Glue Helicity ΔG in The Nucleon

Raza Sabbir Sufian

in collaboration with -

Michael Glatzmaier, Yi-Bo Yang, Keh-Fei Liu, Mingyang Sun

χ_{QCD} Collaboration

University of Kentucky

OVERVIEW

- Motivation for Calculating Glue Helicity in Nucleon
- Gauge invariant ΔG operator
- Lattice Setup for Calculation of Glue Helicity in Longitudinally Polarized Nucleon
- Numerical Results

Motivation for Calculating Glue Helicity in Nucleon

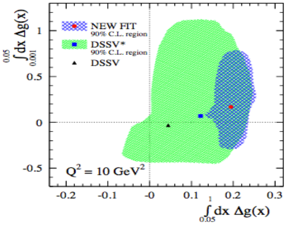
⊕ 'Nucleon Spin Puzzle' raised by EMC Collaboration in 1987, recent experiments by COMPASS [Alexakhin et al., PLB (2007)], HEREMES [Airapetian et al., PRD (2007)] found portion of nucleon spin coming from intrinsic quark spin $\approx \frac{1}{3}$

⊕ What carries rest $\frac{2}{3}$ of the proton spin?

⊕ Lattice calculation [Deka et al. χ_{QCD} Collaboration)] (in quench approximation with Wilson fermions) based on Ji's nucleon spin decomposition finds quark spin constitutes 25% , glue angular momentum 28% and quark orbital angular momentum contribute 47% of proton spin

Experimental Results for Nonzero Gluon Polarization

• Recent(2009 RHIC) experimental data show evidence of nonzero polarization of gluon in the proton [Florian et. al, arXiv:1404.4293]



• STAR Collaboration [arXiv: 1405.5134] indicates preference for positive gluon helicity contribution in the region $x > 0.05$

Gauge invariant ΔG operator

• Total gluon helicity, $\Delta G = \int_0^1 \Delta g(x) dx$

• $\Delta g(x)$ is polarized gluon parton helicity disctribution

Definition of ΔG operator from QCD factorization theorem [Manohar, PRL. (1991)]:

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}_{\alpha,b}^{\ +}(0) | PS \rangle$$

$$\mathcal{L}(\xi^-) = \mathcal{P} \exp[-ig \int_0^{\xi^-}] \mathcal{A}^+(\eta^-, 0_\perp) d\eta^-]$$

$$\mathcal{A}^+ \equiv T^c A_c^+,\ \xi^\pm = (\xi^t \pm \xi^z)\sqrt{2}\ ,\ \tilde{F}^{\alpha\beta} \sim \tfrac{1}{2}\epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$$

⊕ Gauge invariant but partonic interpretation only in LCG

⊕ Does not look like gluon helicity operator

•Carrying out integration of longitudinal momentum reduces to gauge invariant gluon spin operator [Ji, Zhang, Zhao, PRL(2013)]:

$$\hat{S}_g^{\rm inv}(0) = \left[\vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-, 0)\right)\right]^3 ,$$

- Similiar structure to $\vec{E} \times \vec{A}$
- How does it transform under gauge transformation?

•Chen, L$\ddot{u}$, Sun, Wang, Goldman [PRL. (2008), PRL. (2009)]: Decomposed **A** as:

$$\mathbf{A}(\mathbf{x}) = \mathbf{A}_{\mathbf{phys}}(\mathbf{x}) + \mathbf{A}_{\mathbf{pure}}(\mathbf{x})$$

and proposed complete decomposition of nucleon spin

- Motivated by EM, one would like to have $\vec{A}_\perp$ transform covariantly:

$$\vec{A}_\perp \to U(x)\vec{A}_\perp U^\dagger(x)$$

- $A^i_\perp$ satisfies a generalized Coulomb condition,

$$\partial^i A^i_\perp = ig[A^i, A^i_\perp]$$

In large momentum frame, $\vec{A}_{||}$ required to produce null magnetic field:

$$\partial^i A^{j,a}_{||} - \partial^j A^{i,a}_{||} - g f^{abc} A^{i,b}_{||} A^{j,c}_{||} = 0$$

- Solving for $A_{||}$:

$$A^{i,a}_{||}(\xi^-) = \frac{1}{\nabla^+}\Big((\partial^i A^{+,b})\mathcal{L}^{ba}(\xi'^-, \xi^-)\Big)$$

• Using the fact that, $A_\perp = A - A_\parallel$

In the IMF:

$$\mathbf{A}_\perp \quad \rightarrow \quad \left(\mathbf{A^a}(0) - \frac{1}{\nabla^+}(\nabla A^{+,b})\mathcal{L}^{ba}(\xi^-, 0) \right)$$

$$(\Delta G)_z \rightarrow (\vec{E}^a \times \vec{A}^a_\perp)_z$$

•The previous results rely on solving, order-by-order in the coupling, for the perp and parallel components of the gauge-field.

