Kaon semileptonic vector form factor with $N_f = 2 + 1 + 1$ Twisted Mass fermions

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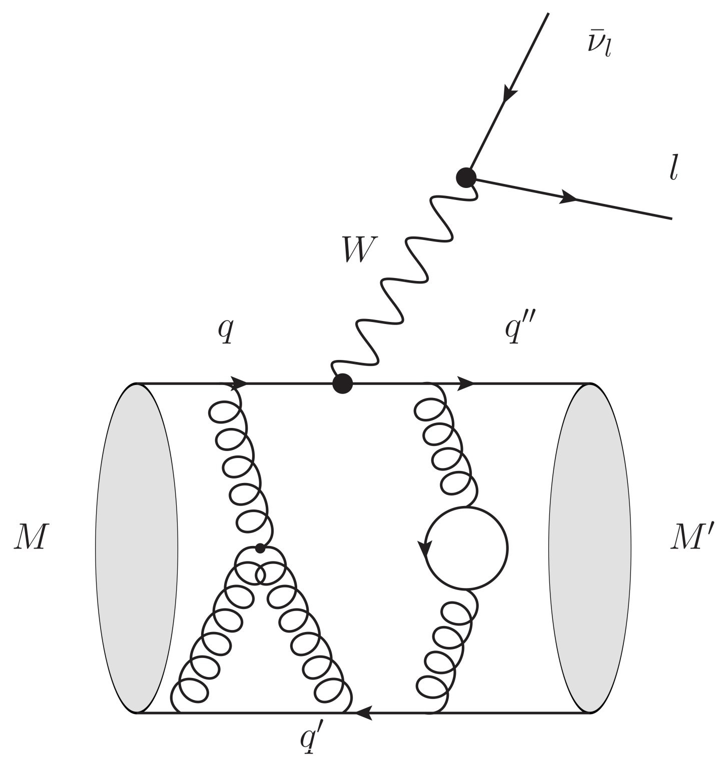
Motivation

Precise determination of the CKM matrix element is important to test the SM



Semileptonic decay rate :

$$\Gamma(K \to \pi l \nu) \propto \left(|V_{us}| f_+(0)\right)^2$$

Experimental average

$$|V_{us}| f_+(0) = 0.2163(5)$$

Extracting f+(0) from Lattice QCD allows us to estimate $|V_{us}|$

Outline

- Simulation Details
- General strategy
- Plateaux
- Interpolation of f_+ and f_0 to q^2=0
- Chiral and continuum extrapolation of f(0)
- Results and CKM unitarity tests
- Summary and Conclusions

Simulation Details

Something on the action:

- Wilson Twisted Mass action at maximal twist with Nf=2+1+1 sea quarks
- Osterwalder-Seiler valence quark action
- Iwasaki gluon action

Simulation Details

PRA027
"QCD simulations for flavor physics in the Standard Model and beyond"
(35 millions of core-hours at the BG/P system in Julich from December 2010 to March 2011)

Details of the ensembles used in this N_f =2+1+1 analysis

The valence light quark mass is put equal to the sea quark mass

ensemble	β	V/a^4	$a\mu_{sea} = a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$	N_{cfg}	$a\mu_s$	$a\mu_c$
$A30.32$	1.90	$32^3 \times 64$	0.0030	0.15	0.19	150	0.0145,	0.1800, 0.2200,
$A40.32$			0.0040			90	0.0185,	0.2600, 0.3000,
$A50.32$			0.0050			150	0.0225	0.3600, 0.4400
$A40.24$	1.90	$24^3 \times 48$	0.0040	0.15	0.19	150		
$A60.24$			0.0060			150		
$A80.24$			0.0080			150		
$A100.24$			0.0100			150		
$B25.32$	1.95	$32^3 \times 64$	0.0025	0.135	0.170	150	0.0141,	0.1750, 0.2140,
$B35.32$			0.0035			150	0.0180,	0.2530, 0.2920,
$B55.32$			0.0055			150	0.0219	0.3510, 0.4290
$B75.32$			0.0075			75		
$B85.24$	1.95	$24^3 \times 48$	0.0085	0.135	0.170	150		
$D15.48$	2.10	$48^3 \times 96$	0.0015	0.12	0.1385	60	0.0118,	0.1470, 0.1795,
$D20.48$			0.0020			90	0.0151,	0.2120, 0.2450,
$D30.48$			0.0030			90	0.0184	0.2945, 0.3595

Range of the simulated pion masses

β	$L(fm)$	M_π(MeV)	$M_\pi L$
1.90	2.84	245.41	3.53
		282.13	4.06
		314.43	4.53
1.90	2.13	282.13	3.05
		343.68	3.71
		396.04	4.27
		442.99	4.78
1.95	2.61	238.67	3.16
		280.95	3.72
		350.12	4.64
		408.13	5.41
1.95	1.96	434.63	4.32
2.10	2.97	211.18	3.19
		242.80	3.66
		295.55	4.46

Lattice Spacings	
a($\beta = 1.90$)	0.0885(36)fm
a($\beta = 1.95$)	0.0815(30)fm
a($\beta = 2.10$)	0.0619(18)fm

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Three different values of the lattice spacing: 0.06 *fm* ÷ 0.09 *fm*

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Different volumes: $2\ fm \div 3\ fm$

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Pion masses in range 210 $\div$ 440 MeV

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To inject momenta we used non-periodic boundary conditions

β	V/a^4	θ
1.90	$32^3 \times 64$	0.0, ±0.400, ±0.933, ±1.733
	$24^3 \times 48$	0.0, ±0.300, ±0.700, ±1.300
1.95	$32^3 \times 64$	0.0, ±0.366, ±0.854, ±1.588
	$24^3 \times 48$	0.0, ±0.275, ±0.641, ±1.191
2.10	$48^3 \times 96$	0.0, ±0.424, ±0.986, ±1.832

General strategy

$$\langle \pi(p') | \; V_\mu \; | K(p) \rangle = (p_\mu + p'_\mu)\, f_+(q^2) + (p_\mu - p'_\mu)\, f_-(q^2)$$

extract the matrix element $\langle \pi(p') | V_\mu | K(p) \rangle$ from appropriate ratio of three-points correlation function to build $f_0(q^2)$ and $f_+(q^2)$

- we used the ratio $R_\mu = \dfrac{C_\mu^{K\pi}(t,\vec{p},\vec{p}')\;\; C_\mu^{\pi K}(t,\vec{p}',\vec{p})}{C_\mu^{\pi\pi}(t,\vec{p}',\vec{p}')\;\; C_\mu^{KK}(t,\vec{p},\vec{p})}$

Fit simultaneously $f_0(q^2)$ and $f_+(q^2)$ to get $f_0(0)=f_+(0)$

- z expansion
- Polynomial fit

Perform the Chiral and continuum extrapolation of f(0)

- SU(2) ChPT
- SU(3) ChPT

General strategy

The matrix element of the vector current between two PS mesons decomposes into two form factors

$$\langle \pi(p')|\ V_\mu\ |K(p)\rangle = \left(p_\mu + p'_\mu\right) f_+\left(q^2\right) + \left(p_\mu - p'_\mu\right) f_-\left(q^2\right)$$

depending on the momentum transfer

$$q^2 = \left(E - E'\right)^2 - \left(p_i - p'_i\right)^2$$

The matrix element can be derived in lattice QCD from a combination of Euclidean three-point functions

