Conformality in twelve-flavor QCD

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Introduction

We study a SU(3) gauge theory with 12 fundamental Dirac fermions on the lattice using the HISQ fermion discretization with Symanzik tree-level gauge action. Our goal is to determine, using spectral quantities, if the continuum theory described by our lattice model is inside the conformal window. In such case we calculate the anomalous mass dimension γ_* characteristic of the infrared conformal fixed point.

Coupling ➡ β	Lattice Size ➡ L×T	Fermion Mass ➡ m
3.7	18x24, 24x32, 30x40, 36x48	0.03 ➩ 0.2
4.0	18x24, 24x32, 30x40, 36x48	0.04 ➩ 0.2

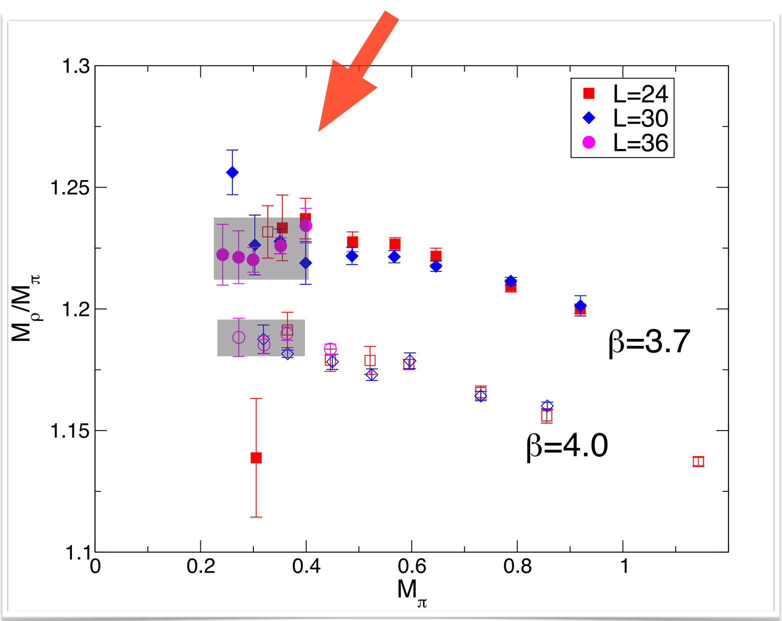
Hyperscaling *[1]* dictates the dynamics for all theories in the conformal window and the the leading mass dependence of the spectrum is:

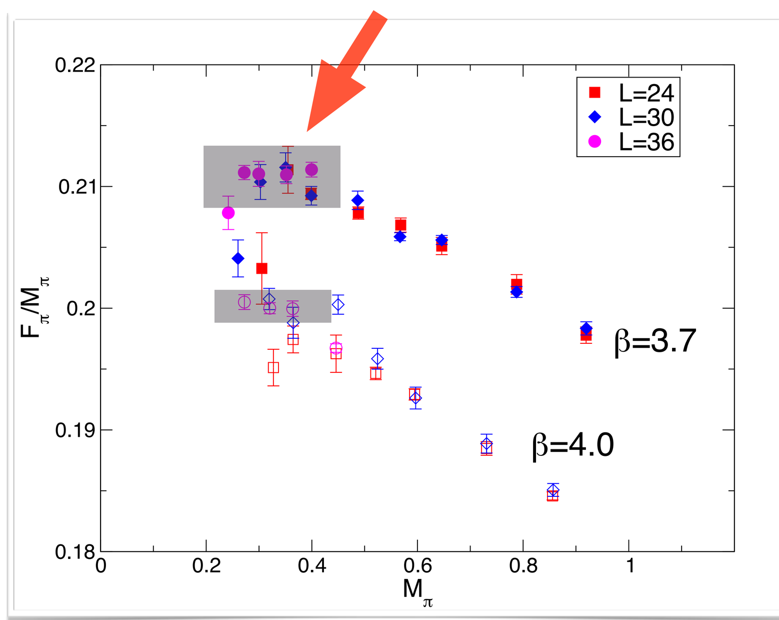
$$M_H \propto m_f^{1/(1+\gamma)}, \quad F_H \propto m_f^{1/(1+\gamma)}$$

The computer simulations are performed using the LatKMI modified version of the publicly available MILC (v7) code. Expensive measurements of disconnected diagrams are made possible thanks to GPU-accelerated code based on QUDA. The main results of this work were obtained on the cluster system "**φ**" at KMI, CX400 at Nagoya University and CX400, plus HA8000 at Kyushu University.