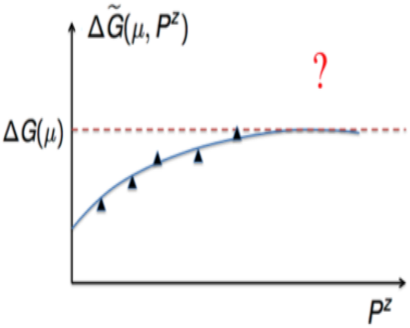
• At zeroth order of coupling:

$$\partial^i A^i_\perp = ig\left[A^i, A^i_\perp\right] \longrightarrow \partial^i A^i_\perp = 0$$

• Also according to X. Chen:

$$\vec{\nabla}.\vec{A}_{phys} = 0$$

- Up-to 1-loop order, Coulomb gauge is a good choice
- Dynamically depends on the momentum of the external particle



Lattice Calculation of Glue Helicity in Longitudinaaly Polarized Proton

- IR physics on lattice and in continuum similar but due to lattice cut off$\sim \frac{1}{a}$, lattice UV result different from continuum UV result
- Therefore LPT required for renormalization which will presented by Michael J. Glatzmaier in his talk (Parallel 9E)
- $\Delta G \to \vec{E} \times \vec{A}_{\perp}$ function of external momentum

Construction of $F_{\mu\nu}$ from D^{ov} operator

Liu, Alexandru, Horváth [PLB(2008)]:

$$\mathrm{tr}_s\, \sigma_{\mu\nu}\, D^{ov}_{0,0}\big(U(a)\big) = c^T a^2 F_{\mu\nu}(0) + \mathcal{O}(a^3)$$

- $c^T = c^T(\rho) = 0.11157$ independent of $A_\mu(x)$, $\kappa = 0.19$

•Non-untralocal beahvior of D^{ov} serves as efficient filter of UV fluctuations through chiral smearing

• QCD vacuum structure with topological charge density defined from D^{ov} has been observed to produce good signals with only a handful configurations

$$E_i = -F_{4i}$$

$A_\mu(x)$ in Coulomb gauge calculated from gauge links:

$$A^c_\mu(x) = \left[\frac{U^c_\mu(x) - U^{c\dagger}_\mu(x)}{2iag_0}\right]_{traceless} \tag{1}$$

$$(\vec{E} \times \vec{A})_i = tr\left(\epsilon_{ijk} E_j A_k\right)$$

Simulation Details

•Valence overlap fermion on $(2+1)$ flavor RBC/UKQCDC DWF 200 gauge configurations ($24^3 \times 64$ lattice)

•Sea quark mass $am_l = 0.005$, $am_s = 0.04$ (pion mass 331 MeV), $a^{-1} = 1.77 GeV$

•2-pt function constructed from grid-8 smeared source with Z_3 noise and with source time slices at t $=$ 0 and t $=$ 32

•2-pt function with low-mode substitution (Talk given by Keh-Fei Liu, Mingyang Sun)

• Sink momenta used $p = 0$, $p = 1$, $p = 2$

•Loop data $L_i(t_1) \equiv (\vec{E} \times \vec{A_c})_i(t_1)$, t_1 insertion time, i-configuration index

•2-pt function $C_i^2(t_2)$, t_2 sink time

•Disconnected 3-pt function

$$C_i^3(t_2, t_1) = \left(C_i^2(t_2))(L_i(t_1)) - < (C^2(t_2) >< L(t_1) > \right)$$