We define the ratio:

$$R_\mu = \frac{C_\mu^{K\pi}\left(t,\vec{p},\vec{p}\,'\right)\ C_\mu^{\pi K}\left(t,\vec{p}\,',\vec{p}\right)}{C_\mu^{\pi\pi}\left(t,\vec{p}\,',\vec{p}\,'\right)\ C_\mu^{KK}\left(t,\vec{p},\vec{p}\right)}$$

in which the renormalization Z_V and Z_K and Z_π cancels

$$C_\mu^{K\pi}\left(t_x, t_y, \vec{p}, \vec{p}\,'\right) \xrightarrow[t_x \to \infty \quad (t_x - t_y)\to\infty]{} \frac{\sqrt{Z_K Z_\pi}}{4 E_K E_\pi} \langle \pi(p')|\ \hat{V}_\mu\ |K(p)\rangle\, e^{-E_K t_x - E_\pi (t_x - t_y)}$$

$$\hat{R}_\mu = \frac{\left(\langle \pi(p')|\, V_\mu\, |K(p)\rangle\right)^2}{4 p_\mu p'_\mu}$$

General strategy

The matrix element of the vector current between two PS mesons decomposes into two form factors

$$\langle \pi(p') | \; V_\mu \; | K(p) \rangle = (p_\mu + p'_\mu) f_+(q^2) + (p_\mu - p'_\mu) f_-(q^2)$$

$$\hat{R}_\mu = \frac{\left(\langle \pi(p') | V_\mu | K(p) \rangle \right)^2}{4 p_\mu p'_\mu}$$

We can define V_0 and V_1 related to the form factors by the relations

$$V_0 = 2\sqrt{R_0}\sqrt{EE'} = \left((E+E') f_+(q^2) + (E-E') f_-(q^2) \right)$$

$$V_i = 2\sqrt{R_i}\sqrt{p_i p'_i} = \left((p_i + p'_i) f_+(q^2) + (p_i - p'_i) f_-(q^2) \right)$$

and resolving the system we obtain:

$$f_+(q^2) = \frac{(E-E')V_i - (p_i - p'_i)V_0}{2Ep'_i - 2E'p_i}$$

$$f_-(q^2) = \frac{(p_i + p'_i)V_0 - (E+E')V_i}{2Ep'_i - 2E'p_i}$$

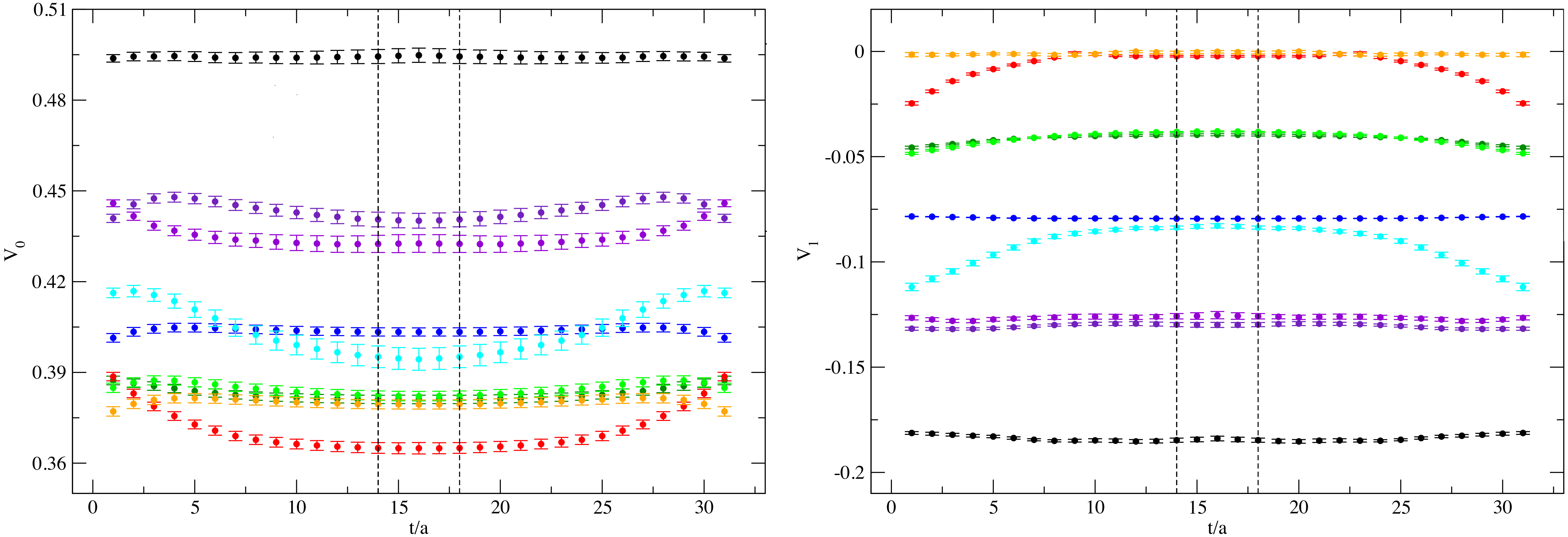
$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_K^2 - m_\pi^2} f_-(q^2)$$

- V_0 and V_i are extracted from the double ratio of the three-points correlation function
- momenta are fixed by the non-periodic boundary conditions
- energies are extracted from the dispersion relation with the masses obtained fitting the two-points correlation function at rest

Plateaux

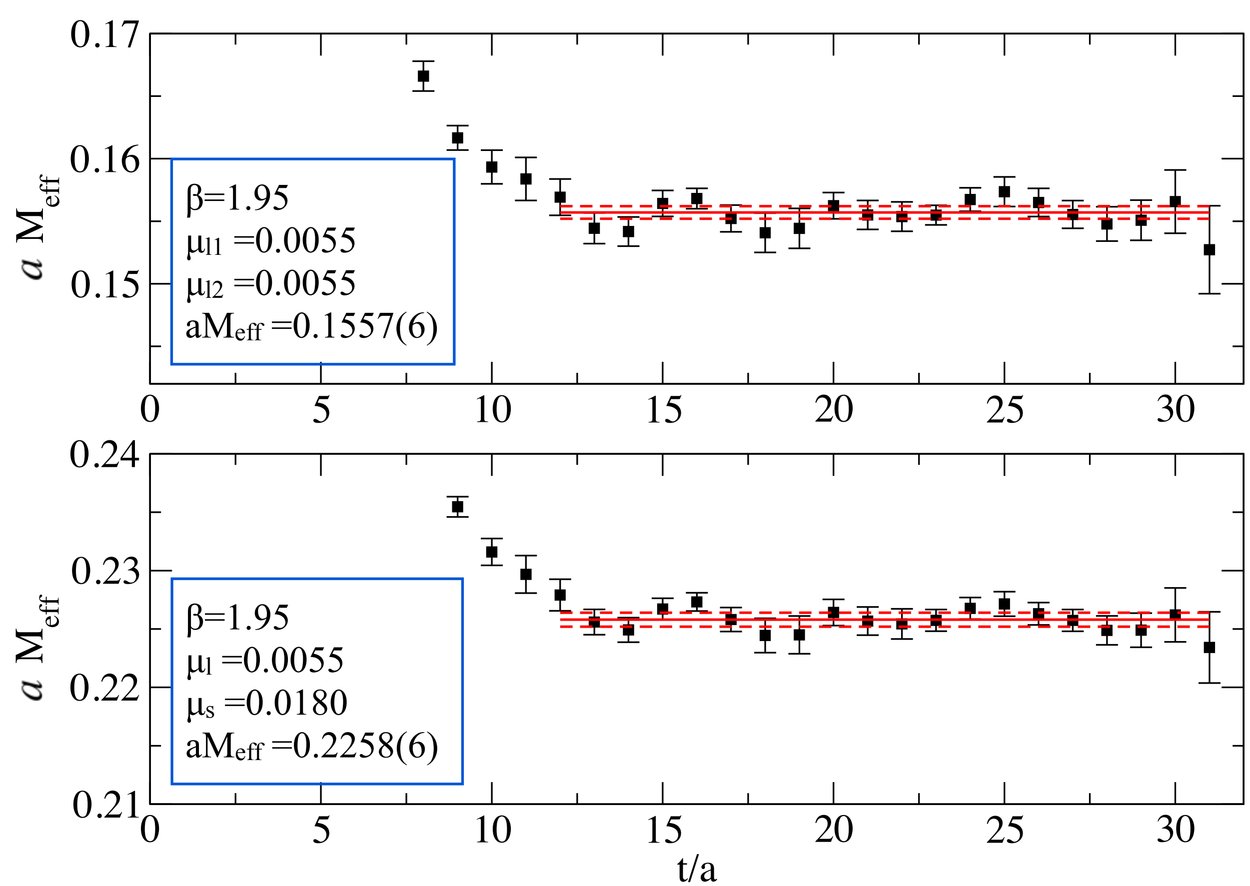
example of plateaux of V_0 and V_1 for all the selected kinematics

