Towards the physical point hadronic vacuum polarisation from Möbius DWF

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Hadronic vacuum polarisation

Can be computed in Euclidean space-time [Blum ’02 ]

\[ \Pi_{\mu\nu} = a^4 \sum_x e^{iQx} \langle J_{\mu}^{em}(x)J_{\nu}^{em}(0) \rangle \]

- \[ \Pi_{\mu\nu}(Q) = (Q^2 \delta_{\mu\nu} - Q_{\mu} Q_{\nu})\Pi(Q^2) \]
- \[ \hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0) \]
- \[ a^{HLO}_\mu = (\frac{\alpha}{\pi})^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2) \]

Systematic uncertainties to be controlled - general

1. Simulations at physical \( m_\pi \)
2. Controlled continuum limit, FV effects
3. Disconnected diagrams [V. G"ulpers, Mon, 14.55 ] [Della Morte et al. ’10 ]
4. Obtaining a real world result: charm quark, isospin effects . . .
Systematic uncertainties to be controlled - HVP related

- Conventional simulations do not allow access to sufficiently low Fourier momenta
- Integral is dominated in the region where relative errors are enhanced
- Structure of HVP tensor is such that $\Pi(0)$ is not directly accessible
- Systematic uncertainty introduced by extrapolation

Conventional procedure

- $\Pi(Q^2) = \frac{\Pi_{\mu\nu}(Q^2)}{Q_\mu Q_\nu - \delta_{\mu\nu} Q^2}$
- Transverse projection: $Q_\mu = 0$
- Take only diagonal components $\Pi_{\mu\mu}$
- $a^{HLO}_\mu = (\frac{\alpha}{\pi})^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$
Improving the systematics of connected HVP

Several new methods on the market

- R123 procedure \((\Pi(Q^2 = 0),\text{utilising twisted BC formalism})\) [de Divitiis et al.'12]
- Padé approximants [Aubin et al.'12]
- Dispersive model study [Golterman et al.'13]
- Hybrid strategy [Golterman et al.'14] [Mon, 14.15, Sess 1D]
- HPQCD time moments [Chakraborty et al.'14] [Mon, 15.15, Sess 1D]
- …

Challenge: Apply the optimal procedure to physical point data

This work: Fitting Padé approximants on the fresh DWF physical point data

inspired by [Aubin et al. '13]
Improving the systematics of connected HVP

Several new methods on the market

- **R123 procedure** \((\prod(Q^2 = 0)\), utilising twisted BC formalism) \cite{deDivitiis2012}
- **Padé approximants** \cite{Aubin2012}
- **Dispersive model study** \cite{Golterman2013}
- **Hybrid strategy** \cite{Golterman2014} \[Mon, 14.15, Sess 1D\]
- **HPQCD time moments** \cite{Chakraborty2014} \[Mon, 15.15, Sess 1D\]
- ...

**Challenge:** Apply the optimal procedure to physical point data

**This work:** Fitting Padé approximants on the fresh DWF physical point data

*inspired by* \cite{Aubin2013}
Previous RBC-UKQCD computation of $a_{\mu}^{HLO}$ [Boyle et al’11]

Non physical $m_\pi$, $a^{-1} \approx 1.3, 1.7, 2.3$ GeV

- Local current at source, conserved at sink
- DWF (Möbius scale=1.0), Iwasaki/DSDR gauge action
- Fitting $Q^2$- dependence of $\Pi(Q^2)$ up to $Q_C^2 \approx 2.5 - 9$ GeV^2

**Strong $m_\pi$ dependence**

- Eliminate the systematics of chiral extrapolation: computing HVP at $m_\pi^{phys}$

![Graph showing the behavior of $a_{\mu}$ as a function of $m_\pi^2$.](image-url)
RBC-UKQCD $N_f = 2 + 1$ Domain Wall ensembles

Boyle et al. '11

$$m_\pi$$

Boyle et al. '11

$m_\pi$ [Boyle et al. '11]

$a^{HLO}_H$ from DWF for non-physical $m_\pi$ [Boyle et al. '11]

physical point HVP (●) recently measured → preliminary results!
$a^{HLO}_\mu$ from DWF at physical pion mass

