Chiral dynamics in the low temperature phase of QCD

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Based on the publication by Bastian B. Brandt, Anthony Francis, Harvey B. Meyer and D.R., *"Chiral dynamics in the low temperature phase of QCD"* [arXiv:1406.5602]

Introduction...

Initial thoughts...

- The theory of strong interactions put into thermal media may behave completely different than at zero temperature (states, vacuum, ...).
- $\rightarrow$ In particular, it is worth asking what becomes out of the relevant degrees of freedom that dominate the low-temperature regime $(T < T_C)$ $\rightarrow \pi$ states

Goals...

- Study the dispersion relation of (quasi)-particles carrying pion quantum numbers.
- Test the chiral expansion around $(T = 0, m_q = 0)$ (Ordinary ChPT at finite T)

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Pion dispersion relation...

- The ordinary pion dispersion relation dictated by Lorentz symmetry is $E = \sqrt{\mathbf{k}^2 + m_\pi^2}$.
- We want to derive and study a modified dispersion relation that takes the following form:

$$\omega_{\mathbf{k}}^2 = u^2(T)(\mathbf{k}^2 + m_\pi^2)$$

where $u(T)$ may be interpreted as the *pion velocity* in the chiral limit (massless pions) and m_π is the *screening mass*.

Notice...

- $\omega_{\mathbf{k}}$ should be understood as a pole in the retarded Green-function carrying pion quantum numbers. Equivalently as a "narrow" peak in the corresponding spectral function.

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Lattice approach

A priori it looks like a very simple problem...

- If $\mathbf{k} = 0$ we should measure nothing but the pion mass at finite temperature.

But the real problem is...

- Because of how finite temperature is implemented on the Lattice, there is no hope on doing spectroscopy along the very short time direction ($N_\tau \sim 16$).
- The kernel appearing in equal-time correlators $K(\omega, x_0) = \frac{\cosh(\omega(\beta/2 - x_0))}{\sinh(\omega\beta/2)}$ falls off very slowly with x_0. $\beta \equiv 1/T$

Solutions...

- $\rightarrow$ Use *screening correlators*
- Other approaches like spectral function reconstruction: Maximum Entropy Method (MEM).

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Pion dispersion relation derivation: D.T. Son *et al.*

We got inspired by the approach from D.T. Son and M. Stephanov:

> *Real-time pion propagation in finite temperature QCD,*
> Phys Rev. D **66** (2002), [arXiv:hep-ph/0204226]

- They demonstrate that the dispersion relation of the pion is fully determined by *static quantities*.
- The results are enforced with both a lagrangian and hydrodynamic approach.

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Our work: logic

Ingredients

1. We exploit Ward Identities arising from the PCAC relation.
2. $\rightarrow$ This enables us to fully determine correlators in the chiral limit (its residues).
3. The idea is then to use those at small but finite quark mass supported by Goldstone's theorem.

Limitations...

- Well separated from the crossover region $T < T_C$
- Quark condensate is assumed to be different from zero.
- The quark mass has to be small.
- Correlation functions containing pion states have to be dominated by the pion itself.

<u>Nothing is said about temperature but $T < T_C$.</u>

A more detailed derivation ...

In the massless theory ($m = 0$) we notice that $\langle P\mathbf{A}\rangle$ is fully determined by WI's:

$$G_{\mathrm{AP}}(x_0, \mathbf{0}) = \int d^3x \, \langle P(x) A_0(0) \rangle = \frac{\langle \bar{\psi}\psi \rangle}{2\beta}(x_0 - \beta/2)$$

$$G_{\mathrm{AP}}(x_0, \mathbf{k}) = \int d^3x e^{-i\mathbf{kx}} \langle P(x) A_0(0) \rangle = \int_0^\infty d\omega \rho_{\mathrm{AP}}(\omega, k) \frac{\sinh(\omega(\beta/2 - x_0))}{\sinh(\omega\beta/2)}$$

One conclude easily that at zero momentum:

$$\rho_{\mathrm{AP}}(\omega, 0) = -\frac{\langle \bar{\psi}\psi \rangle}{2}\delta(\omega)$$

a massless excitation persists at finite temperature for any temperature below T_{C}.

