B meson decay constants and $\Delta B = 2$ matrix elements with static heavy and domain-wall light quarks

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RBC/UKQCD collaborations

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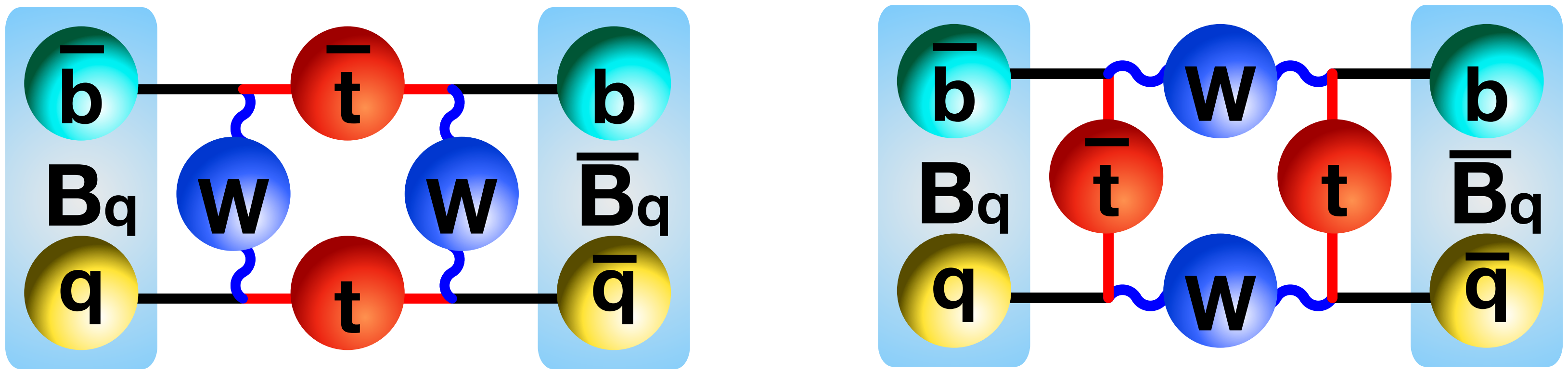
$B^0 - \bar{B}^0$ mixing: constrains on CKM

- $B^0 - \bar{B}^0$ mixing



$q = \{d, s\}$

- Neutral mesons are not eigenstates of the weak interactions.
- New Physics comes through loop diagrams.
- Mass difference between physical eigenstates:

$$\Delta m_q = \frac{G_F^2 m_W^2}{16\pi^2 m_{B_q}} |V_{tq}^* V_{tb}|^2 S_0 \left(\frac{m_t^2}{m_W^2} \right) \eta_B \mathcal{M}_{B_q}$$

$\longrightarrow$ constraints to V_{td}, V_{ts}

- $\Delta B = 2$ mixing matrix elements (non-perturbative hadronic)

$$\mathcal{M}_{B_q} = \langle \overline{B}_q^0 | [\bar{b}\gamma_\mu P_L q][\bar{b}\gamma_\mu P_L q] | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}$$

$B^0 - \bar{B}^0$ mixing: constrains on CKM

SU(3) breaking ratio ξ

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi\sqrt{\frac{\Delta m_d}{\Delta m_s}\frac{m_{B_s}}{m_{B_d}}} \qquad \xi = \frac{m_{B_d}}{m_{B_s}}\sqrt{\frac{\mathcal{M}_{B_s}}{\mathcal{M}_{B_d}}}$$

- The most attractive quantity in the mixing phenomena
- Many of the uncertainties are canceled in the ratio.
- In the simulation, fluctuations are largely canceled in the ratio.

Other important quantities

- B meson decay constants

$$f_{B_d}, f_{B_s}$$

- B-parameters

$$B_q = \frac{3}{8}\frac{\mathcal{M}_{B_q}}{m_{B_q}^2 f_{B_q}^2}$$

RBC/UKQCD Static B Physics

- V. Gadiyak and O. Loktik, *Lattice calculation of SU(3) flavor breaking ratios in* $B^0 - \bar{B}^0$ *mixing*, Phys. Rev. D 72 (2005) 114504.
- O. Loktik and T. Izubuchi, *Perturbative renormalization for static and domain-wall bilinears and four-fermion operators with improved gauge actions*, Phys. Rev. D 75 (2007) 034504.
- C. Albertus, Y. Aoki, P. A. Boyle, N. H. Christ, T. T. Dumitrescu, J. M. Flynn, T. I, T. Izubuchi, O. Loktik, C. T. Sachrajda, A. Soni, R. S. Van de Water, J. Wennekers and O. Witzel, *Neutral B-meson mixing from unquenched lattice QCD with domain-wall light quarks and static b-quarks*, Phys. Rev. D 82 (2010) 014505.
- T. I, Y. Aoki, J. M. Flynn, T. Izubuchib, and O. Loktik, *One-loop operator matching in the static heavy and domain-wall light quark system with O(a) improvement*, JHEP 05 (2011) 040.
- Y. Aoki, T. I, T. Izubuchi, C. Lehner and A. Soni, *Neutral B meson mixings and B meson decay constants with static heavy and domain-wall light quarks,* [arXiv:1406.6192].

Static limit

- **Static approximation (leading order of HQET)**
  - Easy to implement (Static quark propagator is almost free.)
  - Symmetries (HQ spin symmetry + chiral symmetry)

    reduced unphysical operator mixing
  - Continuum limit exists even in the perturbative renormalization.
  - But, we always have the error coming from static approx.

$$O(\Lambda_{\rm QCD}/m_b) \sim 10\%$$

- **Ratio quantities (ξ , f_{B_s}/f_{B_d}) in the static limit**
  - Error coming from static approximation is reduced to:

$$O\left(\frac{m_s - m_d}{\Lambda_{\rm QCD}} \times \frac{\Lambda_{\rm QCD}}{m_b}\right) \sim 2\%$$

Static limit

Static limit as a valuable anchor point

- HQ expansion:

$$\Phi_{\rm hl}(1/m_Q) = \Phi_{\rm hl}(0)\exp\left[\sum_{p=1}^{\infty}\gamma_p\left(\frac{\Lambda_{\rm QCD}}{m_Q}\right)^p\right].$$