Spectrum

We measure the lightest mesonic I=1 resonances that are present in the spectrum when a non-zero fermion mass term is included in the simulations. The pseudoscalar (π) mass and decay constant are measured, together with the vector (ρ) mass. We also measure the lightest I=0 scalar meson (σ) for our most continuum-like coupling.
The first indication that this theory is conformal in the chiral and infinite volume limit comes from the ratio of hadronic scales: it approaches a constant in the chiral limit.





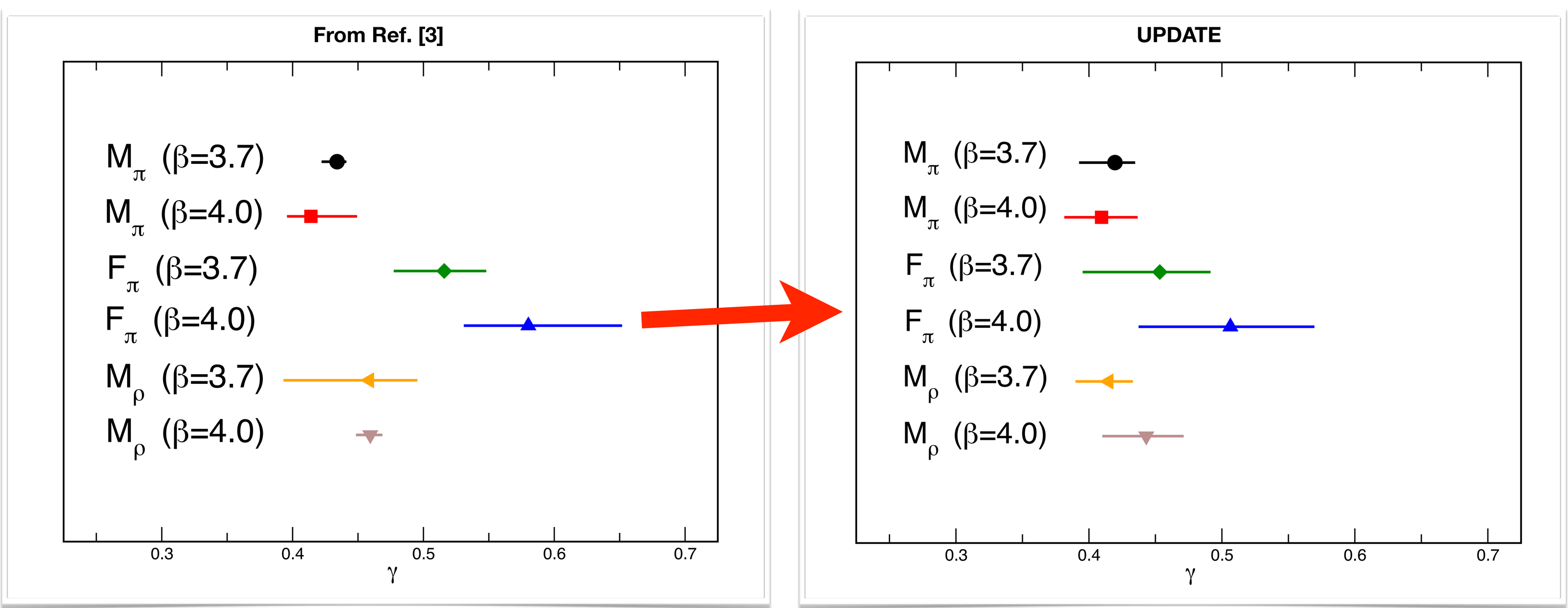
Finite-size scaling

At finite fermion mass and finite lattice size, the presence of a IR conformal fixed point can be studied using universal scaling relations (FSHS) *[2]* for the hadronic masses and decay constants as a function of the scaling variable:

$$x = L m^{1/(1+\gamma_\star)}$$

$$\xi = L \cdot M_H = f_M(x)\,, \quad \xi = L \cdot F_H = f_F(x)$$

The function f(x) is universal but unknown and it depends on the mass anomalous dimension at the fixed point, γ_*. By finding the function that describes an observable for different values of L and m, one can determine the mass anomalous dimension.
This value should be independent of the observable and coupling constant (at leading order).



