Nucleon and pion structure in $N_\mathrm{f} = 2$ QCD

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Outline

- Motivation
- Set-up: simulation parameters
- Proton g_A and $\langle x\rangle_{u-d}$
- $\langle x\rangle_u^{\mathrm{con}}$ in the pion and σ-terms
- Mixed boundary conditions
- Summary

Proton structure calculations are...

- ... essential to exclude beyond-the-Standard-Model (BSM) dark matter candidates, relating predictions to experimental limits.
- ... important to predict cross-sections for processes on the quark-gluon level. Experiment e.g. unable to directly measure strangeness and gluon PDFs.
- ... needed to relate QCD to low energy effective theories that are also relevant for precision experiments.

Here I concentrate on

- How is the mass distributed among the partons? (scalar couplings)
- How is the spin distributed? (axial couplings)
- How is the momentum distributed? (moments of PDFs)

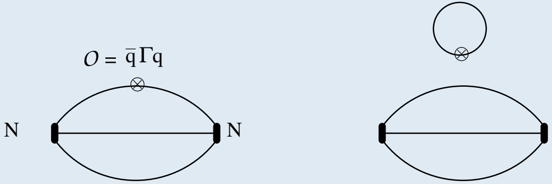
Action and configurations

- $N_{\rm f} = 2$ NP improved Sheikholeslami-Wilson fermions, Wilson glue.
- $m_\pi L$ up to 6.7, a down to 0.06 fm, m_π down to 150 MeV.
- Two lattice spacings around $m_\pi \approx 280$ MeV, three around 430 MeV.
- 300–600 Wuppertal=Gauss smearing iterations on top of APE smearing.

β	a/fm	κ	V	m_π/MeV	Lm_π	$n_{\rm conf}$	$t_{\rm sink}/a$
5.20	0.081	0.13596	$32^3 \times 64$	280	3.69	1986(4)	13
5.29	0.071	0.13620	$24^3 \times 48$	428	3.71	1999(2)	15
		0.13620	$32^3 \times 64$	423	4.89	1998(2)	15,17
		0.13632	$32^3 \times 64$	294	3.42	2023(2)	7,9,11,13,15,17
			$40^3 \times 64$	290	4.19	2025(2)	15
			$64^3 \times 64$	289	6.70	1232(2)	15
		0.13640	$48^3 \times 64$	160	2.77	3442(2)	15
			$64^3 \times 64$	151	3.49	1593(3)	9,12,15
5.40	0.060	0.13640	$32^3 \times 64$	491	4.81	1123(2)	17
		0.13647	$32^3 \times 64$	427	4.18	1999(2)	17
		0.13660	$48^3 \times 64$	261	3.82	2177(2)	17

Three point functions

Evaluate $\langle N|\bar{q}\Gamma q|N\rangle$ (Lines: quark "propagators" M^{-1}_{xy}, $M = \not{D} + m_q$)



connected | disconnected

$q \in \{u, d\}$: both quark-line connected and disconnected terms.

$q = s$: only the disconnected term.

χ symmetry explicitly broken: mixing under renormalization.

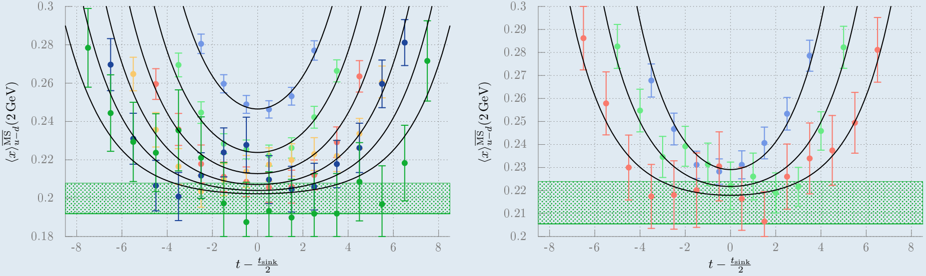
"Connected" requires only 12 rows of M^{-1}.

"Disconnected" $12N^3$ rows (timeslice): stochastic "all-to-all" methods.

"Disconnected" cancels ($m_u = m_d$, $\cancel{QED}$) from isovector combinations: "proton minus neutron", i.e. $\langle N|(\bar{u}\Gamma u - \bar{d}\Gamma d)|N\rangle$.