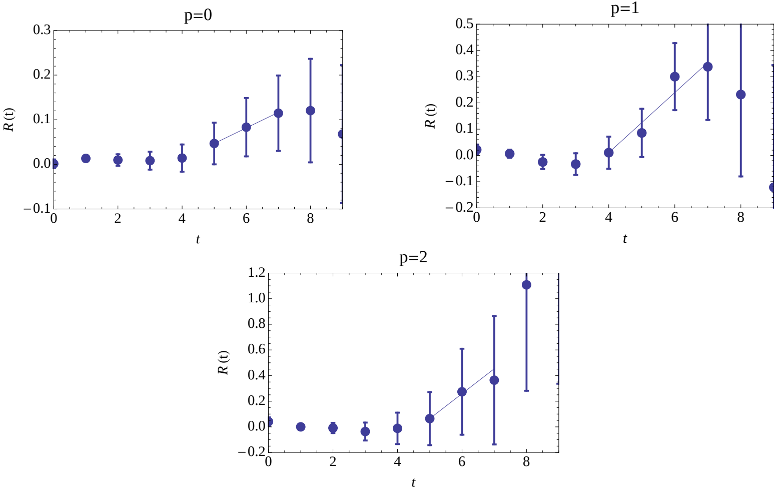
•Jackknife both C^2 and C^3 and use sum method [L. Maiani et al., Nucl. Phys. B293,420 (1987)]:

$$R_j(t_2, t_1) = \frac{\langle \tilde{C}_j^3(t_2, t_1) \rangle}{\langle \tilde{C}_j^2(t_2) \rangle}$$

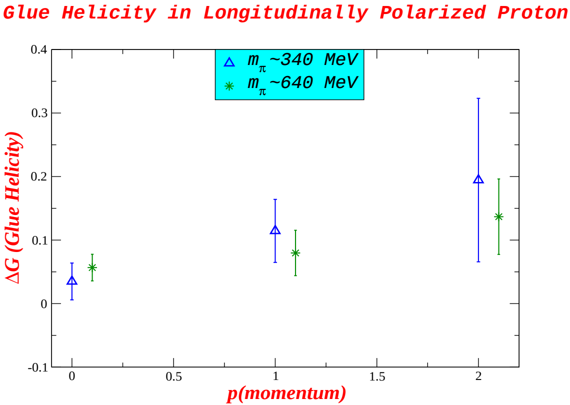
$$S_j(t_2) = \sum_{t_1} R_j(t_2, t_1)$$

Numerical Results

$m_q = 0.0203 \mid m_\pi = 380$ MeV



Fit Results



m_q in Lattice Units	$p = 0$	$p = 1$	$p = 2$
0.0203	0.03489162±0.0289	0.11445934±0.0496	0.19448887±0.1286
0.0576	0.056713±0.0209	0.079707±0.0357	0.136807±0.0594

Future Developments

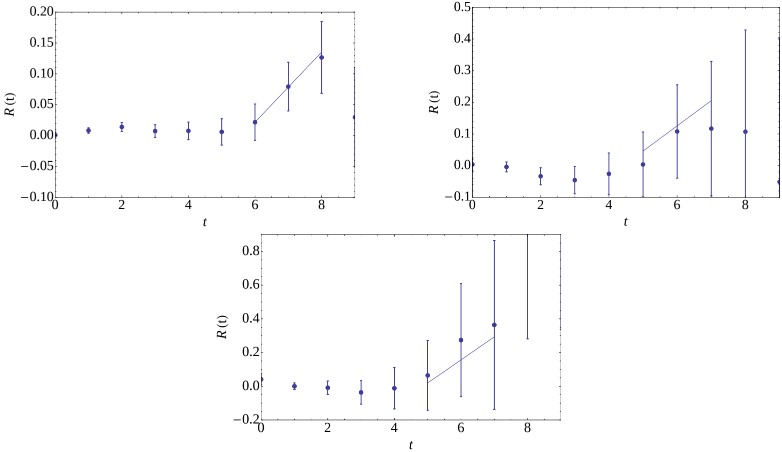
- Shifting source in time construct 2-pt function to increase statistics.... expected to reduce errorbar by $\frac{1}{2}$
- Obtain correct renormalization factor
- Use $32^3 \times 64$ RBC/UKQCD Lattice
- Calculation with other gauge fixing choices
- Plan to construct A_{phys} on lattice

Thank You!

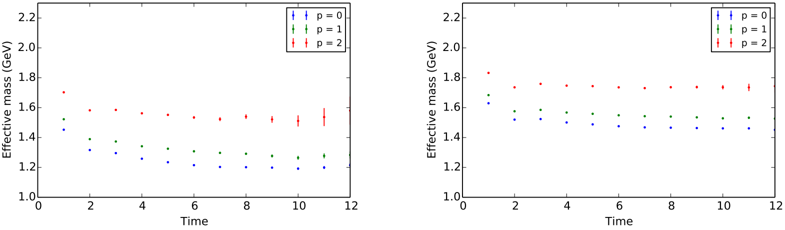
Backup

Numerical Results

$m_q = 0.0576 \mid m_\pi = 640$ MeV



Effective Mass Plots



$$\Delta\tilde{G}(P^z,\mu) = Z_{gg}(P^z/\mu)\Delta G(\mu) + Z_{gq}(P^z/\mu)\Delta\Sigma(\mu)\ , \qquad (1)$$