We define the screening mass m_π at *small but finite quark mass*, by making use of the results for the $\langle P\mathbf{A}\rangle$ correlator and the GOR relation:

$$f_\pi^2 m_\pi^2 = -m \left\langle \bar{\psi}\psi \right\rangle$$

Chiral Ward Identities imply for the static $\langle PP\rangle$ correlator:

$$\int dx_0 \, \langle P(0)P(x)\rangle = -\frac{\left\langle \bar{\psi}\psi \right\rangle^2}{4 f_\pi^2} \frac{\exp(-m_\pi r)}{4\pi r} \qquad r \to \infty$$

Now, we use the following Ansatz

$$\rho_{\mathrm{P}}(\omega, k) = \mathrm{sgn}(\omega) C(k^2)\delta(\omega^2 - \omega_{\mathbf{k}}^2) + \ldots$$

$$\begin{aligned} \int dx_0 \, \langle P(0)P(x)\rangle &= 2 \lim_{\epsilon \to 0} \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{kx}} \int_0^\infty \frac{d\omega}{\omega} e^{-\epsilon\omega} \rho_{\mathrm{P}}(\omega, k) \\ &= \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{kx}} \frac{C(k^2)}{\omega_{\mathbf{k}}^2} + \ldots \end{aligned}$$

One last observation ...

By comparing the last two equation one concludes easily that

$$\omega_{\mathbf{k}}^2 \propto (\mathbf{k}^2 + m_\pi^2)$$

with

$$C(k^2) = -\frac{\langle \bar{\psi}\psi \rangle^2 u^2}{4 f_\pi^2}$$

and f_π is defined by

$$\int dx_0 d^2 x_\perp \langle A_3(x) A_3(0) \rangle = \frac{1}{2} f_\pi^2 m_\pi e^{-m_\pi |x_3|} \qquad |x_3| \to \infty$$

<u>Conclusion:</u>
We have proven our formula for the modified dispersion relation and showed that it is compatible with chiral WI's in the limit of small quark mass.

Lattice estimators for $u(T)$

Working a little bit harder one can calculate the residue for the last correlator:

$$\rho_{\mathrm{A}}(\omega,0) \quad = \quad \mathrm{sgn}(\omega) f_\pi^2 m_\pi^2 \delta(\omega^2 - \omega_{\mathbf{0}}^2) + \ldots$$

and finally by using:

$$\omega_{\mathbf{0}}^2 = \left. \frac{\partial_0^2 G_{\mathrm{A}}(x_0,\mathbf{0})}{G_{\mathrm{A}}(x_0,\mathbf{0})} \right|_{x_0=\beta/2} = -4m^2 \left. \frac{G_{\mathrm{P}}(x_0,\mathbf{0})}{G_{\mathrm{A}}(x_0,\mathbf{0})} \right|_{x_0=\beta/2}$$

$$\begin{aligned} u_f &= \frac{f_\pi^2 m_\pi}{2G_{\mathrm{A}}(\beta/2,\mathbf{0})\sinh(u_f m_\pi \beta/2)} \\ u_m &= -\frac{4m^2}{m_\pi^2} \left. \frac{G_{\mathrm{P}}(x_0,\mathbf{0})}{G_{\mathrm{A}}(x_0,\mathbf{0})} \right|_{x_0=\beta/2} \end{aligned}$$

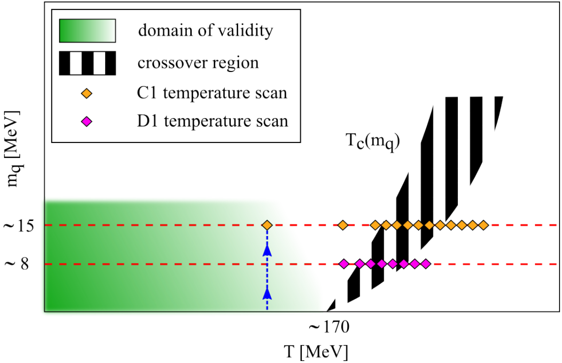
Relevant quantities...

- f_π, m_π
- $m \leftrightarrow \overline{m}^{\overline{\mathrm{MS}}}(\mu = 2\mathrm{GeV})$
- $G_{\mathrm{A}}(\beta/2,\mathbf{0})$, $G_{\mathrm{P}}(\beta/2,\mathbf{0})$

$\longrightarrow$ $u(T)$ is a RGI quantity!!

Lattice setup ...

- Two temperature scans (**C_1**, **D_1**) at constant renormalized awi-mass with $N_f = 2$ $\mathcal{O}(a)$ improved Wilson fermions.
- Lattice sizes are 16×32^3 covering a temperature range from 150 MeV to 235 MeV.



$\longrightarrow$ Additional CLS zero temperature ensemble (**A_5**) 64×32^3 equivalent to **C_1** at $m_\pi = 290$MeV: *test ensamble*.

Pion velocity results in the $\mathbf{C_1}$ scan. ...