Excited states

Simultaneous fit of $C_{\rm 3pt}(t, t_{\rm sink})/(A_0 e^{-m_N t_{\rm sink}})$ (renormalized to $\overline{\rm MS}$) for $\langle x \rangle_{u-d}$ at $m_\pi \approx 290, 150$ MeV, $a \approx 0.071$ fm, $m_\pi L \approx 3.5$ [S Collins]:



Excited states were e.g. also investigated by Dinter et al, arXiv:1108.1076; Owen et al, arXiv:1212.4668; Capitani et al, arXiv:1205.0180; Green et al, arXiv:1209.1687; Bhattacharya et al, 1306.5435; Alexandrou et al, 1312.2874.

Fit function

$$C_{\rm 2pt}(t_{\rm sink}) = e^{-m_N t_{\rm sink}} \left[A_0 + A_1 e^{-\Delta m_N t_{\rm sink}} \right] + \cdots$$

$$C_{\rm 3pt}(t_{\rm sink}, t) = A_0 e^{-m_N t_{\rm sink}} \Big\{ B_0 + B_1 \left[e^{-\Delta m_N (t_{\rm sink} - t)} + e^{-\Delta m_N t} \right] + B_2 e^{-\Delta m_N t_{\rm sink}} \Big\} + \cdots ,$$

$B_0 = \langle N|O|N\rangle$, $B_1 \propto \langle N'|O|N\rangle$, $B_2 \propto \langle N'|O|N'\rangle$, $\Delta m_N = m_{N'} - m_N$.

Fit $C_{\rm 2pt}$ and $C_{\rm 3pt}$ simultaneously for all $t_{\rm sink}, t$ with $t \in [\Delta t, t_{\rm sink} - \Delta t]$, varying Δt, and compare with constant fit to

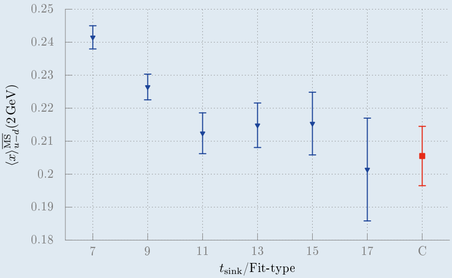
$$\frac{C_{\rm 3pt}(t_{\rm sink}, t)}{C_{\rm 2pt}(t_{\rm sink})} = B_0 + \cdots .$$

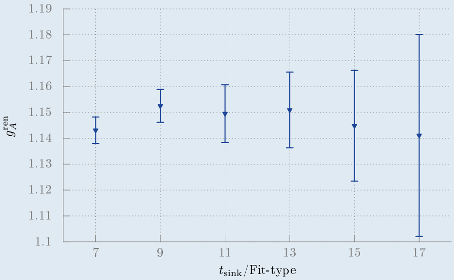
B_2 can only be identified, varying $t_{\rm sink}$.

B_1, corresponding to a change of nodes of the “wavefunction”, may be enhanced if O contains a derivative.

Comparison between constant and combined fits

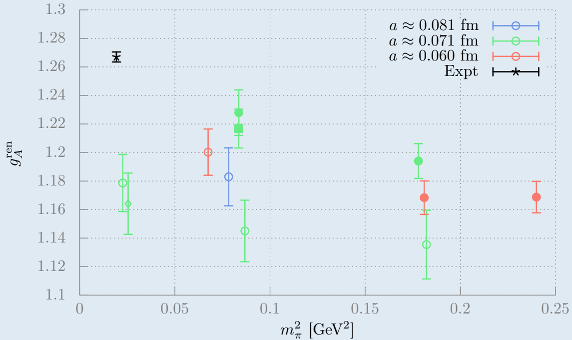
$m_\pi \approx 290$ MeV [S Collins, R Rödl]:





Using our smearing function, the excited state contributions to g_A almost cancel in $C_{\rm 3pt}/C_{\rm 2pt}$.

Results: g_A



$m_\pi L \approx 6.7$:

$m_\pi L > 4.1$:

$m_\pi L > 3.4$:

$m_\pi L \approx 2.8$:

[S Collins, R Rödl]

Comparing similar volumes: no significant discretization effects.

