TWO-BARYON SYSTEMS WITH TWISTED BOUNDARY CONDITIONS

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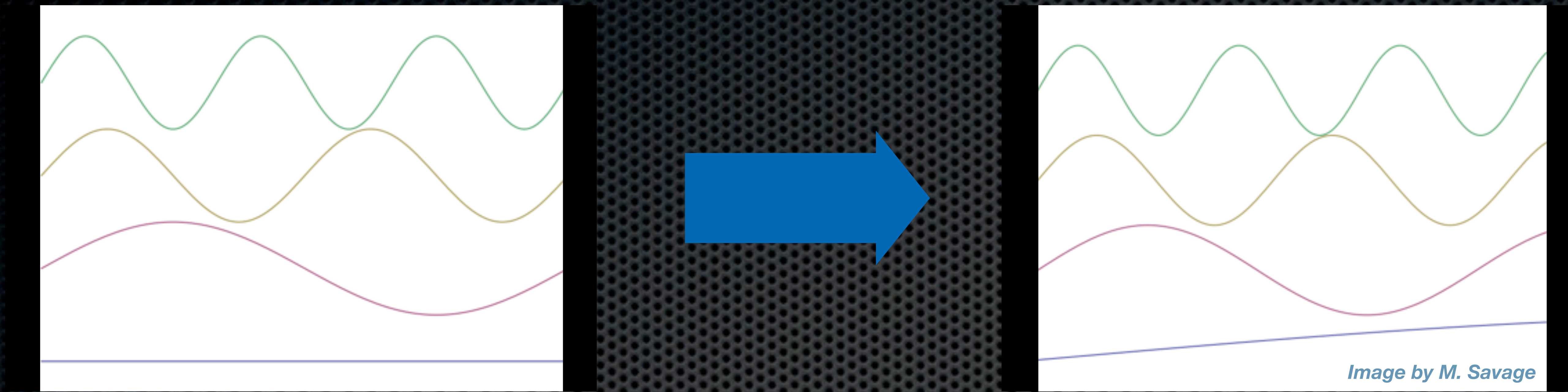
In collaboration with:
Raul Briceno (JLab), Thomas Luu (Julich) and Martin Savage (UW/INT)

TWISTED BOUNDARY CONDITIONS



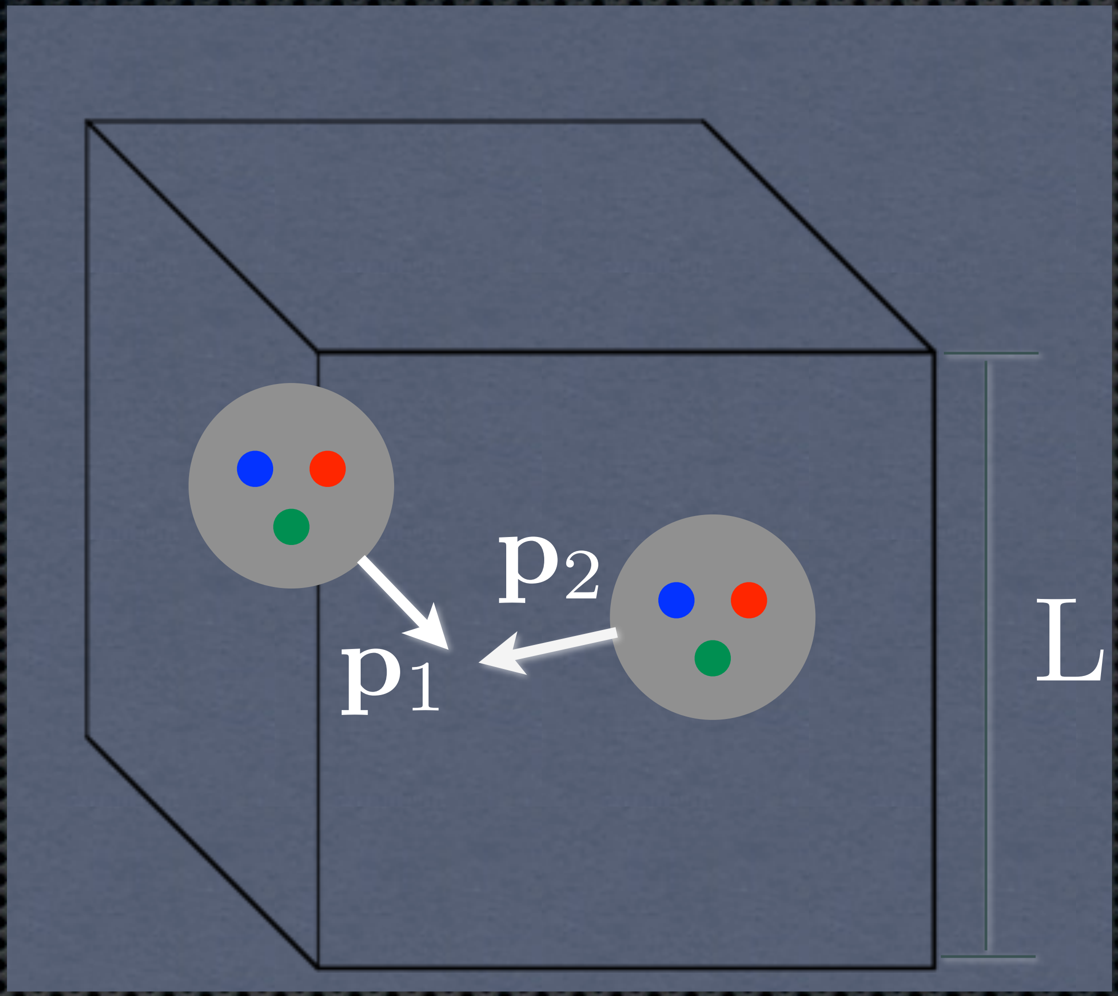
$$\psi(\mathbf{x}+\mathbf{n}L) = e^{i\phi\cdot\mathbf{n}}\psi(\mathbf{x})$$

$$\mathbf{p} = \frac{2\pi}{L}\mathbf{n} + \frac{\phi}{L}, 0 \leq \phi_i \leq 2\pi$$

Bedaque, Phys.Lett. B593 (2004) 82-88.

Tiburzi, Phys.Lett. B617, 40 (2005).
Jiang and Tiburzi, Phys.Lett. B645, 314 (2007).
Boyle, Flynn, Juttner, Sachrajda, and Zanotti, JHEP 0705, 016 (2007).
Simula (ETMC Collaboration), PoS LAT2007, 371 (2007).
Boyle, Flynn, Juttner, Kelly, de Lima, et al., JHEP 0807, 112 (2008).
Aoki, Horsley, Izubuchi, Nakamura, Pleiter, et al. (2008), 0808.1428.
Boyle, Flynn, Juttner, Sachrajda, Sivalingam, et al., PoS LATTICE2012, 112 (2012).
Brandt, Juttner, and Wittig, PoS ConfinementX, 112 (2012).
de Divitiis and Tantalo (2004), hep-lat/0409154.
Sachrajda and Villadoro, Phys.Lett. B609, 73 (2005).
Jiang and Tiburzi, Phys.Rev. D78, 114505 (2008).
Bernard, Lage, Meißner and Rusetsky, JHEP 1101 (2011) 019.
D¨oring, Meißner, Oset and Rusetsky, Eur. Phys. J. A 47 (2011) 139.
D¨oring, Meißner, Oset and Rusetsky, Eur. Phys. J. A 48 (2012) 114.

