Towards the physical point hadronic vacuum polarisation from Möbius DWF

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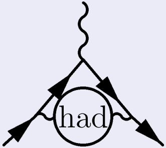
Hadronic vacuum polarisation

Can be computed in Euclidean space-time [Blum '02]

$$\Pi_{\mu\nu} = a^4 \sum_x e^{iQx} \langle J_\mu^{em}(x) J_\nu^{em}(0) \rangle$$



- $\Pi_{\mu\nu}(Q) = (Q^2\delta_{\mu\nu} - Q_\mu Q_\nu)\Pi(Q^2)$
- $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$
- $a_\mu^{HLO} = (\frac{\alpha}{\pi})^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$

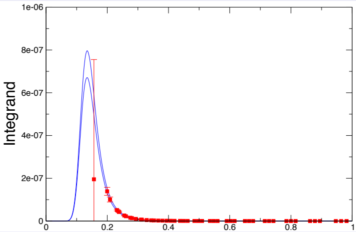
Systematic uncertainties to be controlled - general

1. Simulations at physical m_π
2. Controlled continuum limit, FV effects
3. Disconnected diagrams [V. Gülpers, Mon, 14.55] [Della Morte et al. '10]
4. Obtaining a real world result: charm quark, isospin effects ...

Systematic uncertainties to be controlled - HVP related

- Conventional simulations do not allow access to sufficiently low Fourier momenta
- Integral is dominated in the region where relative errors are enhanced
- Structure of HVP tensor is such that $\Pi(0)$ is not directly accessible
- Systematic uncertainty introduced by extrapolation

Conventional procedure



- $\Pi(Q^2) = \frac{\Pi_{\mu\nu}(Q^2)}{Q_\mu Q_\nu - \delta_{\mu\nu} Q^2}$
- Transverse projection: $Q_\mu = 0$
- Take only diagonal components $\Pi_{\mu\mu}$
- $a_\mu^{HLO} = (\frac{\alpha}{\pi})^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$

Improving the systematics of connected HVP

Several new methods on the market

- R123 procedure ($\Pi(Q^2 = 0)$, utilising twisted BC formalism) [de Divitiis et al '12]
- Padé approximants [Aubin et al '12]
- Dispersive model study [Golterman et al '13]
- Hybrid strategy [Golterman et al '14] [Mon, 14.15,Sess 1D]
- HPQCD time moments [Chakraborty et al '14] [Mon, 15.15, Sess 1D]
- ...

Challenge: Apply the optimal procedure to physical point data

This work: Fitting Padé approximants on the fresh DWF physical point data

inspired by [Aubin et al. '13]

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Challenge: Apply the optimal procedure to physical point data

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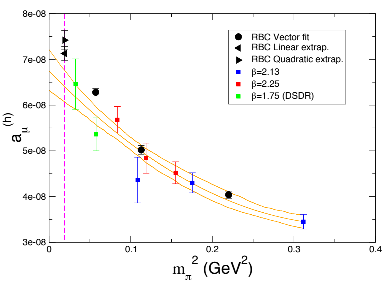
Previous RBC-UKQCD computation of a_μ^{HLO} [Boyle et al'11]

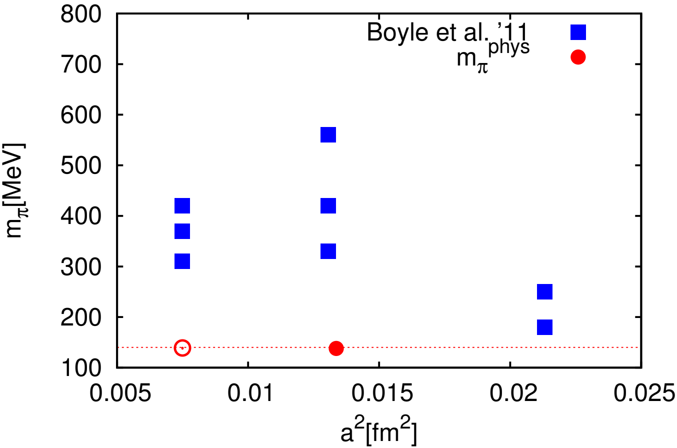
Non physical m_π, $a^{-1} \approx 1.3, 1.7, 2.3\ \mathrm{GeV}$

- Local current at source, conserved at sink
- DWF (Möbius scale=1.0), Iwasaki/DSDR gauge action
- Fitting Q^2- dependence of $\Pi(Q^2)$ up to $Q_C^2 \approx 2.5 - 9\ \mathrm{GeV}^2$



- Strong m_π dependence
- Eliminate the systematics of chiral extrapolation: computing HVP at m_π^{phys}

RBC-UKQCD $N_f = 2+1$ Domain Wall ensembles



- a_μ^{HLO} from DWF for non-physical m_π [Boyle et al '11]
- physical point HVP (•) recently measured → preliminary results!

