Initial Nucleon Structure Results with Chiral Quarks at the Physical Point

Sergey Syritsyn (RIKEN/BNL Research Center)
for RBC & LHP Collaborations:
T. Blum, M. Engelhardt, J. Green, T.Izubuchi, C. Jung, S. Krieg,
M. Lin, S. Meinel, J. Negele, S. Ohta, A. Pochinsky, E. Shintani

LATTICE 2014
Columbia University, New York, USA,
June 23-28, 2014

BERKELEY LAB
Lawrence Berkeley National Laboratory

BROOKHAVEN
NATIONAL LABORATORY

RBRC

Outline

- Techniques: CG deflation, All-Mode Averaging
- Vector Form Factors
- Axial Vector Form Factors
- Quark Momentum Fraction
- Summary & Outlook

Dynamical Möbius (DW) Fermions

- RBC/UKQCD-generated ensemble :

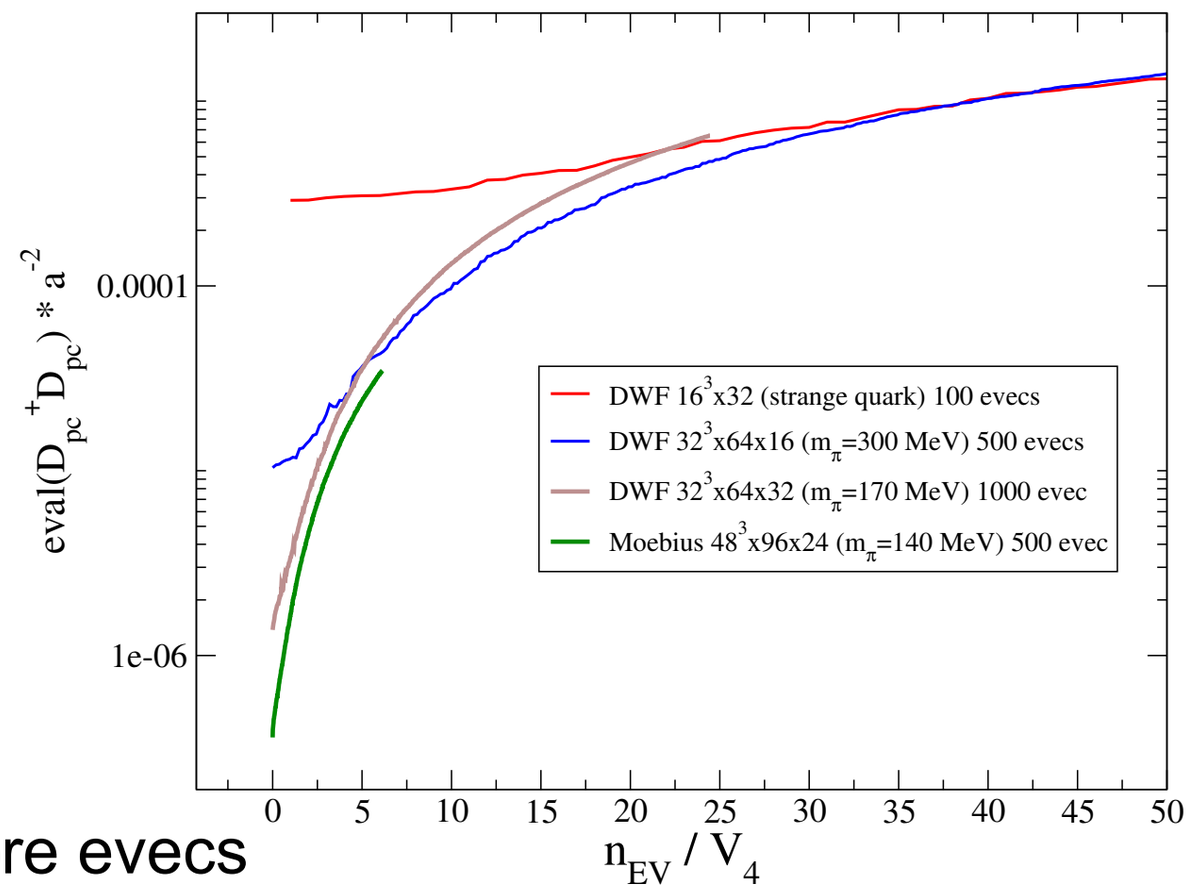
  $a \approx 0.113 \text{ fm} = (1.75 \text{ GeV})^{-1} \quad 48^3 \times 96 = (5.4 \text{ fm})^3 \times 10.8 \text{ fm}, \quad m_\pi L_x \approx 3.84$

- 2+1 dynamical Möbius fermions [R.Brower et al, hep-lat/0409118] $L_5 = 24$

Deflation: $M^\dagger_{\text{eopc}} M_{\text{eopc}}$

- deflate between $m_{u,d}$ and m_s
- PARPACK with n=200 poly.acc.
- condition number x 1/100,

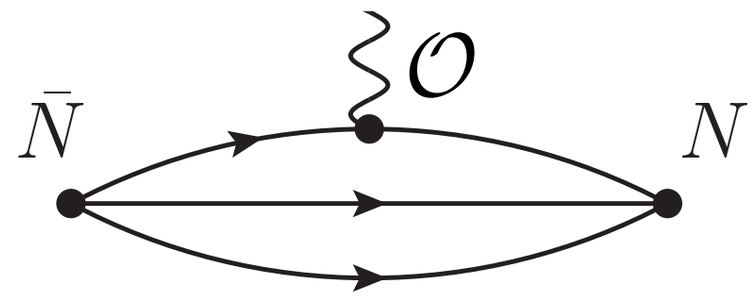
  CG convergence speed x10
- 1 evec=11.39GB; 500 evecs=5.7TB
- Limited by memory & IO; want x3 more evecs