A GENERAL TWO-BARYON FINITE-VOLUME QUANTIZATION CONDITION



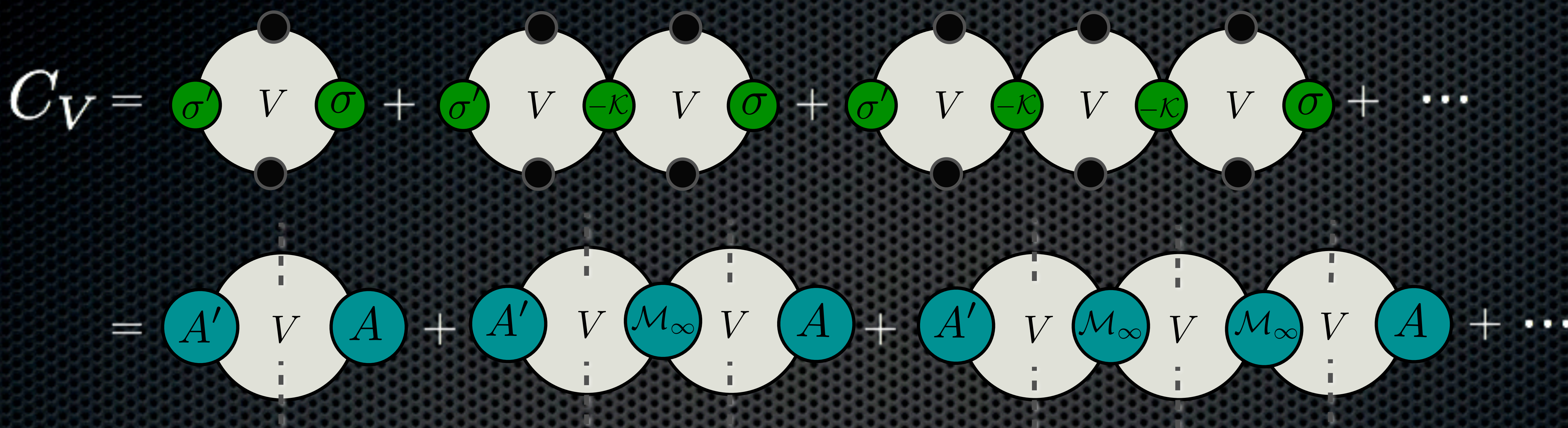
Arbitrary CM momentum, twisted boundary conditions, arbitrary masses and relativistic kinematics.

Briceno, ZD and Luu, Phys. Rev. D 88, 034502.
Briceno, ZD, Luu and Savage, Phys. Rev. D 89, 074509.

See R. Briceno, Phys. Rev. D 89, 074507 for generalization to arbitrary spin.

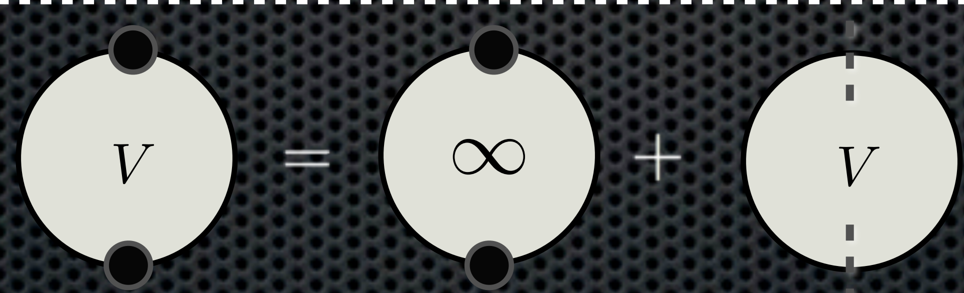
TWO-NUCLEON CORRELATION FUNCTION

$$T \to \infty,\ a \to 0$$



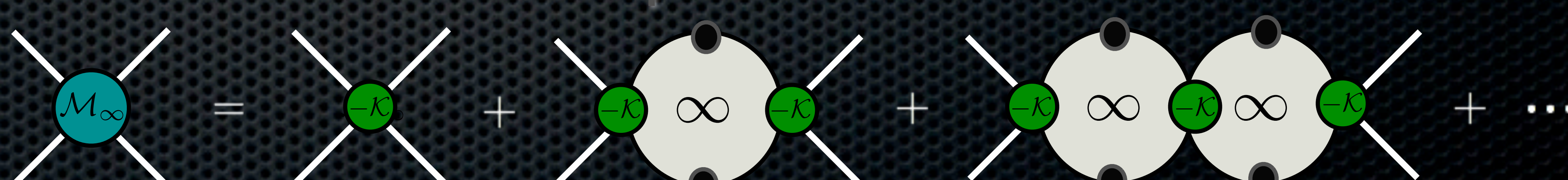
Kim, Sachrajda, Sharpe, et al, Nucl.Phys. B727 (2005) 218-243.

(1)



$$\int \frac{dk^0}{(2\pi)} \int \frac{d^3k}{(2\pi)^3} f(\mathbf{k}) \to \int \frac{dk^0}{(2\pi)} \frac{1}{L^3} \sum_{\mathbf{k}} f(\mathbf{k})$$

(2)



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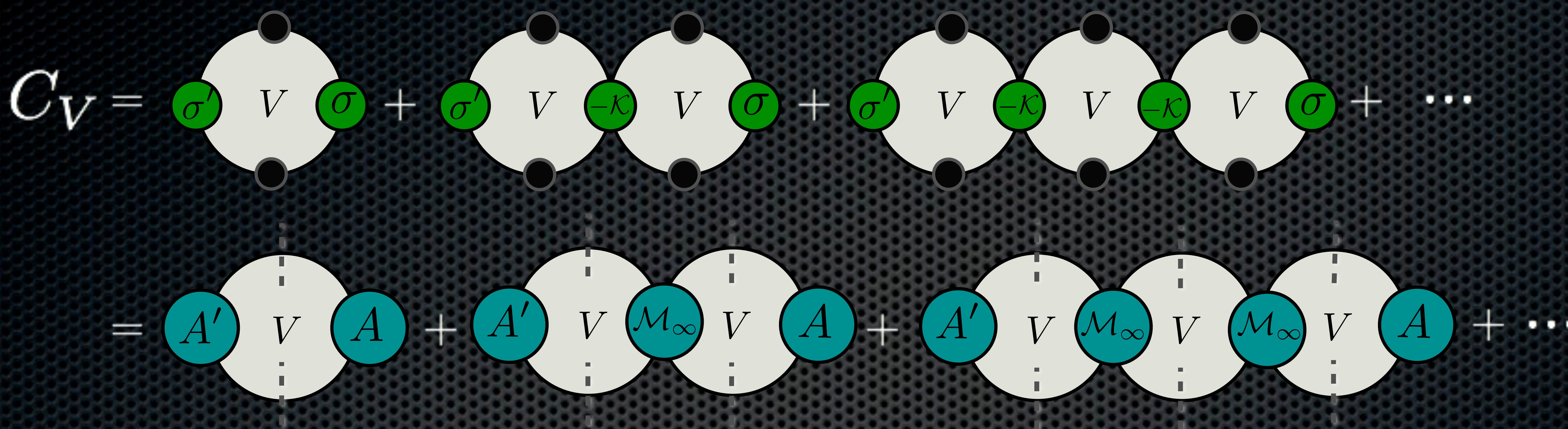

$$\det\left(\delta\mathcal{G}^V + \mathcal{M}^{-1}\right) = 0$$

Briceno, ZD and Luu, Phys. Rev. D 88, 034502.

Non-diagonal in both L and J basis

A function of energy and volume

Parametrized by phase shifts and mixing angles

Diagonal in J basis

TWO-NUCLEON CORRELATION FUNCTION

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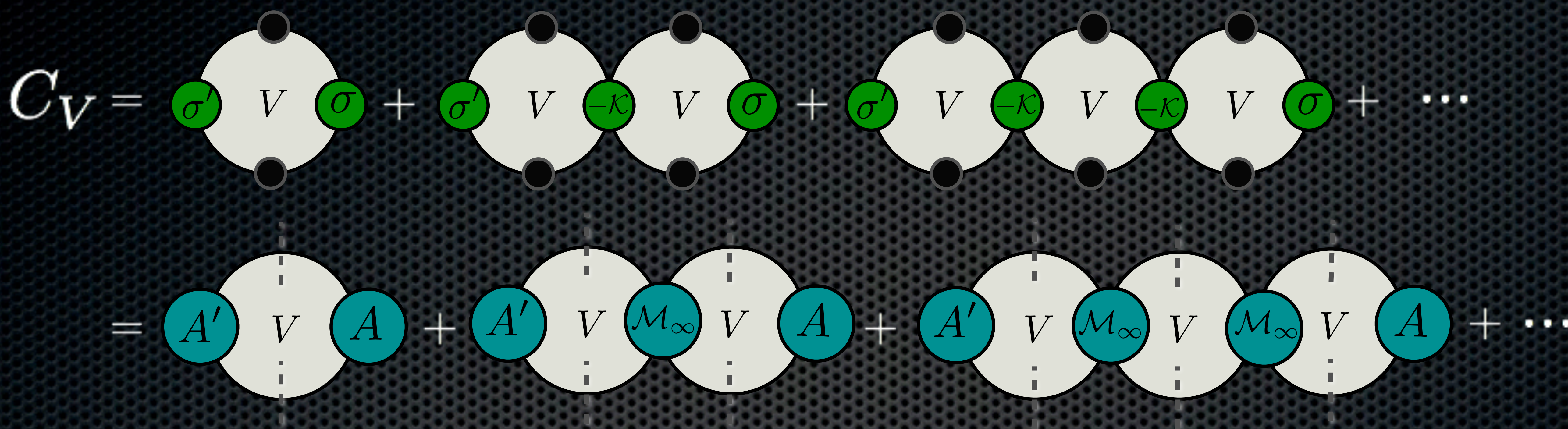

Luscher, Nucl.Phys. B354, 531-578 (1991).
Luscher, Commun.Math.Phys., 105:153 (1986).