Problems arising from this analysis are related to volume effects and non-universal corrections. Our latest analysis only considers data with LM_π>8.5 to reduce the finite volume effects. We also consider corrections to FSHS suggested by Schwinger-Dyson analysis, lattice artifacts and the gauge coupling (as a near-marginal operator) *[3][4]*:

$$\xi = c_0 + c_1 L m^{1/(1+\gamma)} \cdots \text{(no corrections)}$$

$$\xi = c_0 + c_1 L m^{1/(1+\gamma)} + c_2 L m^{\alpha} \qquad \begin{matrix}\alpha = (3-2\gamma)/(1+\gamma)\ \text{(Schwinger-Dyson)} \\ \alpha = 2\ \text{(lattice artifacts)}\end{matrix}$$

$$\xi = (c_0 + c_1 L m^{1/(1+\gamma)})(1 + c_2 m^{\omega})$$

Our conclusion from this analysis are:

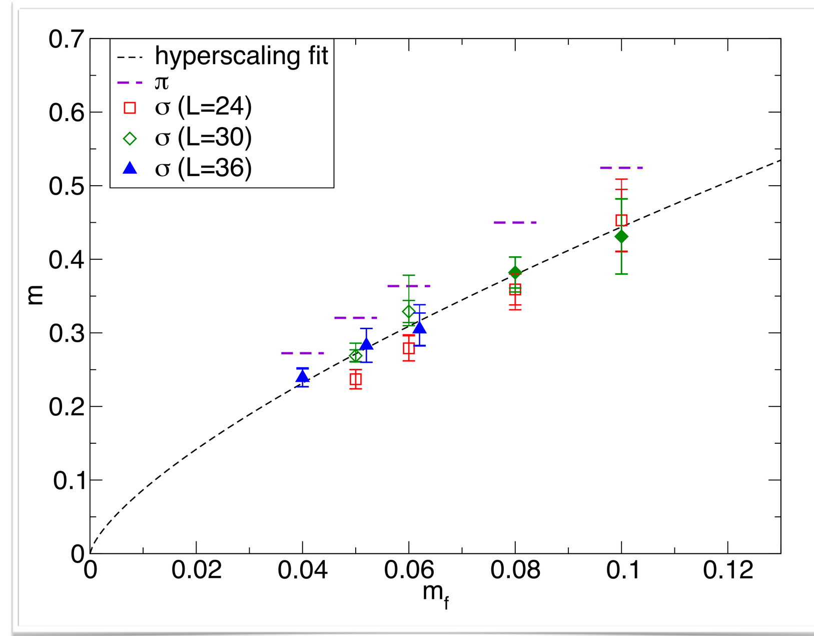
✤the hyperscaling relations are very well consistent with the lattice data in the small fermion mass region.

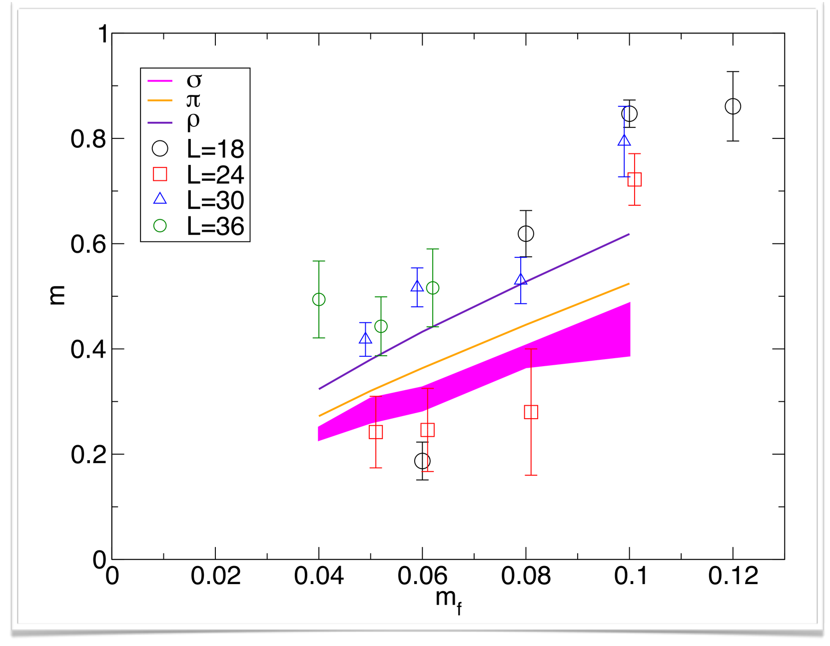
✤<u>the extracted γ_*=0.4-0.5 is consistent across different observables and coupling constants when small volume data are excluded (LM_π>8.5; LF_π>2.)</u>

✤adding corrections due to near-marginal operators near the fixed point (ω≠0) gives compatible results, but improves the fit quality in the heavy mass region.

Iso-singlet scalar channel

In a mass-deformed conformal theory, the lightest flavor-singlet scalar excitation in the spectrum may be parametrically light with respect to the other resonances.
We measure the ground state in the iso-singlet scalar channel using a fermionic bilinear operator and a gluonic operator basis *[5]*.
The signal for the disconnected diagrams crucial to this calculation is extracted using 64 stochastic sources, a variance reduction technique, and a large number of gauge configurations at β=4.0.





