Our conclusion from this analysis are:

Problems arising from this analysis are related to volume effects and non-universal corrections. Our latest analysis only considers data with non-universal corrections. The computer simulations are performed using the LatKMI modified version of the publicly available MILC (v7) code. Expensive measurements of disconnected diagrams are made possible thanks to GPU-accelerated code based on QUDA. The main results of this work were obtained on the cluster system “pi” at KMI, CX400 at Nagoya University and CX400, plus HA8000 at Kyushu University.

Introduction
We study a SU(3) gauge theory with 12 fundamental Dirac fermions on the lattice using the HISQ fermion discretization with Symanzik tree-level gauge action. Our goal is to determine, using spectral quantities, if the continuum theory described by our lattice model is inside the conformal window. In such case we calculate the anomalous mass dimension $\gamma\text{y}$ characteristic of the infrared conformal fixed point.

<table>
<thead>
<tr>
<th>Coupling $\beta$</th>
<th>Lattice Size $= L \times T$</th>
<th>Fermion Mass $= m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>19x24, 24x32, 30x40, 36x48</td>
<td>0.05 $\pm$ 0.02</td>
</tr>
<tr>
<td>4.0</td>
<td>19x24, 24x32, 30x40, 36x48</td>
<td>0.04 $\pm$ 0.02</td>
</tr>
</tbody>
</table>

Hyperscaling [7] dictates the dynamics for all theories in the conformal window and the the leading mass dependence of the spectrum is:

$$M_H \propto m^{\frac{1}{1+y}}$$

The computer simulations are performed using the LatKMI modified version of the publicly available MILC (v7) code. Expensive measurements of disconnected diagrams are made possible thanks to GPU-accelerated code based on QUDA. The main results of this work were obtained on the cluster system “pi” at KMI, CX400 at Nagoya University and CX400, plus HA8000 at Kyushu University.

Spectrum
We measure the lightest mesonic l=1 resonances that are present in the spectrum when a non-zero fermion mass term is included in the simulations. The pseudoscalar (t) mass and decay constant are measured, together with the vector (p) mass. We also measure the lightest l=0 scalar meson (D) for our most continuum-like coupling. This is in agreement with the possibility that (quasi-)conformal dynamics makes the scalar state lighter than the rest of the spectrum.

Finite-size scaling
At finite fermion mass and finite lattice size, the presence of a IR conformal fixed point can be studied using universal scaling relations (FSHS) [2] for the hadronic masses and decay constants as a function of the scaling variable:

$$x = L^{-\frac{1}{1+y}}$$

The function $f(x)$ is universal but unknown and it depends on the mass anomalous dimension at the fixed point, $\gamma\text{y}$. By finding the function that describes an observable for different values of $L$ and $m$, one can determine the mass anomalous dimension. This value should be independent of the observable and coupling constant (at leading order).

Conclusions and future work
Finite-size hyperscaling formulae describe the lattice data for the pseudoscalar mass and decay constant and for the vector mass, resulting in a universal value for the anomalous mass dimension. Both the flavor-singlet scalar mass and the string tension are compatible with these predictions.

Iso-singlet scalar channel
In a mass-deformed conformal theory, the lightest flavor-singlet scalar excitation in the spectrum may be parameterically light with respect to the other resonances. We measure the ground state in the iso-singlet scalar channel using a fermionic bilinear operator and a gluonic operator basis [5]. The signal for the disconnected diagrams crucial to this calculation is extracted using 64 stochastic sources, a variance reduction technique, and a large number of gauge configurations at $\beta=4.0$.

The lightest iso-singlet scalar state is found to be lighter than the pseudoscalar one in the whole fermion mass region explored.

We also measure another purely gluonic observable to extend our hyperscaling analysis to more observables which are affected by different systematic effects compared to the fermionic ones.

The string tension is obtained using two well-known methodologies:

- • from smeared Polyakov loop correlators coupling to toroid excitations on the torus, whose mass as a function of the loop length $L$ is:
  
  $$m_{\text{tor}}(L) = a^2 \pi L - \pi L^2/3 - \pi L^3/12$$

- • from APE smeared Wilson loop operators used to construct Creutz ratios for the static quark-antiquark potential:
  
  $$\langle F(r) \rangle = \langle F(r) \rangle_0 - \alpha_s(r) r + \sigma r$$

- • The two methods agree in the full range of parameters explored
- • The hyperscaling fit of the data gives $\gamma_y = 0.3(1)$, in broad agreement with the rest of the spectrum.

References
[2] Del Debbio, Zwicky, "Hyperscaling relations in mass-deformed conformal gauge theories", PRD 82, 014502