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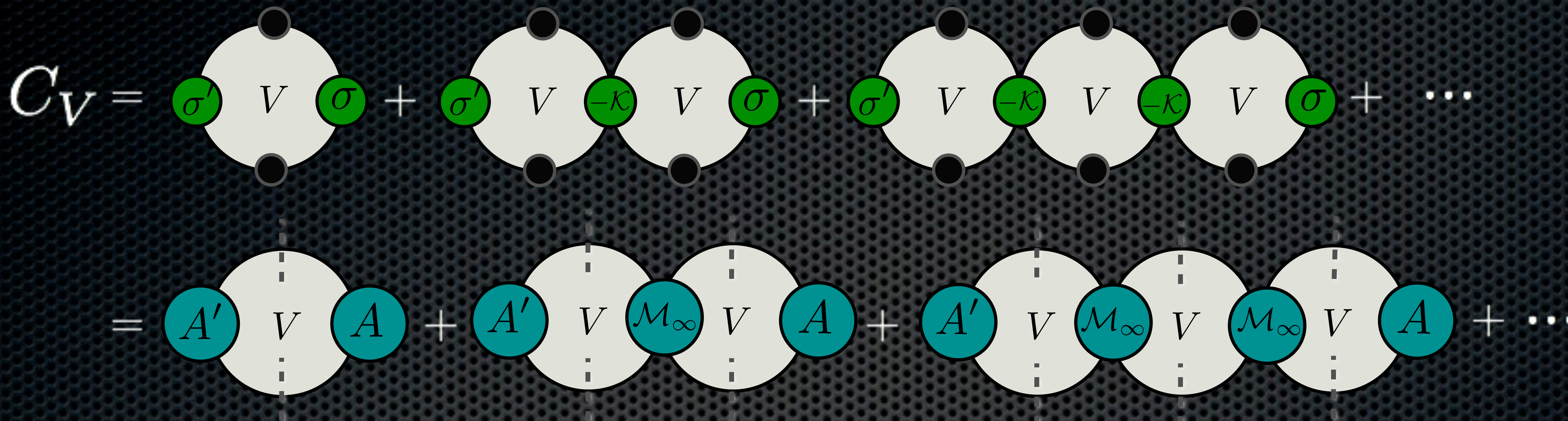

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ASSUMPTION

Neglecting scattering with $l > 3$

Example: Deuteron

$$\boxed{1^+ \ (I=0)} \Rightarrow \boxed{L=0, L=2}$$

$$B_d^\infty = 2.224644(34) \text{ MeV}$$

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{1,S} & \mathcal{M}_{1,SD} & 0 & 0 \\ \mathcal{M}_{1,SD} & \mathcal{M}_{1,D} & 0 & 0 \\ 0 & 0 & \mathcal{M}_{2,D} & 0 \\ 0 & 0 & 0 & \mathcal{M}_{3,D} \end{pmatrix}$$