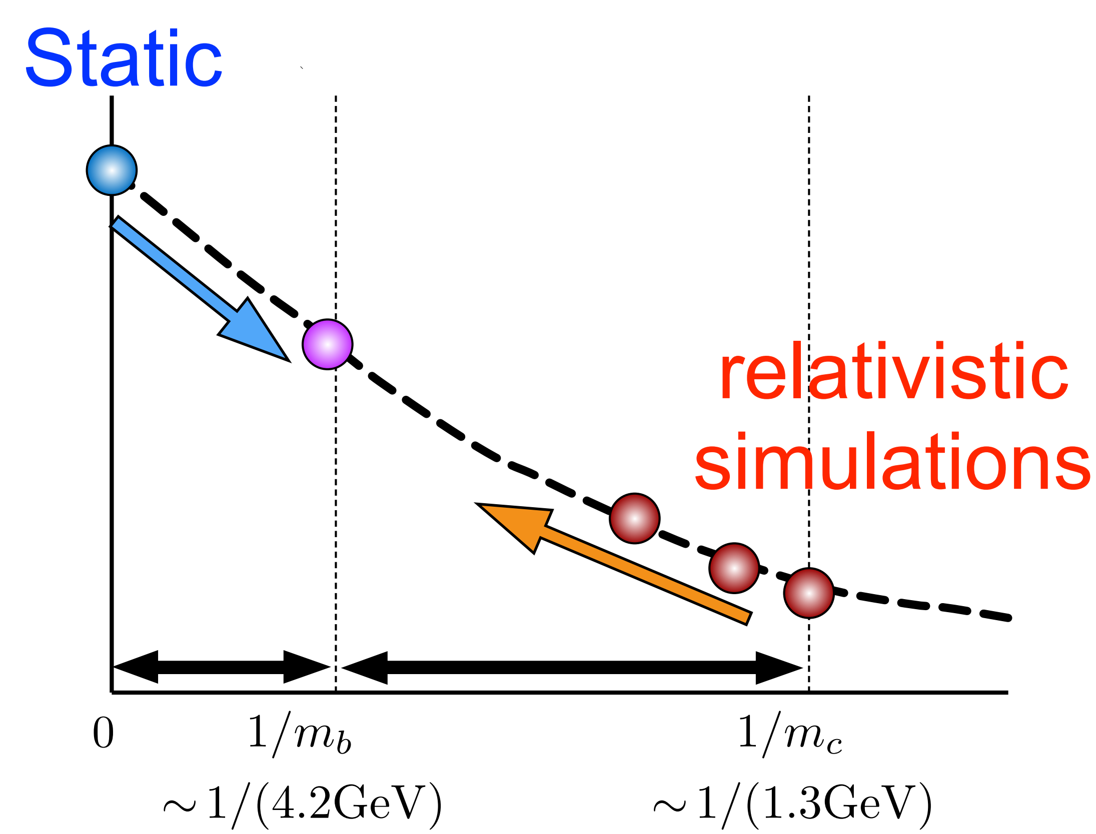
- Equivalent to:

$$\Phi_{\rm hl}(1/m_Q) = \Phi_{\rm hl}(1/m_{Q_A})\exp\left[\sum_{p=1}^{\infty}\gamma_p\left\{\left(\frac{\Lambda_{\rm QCD}}{m_Q}\right)^p - \left(\frac{\Lambda_{\rm QCD}}{m_{Q_A}}\right)^p\right\}\right].$$

m_{Q_A} : anchor point

- Once γ_p is determined, what we need is the overall factor at some anchor point.

- Static limit $m_Q \to \infty$ is close to target point m_b in terms of $1/m_Q$.



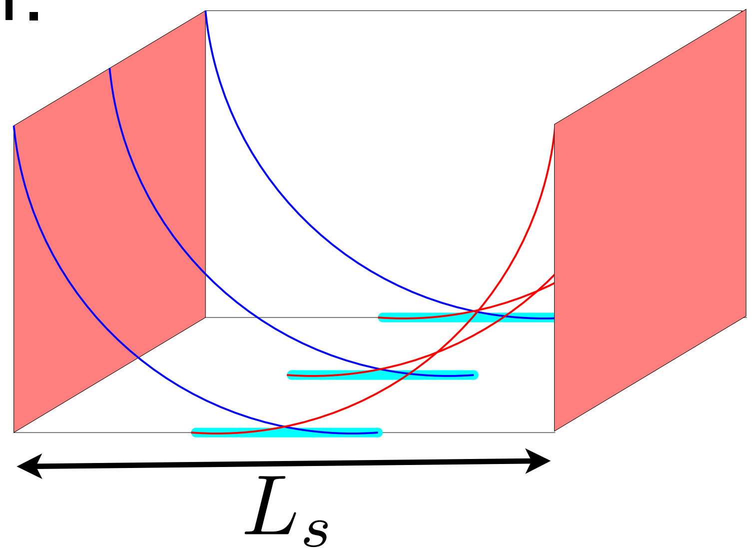
Lattice action setup

- Standard static action with link smearing

$$S_{\rm stat} = \sum_{\vec{x},t} \overline{\Psi}_h(\vec{x},t)\left[\Psi_h(\vec{x},t) - U_0^\dagger(\vec{x},t-a)\Psi_h(\vec{x},t-a)\right]$$

  - Reduced 1/a power divergence.
    - HYP1 [Hasenfratz and Knechtli, 2001]
    - HYP2 [Della Morte et al.(ALPHA), 2004]

- Domain-wall light quark action
  - 5 dimensional, controllable approximate chiral symmetry
  - Unphysical operator mixing does not occur.



- Iwasaki gluon action

Measurement

Gluon ensemble

- Nf=2+1 dynamical DWF + Iwasaki gluon (RBC-UKQCD)

[Phys. Rev. D 83, 074508 (2011)]

label	β	$L^3 \times T \times L_s$	a^{-1} [GeV]	a [fm]	$am_{\rm res}$	m_l/m_h	m_π [MeV]	$m_\pi aL$
24c1	2.13	$24^3 \times 64 \times 16$	1.729(25)	0.114	0.003152(43)	0.005/0.04	327	4.54
24c2						0.01/0.04	418	4.79
32c1	2.25	$32^3 \times 64 \times 16$	2.280(28)	0.0864	0.0006664(76)	0.004/0.03	289	4.05
32c2						0.006/0.03	344	4.83
32c3						0.008/0.03	393	5.52

Measurement parameters

label	am_q	Measured MD traj	# of data	# of src	Δt
24c1	0.005, 0.034, 0.040	900–8980 every 40	203	4	20
24c2	0.010, 0.034, 0.040	1460–8540 every 40	178	2	
32c1	0.004, 0.027, 0.030	520–6800 every 20	315	1	24
32c2	0.006, 0.027, 0.030	1000–7220 every 20	312	1	
32c2	0.008, 0.027, 0.030	520–5540 every 20	252	1	

- Gaussian smearing on fermion field (width ~ 0.45 fm)

Measurement

Operators

- 2PT correlation functions

$$C^{\tilde{L}S}(t) = \sum_{\vec{x}} \langle A_0^L(\vec{x}, t) A_0^S(\vec{x}_0, 0)^\dagger \rangle,$$

$$C^{\tilde{S}S}(t) = \sum_{\vec{x}} \langle A_0^S(\vec{x}, t) A_0^S(\vec{x}_0, 0)^\dagger \rangle,$$

$$C^{SS}(t) = \langle A_0^S(\vec{x}_0, t) A_0^S(\vec{x}_0, 0)^\dagger \rangle.$$

- 3PT correlation functions

$$C_L(t_f, t, t_0) = \sum_{\vec{x}} \langle A_0^S(\vec{x}_0, t_f)^\dagger O_{VV+AA}(\vec{x}, t) A_0^S(\vec{x}_0, t_0)^\dagger \rangle,$$

$$C_S(t_f, t, t_0) = \sum_{\vec{x}} \langle A_0^S(\vec{x}_0, t_f)^\dagger O_{SS+PP}(\vec{x}, t) A_0^S(\vec{x}_0, t_0)^\dagger \rangle.$$

