B meson decay constants and $\Delta B = 2$ matrix elements with static heavy and domain-wall light quarks

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RBC/UKQCD collaborations

Collaborators:
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$B^0 - \bar{B}^0$ mixing: constrains on CKM

- Neutral mesons are not eigenstates of the weak interactions.
- New Physics comes through loop diagrams.
- Mass difference between physical eigenstates:

\[
\Delta m_q = \frac{G_F^2 m_W^2}{16\pi^2 m_{B_q}} |V_{tb}^* V_{tq}|^2 S_0 \left( \frac{m_t^2}{m_W^2} \right) \eta_B M_{B_q}
\]

\[ q = \{d, s\} \]

\[ \rightarrow \text{constraints to } V_{td}, V_{ts} \]

- $\Delta B = 2$ mixing matrix elements (non-perturbative hadronic)

\[
M_{B_q} = \langle \bar{B}_q^0 | [\bar{b}\gamma_{\mu} P_L q] [\bar{b}\gamma_{\mu} P_L q] | B_q^0 \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 B_{B_q}
\]
$B^0 - \bar{B}^0$ mixing: constrains on CKM

- **SU(3) breaking ratio $\xi$**

  \[
  \frac{|V_{td}|}{|V_{ts}|} = \frac{\xi}{\sqrt{\frac{\Delta m_d}{\Delta m_s} \frac{m_{B_s}}{m_{B_d}}}} \quad \xi = \frac{m_{B_d}}{m_{B_s}} \sqrt{\frac{M_{B_s}}{M_{B_d}}}
  \]

  - The most attractive quantity in the mixing phenomena
  - Many of the uncertainties are canceled in the ratio.
  - In the simulation, fluctuations are largely canceled in the ratio.

- **Other important quantities**

  - B meson decay constants
    \[
    f_{B_d}, f_{B_s}
    \]
  - B-parameters
    \[
    B_q = \frac{3}{8} \frac{M_{B_q}}{m_{B_q}^2 f_{B_q}^2}
    \]
RBC/UKQCD Static B Physics


Static limit

- **Static approximation (leading order of HQET)**
  - Easy to implement (Static quark propagator is almost free.)
  - Symmetries (HQ spin symmetry + chiral symmetry)
    reduced unphysical operator mixing
  - Continuum limit exists even in the perturbative renormalization.
  - But, we always have the error coming from static approx.
    \[ O(\frac{\Lambda_{\text{QCD}}}{m_b}) \sim 10\% \]

- **Ratio quantities** \( (\xi, f_{B_s}/f_{B_d}) \) in the static limit
  - Error coming from static approximation is reduced to:
    \[ O \left( \frac{m_s - m_d}{\Lambda_{\text{QCD}}} \times \frac{\Lambda_{\text{QCD}}}{m_b} \right) \sim 2\% \]
Static limit

- Static limit as a valuable anchor point

  - HQ expansion:

    \[ \Phi_{hl}(1/m_Q) = \Phi_{hl}(0) \exp \left[ \sum_{p=1}^{\infty} \gamma_p \left( \frac{\Lambda_{QCD}}{m_Q} \right)^p \right]. \]

  - Equivalent to:

    \[ \Phi_{hl}(1/m_Q) = \Phi_{hl}(1/m_{QA}) \exp \left[ \sum_{p=1}^{\infty} \gamma_p \left\{ \left( \frac{\Lambda_{QCD}}{m_Q} \right)^p - \left( \frac{\Lambda_{QCD}}{m_{QA}} \right)^p \right\} \right]. \]

  \[ m_{QA} : \text{anchor point} \]

  - Once \( \gamma_p \) is determined, what we need is the overall factor at some anchor point.