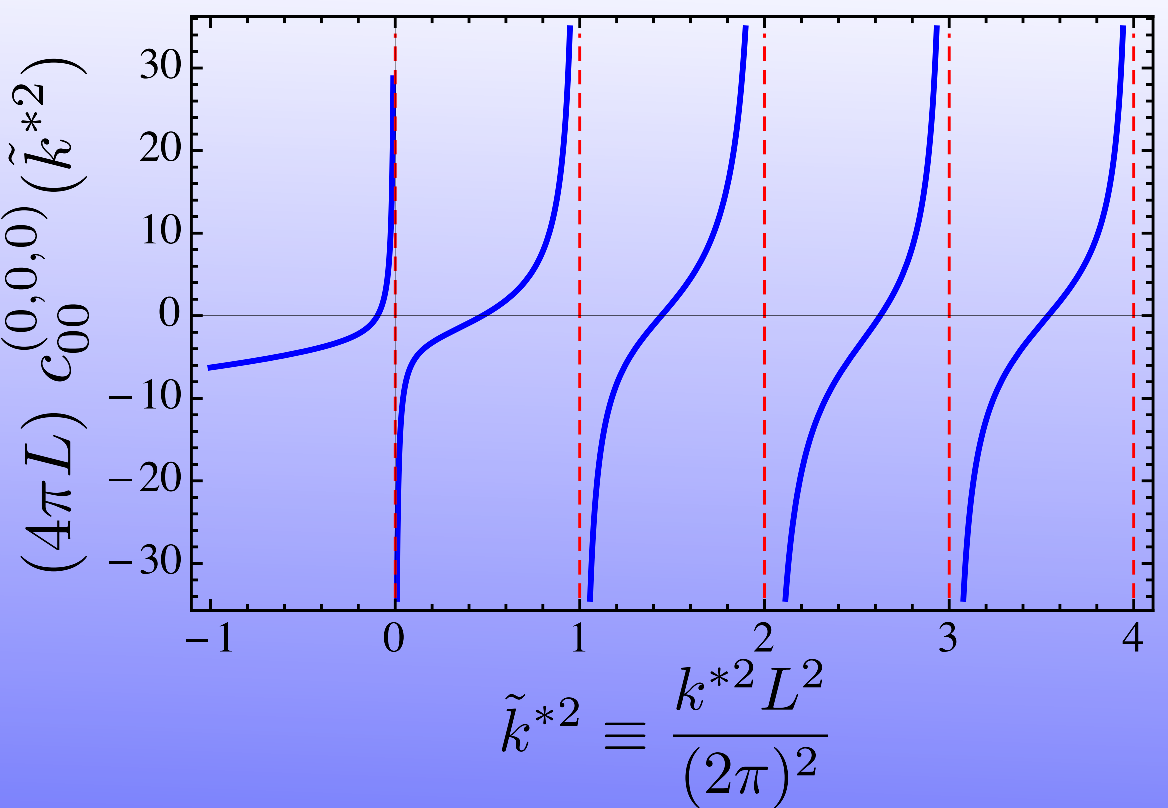
FINITE-VOLUME FUNCTION

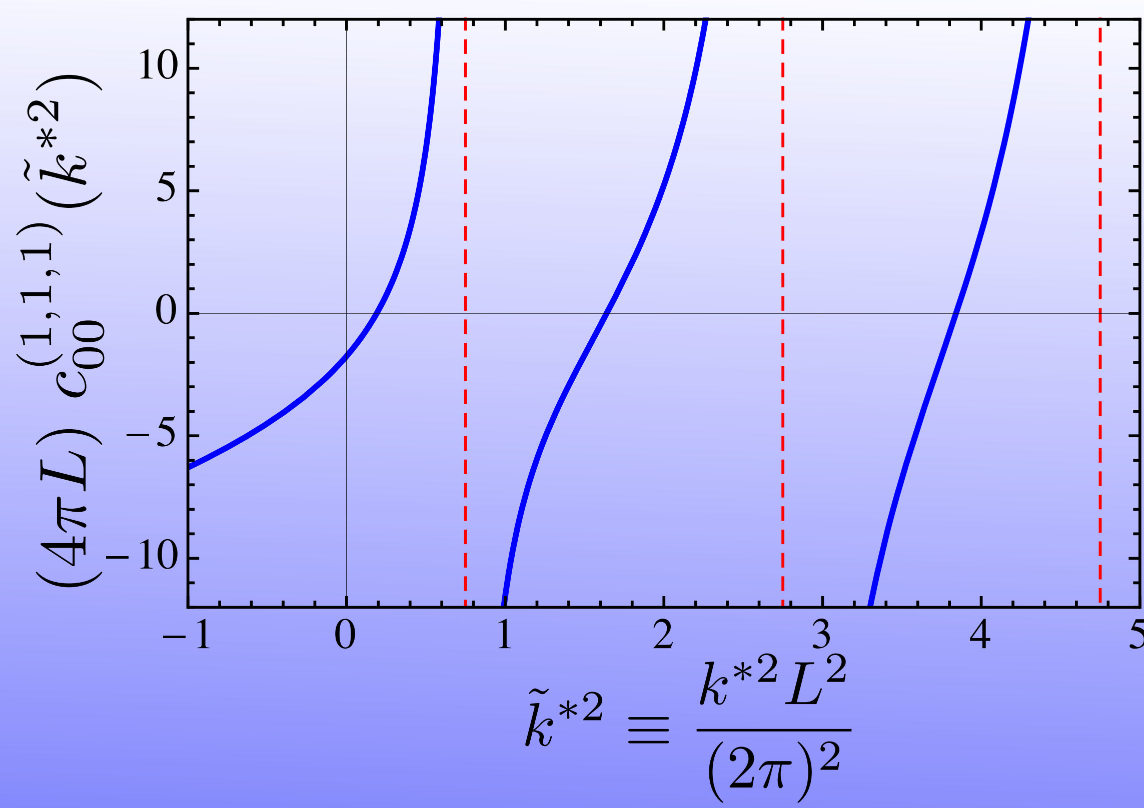
$$[\delta\mathcal{G}^V]_{JM_J,LS;J'M_J',L'S'} = i\eta\frac{k^*}{8\pi E^*}\delta_{SS'}\Big[\delta_{JJ'}\delta_{M_JM_J'}\delta_{LL'} + i\sum_{l,m}\frac{(4\pi)^{3/2}}{k^{*l+1}}c_{lm}^{\mathbf{d},\phi_1,\phi_2}(k^{*2};L)$$

$$\times\sum_{M_L,M_L',M_S}\langle JM_J|LM_L,SM_S\rangle\langle L'M_L',SM_S|J'M_J'\rangle\int d\Omega\, Y^*_{LM_L}Y^*_{lm}Y_{L'M_L'}\Big]$$

Briceno, ZD and Luu, Phys. Rev. D 88, 034502.

Ishizuka, PoS, LAT2009, 119 (2009).





$$c_{lm}^{\mathbf{d},\phi_1,\phi_2}(k^{*2};L) = \frac{\sqrt{4\pi}}{\gamma L^3}\left(\frac{2\pi}{L}\right)^{l-2}\mathcal{Z}_{lm}^{\mathbf{d},\phi_1,\phi_2}[1;(k^*L/2\pi)^2]$$

$$\mathbf{d} = \mathbf{P}L/2\pi$$

with $\mathcal{Z}_{lm}^{\mathbf{d},\phi_1,\phi_2}[s;x^2] = \sum_{\mathbf{r}\in\mathcal{P}_{\mathbf{d},\phi_1,\phi_2}}\frac{|\mathbf{r}|^l\ Y_{lm}(\mathbf{r})}{(\mathbf{r}^2-x^2)^s}$

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$$\times \sum_{M_L,M'_L,M_S}\langle JM_J|LM_L,SM_S\rangle\langle L'M'_L,SM_S|J'M'_J\rangle\int d\Omega\, Y^*_{LM_L}Y^*_{lm}Y_{L'M'_L}]$$

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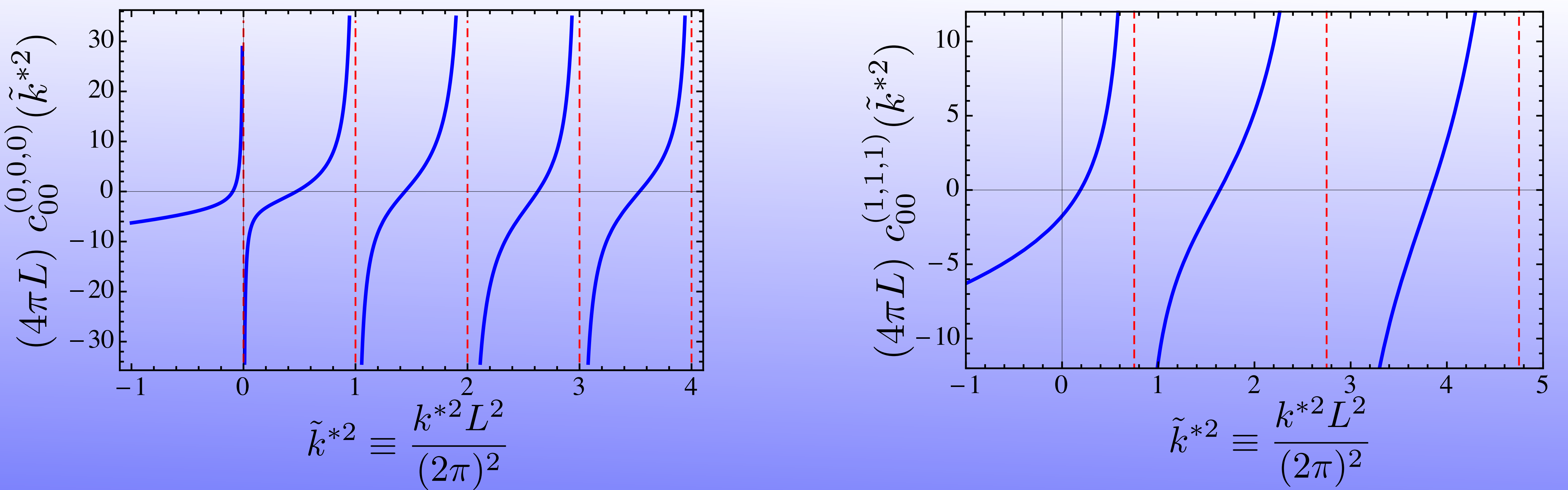

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ON-SHELL KINEMATICS AND FINITE-VOLUME FUNCTION

$\mathbf{r}^2$ depends on:

Volume and its shape, momentum of the CM, and the boundary conditions

Li and Liu, Phys.Lett. B587, 100 (2004).
Detmold W and Savage M J 2004 Nucl.Phys. A743 170–193.
Feng X, Li X and Liu C 2004 Phys.Rev. D70 014505.

$$\psi_{\mathrm{Lab}}(\mathbf{x}_1 + L\mathbf{n}_1, \mathbf{x}_2 + L\mathbf{n}_2) = e^{i\phi_1\cdot\mathbf{n}_1 + i\phi_2\cdot\mathbf{n}_2}\psi_{\mathrm{Lab}}(\mathbf{x}_1, \mathbf{x}_2)$$

$$\mathbf{n}_1, \mathbf{n}_2 \in \mathbb{Z}^3$$

$$\mathbf{r} = \frac{1}{L}\,\hat{\gamma}^{-1}\left[2\pi(\mathbf{n} - \alpha\mathbf{d}) - (\alpha - \frac{1}{2})(\phi_1 + \phi_2) + \frac{1}{2}(\phi_1 - \phi_2)\right]$$

Briceno, ZD, Luu and Savage, Phys. Rev. D 89, 074509.

$$\alpha = \frac{1}{2}\left(1 + \frac{m_1^2 - m_2^2}{E^{*2}}\right)$$

$$\det\left(\delta\mathcal{G}^V + \mathcal{M}^{-1}\right) = 0$$

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Kim, Sachrajda, Sharpe, et al, Nucl.Phys. B727 (2005) 218-243.

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ZD, Savage, Phys.Rev. D84 (2012) 114502.
Fu, Phys.Rev. D85 (2012) 014506.
Leskovec, Prevolesk, Phys.Rev. D85 (2012) 114507.

Bedaque, Phys.Lett. B593 (2004) 82-88.