Physical point lattice parameters:

- Möbius DWF, Iwasaki gauge action
  - $48^3 \times 96 \times 24$, $a^{-1} = 1.73$ GeV - measurements underway
  - $64^3 \times 128 \times 12$, $a^{-1} = 2.31$ GeV

HVP with Möbius DWF

- Möbius scale = 2.0
- Möbius conserved current [see talk by P. Boyle, Mon 6.10p.m., 2.B]
- Local current at source, conserved at sink
- Point source, 12 source positions
Point vs. stochastic source

- Point source, 12 source positions
- $Z(2)$ wall source, 48 source positions
- (one-end trick) \[\text{McNeile et al. '06}\]
Point vs. stochastic source

- Point source, 12 source positions
- Z(2) wall source, 48 source positions
- (one-end trick) [McNeile et al. '06]
- Comparison (12 src. positions each, log scale on y-axis)
- Point src. better in low-$Q^2$ region ($Q^2 \lesssim 0.2 \text{ GeV}^2$)
Physical point HVP from $N_f = 2 + 1$ DWF

Physical point data:

- $L/a = 48^3 \times 94 \times 24, \quad a^{-1} = 1.73 \text{GeV}$
- $\Pi(Q^2)$ convergent sequence of PAs\cite{Aubin et al.13}
  - VMD is unreliable
- Padé approximants $[N,D]$

$$\Pi_{[N,D]}(Q^2) = \frac{\sum_{n=0}^{N-1} a_n Q^{2n}}{1+\sum_{m=1}^{D} b_m Q^{2m}}$$
Physical point HVP from $N_f = 2 + 1$ DWF

- $L/a = 48$, $a^{-1} = 1.73$ GeV, $m_\pi = 138$ MeV
- $Q_C^2 = 1.5$ GeV$^2$
Physical point HVP from $N_f = 2 + 1$ DWF

$L/a = 48$, $a^{-1} = 1.73$ GeV, $m_\pi = 138$ MeV

$Q^2_C = 1.5$ GeV$^2$
Left: Physical point data (Möbius DWF)

Right: Dispersive model study [Golterman et al. '13]

Same qualitative behaviour - Padé [2,2] looks acceptable

Nevertheless, even for Padé [2,2]
- Removing correlations
- Results for different choice of $Q_C^2$ not compatible

Quoting the value for $a_{\mu}^{HLO}$ would be premature
Light and strange contributions separated

Limited statistics (28 meas. config.) with physical $m_\pi$ already gives:

- $\frac{\delta a_\mu^{\text{stat.}}}{a_\mu}$ for light contribution is $O(10)$ larger than for strange HVP
Summary

- Current status with DWF:
  - physical point data with \( \sim 10\% \) stat. errors, measurements underway
  - in addition to the previous non-phys. point computation
- Significant increase signal/noise ratio near \( Q^2 = 0 \) coming from the light sector
- Large systematics with conventional procedure anticipated

Outlook

- Add another lattice spacing with \( m^{phys}_\pi \)
- Hybrid method \[ \text{See talks: K.Maltman (Mon, 14.15, 1D)} \]
- HPQCD time-moment approach \[ \text{See talks: B.Chakraborty (Mon, 15.15,1D)} \] and possible improvements:
  - Discrete moments \[ \text{See talks: K.Maltman (Mon, 14.15, 1D)} \]
  - Large volume limit \[ \text{See talks: C. Lehner (Fri, 15.35, 8D)} \]
- Ultimate goal: \( a^{HLO}_\mu \) with full control over syst. and stat. uncertainties (\(< 1\%\))
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[2, 2] Padé fits for different $Q_C^2$

Take correlations into account

Reference $a_{\mu}^{HLO}(Q_C^2_{\text{ref}})$ subtracted under bootstrap [$Q_C^2_{\text{ref}} = 1.5\,\text{GeV}^2$]

Results for different choice of $Q_C^2$ not combatible $\rightarrow$ uncontrolled systematics