Nucleon Matrix Elements

- Only connected quark contractions

$$C^{\mathcal{O}}_{3pt} =$$



- Matrix elements from C_{3pt}/C_{2pt} ratio

$$R_{\mathcal{O}}(T,\tau;P,P') = \frac{C_{\mathcal{O}}(T,\tau;P,P')}{\sqrt{C_{2\mathrm{pt}}(T,P)C_{2\mathrm{pt}}(T,P')}} \cdot \sqrt{\frac{C_{2\mathrm{pt}}(T-\tau,P)C_{2\mathrm{pt}}(\tau,P')}{C_{2\mathrm{pt}}(T-\tau,P')C_{2\mathrm{pt}}(\tau,P)}}$$

$$\xrightarrow[T,\tau,(T-\tau)\to\infty]{} \langle P'|\mathcal{O}|P\rangle$$

- Employ summation method to "demote" transitional excited state contributions $\mathcal{O}(e^{-\Delta E\cdot\frac{T}{2}}) \longrightarrow \mathcal{O}(e^{-\Delta E\cdot T})$

$$\sum_{\tau}^{T} R_{\mathcal{O}}(T,\tau) = \langle P'|\mathcal{O}|P\rangle \cdot T + O(e^{-\Delta E\cdot T})$$

Improved Stoch.Estimation: All-Mode Averaging

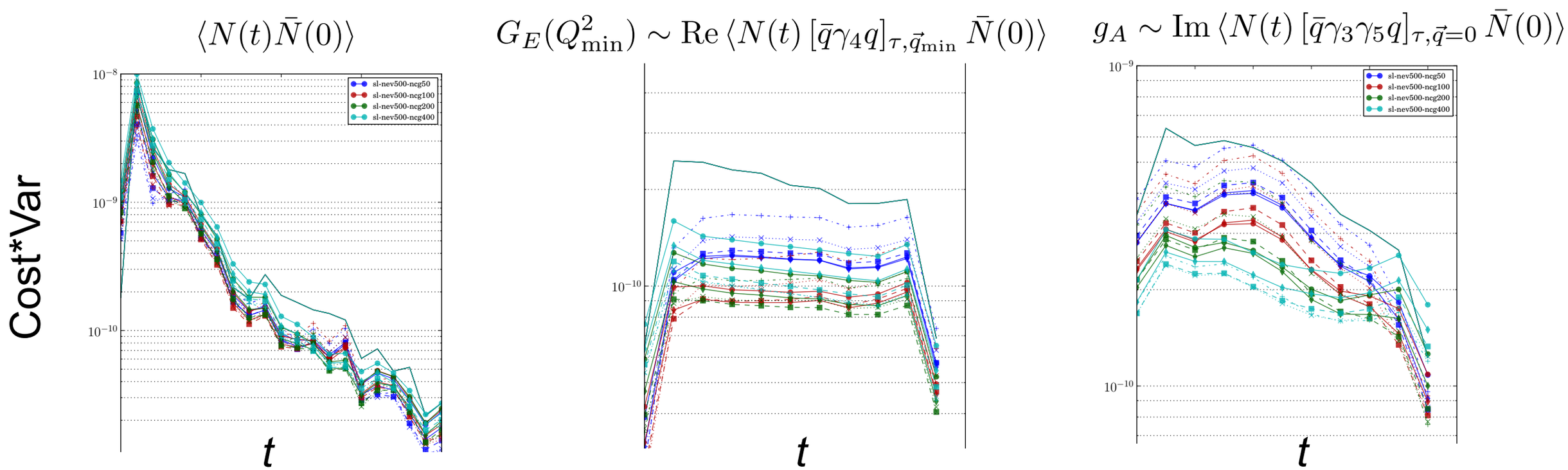
- All-mode averaging [T.Blum et al, PRD88:094503 (arXiv:1208.4349)] :

$$\langle \mathcal{O} \rangle_{\rm imp} = \langle \mathcal{O}_{\rm approx} \rangle_{N_{\rm approx}} + \langle \big(\mathcal{O}_{\rm exact} - \mathcal{O}_{\rm approx} \big) \rangle_{N_{\rm exact}}$$

$$(\delta \mathcal{O}_{\rm imp})^2 \sim \frac{1}{N_{\rm approx}} {\rm Var}\{\mathcal{O}_{\rm approx}\} + \frac{1}{N_{\rm exact}} {\rm Var}\{ \big(\underbrace{\mathcal{O}_{\rm exact} - \mathcal{O}_{\rm approx}}_{{\rm bias}\ \Delta\mathcal{O}} \big) \} \quad (*)$$

- Tune approximation (n^{CG}) and (N_{approx}/N_{exact}) for optimal cost

$${\rm Cost}_{\rm imp} \cdot \big(\delta \mathcal{O}_{\rm imp}\big)^2 \sim \big(1 + \frac{n^{\rm CG}_{\rm approx}}{n^{\rm CG}_{\rm exact}} \cdot \frac{N_{\rm approx}}{N_{\rm exact}}\big) \cdot \Big[{\rm Var}\{\Delta\mathcal{O}\} + \frac{N_{\rm exact}}{N_{\rm approx}} {\rm Var}\{\mathcal{O}_{\rm approx}\} \Big]$$

