Quark spin in the nucleon from Anomalous Ward Identity with Overlap Fermion

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χ QCD Collaboration

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Outline

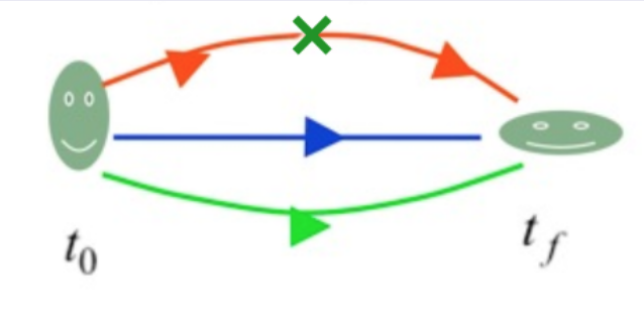
Introduction

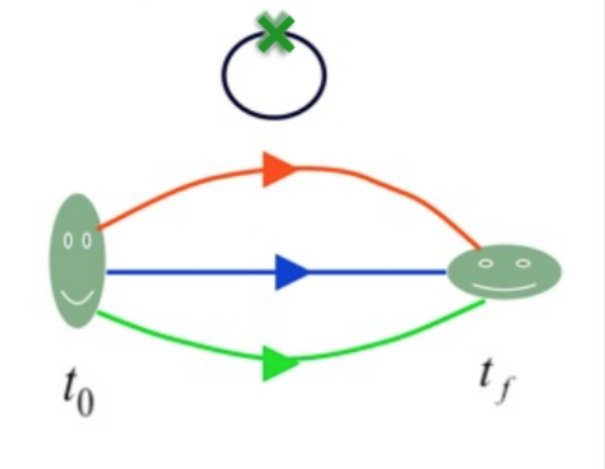
"Spin crisis"

$$\Delta\Sigma = \sum_q \Delta q \sim 0.2 - 0.3$$

- Quark spin?
- Quark orbital angular momentum?
- Glue spin?
- Glue orbital angular momentum?

Lattice approach to the quark spin





The direct method for DI

The forward matrix element of the axial current

$$s_\mu \Delta q = \langle p, s | \bar{q} i \gamma_\mu \gamma_5 q | p, s \rangle$$

- A flavor-singlet renormalization factor should be taken care of.
- The axial loop is not saturated by the low modes and gain little benefit from the LMA technique[a]. More inversions should be done to get satisfactory statistical signals.

[a] M. Gong et al., arXiv:1304.1194

The method with the anomalous Ward identity

The three-point function with pseudo-scalar and anomaly currents

$$\langle p,s|A_\mu|p,s\rangle\, s_\mu = \lim_{q\to 0}\frac{i|s|}{q.s}\left\langle p',s\left|2\sum_{f=1}^{N_f} m_f\bar{q}_f i\gamma_5 q_f + N_f\frac{i}{8\pi^2}G^{\alpha}_{\mu\nu}\tilde{G}^{\alpha\mu\nu}(q)\right|p,s\right\rangle$$

- R. Gupta and J. Mandula first tried this method with quenched configurations.
- The statistical signal is rather poor by involving the gauge links.

The local topological charge density operator

$$D_{ov} = 1 + \gamma_5 \epsilon(\gamma_5 D_w)$$

$$q(x) = tr\gamma_5(1 - \frac{1}{2}D_{ov}) = \frac{1}{16\pi^2} G^{\alpha}_{\mu\nu} \tilde{G}^{\alpha\mu\nu}$$

$$\langle p, s|A_\mu|p, s\rangle s_\mu = \lim_{q\to 0} \frac{i|s|}{q.s} \left\langle p', s \left| 2\sum_{f=1}^{N_f} m_f \bar{q}_f i\gamma_5 q_f + 2iN_f q \right| p, s \right\rangle$$

- g_A^0 is nonperturbatively renormalized by AWI.
- The contribution from anomaly and from each flavor can be investigated separately.
- The low-mode and the high-mode parts are separated and can be improved with different techniques.