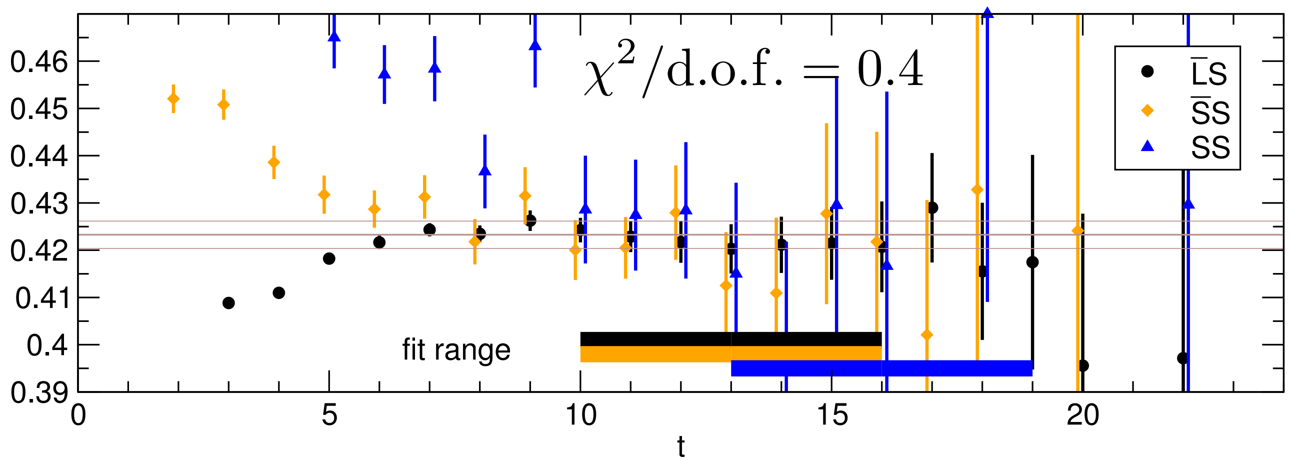
$A_0^L(\vec{x}, t)$: local

$A_0^S(\vec{x}, t)$: smeared both on heavy and light

$A_0^L(\vec{x}, t),\ O_{VV+AA}(\vec{x}, t)$: $O(a)$ improved operators

Data extraction

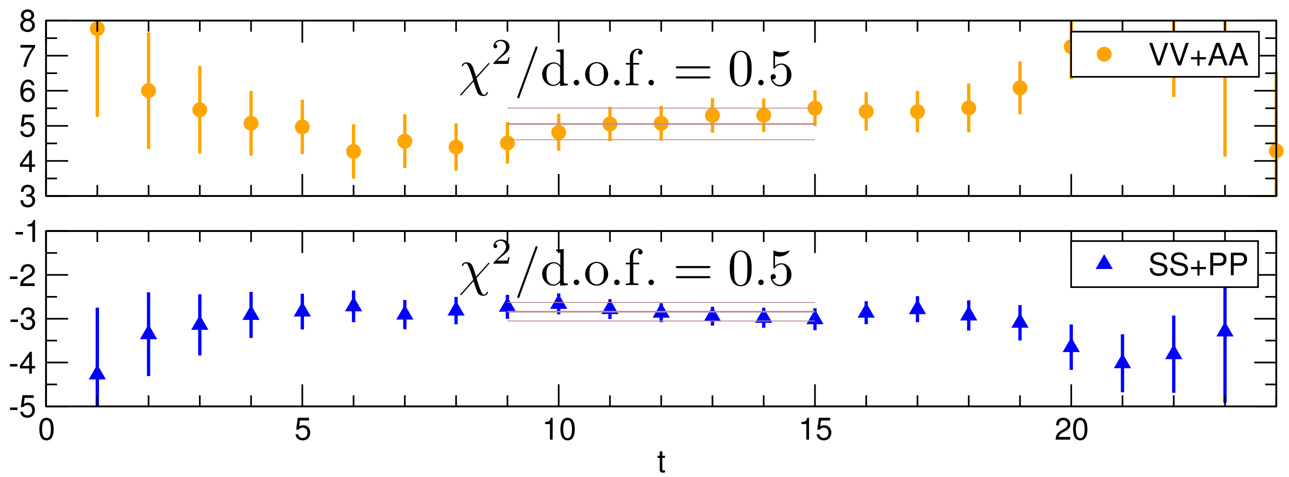
Correlator fitting



32c, HYP1

(m_h, m_l, m_q)

$= (0.03, 0.004, 0.004)$



$$C_{2\mathrm{PT}}(t) = A_{2\mathrm{PT}}(e^{-E_0 t} + e^{-E_0(T-t)})$$

$$C_{3\mathrm{PT}}(t_f, t, t_0) = A_{3\mathrm{PT}}$$

$$\Downarrow$$

$$\Phi^{\mathrm{lat}}_{B_q}, \quad M^{\mathrm{lat}}_{B_q}$$

Matching (continuum QCD and lattice HQET)

- Static with link smearing + DWF, incl. O(a) error, one-loop

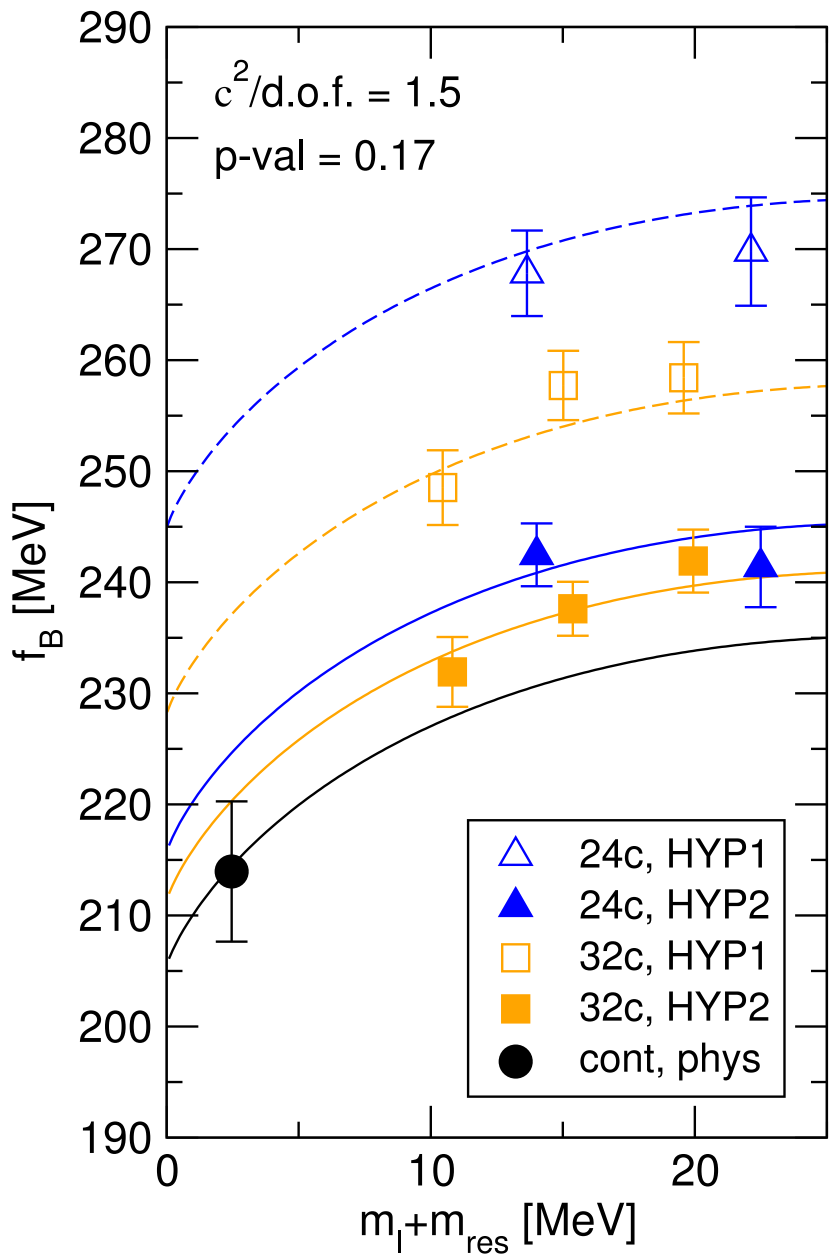
[T.I, Aoki, Flynn, Izubuchi, Loktik (2011)]