$\beta = 1.90$
$\mu_l^{(sea)} = 0.0050$
$\mu_s = 0.0145$



Plateaux

example of plateaux of the effective mass of a two-points correlation function at zero momentum



The fit intervals for the two-point correlation functions at rest are the one reported in [arXiv:1403.4504]

β	V/a^4	$[t_{min}, t_{max}]_{(\ell\ell,\ell s)}/a$
1.90	$24^3 \times 48$	$[12, 23]$
1.90	$32^3 \times 64$	$[12, 31]$
1.95	$24^3 \times 48$	$[13, 23]$
1.95	$32^3 \times 64$	$[13, 31]$
2.10	$48^3 \times 96$	$[18, 40]$

Extracting f(0)

We fitted simultaneously $f_+(q^2)$ and $f_0(q^2)$ to extract f(0) using the z expansion *

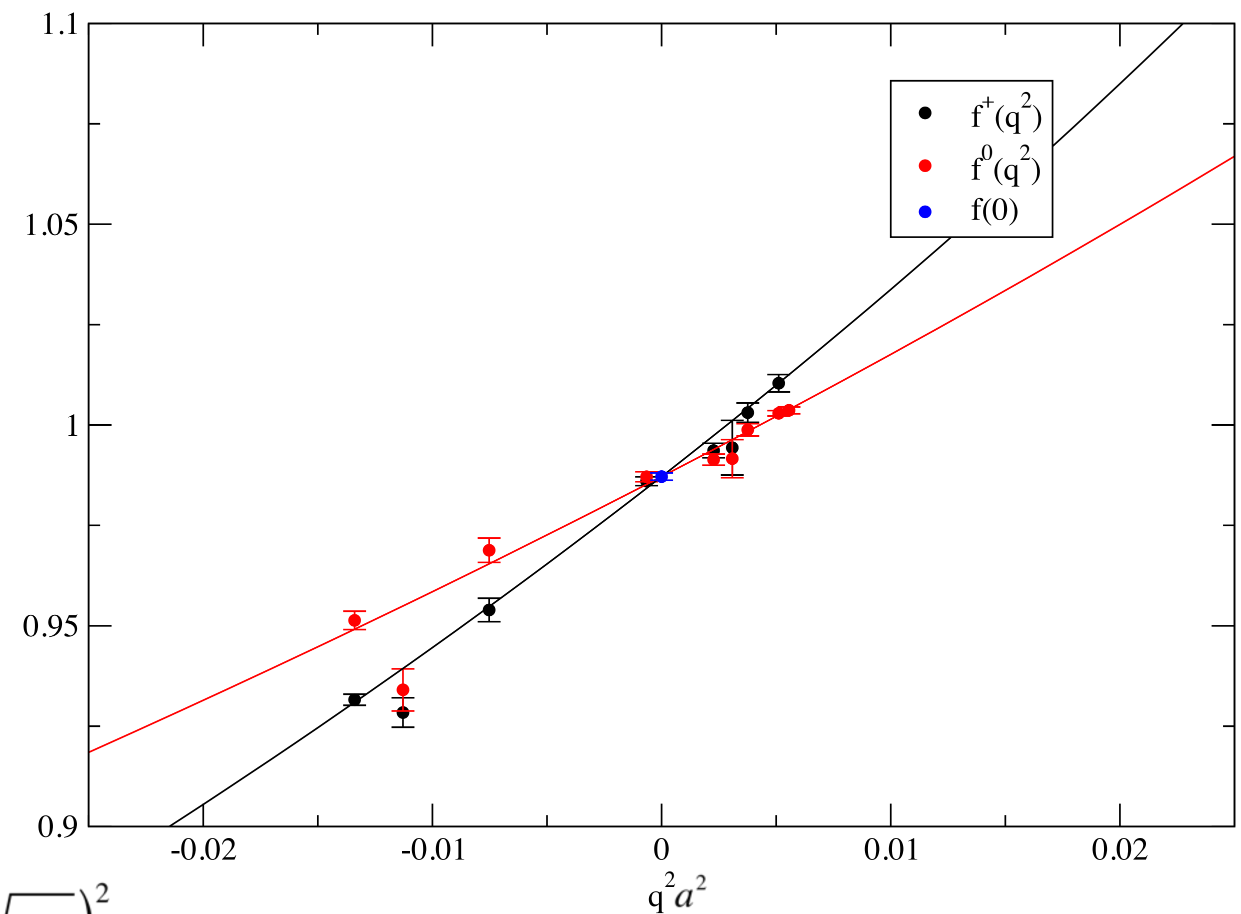
to interpolate at $q^2=0$ we neglect the points corresponding to large negative q^2

*** Bourrely Caprini and Lellouch [Phys.Rev. D79 (2009) 013008]**

$$f_+(q^2) = \frac{a_0 + a_1\left(z + \frac{1}{2}z^2\right)}{1 - \frac{q^2}{M_V^2}}$$

$$f_0(q^2) = \frac{b_0 + b_1\left(z + \frac{1}{2}z^2\right)}{1 - \frac{q^2}{M_S^2}}$$

Ensemble *A.60.24* $a\mu_s=0.0225$



with

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (m_K + m_\pi)^2 \qquad t_0 = (m_K + m_\pi)\left(\sqrt{m_K} - \sqrt{m_\pi}\right)^2$$

Extracting f(0)