Techniques adopted for DI

- Deflated overlap inverter with HYP-smeared DWF configurations.
  - The inversion is sped up by more than 50 times[a].
- Low-mode substitution is adopted to construct the nucleon correlation function with smeared Z_3 noise grid sources.
  - The error bars of nucleon mass are reduced by 14 times.
  - The error bars are further reduced by averaging 32 source time slices.

See plots

- Low-mode average is adopted to construct the quark loop.
  - The pseudo-scalar loop is well saturated by low modes. See plots
- The topological charge density is calculated with diluted Z_4 noise sources.
  - The dilution scheme is $(2, 2, 2, 2)$ with even-odd dilution. $32 \times 12 \times 20 = 7680$ Dirac matrix multiplications per configuration.

[a] A. Li et al., Phys.Rev.D82:114501,2010

Extraction the form factor from the 3pt and 2pt functions

The ratio of 3pt and 2pt functions

$$R(t',t) = \frac{iE_p}{m_N p} e^{(E_p - m_N)(t-t')} \frac{\left\langle C_N^{pol}(t,p)(m_q L_{PS}(t',p) + q(t,p) - V.A.) \right\rangle}{\left\langle C_N^{unpol}(t,0) \right\rangle}$$

Also we have square-root technique to extract the ratio. See formulae

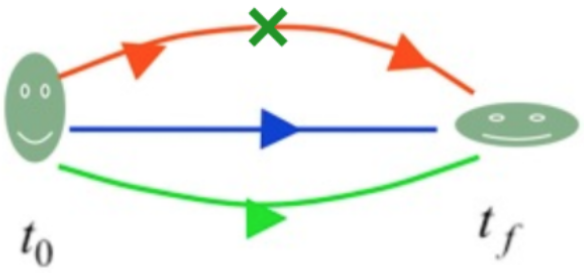
The slope of the summed ratio

$$R'(t) = \sum_{t'=t_0+1}^{t-1} R(t',t) \sim A(Q^2)t + B(Q^2)$$

The momentum extrapolation

$$\Delta q^{disc} = \lim_{Q^2 \to 0} A(Q^2)$$

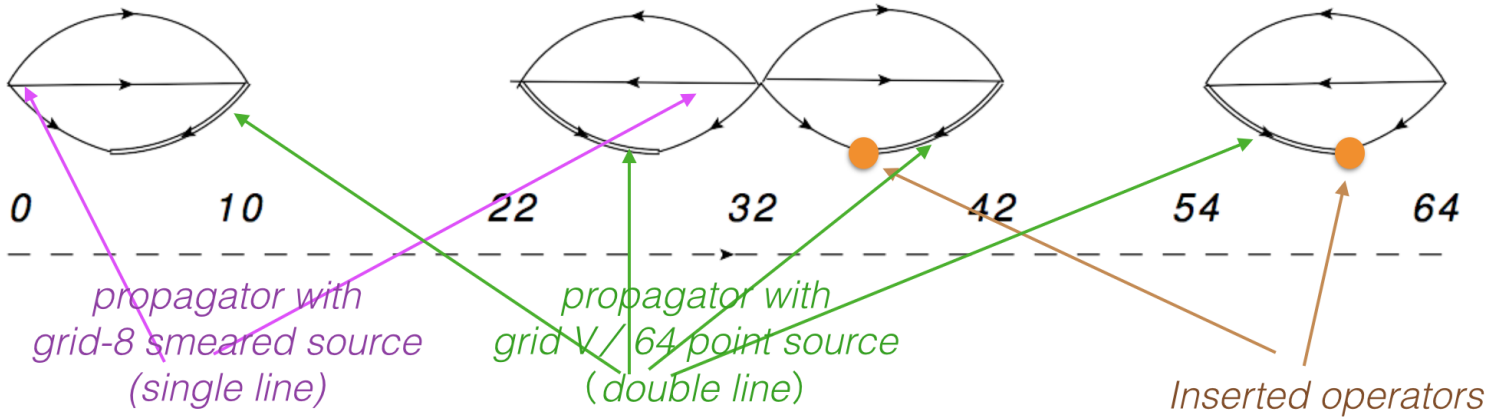
Different methods for CI



Pros and Cons

- The sequential method
  - \+ No artificial noises introduced.
  - \- The momenta, interpolation fields, quark masses have to be fixed on sequential source time slice.
- The two-end method
  - \+ More flexible for different momenta and interpolation fields.
  - \+ Compatible with multi-mass solvers.
  - \- introduce artificial nosies which lead to more statistical errors.

The two-end method



Techniques adopted for CI

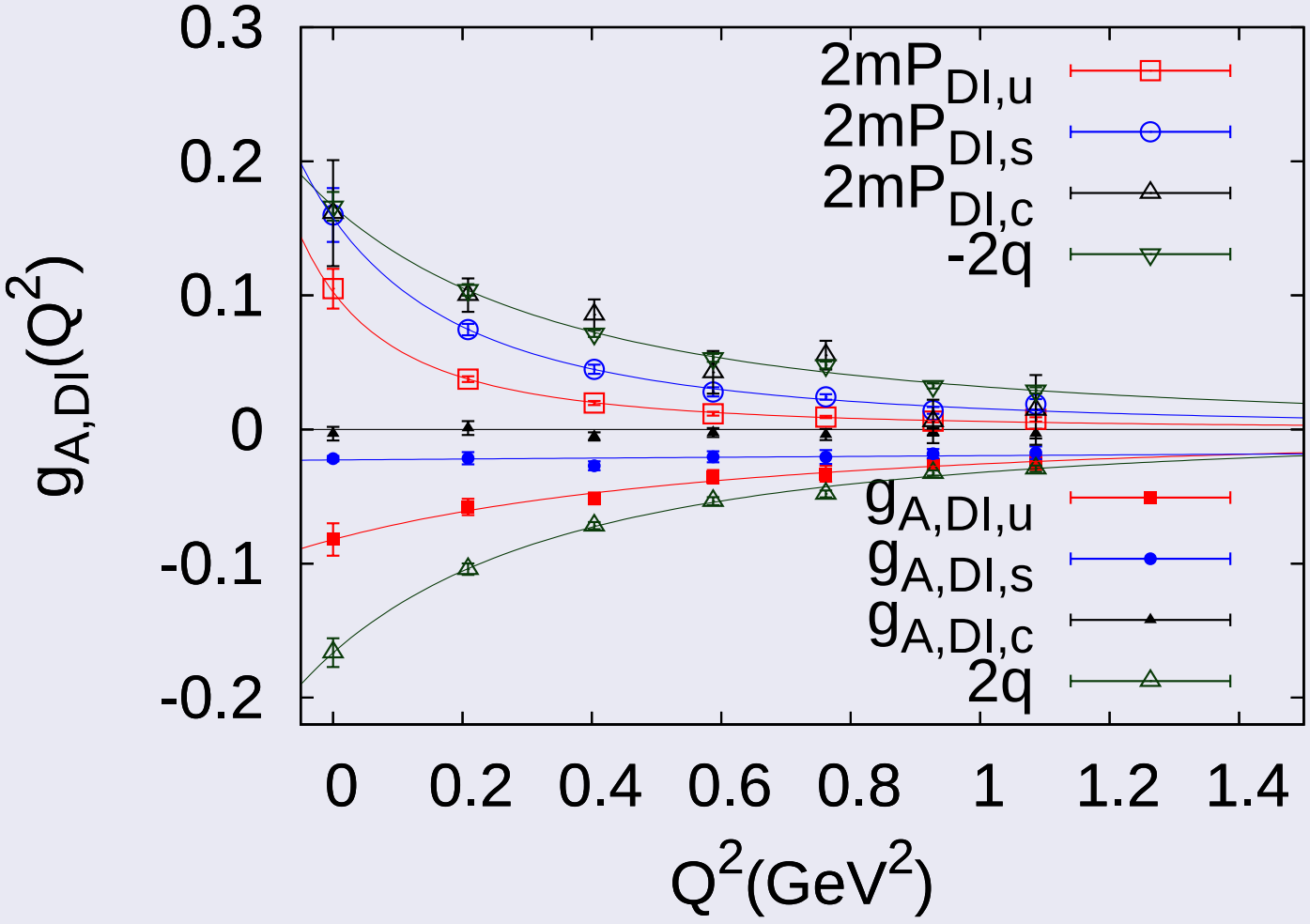
- Smeared grid-8 sources on $t = 0, 32$ and Point grid-64 sources on t=$10, 22, 42, 54$ for the high mode calculations.
- Extended low-mode substitution on 4 propagators. The low mode part of the propagator from current to sink is the all-to-all propagator.