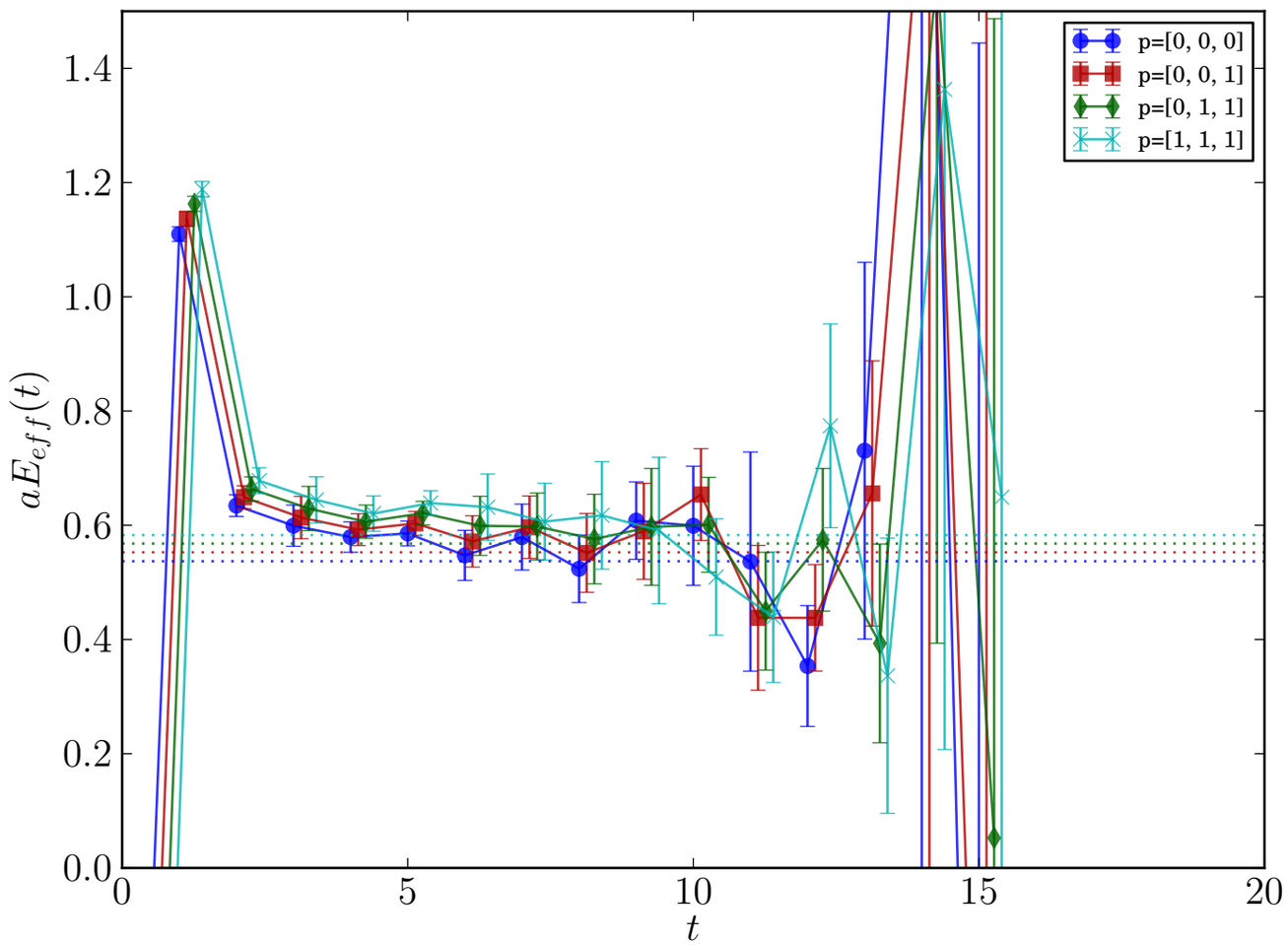


- select n^{CG}=400, N_{approx}/N_{exact}=32 ; x2.5 - x3 noise reduction

Initial Results: Effective Mass

- 10 gauge configs, spaced to span 1/2 of the ensemble
- 320 approx("sloppy") samples, 10 exact (bias) samples
- Gaussian quarks source tuned to approximate nucleon ground state

$$aE_{\rm eff}(t) = \langle \log \frac{C_{\rm 2pt}(t)}{C_{\rm 2pt}(t+1)} \rangle$$