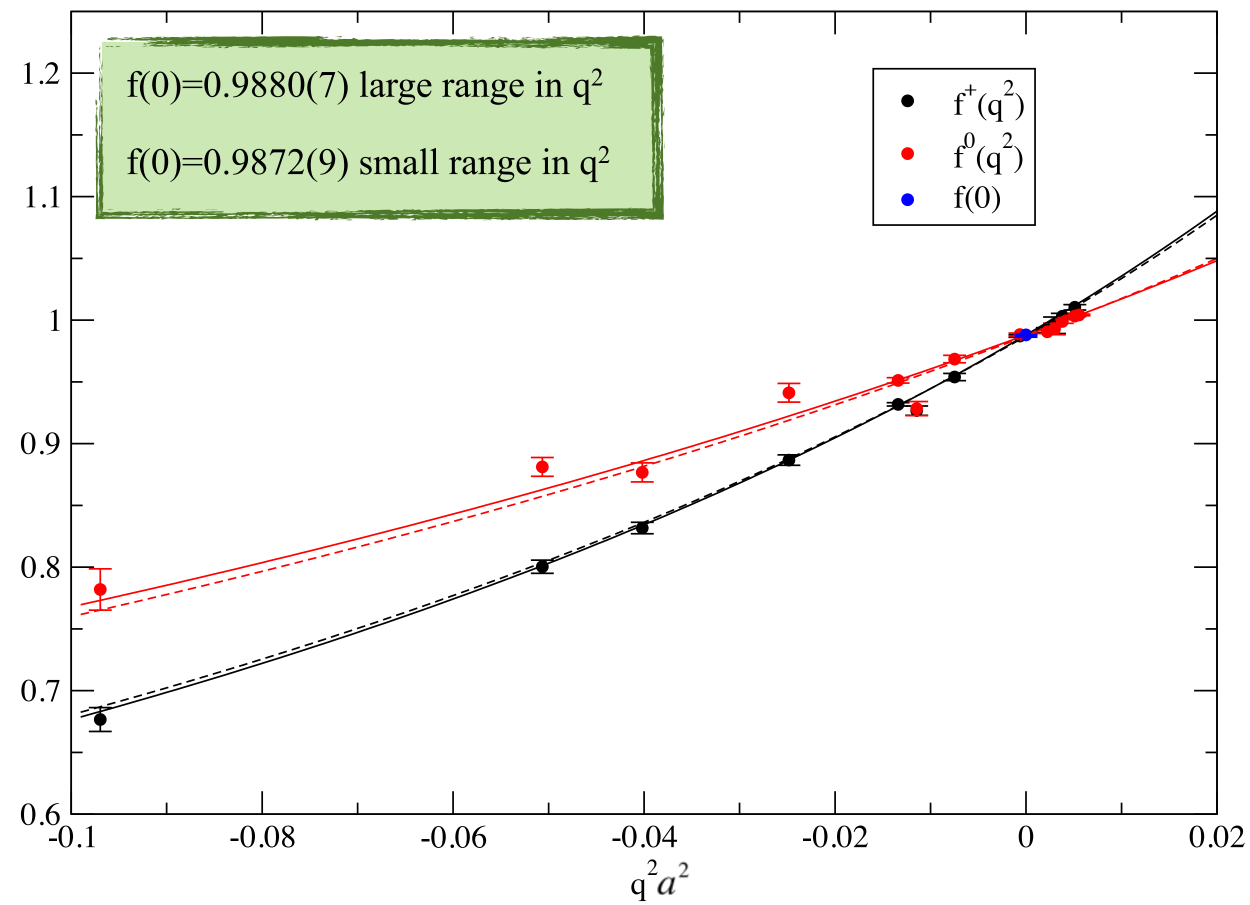
We fitted simultaneously $f_+(q^2)$ and $f_0(q^2)$ to extract f(0) using the z expansion to interpolate at $q^2=0$ we neglect the points corresponding to large negative q^2

$$f_+(q^2) = \frac{a_0 + a_1\left(z + \frac{1}{2}z^2\right)}{1 - \frac{q^2}{M_V^2}}$$

$$f_0(q^2) = \frac{b_0 + b_1\left(z + \frac{1}{2}z^2\right)}{1 - \frac{q^2}{M_S^2}}$$

The fit works well even for a larger range in q^2

Ensemble *A.60.24* $a\mu_s=0.0225$



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$$f_0(q^2) = \frac{b_0 + b_1\left(z + \frac{1}{2}z^2\right)}{1 - \frac{q^2}{M_S^2}}$$

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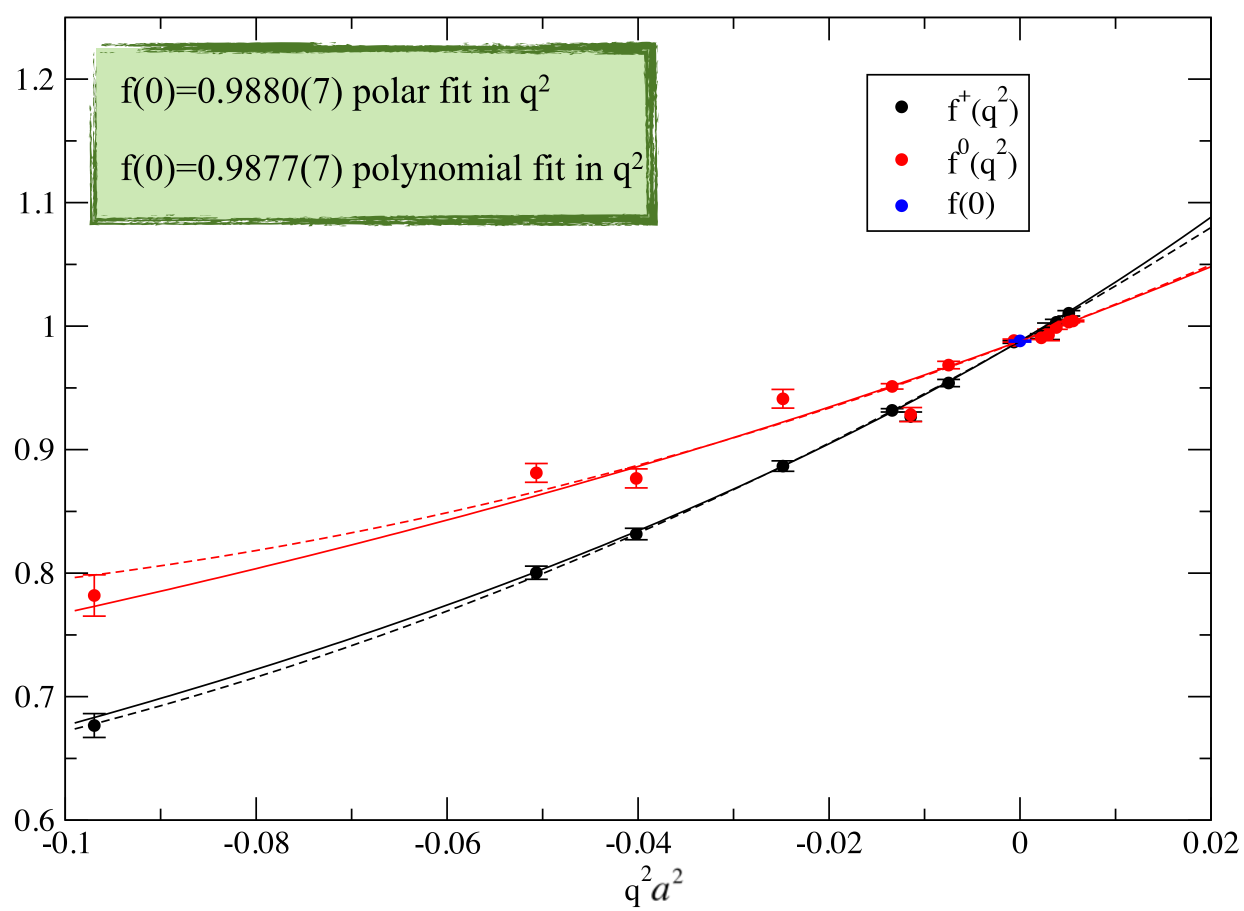