a_μ^{HLO} from DWF at physical pion mass

Physical point lattice parameters:

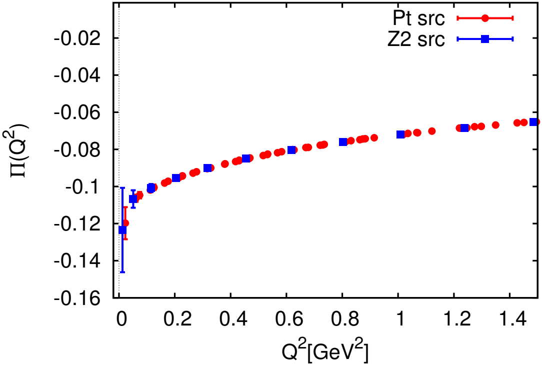
- Möbius DWF, Iwasaki gauge action
  - $48^3 \times 96 \times 24$, $a^{-1} = 1.73\ \mathrm{GeV}$ *-measurements underway*
  - $64^3 \times 128 \times 12$, $a^{-1} = 2.31\ \mathrm{GeV}$

HVP with Möbius DWF

- Möbius scale $=2.0$
- Möbius conserved current [see talk by P.Boyle, Mon 6.10p.m., 2.B]
- Local current at source, conserved at sink
- Point source, 12 source positions

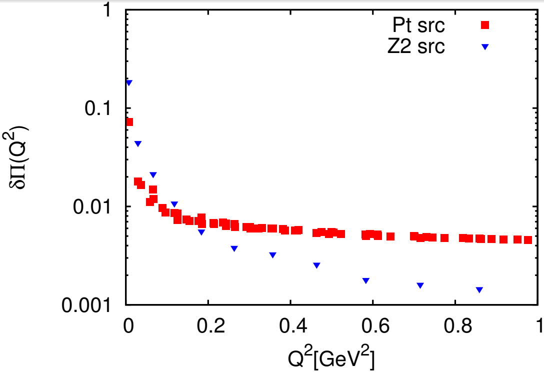
Point vs. stochastic source

- Point source, 12 source positions
- Z(2) wall source, 48 source positions
- (one-end trick) [McNeile et al. '06]



Point vs. stochastic source

- Point source, 12 source positions
- Z(2) wall source, 48 source positions
- (one-end trick) [McNeile et al. '06]
- Comparison (12 src. positions each, log scale on y-axis)
- Point src. better in low-Q^2 region ($Q^2 <\sim 0.2\ GeV^2$)



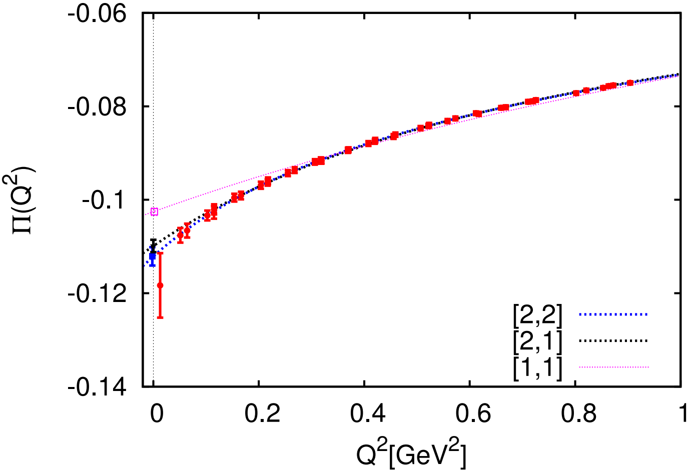
Physical point HVP from $N_f = 2 + 1$ DWF

Physical point data:

- $L/a = 48^3 \times 94 \times 24, \quad a^{-1} = 1.73 GeV$
- $\Pi(Q^2)$ convergent sequence of PAs[Aubin et al,'13]
  - VMD is unreliable
- Padé approximants [N,D]

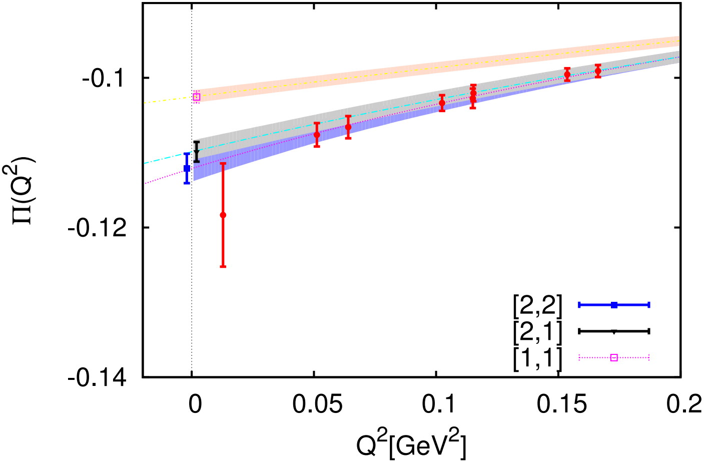
$$\Pi_{[N,D]}(Q^2) = \frac{\sum_{n=0}^{N-1} a_n Q^{2n}}{1+\sum_{m=1}^{D} b_m Q^{2m}}$$