Agadijanov, Meissner and Rusetsky, JHEP01. 103 (2014).

$$\det\left(\delta\mathcal{G}^V + \mathcal{M}^{-1}\right) = 0$$

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THE SPECTRUM OF THE DEUTERON IN A FINITE VOLUME

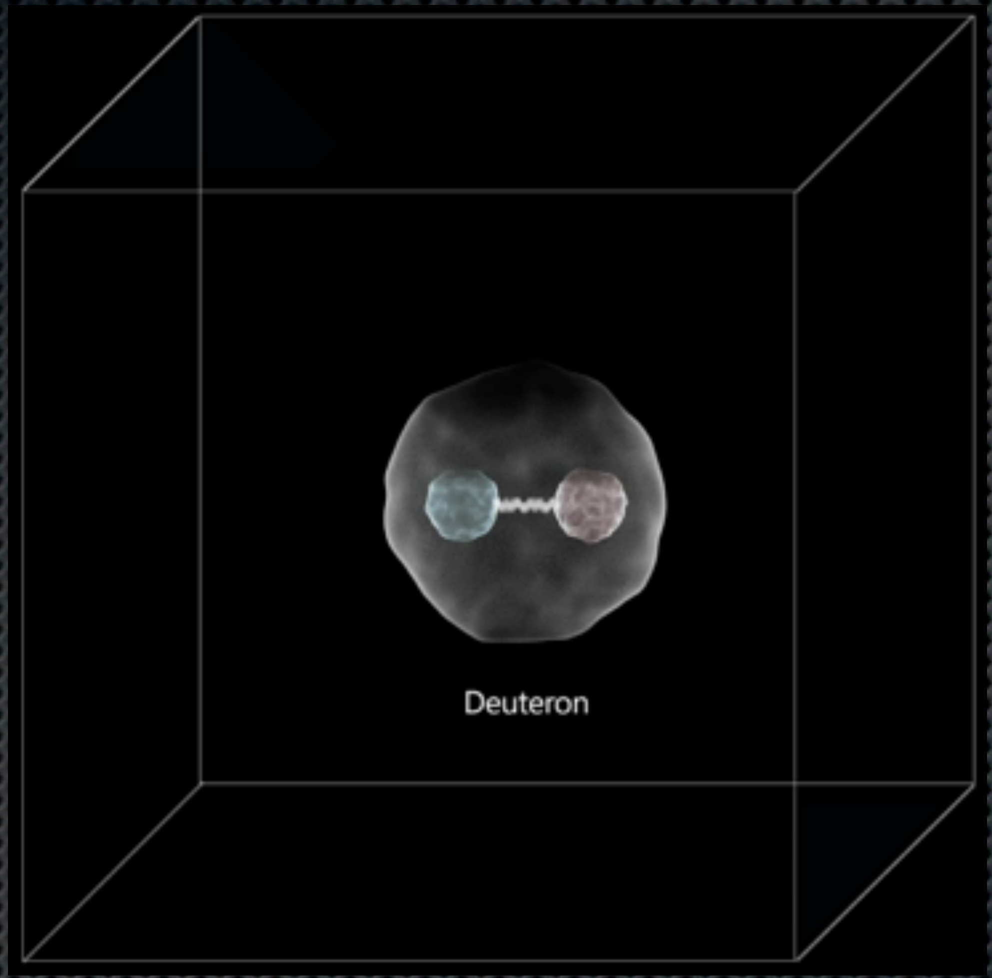


Image by J. Rodriguez.

1) Periodic boundary conditions and varying **the CM momentum**

See Briceno, ZD Luu and Savage, arXiv:1309.1556 (2013).

2) Varying the **twisted boundary conditions** with zero CM momentum

Briceno, ZD, Luu and Savage, Phys. Rev. D 89, 074509.

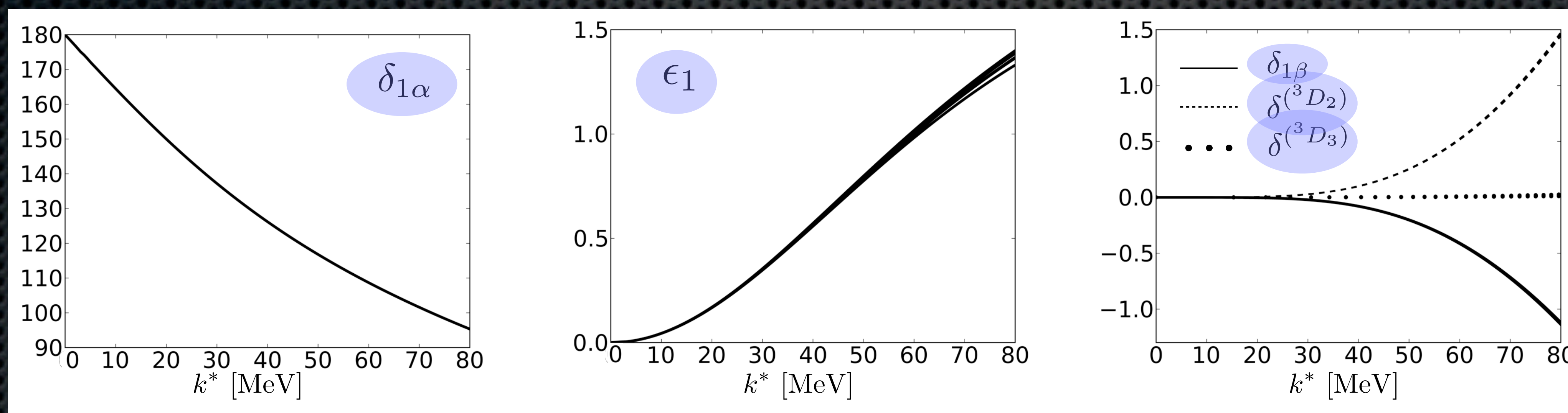
INPUT: SCATTERING PARAMETERS FROM PHENOMENOLOGICAL ANALYSES

$$S_{(J=1)} = \begin{pmatrix} \cos\epsilon_1 & -\sin\epsilon_1 \\ \sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix} \begin{pmatrix} e^{2i\delta_{1\alpha}} & 0 \\ 0 & e^{2i\delta_{1\beta}} \end{pmatrix} \begin{pmatrix} \cos\epsilon_1 & \sin\epsilon_1 \\ -\sin\epsilon_1 & \cos\epsilon_1 \end{pmatrix}$$

NN-OnLine 15 years 1994 - 2009

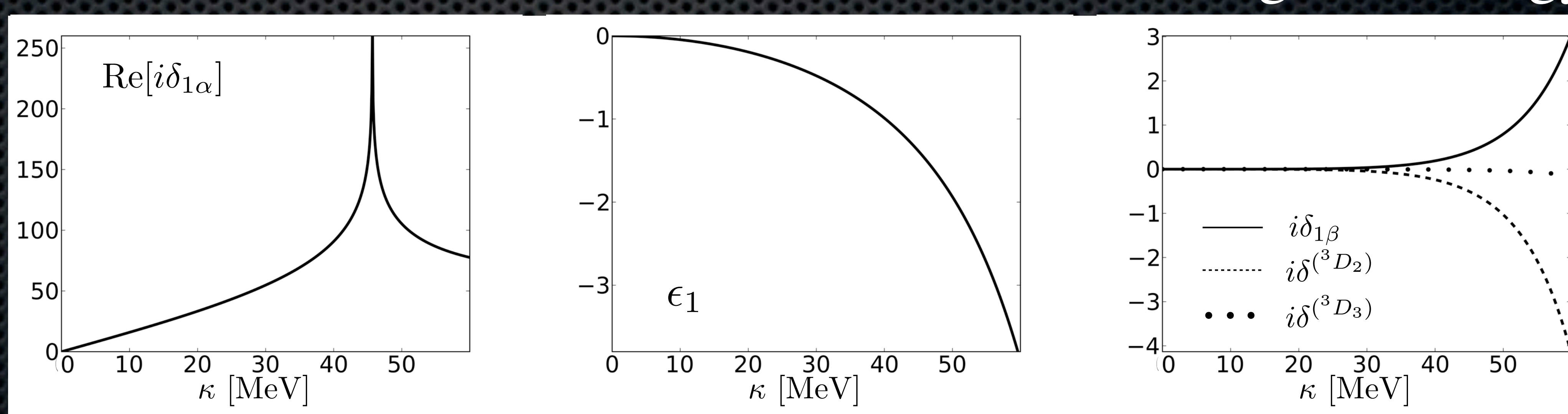
$$\mathcal{M} = \frac{4\pi}{M_N k^*}\frac{S-1}{2i}$$

Biedenharn, Blatt, Phys.Rev. 86, 399 (1952).
Biedenharn, Blatt, Phys.Rev. 93, 1387 (1954).