The results obtained with polynomial fit formulae are also compatible

$$f_+(q^2) = f(0)\left(1 + P_1 q^2 + P_2 q^4\right)$$

$$f_0(q^2) = f(0)\left(1 + P_3 q^2 + P_4 q^4\right)$$

$f_+(0)$ Chiral and continuum extrapolation

Two different approaches for the chiral extrapolation

- SU(2) ChPT **Flynn & Sachrajda [NPB 812 (2009)]**

$$f_+(0) = F_0^+ \left(1 - \frac{3}{4}\xi \log \xi + P_2 \xi + P_3 a^2\right)$$

$$\xi = \frac{2B_0 m_l}{(4\pi f_0)^2}$$

- SU(3) ChPT (at fixed m_s)

$$f_+(0) = 1 + f_2 + \Delta f$$

$$f_2^{\text{fullQCD}} = \frac{3}{2} H_{\pi K} + \frac{3}{2} H_{\eta K}$$

$$H_{PQ} = -\frac{1}{64\pi^2 f_\pi^2}\left[M_P^2 + M_Q^2 + \frac{2M_P^2 M_Q^2}{M_P^2 - M_Q^2}\log\frac{M_Q^2}{M_P^2}\right]$$

$$\Delta f = \left(m_s - m_l\right)^2 \left[\Delta_0 + \Delta_1 m_l\right] + \Delta_2 a^2$$

f+(0): SU(2) Chiral and continuum extrapolation

f+(0)

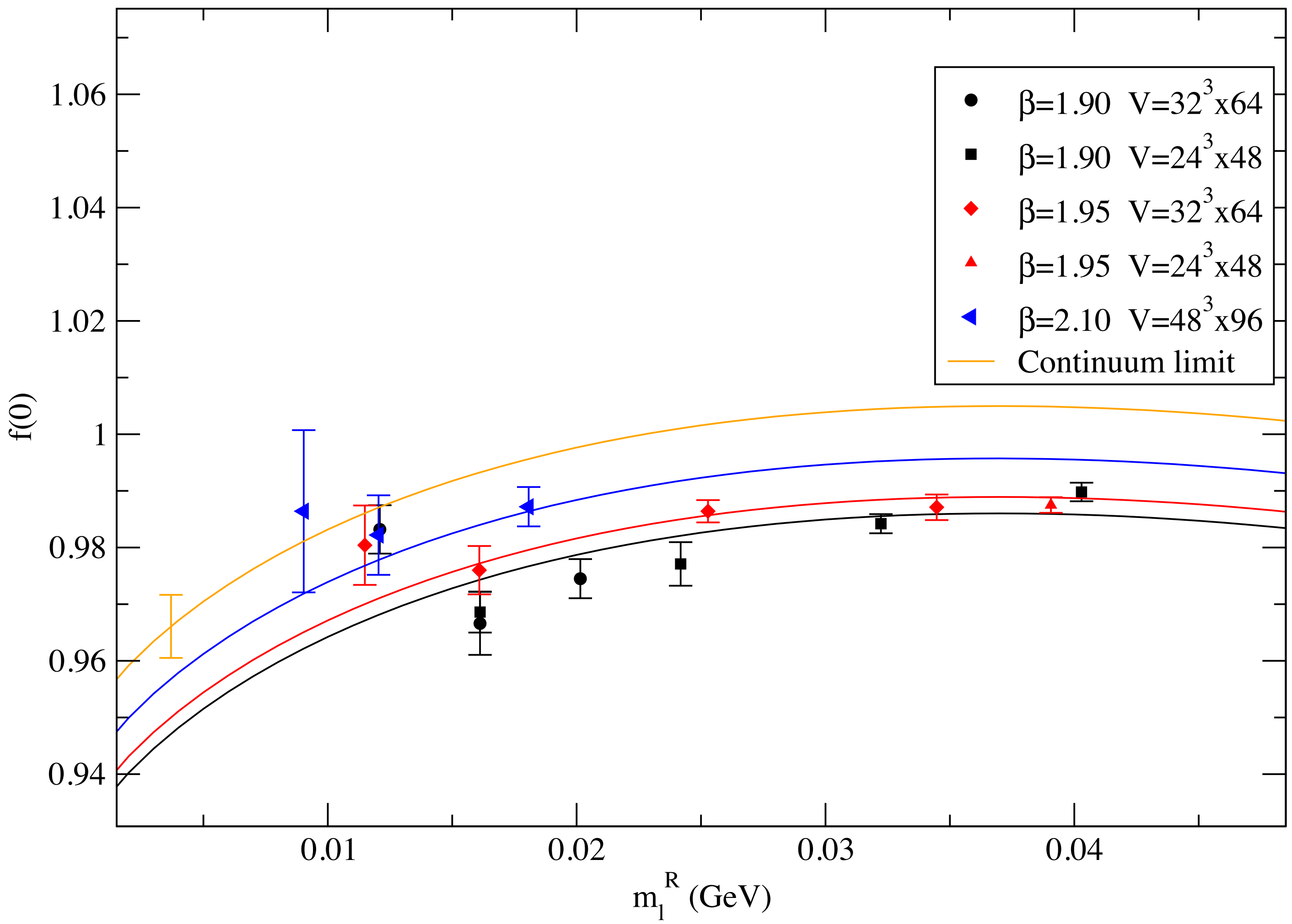
we performed a small interpolation in the data to arrive at $m_{s\ phys}$

fit formula:

SU(2) Chiral fit: $f_+(0) = F_0^+ \left(1 - \frac{3}{4}\xi \log \xi + P_2 \xi + P_3 a^2\right)$

$$\xi = \frac{2B_0 m_l}{(4\pi f_0)^2}$$

SU(2) Chiral fit



The compatibility between ensemble A40.32 and A40.24 shows that FSE are small

To calculate f+(0) we used m_s=99.6(4.1) MeV and $m_{u/d}$=3.70(17) MeV from our previous work [arXiv:1403.4504]

our result:

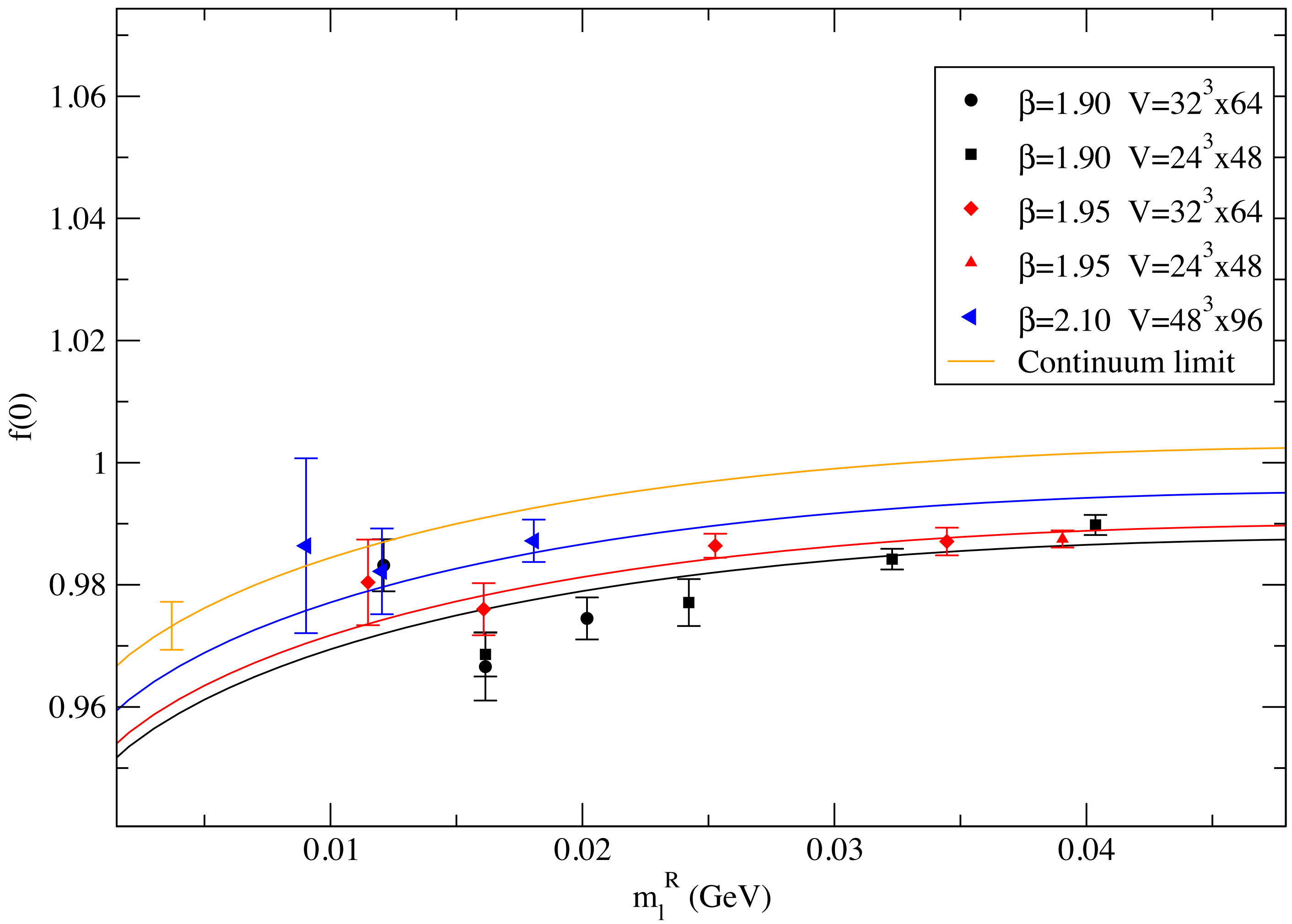
$$f_+(0) = 0.9641(58)$$

$f_+(0)$: SU(3) Chiral and continuum extrapolation

$f_+(0)$

we performed a small interpolation in the data to arrive at $m_{s\ phys}$

SU(3) Chiral fit



fit formula:

SU(3) Chiral fit

$$f_+(0) = 1 + f_2 + \Delta f$$

$$\Delta f = \left(m_s - m_l\right)^2 \left[\Delta_0 + \Delta_1 m_l\right] + \Delta_2 a^2$$

M^2_K, M^2_π and f_π appearing in f_2 are expressed at LO

We also tried the same fit using f_π instead of f_0 in the definition of f_2 obtaining cosistent results $f_+(0)=0.9734(40)$

The compatibility between ensemble A40.32 and A40.24 shows that FSE are small

To calculate $f_+(0)$ we used m_s=99.6(4.1) MeV and $m_{u/d}$=3.70(17) MeV extracted in our previous work [arXiv:1403.4504]

our result:

$$f_+(0) = 0.9725(41)$$

$f_+(0)$ - results and sistematics

The results from our analysis:

$$f_+(0) = 0.9683(65)$$

in particular:

$$f_+(0) = 0.9683(50)_{\text{stat+fit}}(42)_{\text{chiral}}$$

stat+fit is referred to both the statistical uncertainties (including the total error on the light and strange quark mass determination) and the uncertainties due to the fitting procedure

Chiral extrapolation systematic uncertainties have been evaluated comparing the results obtained from two different fit formulae i.e. SU(2) ChPT and SU(3) ChPT

An estimate of the systematic effects associated to the **FSE** has not been performed yet. However comparing ensembles A40.24 and A40.32 we expect these effect to be small compared to the other uncertainties

Testing the CKM unitarity

Testing the first row

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$V_{CKM} = \begin{pmatrix} \boxed{V_{ud} \; V_{us} \; V_{ub}} \\ V_{cd} \; V_{cs} \; V_{cb} \\ V_{td} \; V_{ts} \; V_{tb} \end{pmatrix}$$

	Experimental input [1]	our result	determination of $\|V_{us}\|$
$K_{\ell 2}$	$\left\|\frac{V_{us}}{V_{ud}}\right\| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5)$	$\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.183(17)$ [2]	$\|V_{us}\| = 0.2271(33)$
$K_{\ell 3}$	$\|V_{us}\| f_+(0) = 0.2163(5)$	$f_+(0) = 0.9683(65)$	$\|V_{us}\| = 0.2234(16)$

$|V_{ud}| = 0.97425(22)$ from β-decay [3]

$K_{\ell 2}$
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0007(16)$$

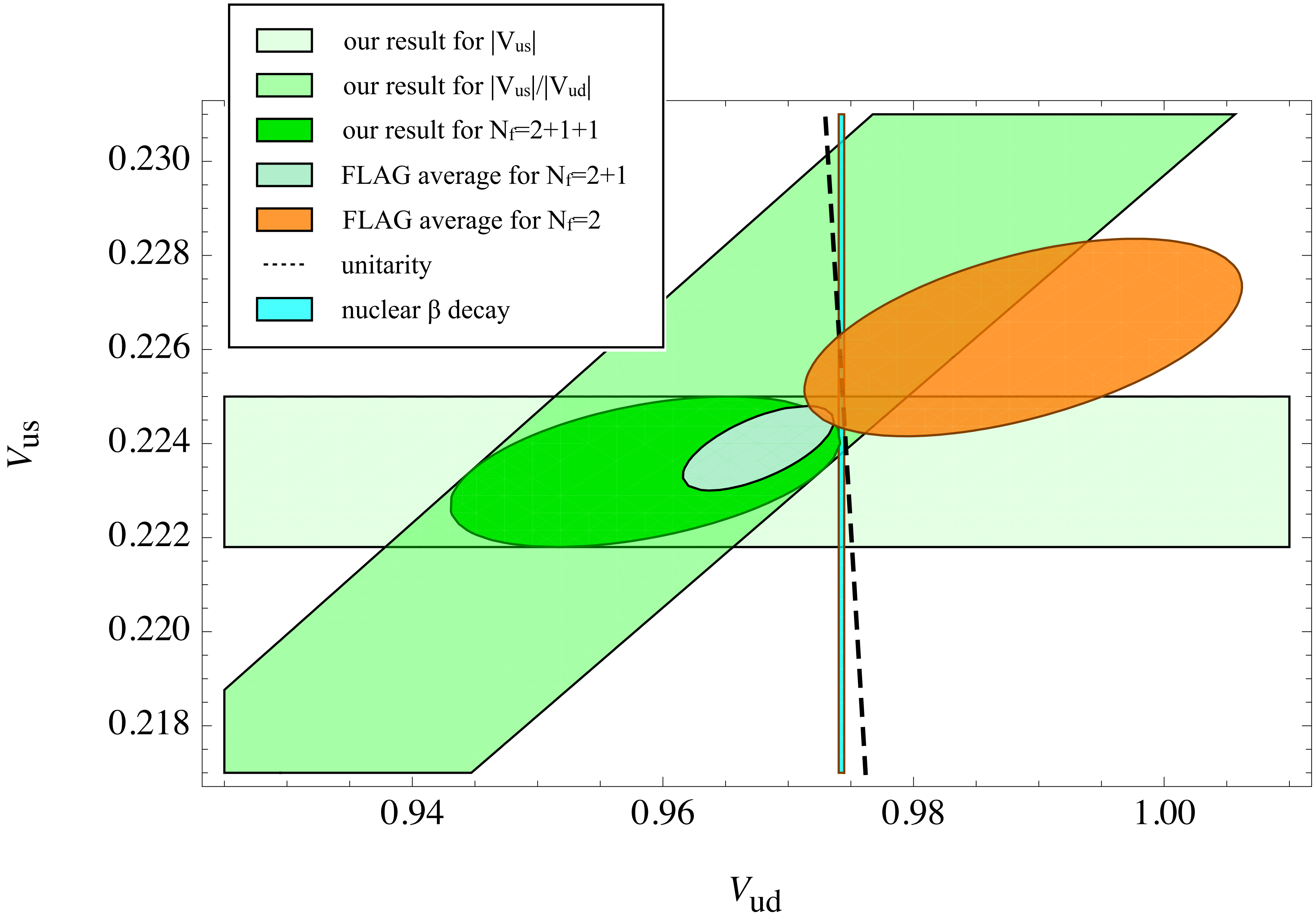
$K_{\ell 3}$
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9991(8)$$

(1) Eur.Phys.J. C69 (2010) 399-424

(2) PoS LATTICE 2013 (Carrasco et al.)