The simulation parameters

Fermion actions	Valence overlap on DWF configurations from RBC
Lattice size	$24^3 \times 64$
Lattice spacing	$1/a = 1.77(5)$ GeV
$m_{ud}^{(sea)}$	0.005
$m_s^{(sea)}$	0.04
$m_\pi^{(sea)}$	≈ 331 MeV
Number of configurations	203

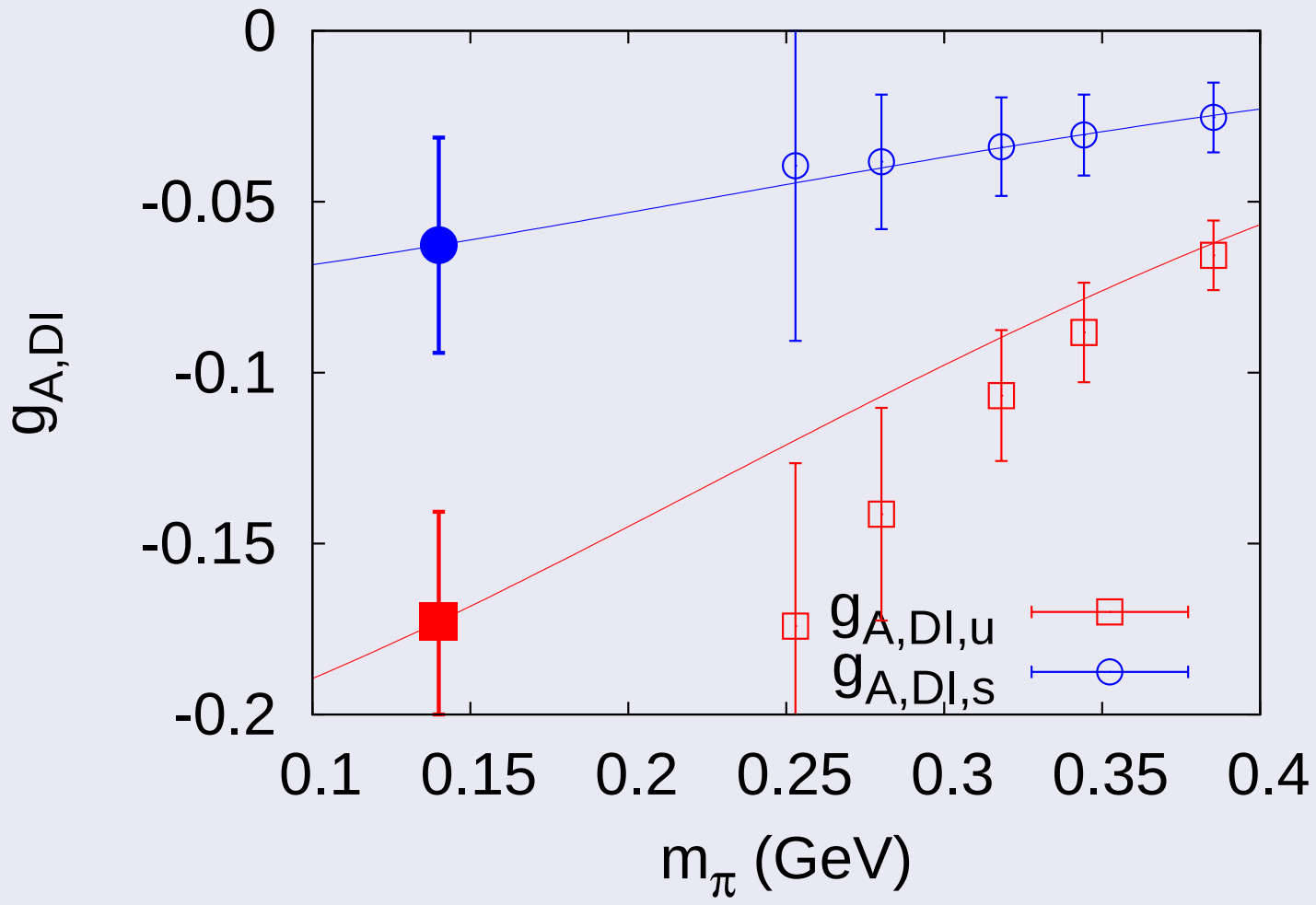
The momentum extrapolation of DI

Extrapolation with the dipole fit

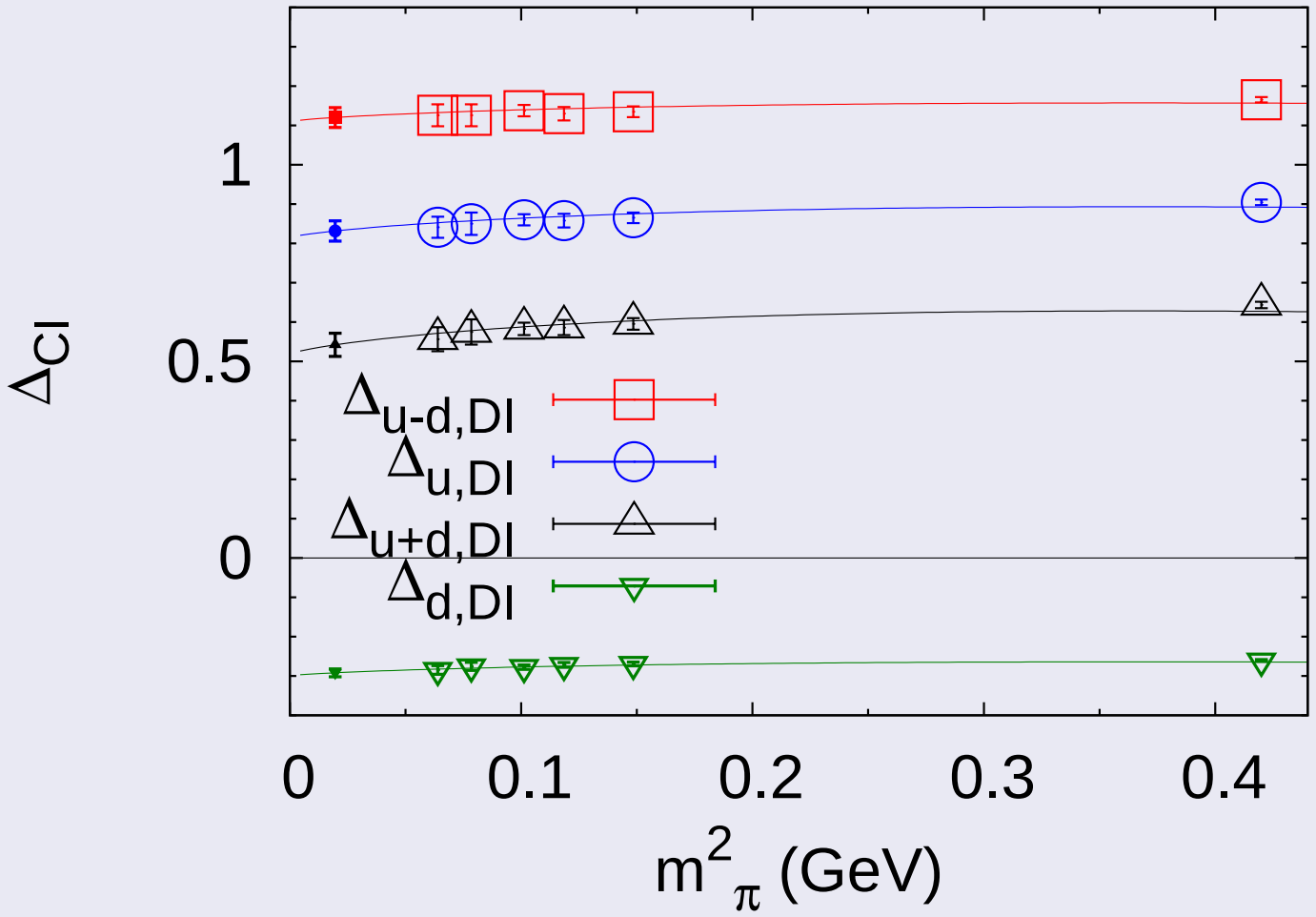


The chiral extrapolation of DI