Positive energy

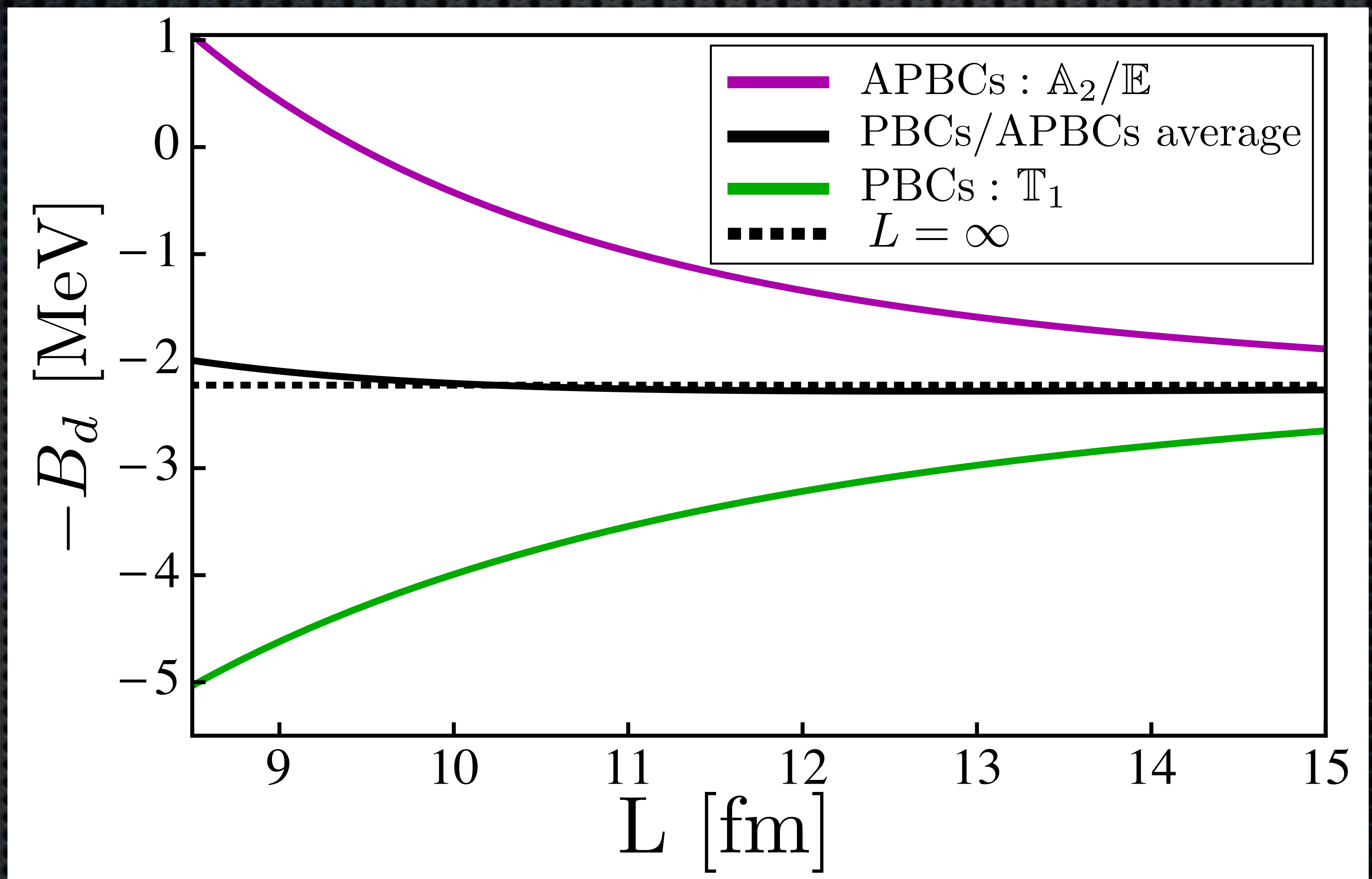
Negative energy



$$\kappa = -ik^*$$

DEUTERON SPECTRUM
TWISTED BOUNDARY CONDITIONS AND VOLUME IMPROVEMENT

$$\phi^p = -\phi^n \equiv \phi = (0,0,0)\ (\mathrm{PBCs}),\ (\pi,\pi,\pi)\ (\mathrm{APBCs}), \mathrm{and}\ (\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2})(\mathrm{i-PBCs})$$



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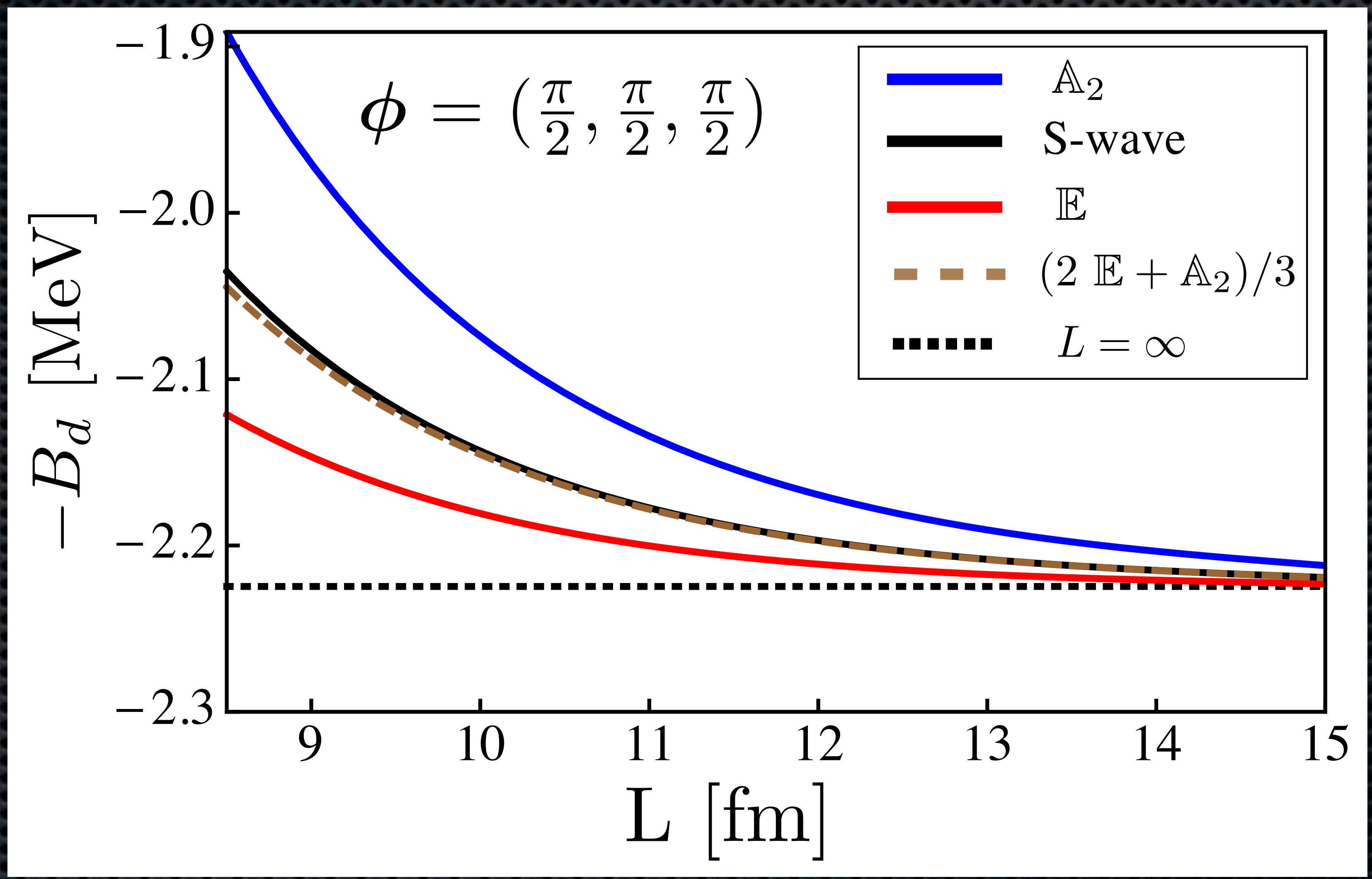