(3) Phys.Rev. C79 (2009) 055502

Testing the CKM unitarity



Conclusions

- We presented N_f=2+1+1 preliminary results for the semileptonic form factor $f_+(0)$

Summary of the results in comparison with FLAG averages:

	Our results	$\text{FLAG}_{N_f=2}$	$\text{FLAG}_{N_f=2+1}$	$\text{FNAL/MILC}^*_{N_f=2+1+1}$
$f_+(0)$	0.9683(65)	0.9560(84)	0.9661(32)	0.9704(32)

Future plans:

- compare with an estimate of $f_+(0)$ coming from the scalar density
- A more detailed analysis of the q^2 dependence of the form factor and a comparison with the experimental data

* PRL 112 (2014) (Bazavov et. al.)

Backup

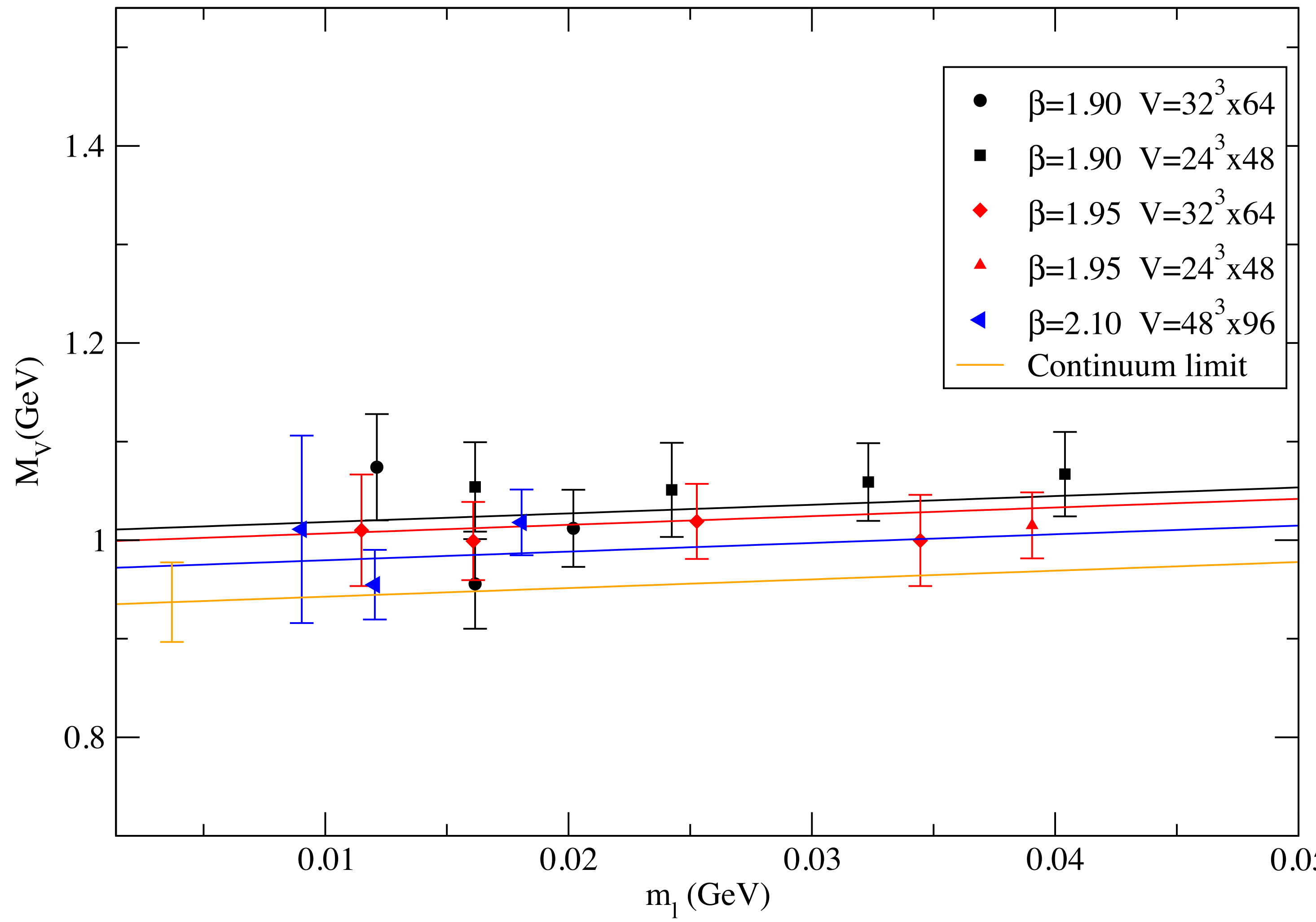
Chiral and continuum extrapolation of M_V

Extrapolating the vector pole mass M_V obtained from the fit in q^2 of f_+ we should get a rough estimate of the K^* mass

Our result:

$$M_V^{Phys} = 937(42)\text{MeV}$$

while K^* has a mass of 892 MeV

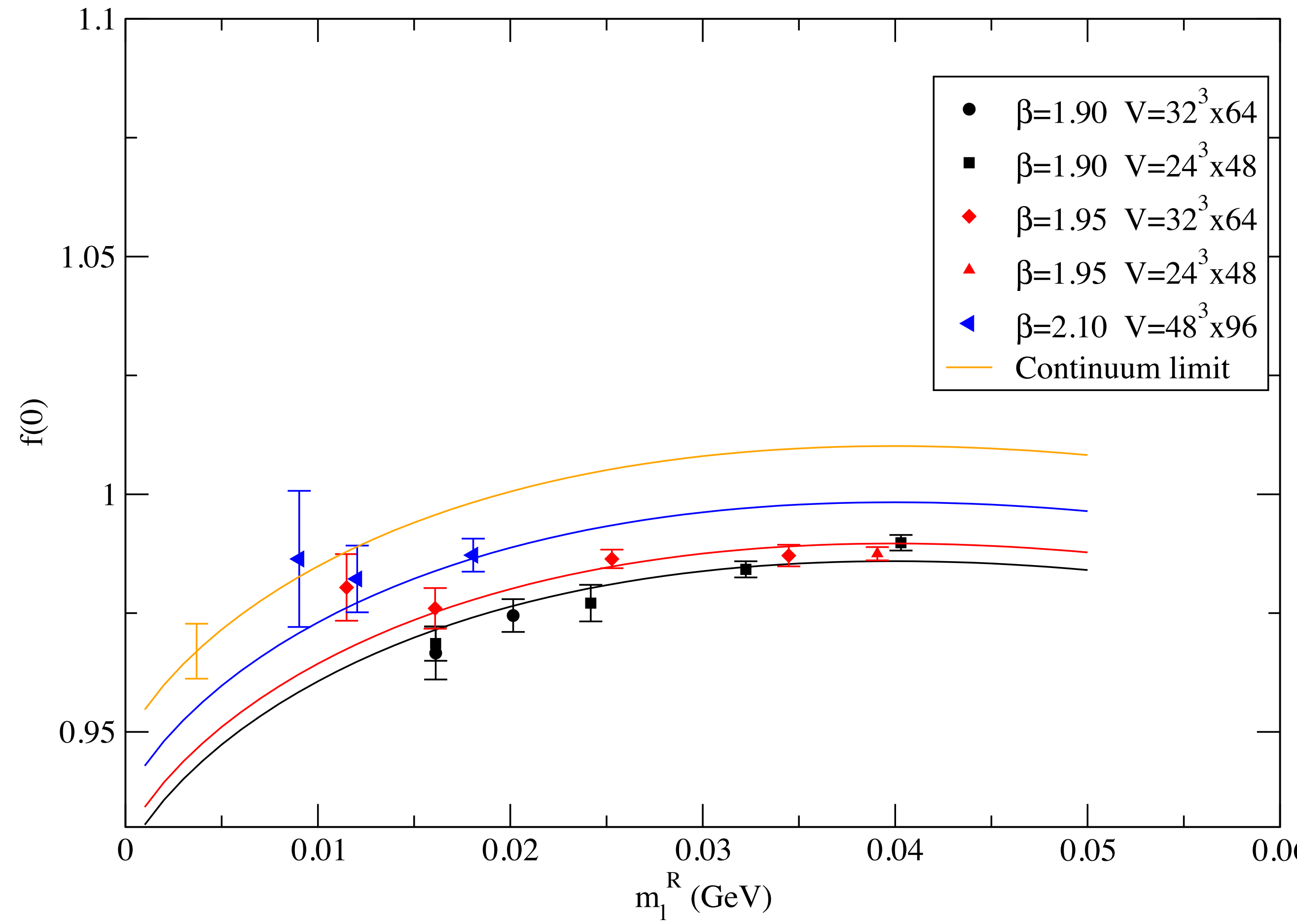


Chiral and continuum extrapolation of M_V

Excluding the ensemble A30.32 from the chiral and continuum extrapolation of f(0) we get

SU(2) result:

$$f_+(0) = 0.9649(61)$$



while with all the points we get

$$f_+(0) = 0.9641(58)$$

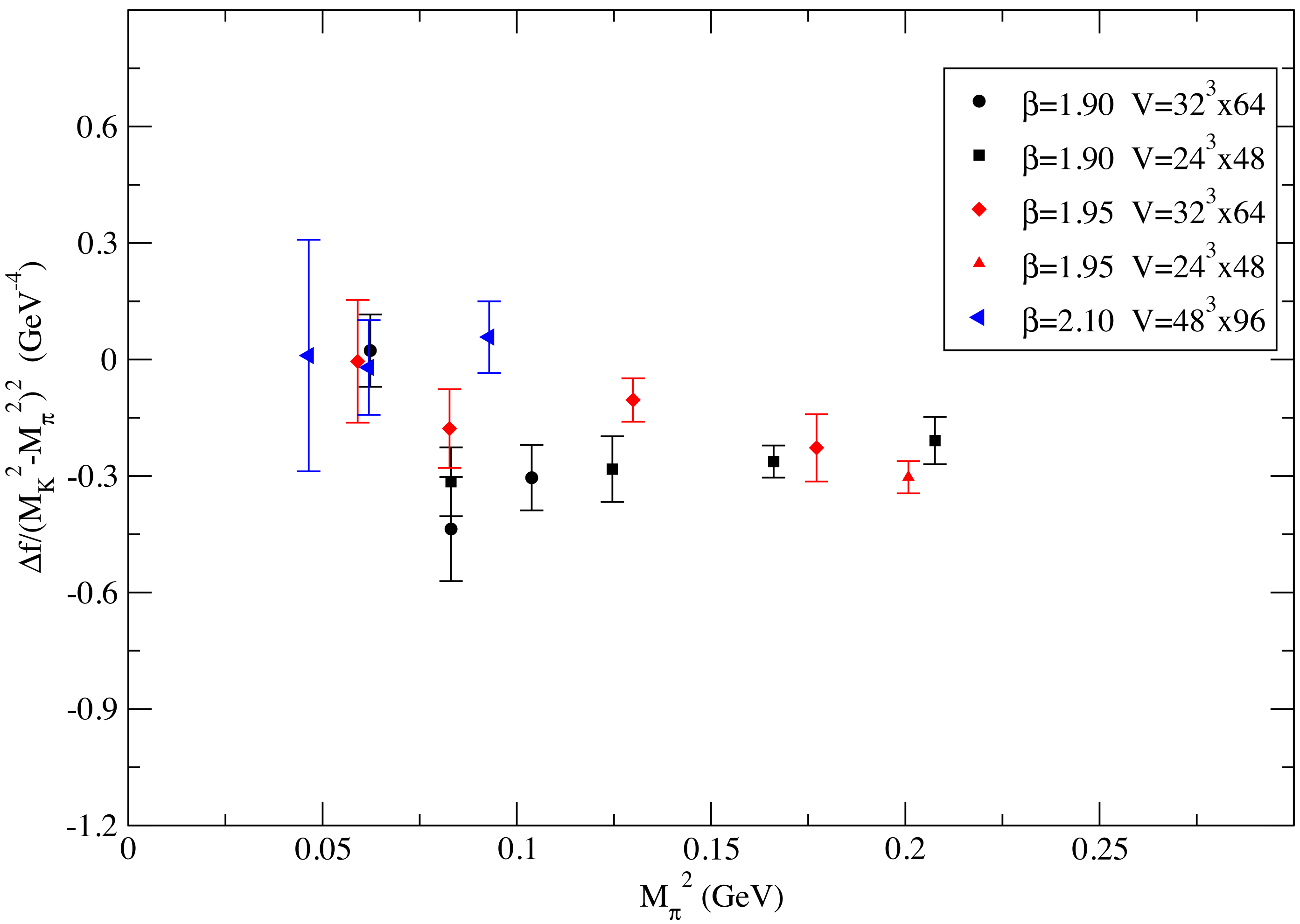
So even if A30.32 seems to be off is overall effect is less then 0.1% and therefore marginal

Ademollo Gatto Theorem

The AG theorem states that in SU(3) limits $f_+(0)=1$ and deviation from unity are of the order of $(m_s-m_l)^2$

$$f_+(0) = 1 + \propto \left(m_s - m_l\right)^2$$

We can test AG theorem plotting $\Delta f=f_+(0)-1-f_2$ divided by $(m_s-m_l)^2$ as a function m_l notice the the data as been extrapolated at m_s^{phys}



On the action

- Wilson Twisted Mass action at maximal twist with Nf=2+1+1 sea quarks

Light degenerate doublet

$$S^l_{tm} = a^4 \sum_x \overline{\psi}(x) \left\{ \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - i\gamma_5\tau^3 \left[M_0 - \frac{a}{2}\nabla_\mu\nabla^*_\mu \right] + \mu_l \right\} \psi(x)$$

Heavy non degenerate doublet

$$S^h_{tm} = a^4 \sum_x \overline{\psi}(x) \left\{ \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - i\gamma_5\tau^1 \left[M_0 - \frac{a}{2}\nabla_\mu\nabla^*_\mu \right] + \mu_\sigma + \mu_\delta\tau^3 \right\} \psi(x)$$

- Osterwalder-Seiler valence quark action

$$S^f_{OS} = a^4 \sum_x \overline{q}_f(x) \left\{ \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - i\gamma_5 r_f \left[M_0 - \frac{a}{2}\nabla_\mu\nabla^*_\mu \right] + \mu_f \right\} q_f(x)$$