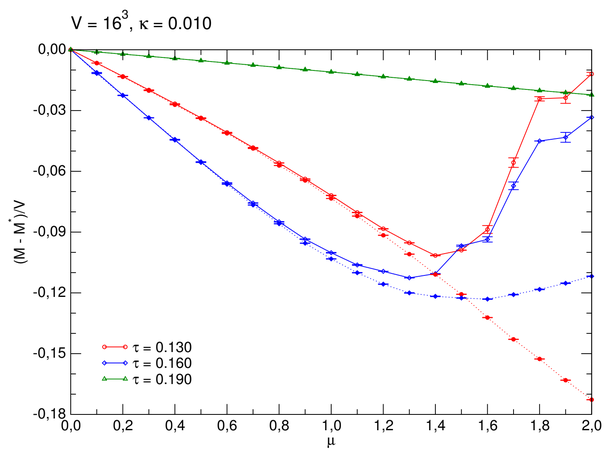
$$\frac{1}{V}(M - M^\star) = \frac{1}{V} \left\langle \sum_x (P_x - P_x^\star) \right\rangle = \frac{1}{V} \frac{\partial \log Z}{\partial(\kappa \sinh(\mu))} ,$$

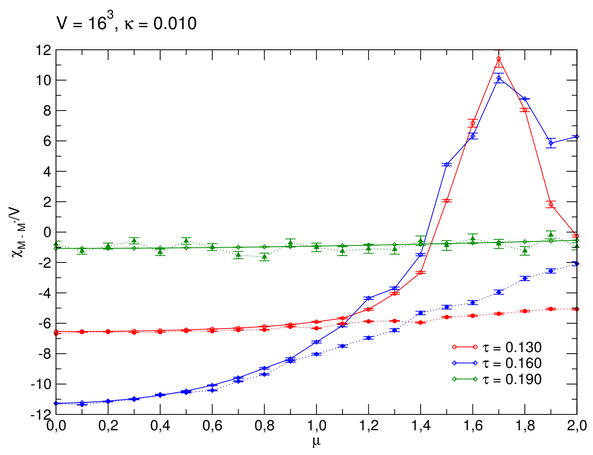
and the corresponding susceptibility $\chi_{M-M^\star}$.

Comparison to the reference data from dual simulations (dotted lines):









References

[1] K. Langfeld, B. Lucini and A. Rago, Phys. Rev. Lett. **109** (2012) 111601
K. Langfeld and B. Lucini, arXiv:1404.7187 [hep-lat].
[2] Y. D. Mercado, H. G. Evertz and C. Gattringer, Phys. Rev. Lett. **106** (2011) 222001