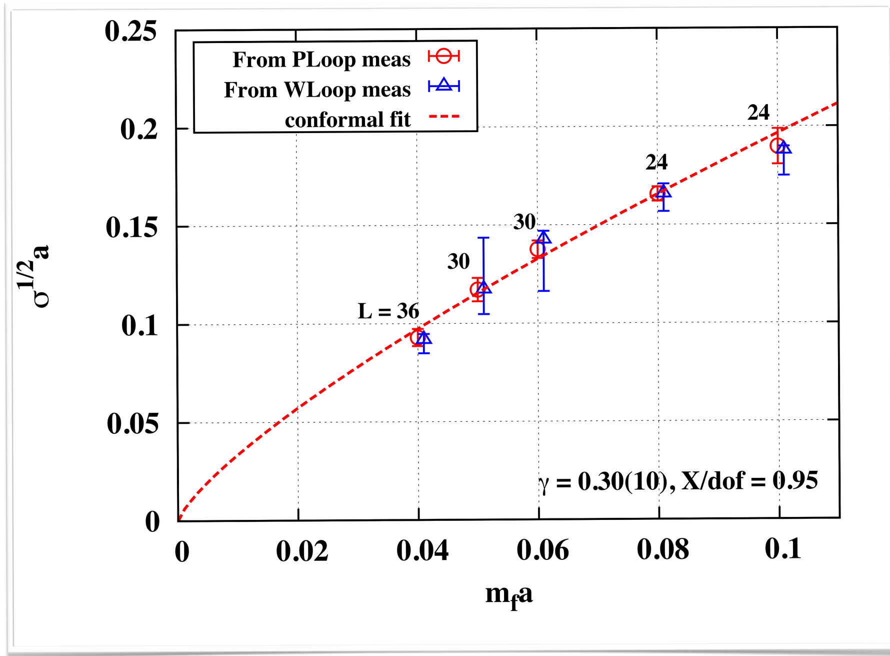
<u>The lightest iso-singlet scalar state is found to be lighter than the pseudoscalar one in the whole fermion mass region explored.</u>

✤This is in agreement with the possibility that (quasi-)conformal dynamics makes the scalar state lighter than the rest of the spectrum.

✤A hyperscaling formula with the anomalous mass dimension obtained from M_π fits to the large volume scalar data.

✤When purely gluonic operators are used to look at the flavor-singlet scalar channel, the extracted ground state mass tends to be higher than the one obtained with fermionic operators. But we find a non-negligible component in the cross-correlation between fermionic and gluonic operators for a particular set of parameters.

String tension



We also measure another purely gluonic observable to extend our hyperscaling analysis to more observables which are affected by different systematic effects compared to the fermionic ones.
The string tension is obtained using two well-known methodologies

- from smeared Polyakov loop correlators coupling to torelon excitations on the torus, whose mass as a function of the loop length L is:

$$am_{\text{tor}}(L) = a^2\sigma L - \pi/3L - \pi^2/(18L^3a^2\sigma)$$

- from APE smeared Wilson loop operators used to construct Creutz ratios for the static quark-antiquark potential:

$$V(r) = v_0 - \alpha/r + \sigma r$$

✤The two methods agree in the full range of parameters explored

✤The hyperscaling fit of the data gives γ_*=0.3(1), in broad agreement with the rest of the spectrum

Conclusions and future work

Finite-size hyperscaling formulae describe the lattice data for the pseudoscalar mass and decay constant and for the vector mass, resulting in a universal value for the anomalous mass dimension. Both the flavor-singlet scalar mass and the string tension are compatible with these predictions.

These are indications that the twelve-flavor SU(3) gauge theory simulated on the lattice corresponds to a conformal theory in the continuum and infinite volume limit.

We also note that chiral perturbation theory does not apply to this context for the reason that the expansion parameter is much larger than unity and that the "pions" are no longer the lightest degrees of freedom of the theory.

<u>The presence of an iso-singlet scalar state parametrically lighter than the rest of the spectrum can be related to the dilatonic nature of the conformal dynamics and it makes a pressing issue to investigate the same feature in candidate models for Walking Technicolor</u> *[6][7]*.

References

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[3] LatKMI Collaboration, "Lattice study of conformality in twelve-flavor QCD", PRD 86, 054506
[4] Cheng et al.,"Finite size scaling of conformal theories in the presence of a near-marginal operator", arxiv:1401.0195
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