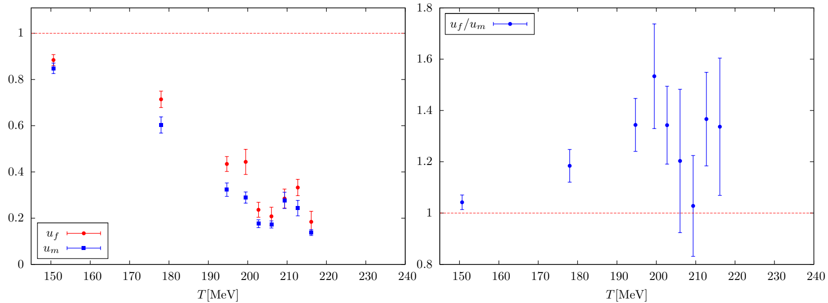
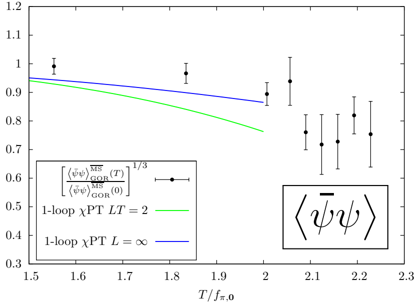


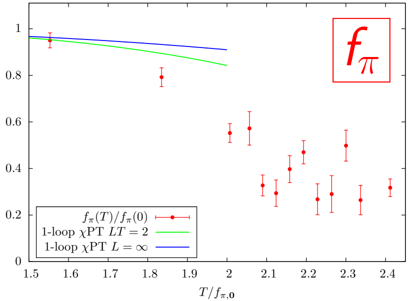
Figure : Left: Pion velocitiy $u(T)$ Lattice estimators. Right: Ratio of estimators to test the chiral approximation as a function of temperature.

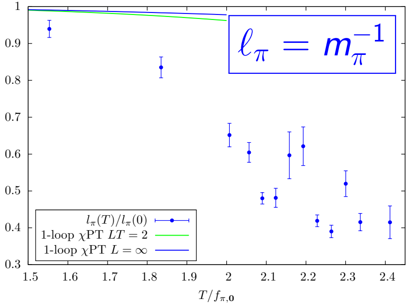
Well in the deconfined phase $u_f/u_m \sim \mathcal{O}(T/m)$.

Test of chiral predictions ($\mathbf{A_5}$ comparison) ...

m_π [MeV]	305(5)
f_π [MeV]	93(2)
$\left\lvert\langle\bar\psi\psi\rangle^{\overline{\mathrm{MS}}}_{\mathrm{GOR}}\right\rvert^{1/3}$ $(\mu = 2\mathrm{GeV})$ [MeV]	364(7)
ω_0 [MeV]	294(4)
$f_{\pi,0}$ [MeV]	97(3)
$\left\lvert\langle\bar\psi\psi\rangle^{\overline{\mathrm{MS}}}_{\mathrm{GOR},0}\right\rvert^{1/3}$ $(\mu = 2\mathrm{GeV})$ [MeV]	368(9)
u_f	0.96(2)
u_m	0.92(6)
u_f / u_m	1.04(4)
ω_0 / m_π	0.96(2)







Cross check. Maximum Entropy Method (MEM) ...

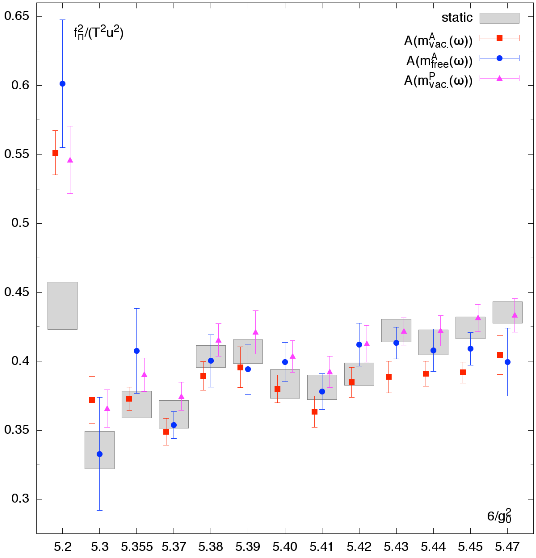
Goal: To reproduce the spectral function from the Euclidean correlator via different models:

Recalling the form of the spectral function for G_A:

$$\rho_A(\omega, \mathbf{0}) = \frac{f_\pi^2 m_\pi}{2u}\delta(\omega - \omega_{\mathbf{0}}) \;+\; \ldots \;\Longrightarrow\; \mathcal{A}(\Lambda) \equiv 2\int_0^\Lambda \frac{d\omega}{\omega}\rho_A(\omega, \mathbf{0}) = \frac{f_\pi^2}{u^2}$$

One introduces a strong systematic with MEM. One has to check the model independency of the results very carefully!

Summary of MEM results ...