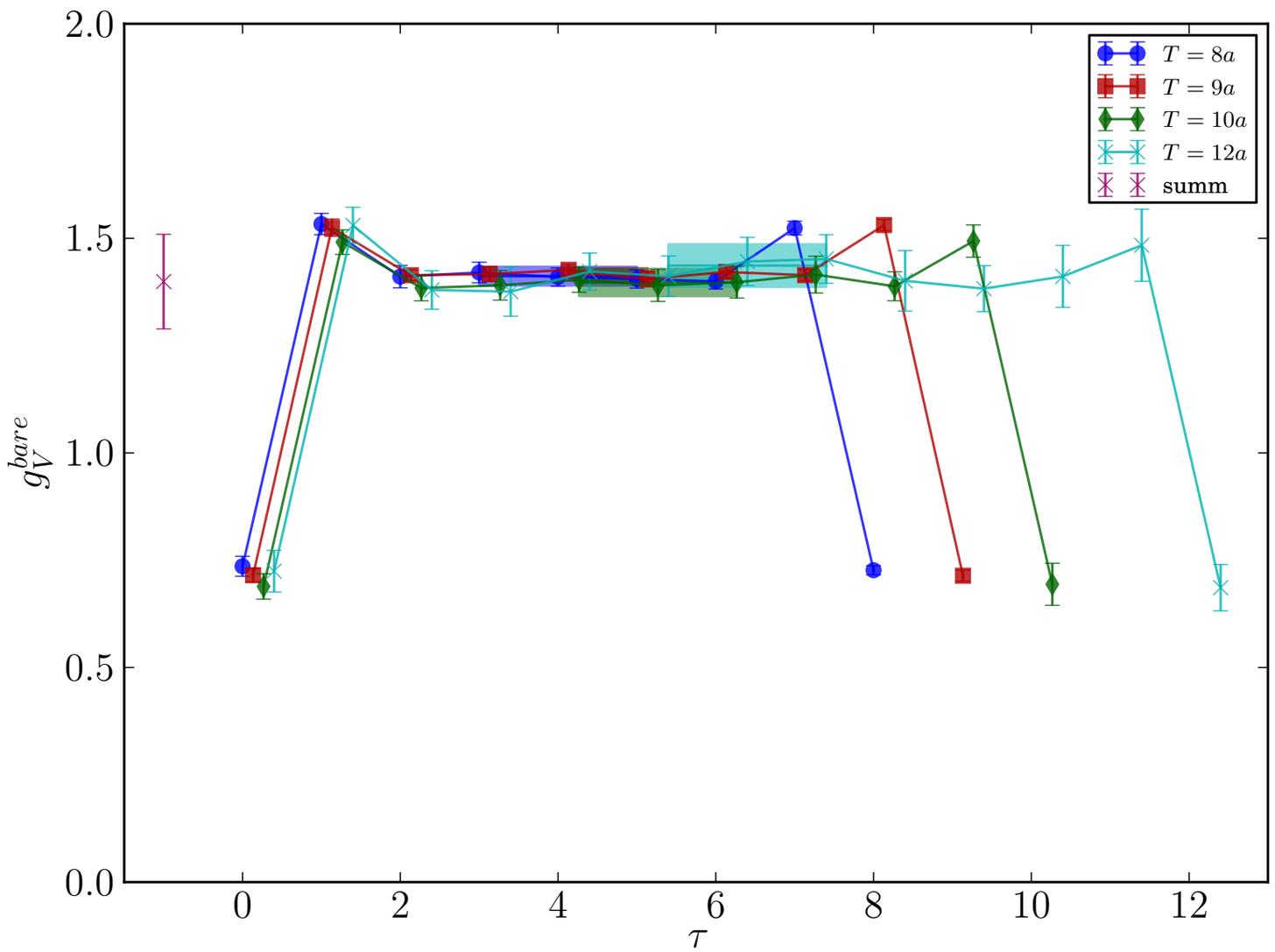
Vector Charge g_V

- Source-Sink separations T=8a,9a,10a,12a
- Employ summation method to “demote” transitional excited states

$$\mathcal{O}\left(e^{-\Delta E\cdot\frac{T}{2}}\right) \longrightarrow \mathcal{O}\left(e^{-\Delta E\cdot T}\right)$$

Isovector *(u-d)* vector charge

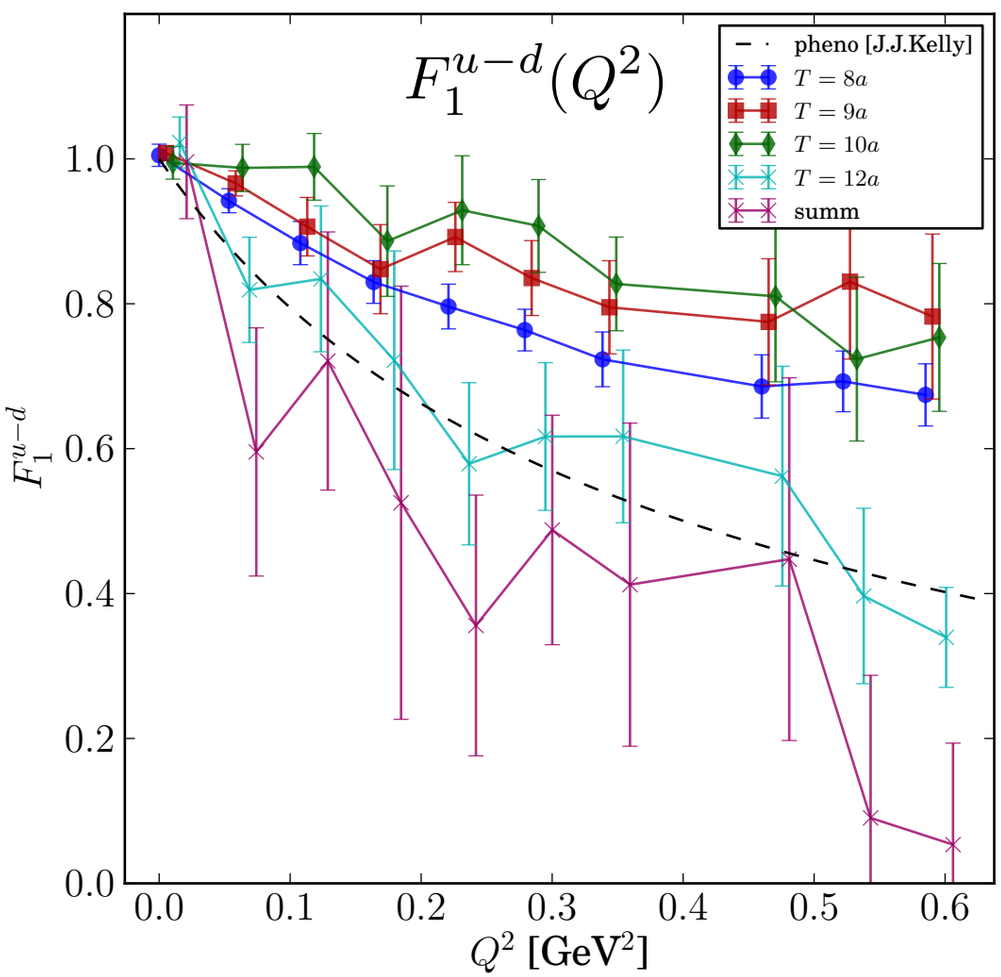
$$g_V^{\text{bare}} = \langle N| \int d^3x\, \bar{q}\gamma_4 q|N\rangle$$