DEUTERON SPECTRUM

TWISTED BOUNDARY CONDITIONS AND VOLUME IMPROVEMENT

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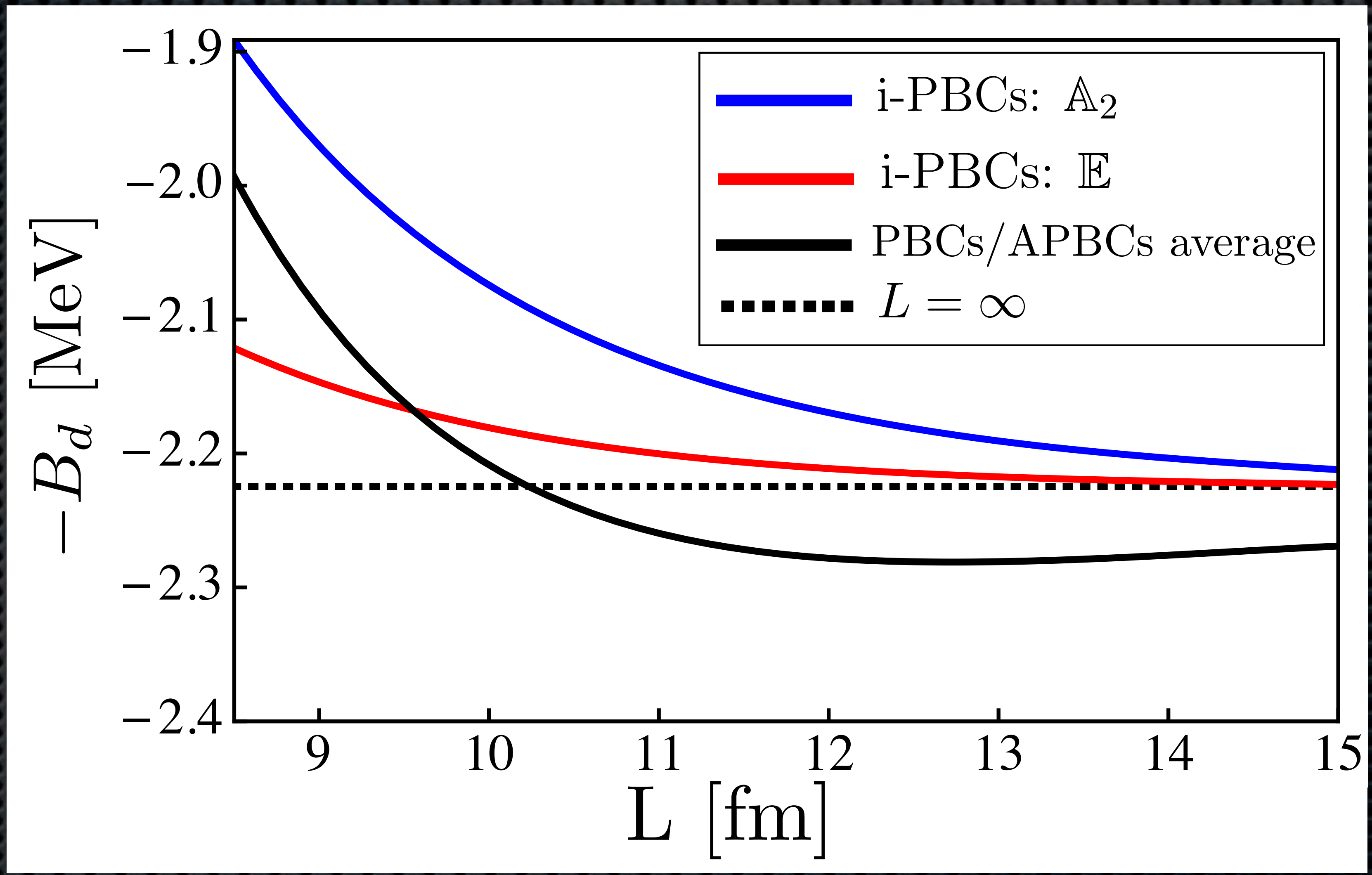

VOLUME-EFFECTS REDUCTION
AN ANALYTICAL INVESTIGATION

The S-wave limit

$$k^* \cot \delta^{(^3S_1)}|_{k^*=i\kappa} + \kappa = \sum_{\mathbf{n}\neq\mathbf{0}} e^{i(\alpha-\frac{1}{2})\mathbf{n}\cdot(\phi^p+\phi^n)} e^{-i\frac{1}{2}\mathbf{n}\cdot(\phi^p-\phi^n)} e^{i2\pi\alpha\mathbf{n}\cdot\mathbf{d}} \; \frac{e^{-|\hat{\gamma}\mathbf{n}|\kappa\mathrm{L}}}{|\hat{\gamma}\mathbf{n}|\mathrm{L}}$$

$\kappa = \sqrt{MB_d} \approx 45.7 \text{ MeV}$

VOLUME-EFFECTS REDUCTION
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$\kappa = \sqrt{MB_d} \approx 45.7$ MeV

$$\sum_{\mathbf{n}\neq \mathbf{0}} e^{-i\mathbf{n}\cdot\phi} \frac{e^{-|\mathbf{n}|\kappa\mathrm{L}}}{|\mathbf{n}|\mathrm{L}}$$

$\phi = (0,0,0)$	$\mathcal{O}(e^{-\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-\sqrt{2}\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-\sqrt{3}\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-2\kappa\mathrm{L}}/\mathrm{L})$
$\phi = (\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2})$	$\mathcal{O}(e^{-\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-\sqrt{2}\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-\sqrt{3}\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-2\kappa\mathrm{L}}/\mathrm{L})$
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Average of PBC and APBC	$\mathcal{O}(e^{-\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-\sqrt{2}\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-\sqrt{3}\kappa\mathrm{L}}/\mathrm{L})$	$\mathcal{O}(e^{-2\kappa\mathrm{L}}/\mathrm{L})$

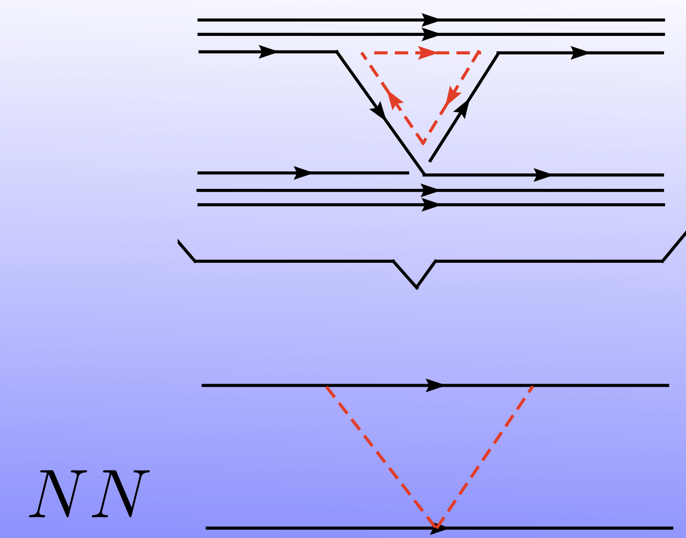
PARTIALLY TWISTED BOUNDARY CONDITIONS

NN sector

Bedaque and Chen, Phys.lett. B616, 208 (2005).

* No s-channel diagram with intermediate mesons containing a sea quark.

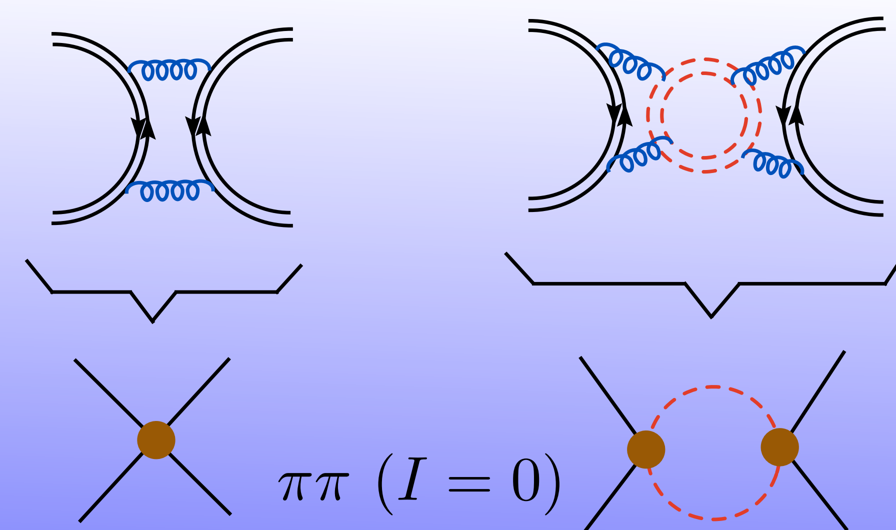
* The quantization condition valid up to exponential corrections in the masses of exchanged bosons.