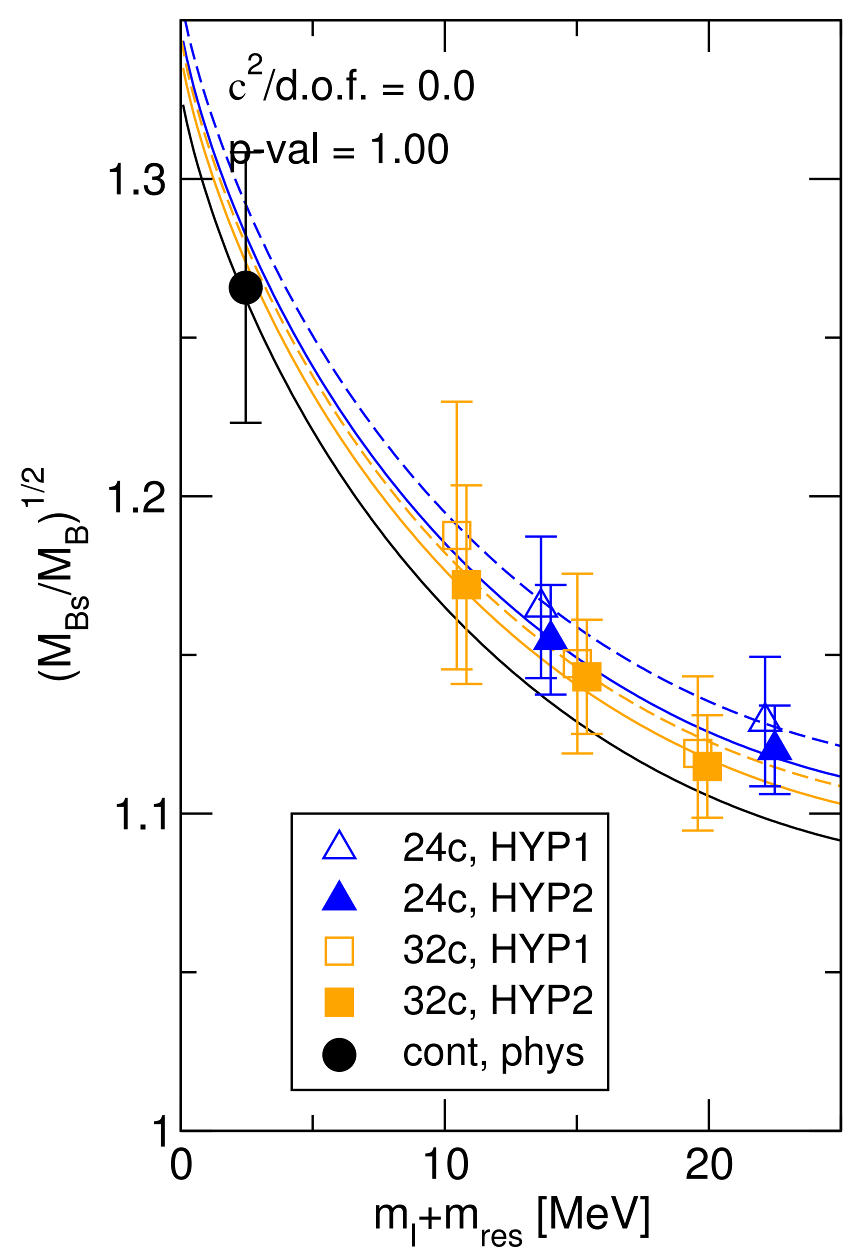
$$f_{B_q} = (\text{matching factor}) \times \frac{\Phi^{\mathrm{lat}}_{B_q}}{\sqrt{m_B}}, \quad \mathcal{M}_{B_q} = (\text{matching factor}) \times m_B M^{\mathrm{lat}}_{B_q}$$

Chiral and continuum extrapolation

- Combined fits

NLO SU(2) HMChPT





Linear fits are also used to estimate an uncertainty from chiral fits.

Results

Final results in the static limit

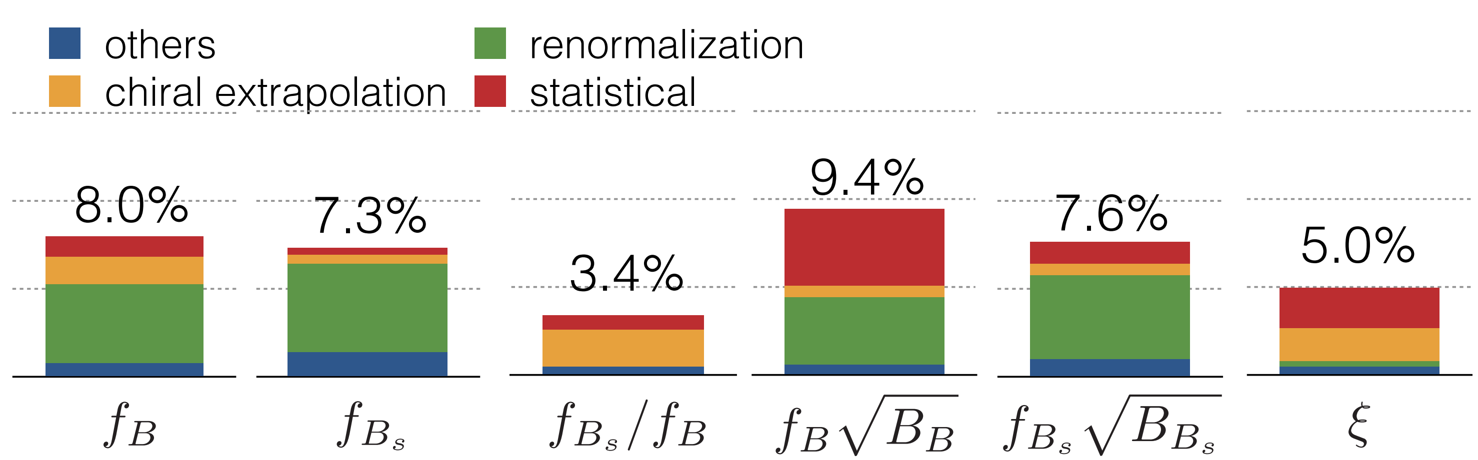
$$f_B = 218.8(6.5)_{\rm stat}(16.1)_{\rm sys}\ \text{MeV},$$
$$f_{B_s} = 263.5(4.8)_{\rm stat}(18.7)_{\rm sys}\ \text{MeV},$$
$$f_{B_s}/f_B = 1.193(20)_{\rm stat}(35)_{\rm sys}.$$