Physical point HVP from $N_f = 2+1$ DWF



- $L/a = 48$, $a^{-1} = 1.73\ \mathrm{GeV}$, $m_\pi = 138\ \mathrm{MeV}$
- $Q_C^2 = 1.5\ \mathrm{GeV}^2$

Physical point HVP from $N_f = 2 + 1$ DWF



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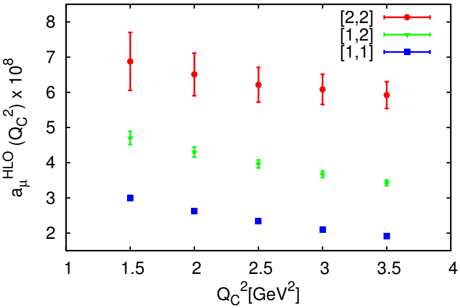
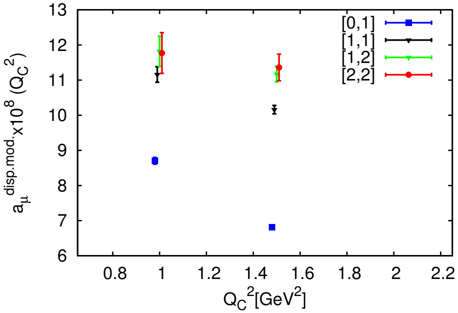
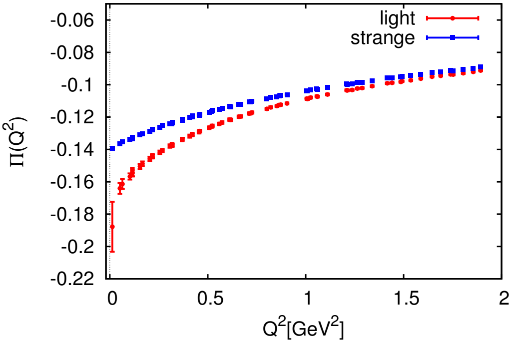




- **Left**: Physical point data (Möbius DWF)
- **Right:** Dispersive model study [Golterman et al. '13]
- Same qualitative behaviour - Padé [2,2] looks acceptable
- Nevertheless, even for Padé [2,2]
  - Removing correlations
  - Results for different choice of Q_C^2 not compatible
- Quoting the value for a_μ^{HLO} would be premature

Physical point HVP from $N_f = 2 + 1$ DWF

Light and strange contributions separated



Limited statistics (28 meas. config.) with physical m_π already gives:

- $\frac{\delta a_\mu^{stat.}}{a_\mu}$ for light contribution is $O(10)$ larger than for strange HVP

Summary and outlook

Summary

- Current status with DWF:
  - physical point data with $\sim$10% stat. errors, measurements underway
  - in addition to the previous non-phys. point computation
- Significant increase signal/noise ratio near $Q^2 = 0$ coming from the light sector
- Large systematics with conventional procedure anticipated

Outlook

- Add another lattice spacing with m_π^{phys}
- Hybrid method [See talks: K.Maltman (Mon, 14.15, 1D)]
- HPQCD time-moment approach [See talks: B.Chakraborty (Mon, 15.15,1D)] and possible improvements:
  - Discrete moments [See talks: K.Maltman (Mon, 14.15, 1D)]
  - Large volume limit [See talks: C. Lehner (Fri, 15.35, 8D)]
- Ultimate goal: a_μ^{HLO} with full control over syst. and stat. uncertainties ($< 1\%$)

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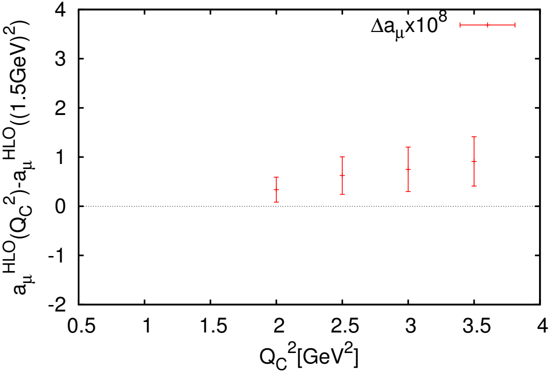
- The calculations reported here have been done on DIRAC Bluegene/Q computer at the University of Edinburgh's Advanced Computing Facility

Physical point HVP



- $[2,2]$ Padé fits for different Q_C^2
- Take correlations into account
- Reference $a_\mu^{HLO}(Q^2_{C\ ref})$ subtracted under bootstrap $[Q^2_{C\ ref} = 1.5 GeV^2]$
- Results for different choice of Q_C^2 not combatible $\rightarrow$ uncontrolled systematics