  - Static limit \( m_Q \to \infty \) is close to target point \( m_b \) in terms of \( 1/m_Q \).
Lattice action setup

- **Standard static action with link smearing**
  
  \[ S_{\text{stat}} = \sum_{\vec{x}, t} \overline{\Psi}_h(\vec{x}, t) \left[ \Psi_h(\vec{x}, t) - U_0^\dagger(\vec{x}, t - a)\Psi_h(\vec{x}, t - a) \right] \]

  - Reduced 1/a power divergence.
    - HYP1 [Hasenfratz and Knechtli, 2001]
    - HYP2 [Della Morte et al.(ALPHA), 2004]

- **Domain-wall light quark action**
  
  - 5 dimensional, controllable approximate chiral symmetry
  - Unphysical operator mixing does not occur.

- **Iwasaki gluon action**
Measurement

- Gluon ensemble
  - Nf=2+1 dynamical DWF + Iwasaki gluon (RBC-UKQCD)

<table>
<thead>
<tr>
<th>label</th>
<th>$\beta$</th>
<th>$L^3 \times T \times L_s$</th>
<th>$a^{-1}$ [GeV]</th>
<th>$a$ [fm]</th>
<th>$am_{res}$</th>
<th>$m_1/m_h$</th>
<th>$m_\pi$ [MeV]</th>
<th>$m_\pi aL$</th>
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<td>24c1</td>
<td>2.13</td>
<td>$24^3 \times 64 \times 16$</td>
<td>1.729(25)</td>
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<td>0.003152(43)</td>
<td>0.005/0.04</td>
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<td>$32^3 \times 64 \times 16$</td>
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<td>0.0864</td>
<td>0.0006664(76)</td>
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<td>5.52</td>
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- Measurement parameters

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<th># of src</th>
<th>$\Delta t$</th>
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<td>520–5540 every 20</td>
<td>252</td>
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</table>

- Gaussian smearing on fermion field (width ~ 0.45 fm)
Measurement

Operators

- 2PT correlation functions

\[ C^{\tilde{L}S}(t) = \sum_{\tilde{x}} \langle A_0^L(\tilde{x}, t) A_0^S(\tilde{x}_0, 0) \rangle, \]

\[ C^{\tilde{S}S}(t) = \sum_{\tilde{x}} \langle A_0^S(\tilde{x}, t) A_0^S(\tilde{x}_0, 0) \rangle, \]

\[ C^{SS}(t) = \langle A_0^S(\tilde{x}_0, t) A_0^S(\tilde{x}_0, 0) \rangle. \]

- 3PT correlation functions

\[ C_L(t_f, t, t_0) = \sum_{\tilde{x}} \langle A_0^S(\tilde{x}_0, t_f) O_{VV+AA}(\tilde{x}, t) A_0^S(\tilde{x}_0, t_0) \rangle, \]

\[ C_S(t_f, t, t_0) = \sum_{\tilde{x}} \langle A_0^S(\tilde{x}_0, t_f) O_{SS+PP}(\tilde{x}, t) A_0^S(\tilde{x}_0, t_0) \rangle. \]

\[ A_0^L(\tilde{x}, t) : \text{ local} \]

\[ A_0^S(\tilde{x}, t) : \text{ smeared both on heavy and light} \]

\[ A_0^L(\tilde{x}, t), O_{VV+AA}(\tilde{x}, t) : O(a) \text{ improved operators} \]
Data extraction

- **Correlator fitting**

32c, HYP1

\[(m_h, m_l, m_q)\] = (0.03, 0.004, 0.004)

\[C_{2\text{PT}}(t) = A_{2\text{PT}}(e^{-E_0 t} + e^{-E_0(T-t)})\]

\[C_{3\text{PT}}(t_f, t, t_0) = A_{3\text{PT}}\]

\[\Phi_{B_q}^{\text{lat}}, \quad M_{B_q}^{\text{lat}}\]

- **Matching (continuum QCD and lattice HQET)**

  - Static with link smearing + DWF, incl. O(a) error, one-loop

  [T.I, Aoki, Flynn, Izubuchi, Loktik (2011)]

\[f_{B_q} = (\text{matching factor}) \times \frac{\Phi_{B_q}^{\text{lat}}}{\sqrt{m_B}}, \quad M_{B_q} = (\text{matching factor}) \times m_B M_{B_q}^{\text{lat}}\]
Chiral and continuum extrapolation