$m_\pi \approx 425$ MeV: g_A increases by $\approx 5\%$ with $m_\pi L \approx 3.7 \to 4.9$

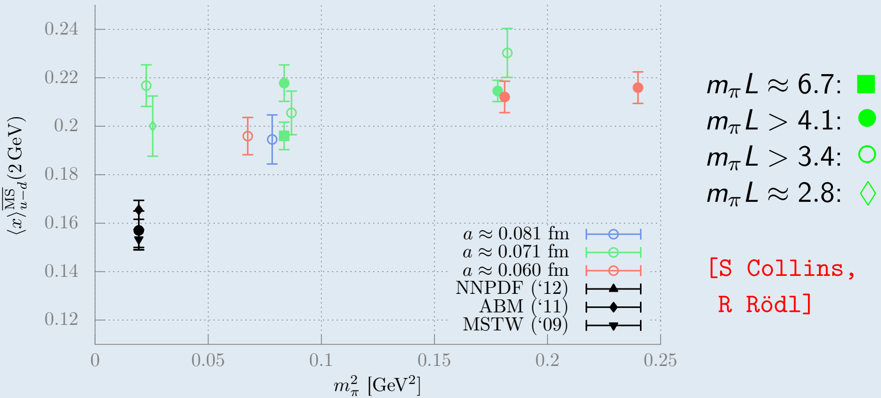
$m_\pi \approx 290$ MeV: g_A up by $\approx 6\%$ with $m_\pi L \approx 3.4 \to 4.2$, then constant.

$m_\pi \approx 150$ MeV: No difference between $m_\pi L \approx 2.8$ and $m_\pi L \approx 3.5$.

$\geq 80^3$ volume would have been interesting.

- With similar FSE as at 290 MeV or 430 MeV the 150 MeV point would have hit the experimental value.
- Unfortunately, we are unable to check this.
- χPT however predicts FSE at constant $m_\pi L$ to decrease with m_π^2.
- $m_\pi L$ may be too small for FSE to be dominated by pion exchange.
- χPT may not yet converge well at our pion masses?
  $\rightarrow$ Plenary talk S Dürr

Results: $\langle x \rangle_{u-d}$



[S Collins, R Rödl]

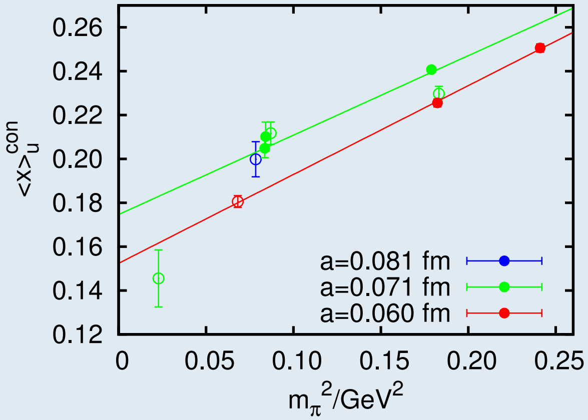
No significant FSE going from $m_\pi L \approx 3.4 \to 6.7$.

No significant lattice spacing effects.

- Physical point is missed.
- NP Renormalization? Under investigation but 20% are a lot.
- Finite-a effects: We only vary a by 25%. Unlike for g_A there will be $\mathcal{O}(a)$ corrections.

$\Rightarrow N_{\mathrm{f}} = 2 + 1$ CLS simulations with open boundary conditions: $a \to 0$.

Results: Pion $\langle x \rangle_u^{\mathrm{con}}$



[N Javadi-Motaghi]

m_π^2 fit to $m_\pi L > 4$ volumes (solid): Finite a-effects?

For the pion ($m_u = m_d$, $\cancel{QED}$) $\langle x \rangle_{u-d} = 0$. Disconnected contribution needs to be included for $\langle x \rangle_u$. Effect could also be due to omitting this.

Decomposition of the proton (and pion) mass I

$$m_N = \underbrace{\sum_{q\in\{u,d,s,\dots\}} m_q \langle N|\bar{q}\,\mathbb{1}\,q|N\rangle}_{\text{quarks}} + \underbrace{\left\langle N\left|\frac{1}{8\pi\alpha_L}(\mathbf{E}^2-\mathbf{B}^2)+\sum_q \bar{q}\mathbf{D}\cdot\gamma q\right|N\right\rangle}_{\text{gluon interactions (Eucl. spacetime)}}$$
$$+\underbrace{\frac{1}{4}\left(m_N - \sum_q m_q\langle N|\bar{q}\,\mathbb{1}\,q|N\rangle\right)}_{\text{trace anomaly}}$$

VEV $\langle 0|\bar{q}q|0\rangle$ is understood to be subtracted from $\langle N|\bar{q}q|N\rangle$.