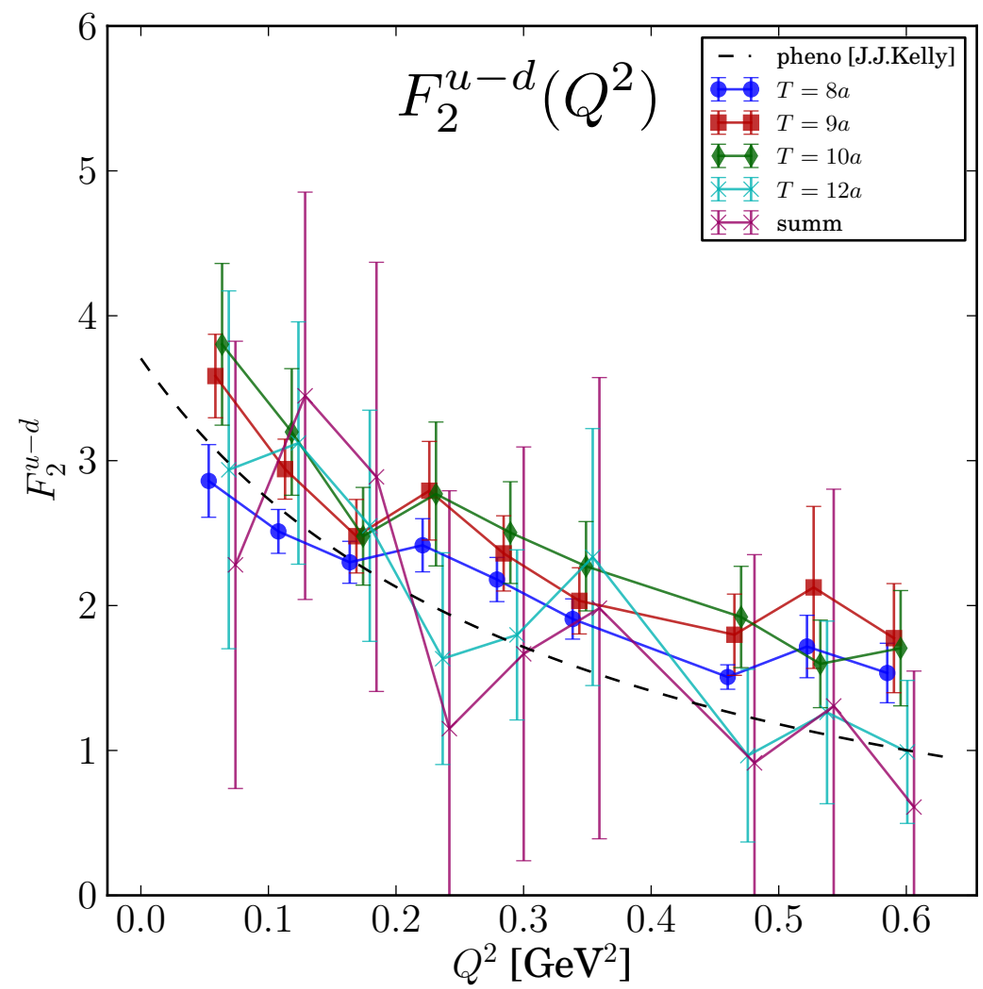


Vector (u-d) Form Factors $F_{1,2}$

- Comparison to phenomenology [J.J.Kelly, PRC70:068202 (2004)]