- Combined fits

Linear fits are also used to estimate an uncertainty from chiral fits.
Results

Final results in the static limit

\[
\begin{align*}
    f_B &= 218.8(6.5)_{\text{stat}}(16.1)_{\text{sys}} \text{ MeV}, \\
    f_{B_s} &= 263.5(4.8)_{\text{stat}}(18.7)_{\text{sys}} \text{ MeV}, \\
    f_{B_s}/f_B &= 1.193(20)_{\text{stat}}(35)_{\text{sys}}. \\
    f_B \sqrt{\hat{B}_B} &= 240(15)_{\text{stat}}(17)_{\text{sys}} \text{ MeV}, \\
    f_{B_s} \sqrt{\hat{B}_{B_s}} &= 290(09)_{\text{stat}}(20)_{\text{sys}} \text{ MeV}, \\
    \xi &= 1.208(41)_{\text{stat}}(44)_{\text{sys}}.
\end{align*}
\]

\[
\begin{align*}
    \hat{B}_B &= 1.17(11)_{\text{stat}}(19)_{\text{sys}}, \\
    \hat{B}_{B_s} &= 1.22(06)_{\text{stat}}(12)_{\text{sys}}, \\
    B_{B_s}/B_B &= 1.028(60)_{\text{stat}}(43)_{\text{sys}}.
\end{align*}
\]

(O(1/m) errors are not included in the error.)

Error budget

- Others: 8.0%
- Chiral extrapolation: 7.3%
- Renormalization: 3.4%
- Statistical: 9.4%
- Others: 7.6%
- Others: 5.0%
Comparison

Results

Decay constants have ~10% deviation from physical b results.

as of Jun 22, 2014
Results

Comparison

\[ \frac{f_{B_s}}{f_B} \]

\[ \xi \]

Ratio quantities do not have a significant deviation.
To more accuracy

- **Improve for next**

  - **All-Mode-Averaging (AMA)** [T. Blum, T. Izubuchi, E. Shintani (2012)]
    improved operator using lattice symmetry → good statistics

  - **Almost physical pion ensemble** (Mobius domain-wall (RBC/UKQCD))

    | action       | 1/a [Gev] | lattice       | size [fm] | $m_\pi$ [MeV] |
    |--------------|-----------|---------------|-----------|--------------|
    | MDWF + IW    | 1.75      | $48^3 \times 96 \times 24$ | 5.5       | 138          |
    | MDWF + IW    | 2.31      | $64^3 \times 128 \times 12$ | 5.5       | 139          |

  - **Non-perturbative renormalization**

    $1/a$ power divergence needs to introduce additional renormalization condition than usual one.

  - **Including $1/m_b$ correction** by simulations in lower mass region
To more accuracy

- AMA
  - 64 source points with sloppy CG
  - Deflated sloppy CG with res $\sim 3e^{-3}$ for ud quark
  - Sloppy CG with res $\sim 1e^{-4}$ for strange quark
To more accuracy

AMA

χ²/d.o.f. = 1.5
p-val = 0.17

χ²/d.o.f. = 2.6
p-val = 0.02

χ²/d.o.f. = 0.3
p-val = 0.92

χ²/d.o.f. = 1.0
p-val = 0.41
- Still on-going calculation to increase statistics and number of mass parameters.
- Currently the cost of AMA is less than the previous one.
Summary and outlook

- B meson decay constants and neutral B meson mixing matrix elements in the continuum limit are obtained using static approximation.

- Decay constants has \( \sim 10\% \) deviation from physical b results, possibly due to 1/mb error.

- Ratio quantities does not have significant deviation from physical b results, because 1/mb error is largely suppressed.

- Reducing statistical and chiral extrapolation error is important to high precision.

- For non-ratio quantities, non-perturbative matching is also important.

- AMA can reduce the statistical error.

- Planning calculations at physical pion.

- Planning non-perturbative renormalization.