Pion-nucleon σ-term: $\sigma_{\pi N} = m_u\langle N|\bar{u}u|N\rangle + m_d\langle N|\bar{d}d|N\rangle = \sigma_u + \sigma_d$.

Scalar particles (Higgs, neutralino etc.) couple $\propto$ quark matrix elements.

Decomposition of the proton (and pion) mass II

$$\sigma_\pi = m_{ud}\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle = m_{ud}\frac{\partial m_\pi}{\partial m_{ud}} \underbrace{= \frac{m_\pi}{2}}_{\text{GMOR}} + \mathcal{O}(m_\pi^3)\,.$$

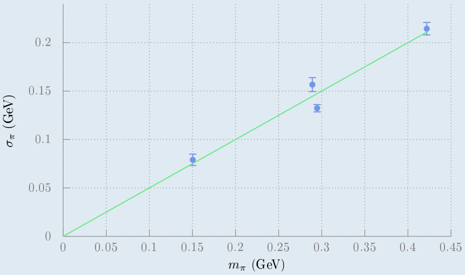
Therefore:

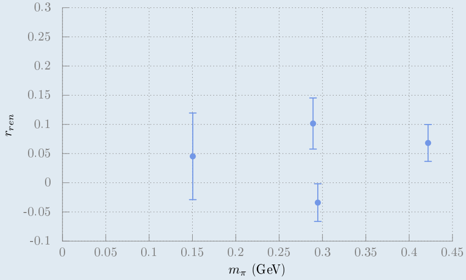
$$m_\pi \approx \underbrace{\frac{1}{2}m_\pi}_{\sigma_\pi} + \underbrace{\frac{3}{8}m_\pi}_{\text{gluon interactions}} + \underbrace{\frac{1}{8}m_\pi}_{\text{trace anomaly}}$$

σ_π can be further decomposed into valence and sea quark contributions.
Wilson fermions: singlet and non-singlet mass renormalization constants differ by $r_m > 1 \Rightarrow$ "valence" > "connected":

$$r := \frac{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle^{\text{sea}}}{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle} = r_m\left(\frac{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle^{\text{dis}}_{\text{lat}}}{\langle\pi|\bar{u}u+\bar{d}d|\pi\rangle_{\text{lat}}} - 1\right) + 1$$

σ_π compared to $m_\pi/2$ and sea quark contrib.





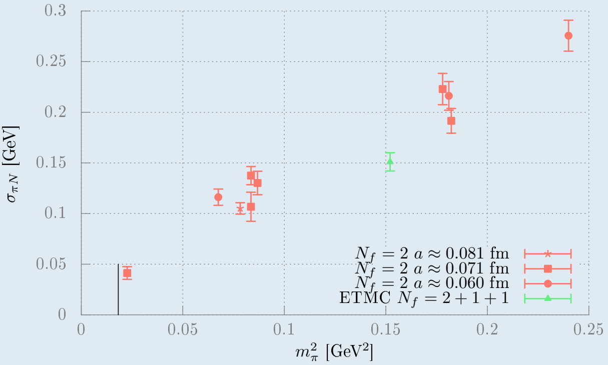
[S Collins, D Richtmann]

The theoretical expectation $\sigma_\pi \approx m_\pi/2$ is confirmed.

Less than $\sim 10\%$ of σ_π is due to sea quarks.

However, for $a \approx 0.071$ fm about 30% of the signal originates from the disconnected contribution.

$\sigma_{\pi N}$ for the nucleon



[S Collins]

The non-vanishing light quark masses are directly responsible for only $\approx$ 35 MeV of the nucleon mass but for 68 MeV of the pion mass!

This may not be too surprising since $m_N \not\to 0$ as $m_{ud} \to 0$ but recently I met someone who believes in "constituent quarks".