$$f_B\sqrt{\hat{B}_B} = 240(15)_{\rm stat}(17)_{\rm sys}\ \text{MeV},$$
$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 290(09)_{\rm stat}(20)_{\rm sys}\ \text{MeV},$$
$$\xi = 1.208(41)_{\rm stat}(44)_{\rm sys}.$$

$$\hat{B}_B = 1.17(11)_{\rm stat}(19)_{\rm sys},$$
$$\hat{B}_{B_s} = 1.22(06)_{\rm stat}(12)_{\rm sys},$$
$$B_{B_s}/B_B = 1.028(60)_{\rm stat}(43)_{\rm sys}.$$

(O(1/m) errors are not included in the error.)

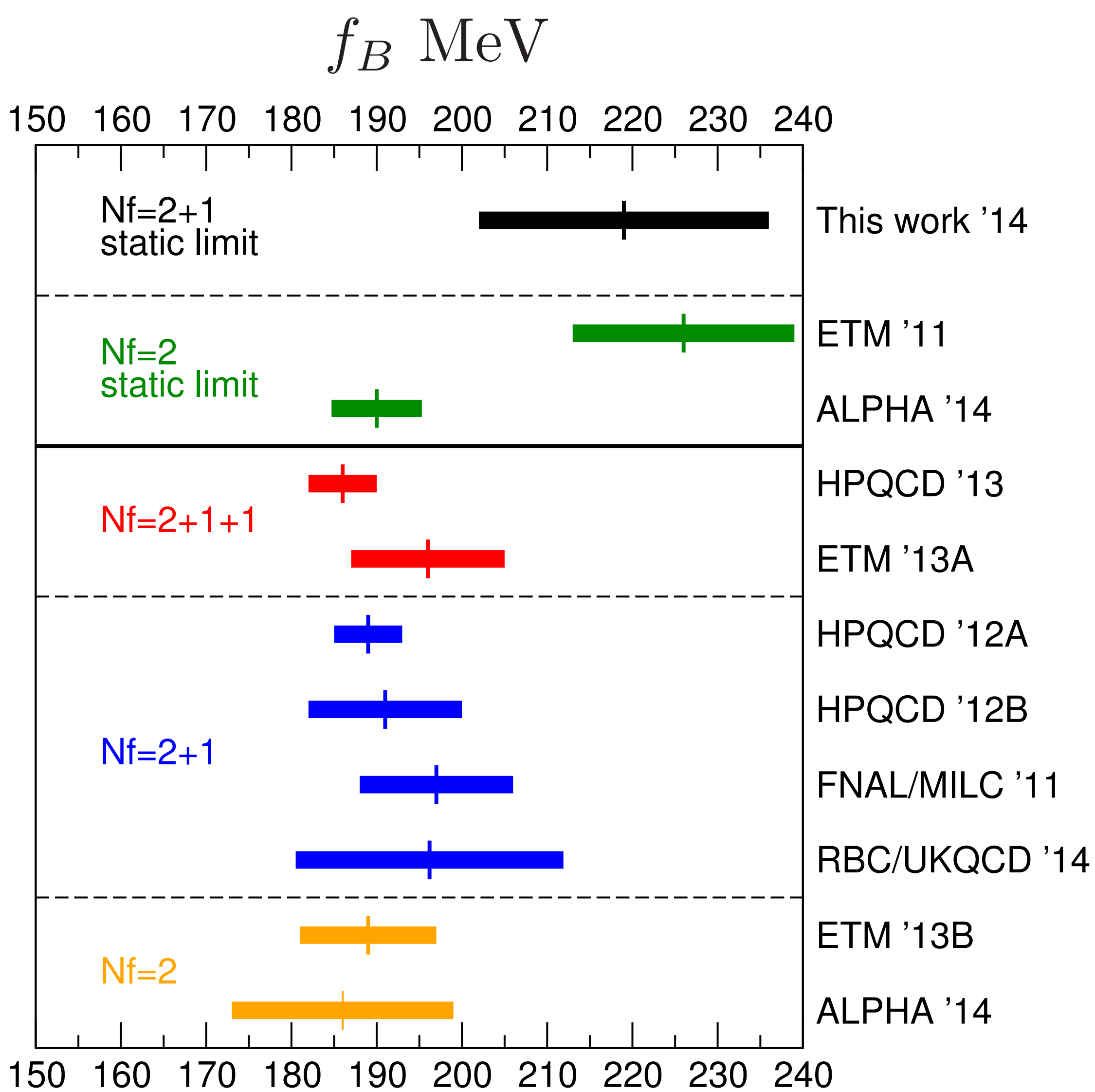
Error budget

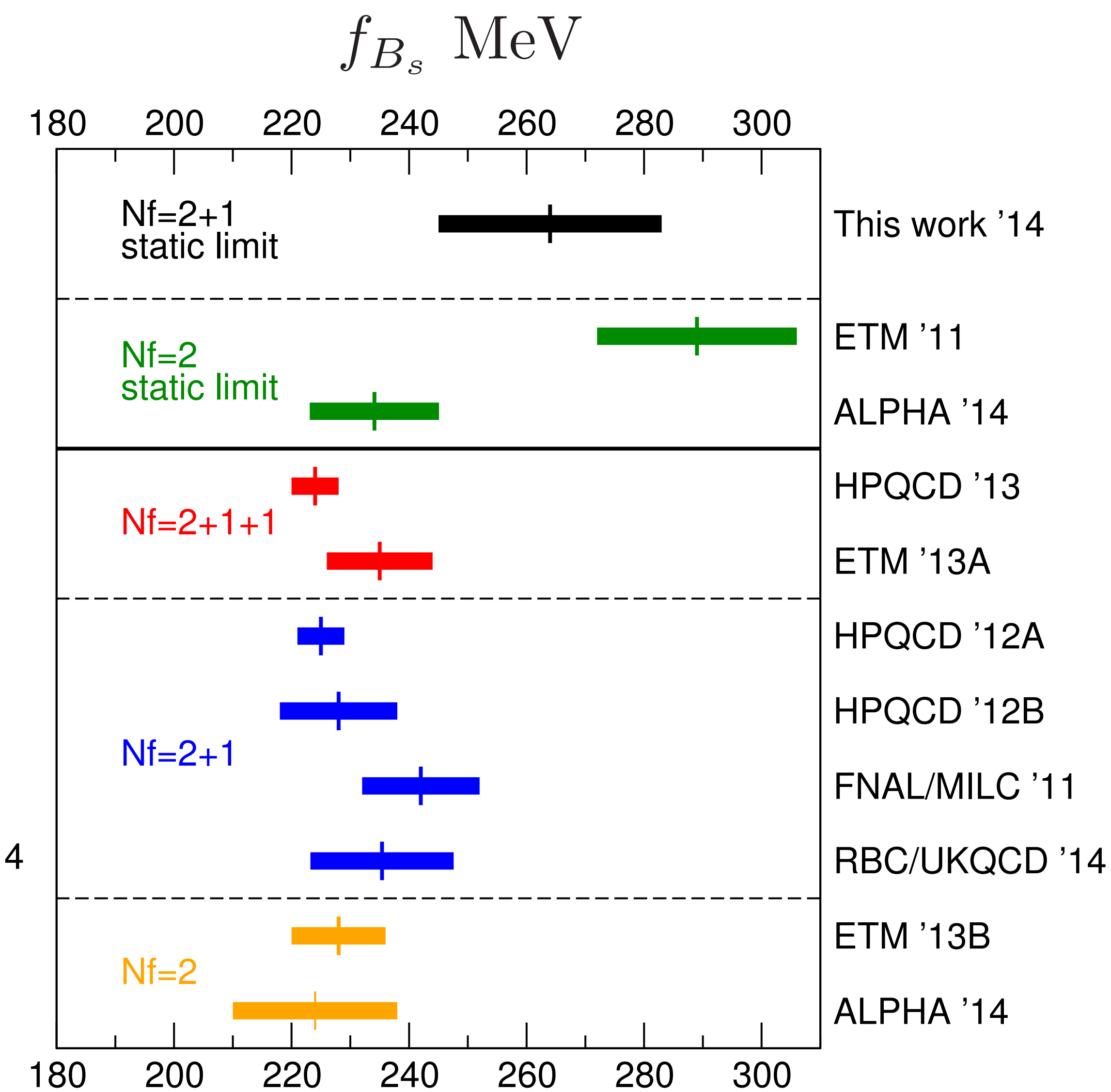


Results

- Comparison

as of Jun 22, 2014





Decay constants have ~10% deviation from physical b results.

Results

Comparison

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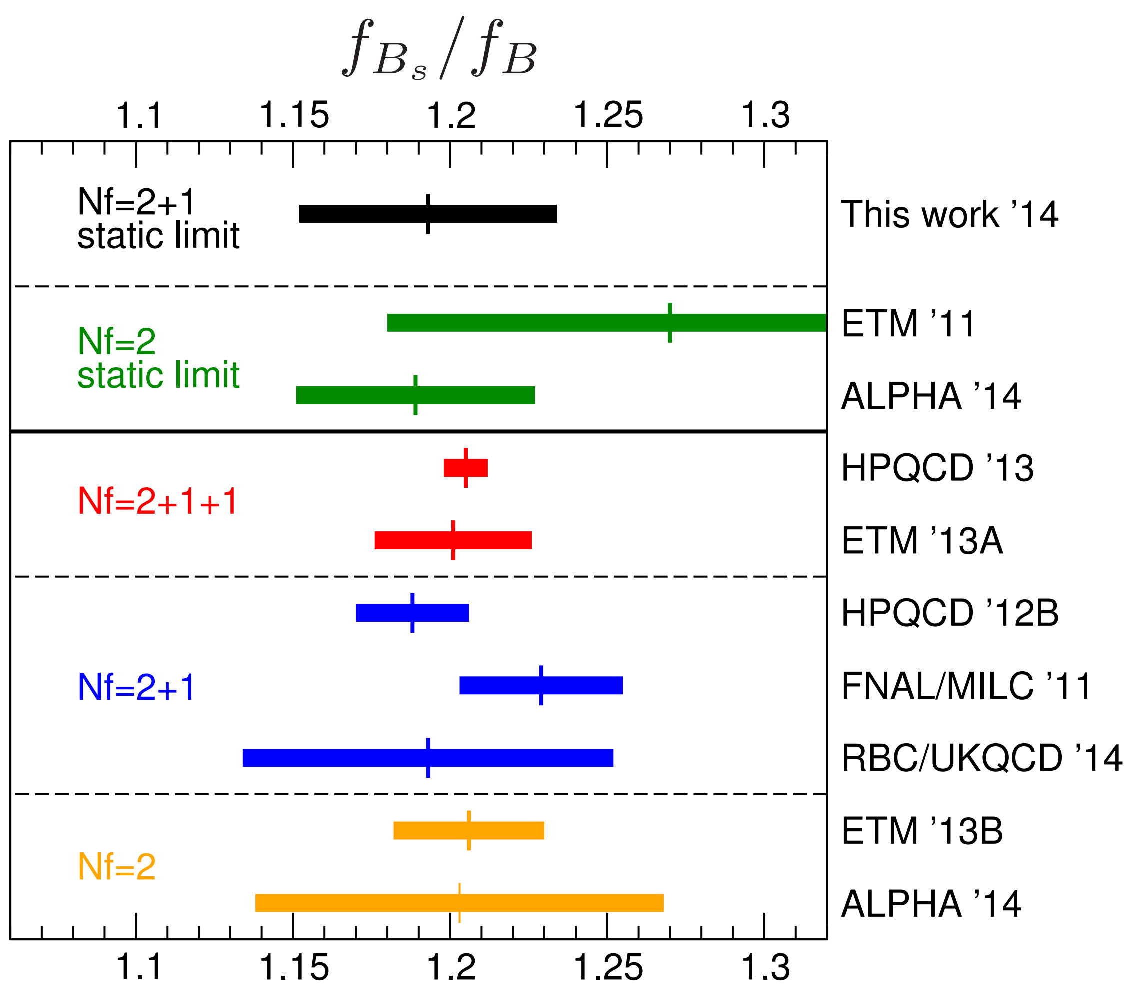
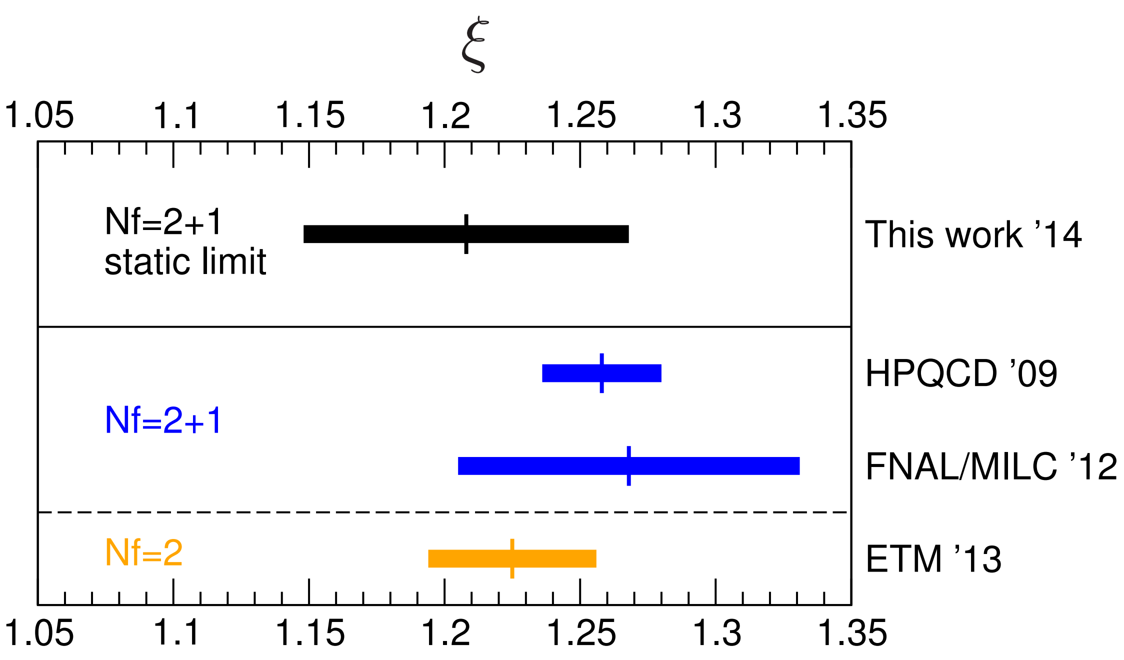