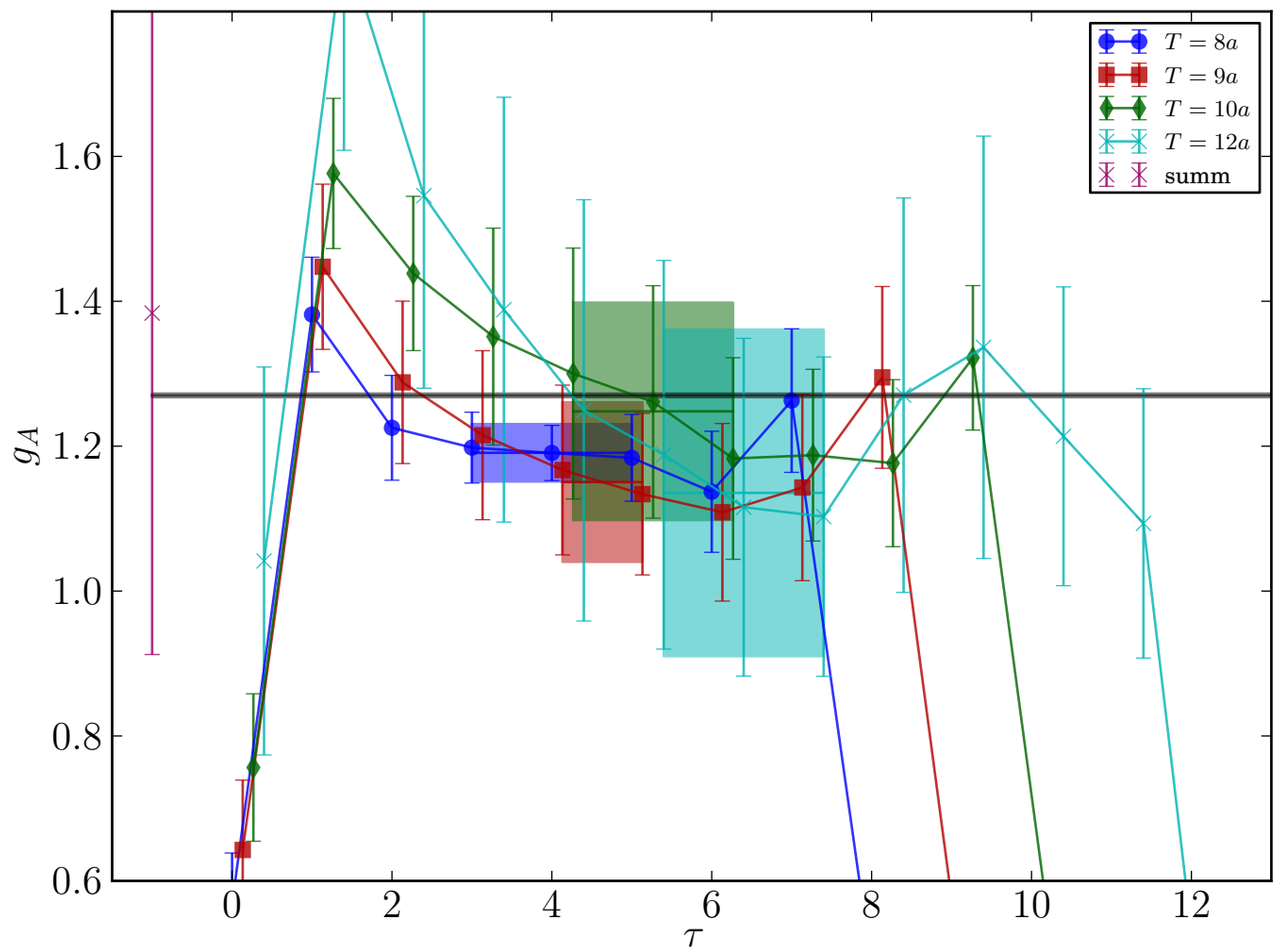
$$\langle P+q|\,\bar{q}\gamma^{\mu}q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[\,F_1(Q^2)\,\gamma^{\mu} + F_2\,(Q^2)\,\frac{i\sigma^{\mu\nu}q_{\nu}}{2M_N}\Big]U_P$$





Axial Charge g_A

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = g_A\ \bar{u}_p\gamma^\mu\gamma^5 u_p\ ,$$