Scalars: even $|n\rangle$, pseudoscalars: odd $|n\rangle$. Vacuum: $|0\rangle$, Pion: $|1\rangle$.
$\hat{O}^\dagger|0\rangle \propto |1\rangle$ creates a pion, S is the scalar current.
(Anti-)periodic boundary conditions:

$$\langle O(t_f)S(t)O^\dagger(0)\rangle = \left[\sum_{m\,\mathrm{even}} \sum_{n,k\,\mathrm{odd}} + \sum_{m\,\mathrm{odd}} \sum_{n,k\,\mathrm{even}}\right] \times \left(\langle m|\hat{O}|n\rangle\langle n|S|k\rangle\langle k|\hat{O}^\dagger|m\rangle e^{-tE_k} e^{-(t_f-t)E_n} e^{-(L_t-t_f)E_m}\right)$$

First sum is OK for the ground state pion since $E_0 = 0$ and $E_2 \gtrsim 2E_1$.
But we are not interested in the σ-term of the scalar/$\pi\pi$ (second sum)!
Neglecting $n \geq 2$ one easily obtains:

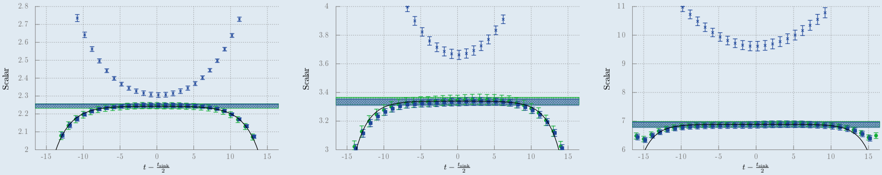
$$\frac{C_{\mathrm{3pt}}(t_f, t)}{C_{\mathrm{2pt}}(t_f)} - \langle 0|S|0\rangle = \underbrace{\left(\langle 1|S|1\rangle - \langle 0|S|0\rangle\right)}_{\sigma-\mathrm{term}} \frac{1}{1 + e^{(2t_f - L_t)E_1}}$$

Unfortunately, $n = 2$ is not always negligible.

Connected contribution to the pion σ-term

We compute two- and three-point functions with antiperiodic and with mixed BCs in time (one propagator antiperiodic, one periodic).

We then add/subtract these depending on whether t is "inside"/"outside", thereby removing wrong-parity contributions.



$m_\pi \approx 425$ MeV
$m_\pi L_t \approx 9.8$

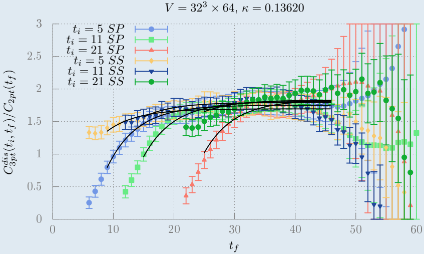
$m_\pi \approx 290$ MeV
$m_\pi L_t \approx 6.7$

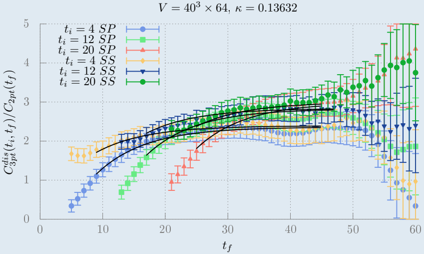
$m_\pi \approx 150$ MeV
$m_\pi L_t \approx 3.5$

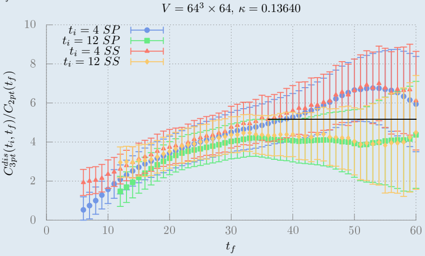
Question to the audience for my write-up: we experimented with this since the late 90s. So it is an old idea but who invented it?

Disconnected contributions

Stochastic sources every 8 timeslices $\Rightarrow$ no overhead for extra t-values.







Summary

- g_A seems to approach the physical value, once $m_\pi L > 4$.
- Finite volume effects for $m_\pi L < 4$ are not well described by χPT.
- Possibly little above $m_\pi = 150$ MeV is well described by χPT.
- $\langle x \rangle_{u-d}$ comes out 20% bigger than expected.
  a-effects? Renormalization?
- At light pion masses, the lattice needs to be "long" for mesonic observables, in particular for the σ-term of the pion.
- We worked with mixed boundary conditions to alleviate this problem.
- The resulting σ-term of the pion agrees with the theoretical expectation.