Two-meson sector

* s-channel diagrams with intermediate mesons containing a sea quark exist.

* Quantization condition? Intricate cancellations in partially quenched QCD?

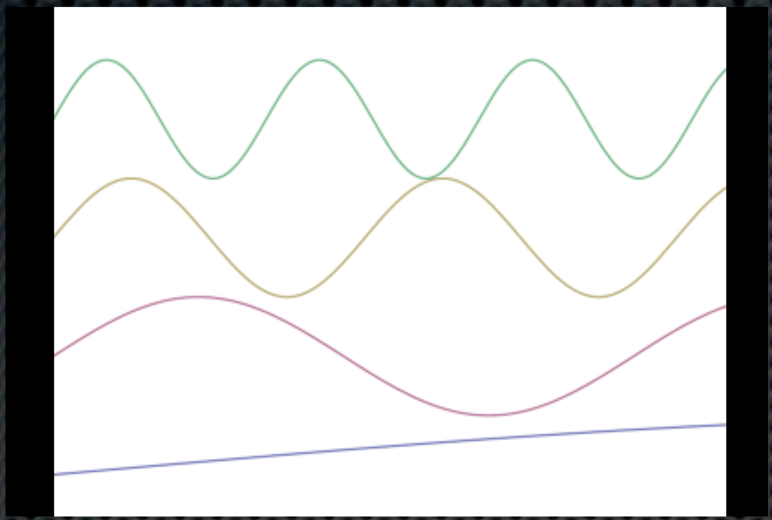


Agadijanov, Meissner and Rusetsky, JHEP01. 103 (2014).

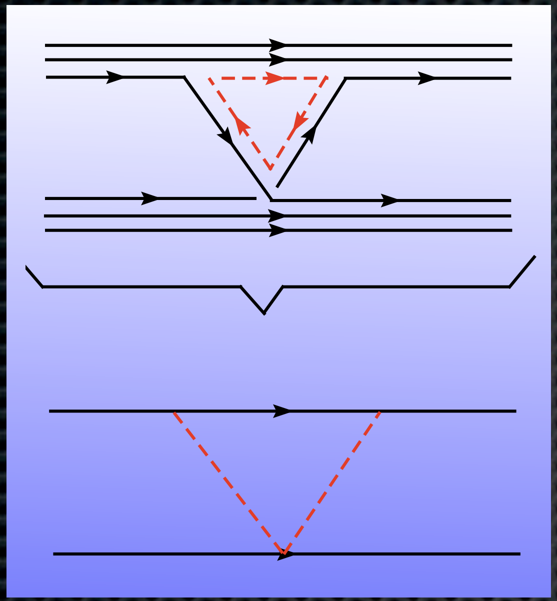
Figures by Bedaque and Chen.

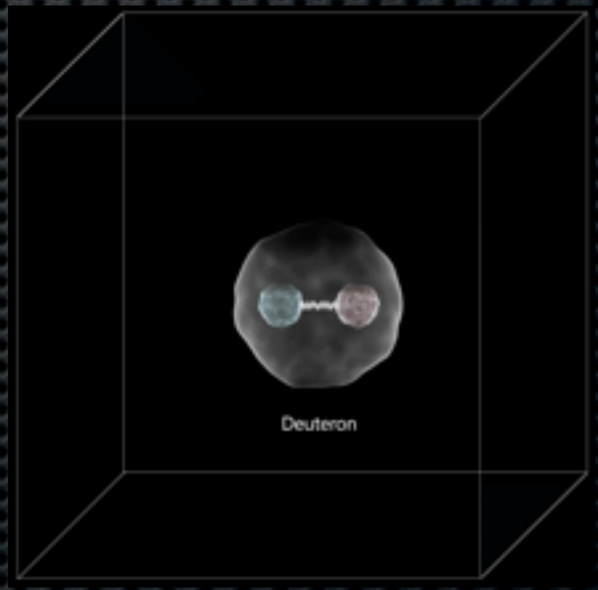
SUMMARY AND CONCLUSIONS

A general quantization condition for the two-hadron systems in a finite volume with twisted boundary conditions is presented.

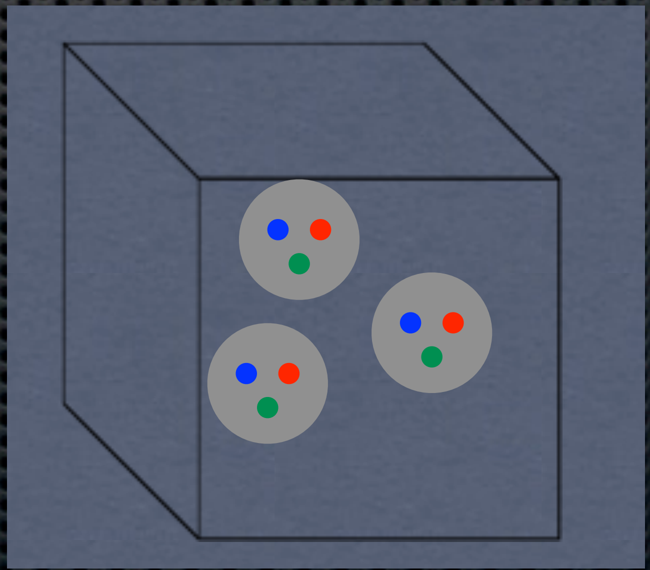


Calculations with optimized TBCs make the extraction of binding energies feasible at smaller volumes.





The quantization condition holds for partial twisting scenario in the two-nucleon sector and makes this approach computationally plausible.



Volume improvement in three (and higher)-particle bound states energies using optimized boundary conditions?

THANK YOU

BACKUP SLIDES

ON-SHELL KINEMATICS AND FINITE-VOLUME FUNCTION

$$\psi_{\rm Lab}(\mathbf{x}_1 + L\mathbf{n}_1, \mathbf{x}_2 + L\mathbf{n}_2) = e^{i\phi_1\cdot\mathbf{n}_1 + i\phi_2\cdot\mathbf{n}_2}\psi_{\rm Lab}(\mathbf{x}_1, \mathbf{x}_2)$$

(1) $$\psi_{\rm Lab}(x_1, x_2) = e^{-iEX^0 + i\mathbf{P}\cdot\mathbf{X}}\varphi_{\rm Lab}(0, \mathbf{x}_1 - \mathbf{x}_2)$$

$$X = \alpha x_1 + (1-\alpha)x_2$$

$$\alpha = \frac{1}{2}\left(1 + \frac{m_1^2 - m_2^2}{E^{*2}}\right)$$

Luscher, Nucl.Phys. B354 (1991) 531-578.
Rummukainen, Gottlieb, Nucl.Phys. B450 (1995) 397-436.

(2) $$\varphi_{\rm Lab}(0, \mathbf{x_1} - \mathbf{x_2}) = \varphi_{\rm CM}(\hat{\gamma}(\mathbf{x}_1 - \mathbf{x}_2))$$

$$\hat{\gamma}\mathbf{x} = \gamma\mathbf{x}_{||} + \mathbf{x}_{\perp}$$

Bour, et al, Phys.Rev. D84 (2011) 091503.
ZD, Savage, Phys.Rev. D84 (2012) 114502.

(3) $$e^{i\alpha\mathbf{P}\cdot(\mathbf{n}_1 - \mathbf{n}_2)L + i\mathbf{P}\cdot\mathbf{n}_2 L}\varphi_{\rm CM}(\mathbf{y}^* + \hat{\gamma}(\mathbf{n}_1 - \mathbf{n}_2)L) = e^{i\phi_1\cdot\mathbf{n}_1 + i\phi_2\cdot\mathbf{n}_2}\varphi_{\rm CM}(\mathbf{y}^*)$$

$$\mathbf{y}^* = \mathbf{x}_1^* - \mathbf{x}_2^*$$

$$\mathbf{r} = \frac{1}{L}\,\hat{\gamma}^{-1}\left[2\pi(\mathbf{n} - \alpha\mathbf{d}) - (\alpha - \frac{1}{2})(\phi_1 + \phi_2) + \frac{1}{2}(\phi_1 - \phi_2)\right]$$

Briceno, ZD, Luu and Savage, Phys. Rev. D 89, 074509.

See also: R. Briceno, Phys. Rev. D 89, 074507.