Ratio quantities do not have a significant deviation.

To more accuracy

Improvements for next

- All-Mode-Averaging (AMA) [T. Blum, T. Izubuchi, E. Shintani (2012)]

  improved operator using lattice symmetry $\longrightarrow$ good statistics

- Almost physical pion ensemble (Mobius domain-wall (RBC/UKQCD))

action	$1/a$ [Gev]	lattice	size [fm]	m_π [MeV]
MDWF + IW	1.75	$48^3 \times 96 \times 24$	5.5	138
MDWF + IW	2.31	$64^3 \times 128 \times 12$	5.5	139

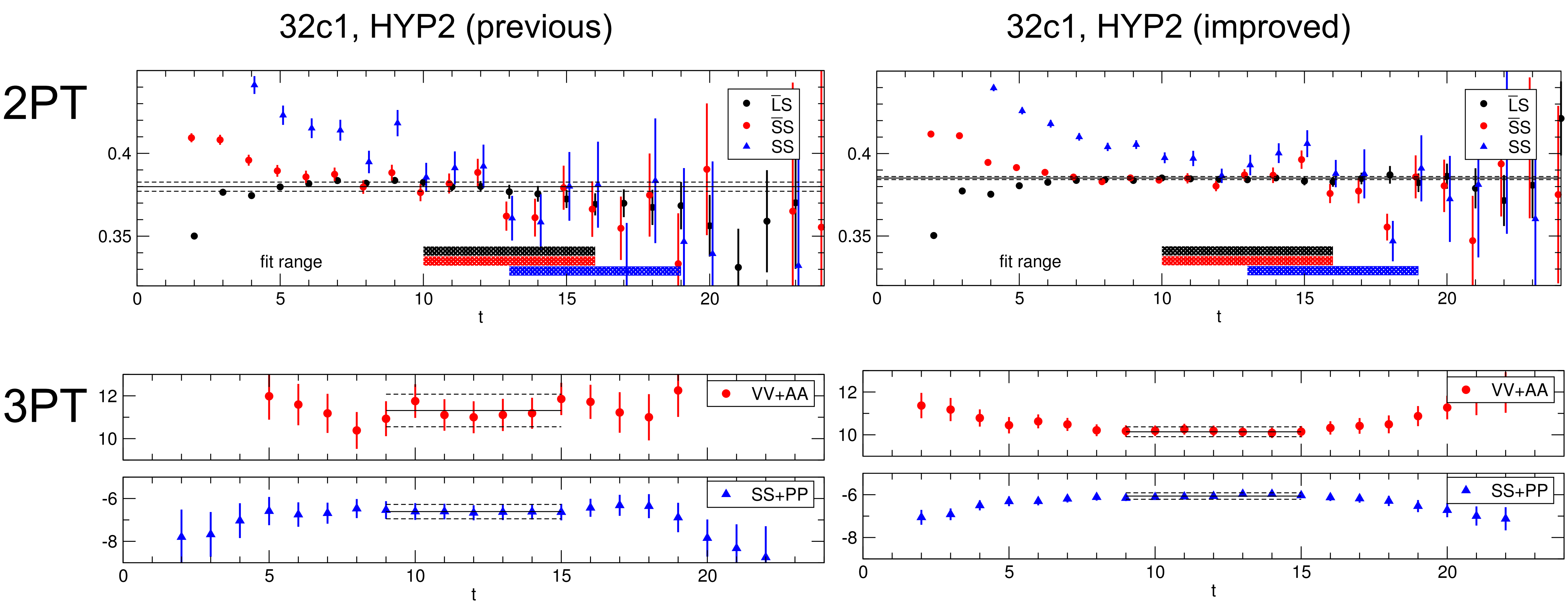
- Non-perturbative renormalization

  $1/a$ power divergence needs to introduce additional renormalization condition than usual one.