where $\Delta\Sigma(\mu)$ is the quark spin, and μ is the renormalization scale. Z_{gg} and Z_{gq} are the matching coefficients calculable in QCD perturbation theory. The operator considered in Ref. [7] was $\vec{E}\times\vec{A}_\perp$, where $\vec{A}_\perp$ is the transverse part of the gauge field, or $\vec{E}\times\vec{A}$ in the Coulomb gauge.

Jaffe-Manohar Decomposition

Jaffe-Manohar sum rule for proton spin [Nucl. Phys. B (1990)]

$$
\begin{aligned}
J^z &= \int d^3\xi\, \psi^\dagger \frac{\Sigma^3}{2}\psi + \int d^3\xi\, \psi^\dagger \left(\vec{\xi}\times(-i\vec{\nabla})\right)^3 \psi \\
&\quad + \int d^3\xi \left(\vec{E}_a \times \vec{A}^a\right)^3 + \int d^3\xi\, E_a^i \left(\vec{\xi}\times\vec{\nabla}\right)^3 A^{i,a}
\end{aligned}
$$

where $E_a^i = F_a^{+i}$
Each term separately is not gauge-invariant, except for quark spin part

Light-cone coordinates used, i.e. $\xi^\pm = (\xi^0 \pm \xi^3)/\sqrt{2}$

Light-cone is not accessible to Lattice QCD calculation which is based on Euclidean path-integral formulation

Chen, L$\ddot{u}$, Sun, Wang, Goldman Decomposition

Chen, L$\ddot{u}$, Sun, Wang, Goldman [PRL. 100, 232002 (2008), PRL. 103, 062001 (2009)]: Decomposed **A** as:

$$\mathbf{A}(\mathbf{x}) = \mathbf{A}_{\mathbf{phys}}(\mathbf{x}) + \mathbf{A}_{\mathbf{pure}}(\mathbf{x})$$

$$\boldsymbol{J}_{QCD} = \boldsymbol{S}'_q + \boldsymbol{L}'_q + \boldsymbol{S}'_G + \boldsymbol{L}'_G,$$

$$\begin{aligned}
\boldsymbol{S}'_q &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma} \psi \, d^3x, \\
\boldsymbol{L}'_q &= \int \psi^\dagger \boldsymbol{x} \times \left(\frac{1}{i} \nabla - g \boldsymbol{A}_{pure} \right) \psi \, d^3x, \\
\boldsymbol{S}'_G &= \int \boldsymbol{E}^a \times \boldsymbol{A}^a_{phys} \, d^3x, \\
\boldsymbol{L}'_G &= \int E^{aj} \left(\boldsymbol{x} \times \boldsymbol{D}_{pure} \right) A^{aj}_{phys} \, d^3x,
\end{aligned}$$

Wakamatsu Decompostion [Phys. Rev. D 81, 114010 (2010)]

→ Quark parts same as Ji decompostion

→Quark and gluon intrinsic spin parts same Chen decomposition

→Both Chen and Wakamtsu deomposition gauge invariant but in non-covariant forms. Not convenient for connecting with high-energy DIS observables

→Non-covariant treatment makes it hard to check out the Lorentz-frame dependence or independence of the nucleon spin sum rule derived on the basis of them

Wakamatsu then proposed another generalization with condition [Mechanical decomposition]:

$$F^{\mu\nu}_{pure} \equiv \partial^\mu A^\nu_{pure} - \partial^\nu A^\mu_{pure} - i\, g\, [A^\mu_{pure}, A^\nu_{pure}] = 0,$$

$$A^\mu_{phys}(x) \to U(x)\, A^\mu_{phys}(x)\, U^{-1}(x),$$
$$A^\mu_{pure}(x) \to U(x) \left(A^\mu_{pure}(x) - \frac{i}{g}\, \partial^\mu \right) U^{-1}(x).$$

→ Shown [M. Wakamatsu, Phys. Rev. D 83, 014012 (2011)] that the quark and gluon intrinsic spin parts coincide with the first moments of the polarized distribution functions appearing in the polarized DIS cross-sections.

$$\Delta q = \int \Delta q(x)\, dx, \quad \Delta g = \int \Delta g(x)\, dx.$$