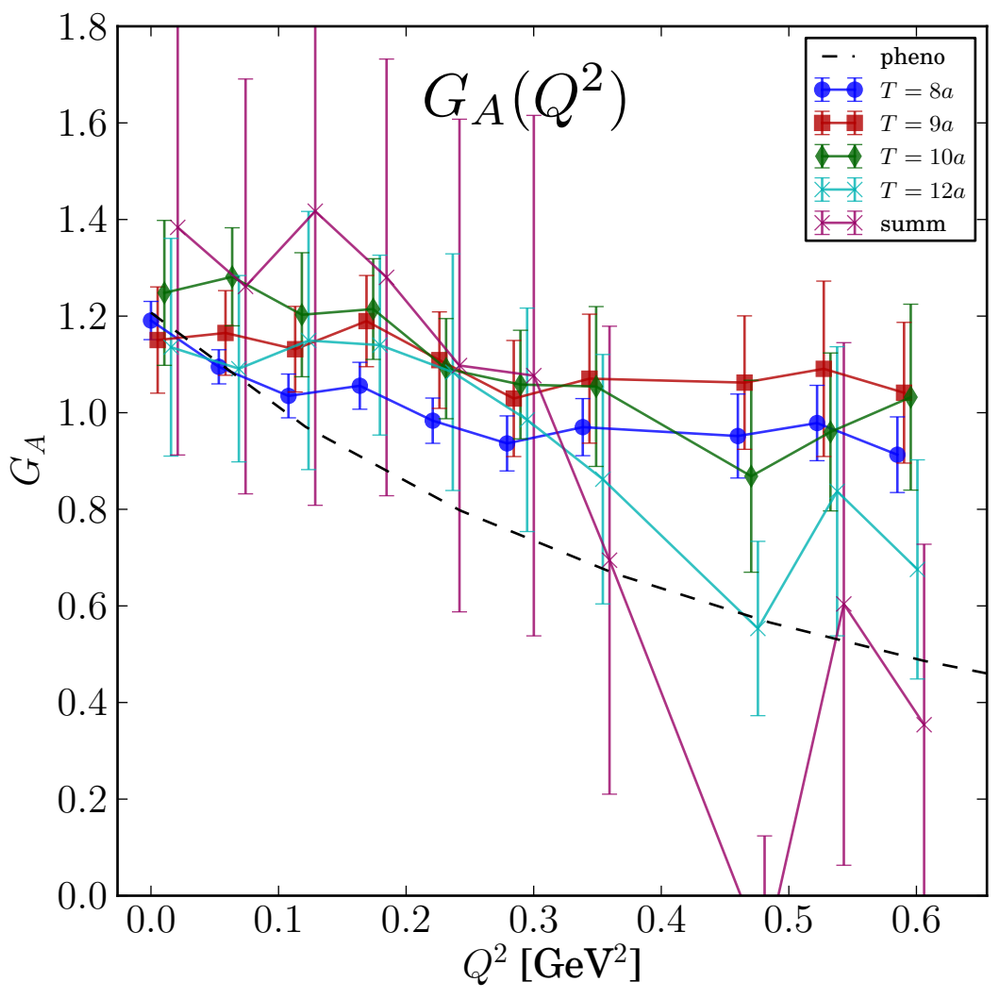
- No excited state contribution seen (yet)

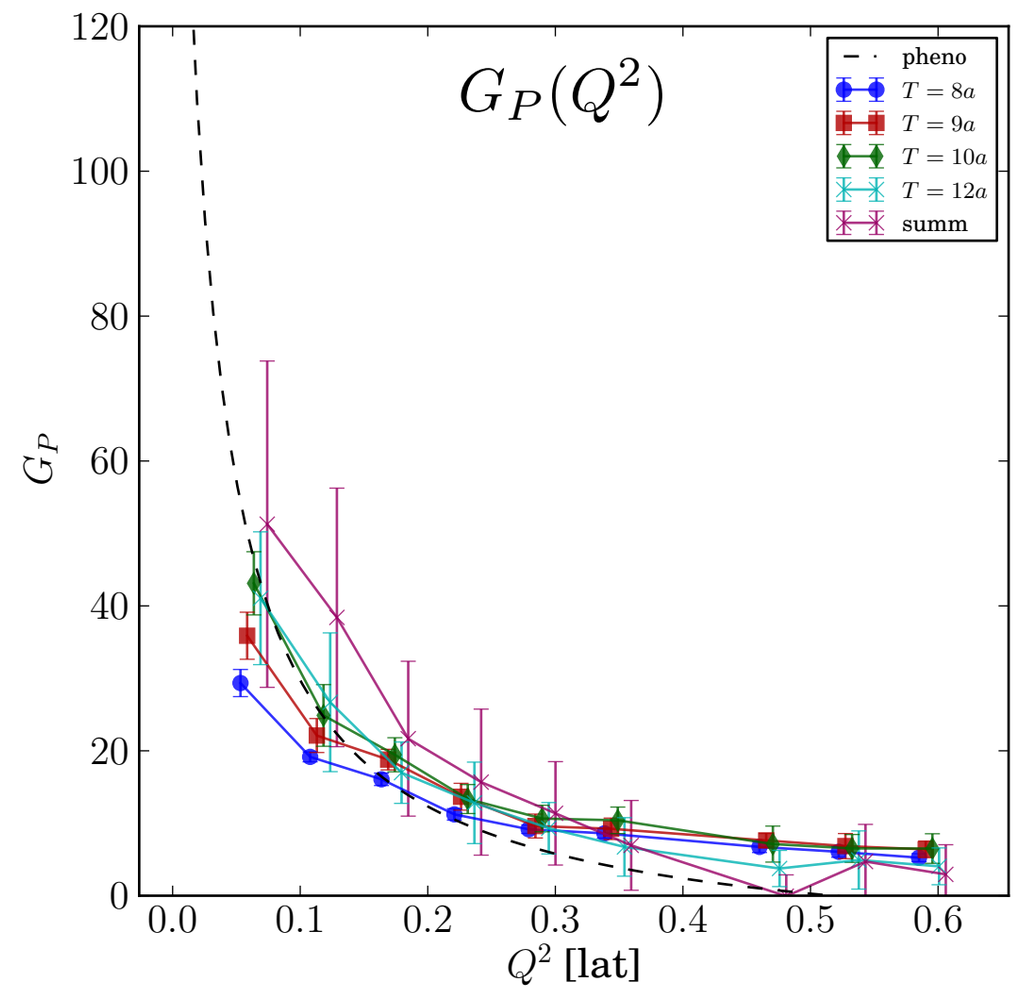
Axial Form Factors

- Experiments with electroweak probes:

  ν scattering, $\pi^{\pm}$ production, e^- & μ^- capture

$$\langle P+q|\,\bar{q}\gamma^{\mu}\gamma^{5}q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[\,G_A(Q^2)\,\gamma^{\mu}\gamma^{5} + G_P(Q^2)\,\frac{\gamma^5 q^{\mu}}{2M_N}\Big]U_P$$





$$G_A(Q^2) = \frac{g_A}{(1+\frac{Q^2}{M_A^2})^2}$$

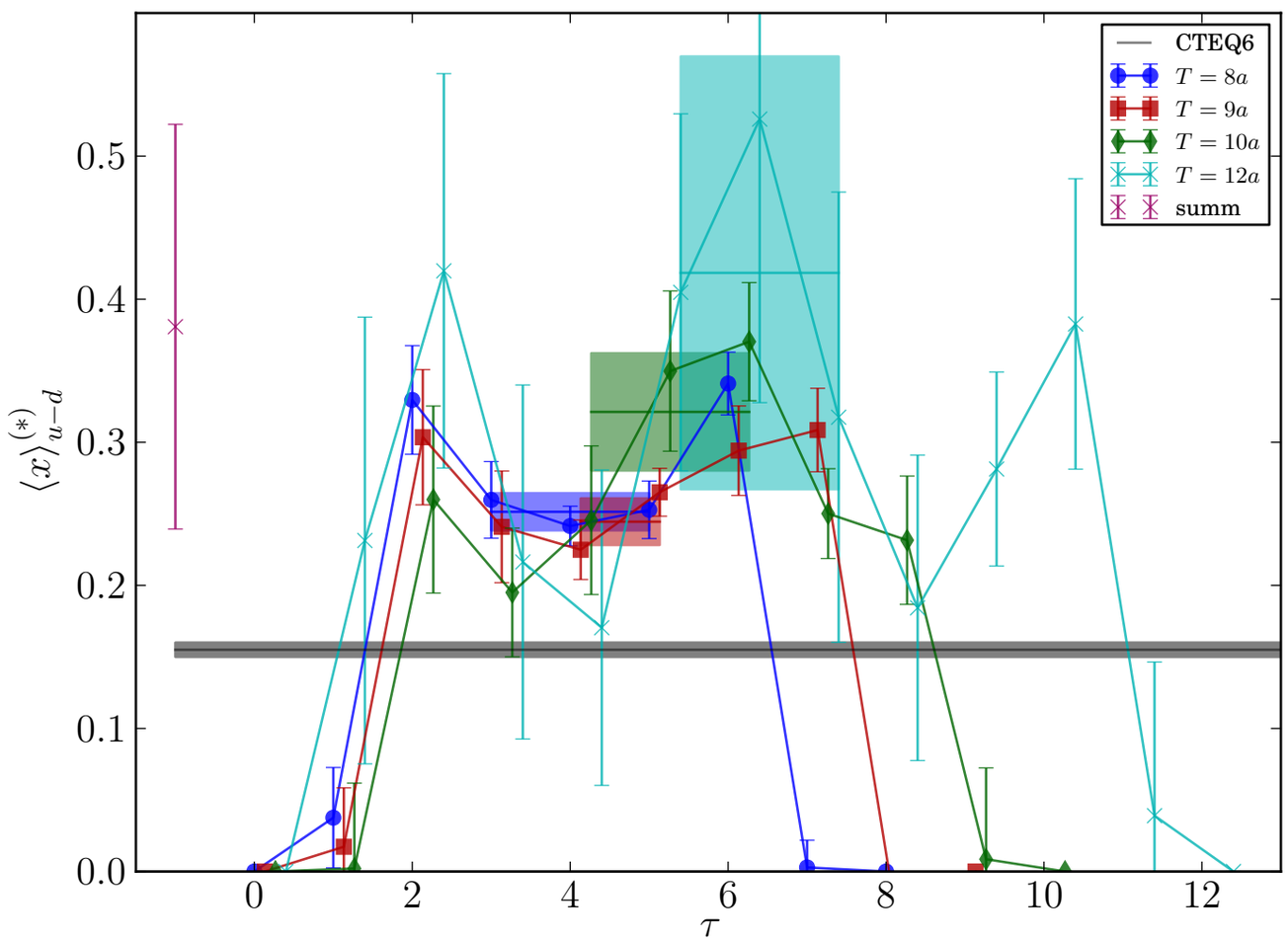
$$G_P(Q^2) = \frac{4M_N g_{\pi N} F_\pi}{m_\pi^2 + Q^2} - \frac{2}{3} g_A M_N^2 \langle r_A^2 \rangle$$

Quark Momentum Fraction

$$\langle x \rangle_{u-d} = \int dx\, x \left(u(x) + \bar{u}(x) - d(x) - \bar{d}(x) \right)$$

Phenomenology: $\langle x \rangle_{u-d}^{\overline{MS}(2\ \mathrm{GeV})} = 0.155(5)$

$$\langle N(p) | \bar{q} \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} q | p \rangle = \langle x \rangle_q\ \bar{u}_p \gamma_{\{\mu} p_{\nu\}} u_p$$



- (*) renormalization from 24^3 lattice with the same action & lattice spacing

Summary & Outlook

Summary

- Initial results (~1/4th - 1/6th of statistics) with chiral quarks at the physical point
- Promising results for vector, axial vector form factors
- Excited states clearly present in G_P

Outlook

- Increase statistics x6 in 2014-2015
- Improve exc.state analysis once statistics is sufficient for reliable fits
- Explore other approximations, e.g Möbius with shortened L5
- Disconnected diagrams with hierarchical probing (S.Meinel)