- Including $1/m_b$ correction by simulations in lower mass region

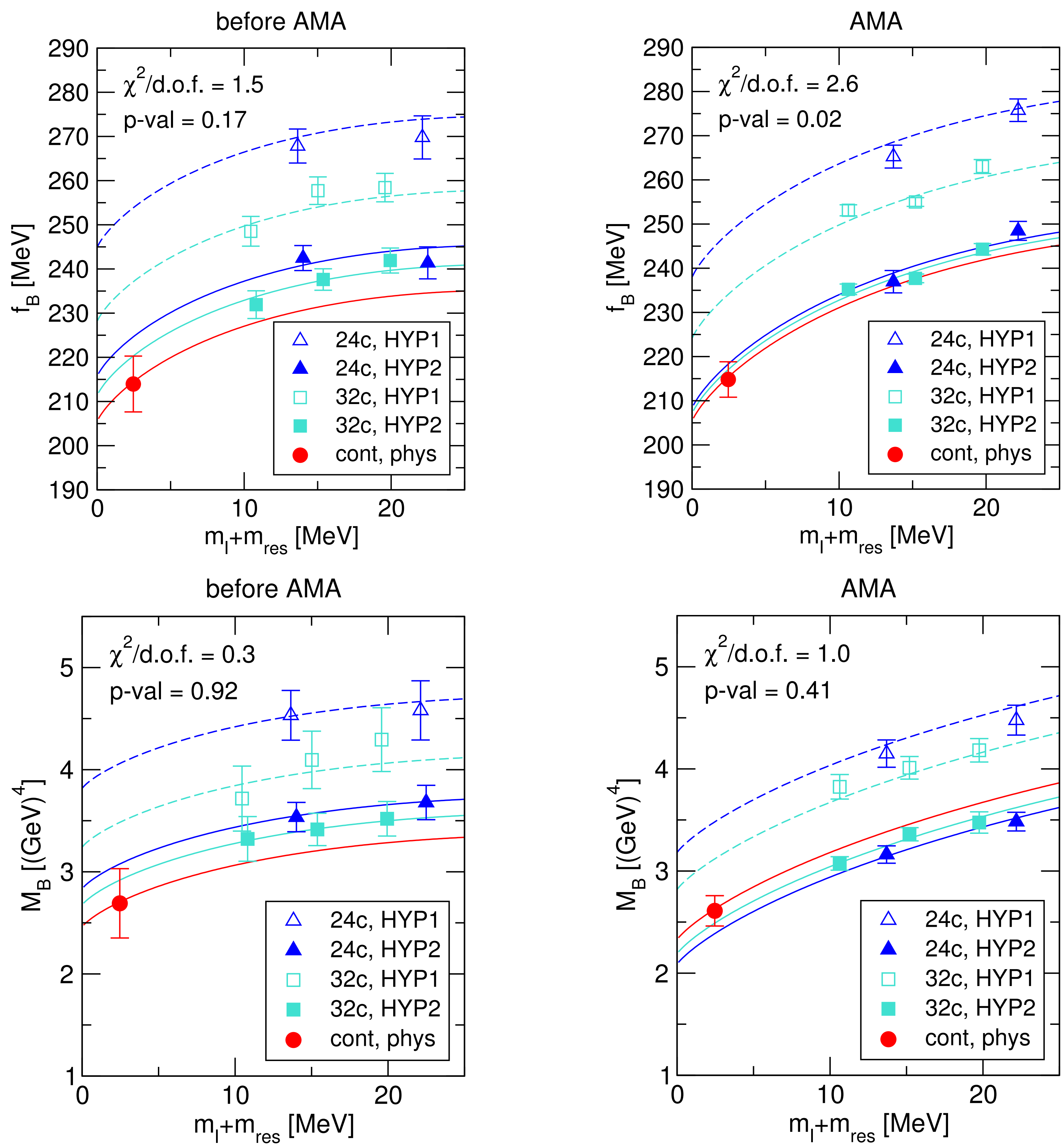
To more accuracy

- AMA
  - 64 source points with sloppy CG
  - Deflated sloppy CG with res ~ 3e-3 for ud quark
  - Sloppy CG with res ~ 1e-4 for strange quark



To more accuracy

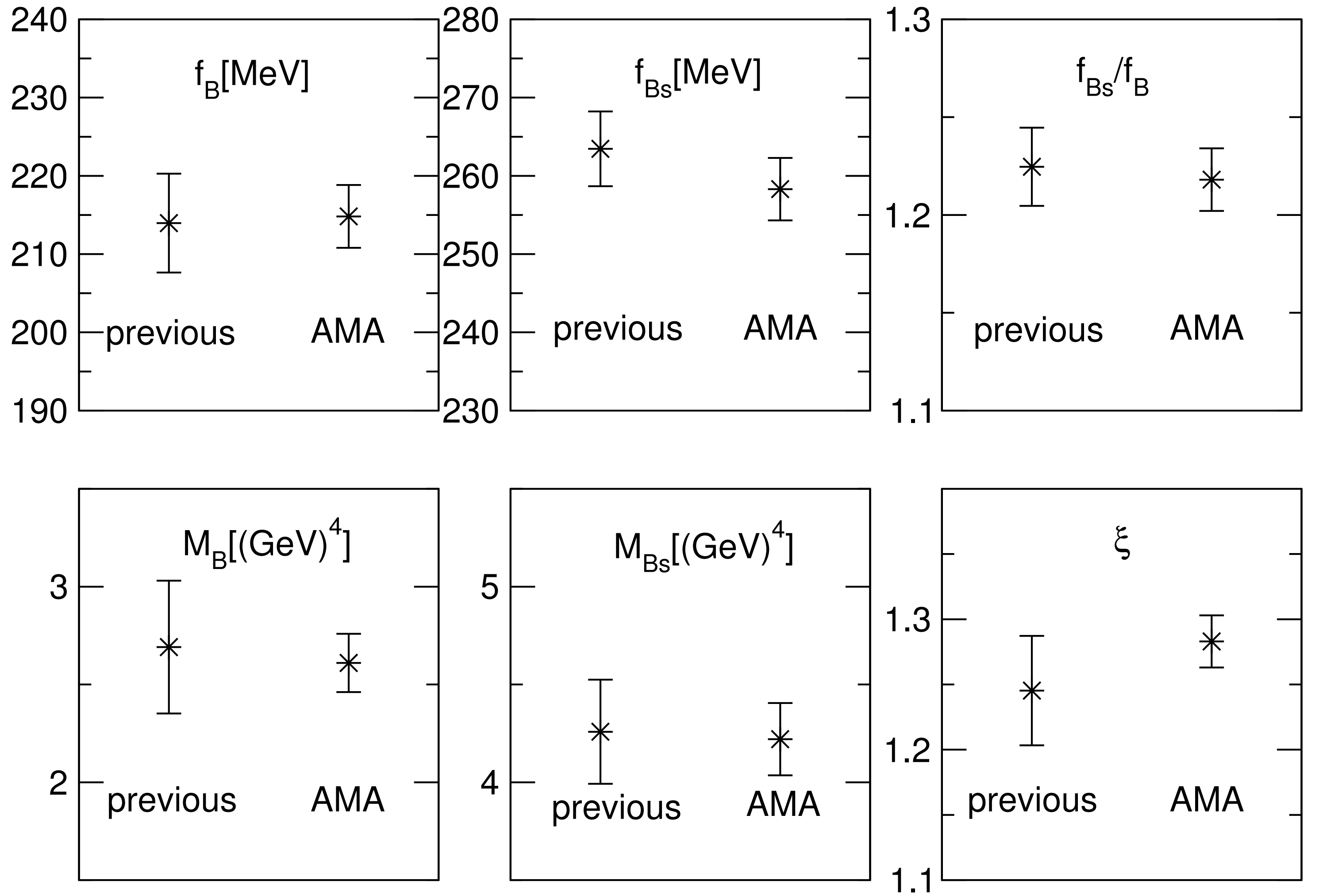
- AMA



To more accuracy

- AMA

very preliminary



SU(2)ChPT only

statistical error only

- Still on-going calculation to increase statistics and number of mass parameters.
- Currently the cost of AMA is less than the previous one.

Summary and outlook

- B meson decay constants and neutral B meson mixing matrix elements in the continuum limit are obtained using static approximation.
- Decay constants has ~10% deviation from physical b results, possibly due to 1/mb error.
- Ratio quantities does not have significant deviation from physical b results, because 1/mb error is largely suppressed.
- Reducing statistical and chiral extrapolation error is important to high precision.
- For non-ratio quantities, non-perturbative matching is also important.
- AMA can reduce the statistical error.
- Planning calculations at physical pion.
- Planning non-perturbative renormalization.