Conclusions & Outlook

- Reasonable agreement of the two $u(T)$ estimators up to $T \simeq 190\text{MeV}$ + MEM cross check indicates the validity of the chiral expansion and of its assumptions.
- $u_f(T \simeq 150\text{MeV}) = 0.88(2)$ suggests a violation of boost invariance because of the presence of the thermal medium.
  - $\rightarrow$ May have implications for the HRG gas model.
- The screening pion decay constant f_π and the mass of the pion quasiparticle $\omega_0(T) = um_\pi$ at $T \simeq 150$ MeV differ very little from the predictions of χPT around $(T = 0, m_q = 0)$.

- Extend the calculation to lighter quark mass and maybe lower temperatures.
- Further test the functional form of the pion dispersion relation by analyzing data at nonzero momentum $\mathbf{k}$.
- Detailed study of finite size effects and cutoff dependence.

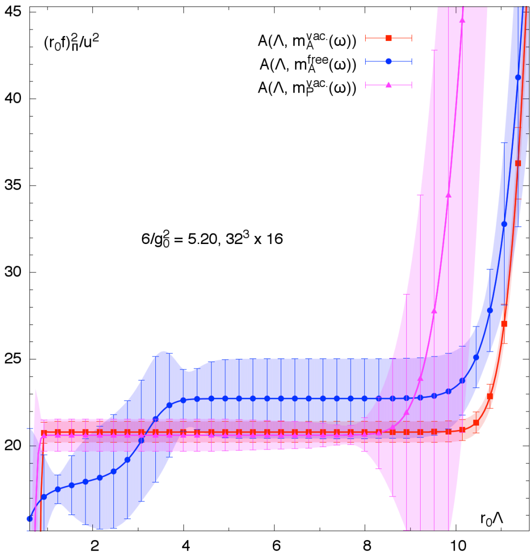


Figure : Cutoff Λ-dependence of $\mathcal{A}(\Lambda, m(\omega))$

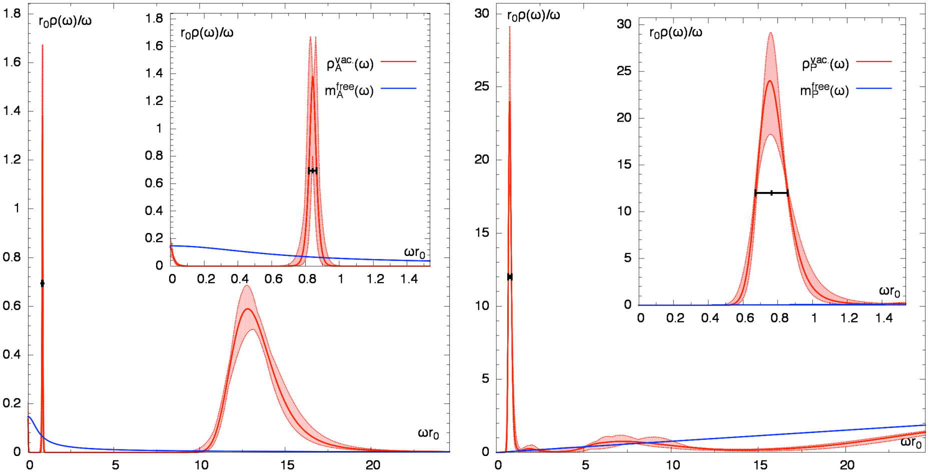


Figure: Left: $\langle PP \rangle (x_0)$ channel. Right: $\langle A_0 A_0 \rangle (x_0)$ channel.

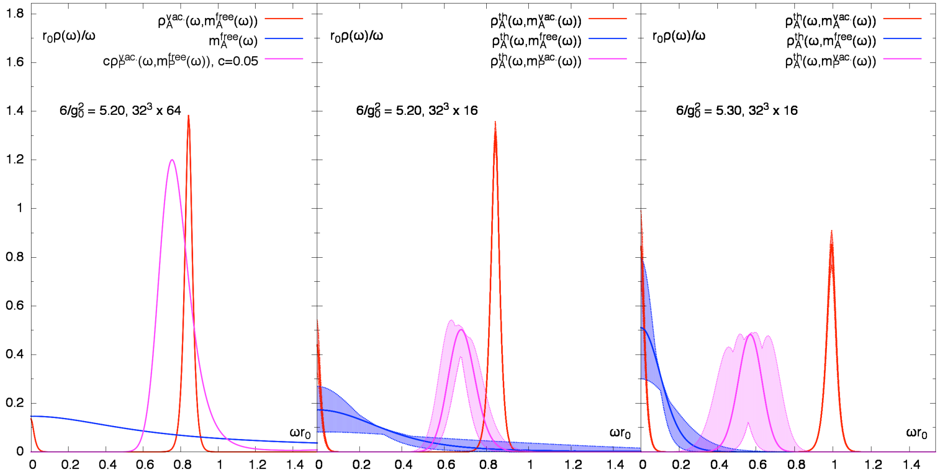


Figure: $\langle A_0 A_0 \rangle$ reconstruction for the 3 different default models.