Chiral extrapolation with $m_\pi^2 ln(m_\pi^2/\Lambda^2)$ fit at $\Lambda = 1.0 GeV$



The chiral extrapolation of CI



The result numbers

The results after chiral extrapolation

u-d (CI)	1.120(28)
u+d (CI)	0.542(30)
u/d (DI)	-0.173(32)
s (DI)	-0.063(32)
u+d+s	0.133(63)
u+d-2s	0.322(70)

Conclusion

Summary

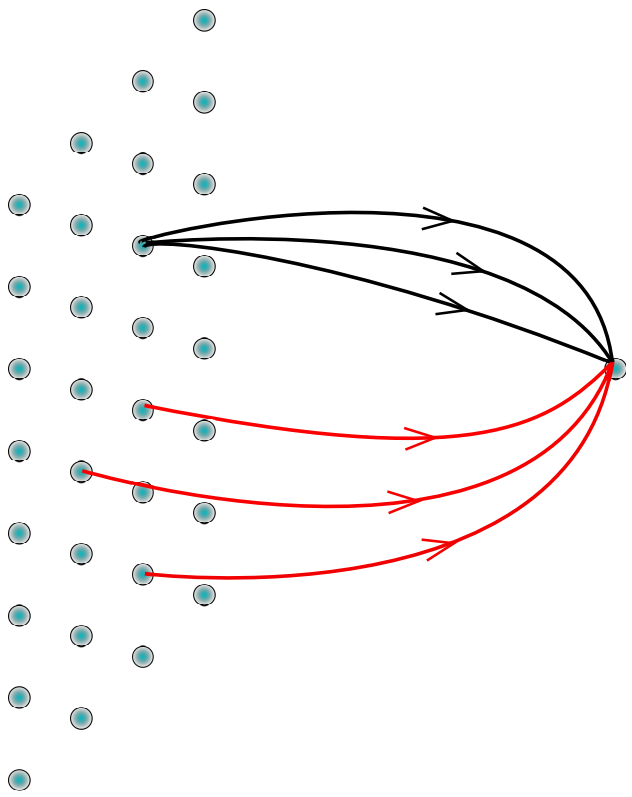
- The connected and disconnected quark contributions to the nucleon spin are calculated with overlap fermions.
- g_A^0 is nonperturbatively renormalized by the anomalous Ward identity.
- The anomaly contribution is negative. For heavy quarks, the anomaly part and the pseudo-scalar part cancel out.
- The disconnected insertion contributions of the u/d/s quarks are large negative.

Future plans

- Data on different lattices and sea quark masses will be done and we will do the full chiral and continuum extrapolations. (The $48^3 \times 96$ lattice with $L = 5.5$fm and physical pion mass is proposed.)
- The systematical errors will be carefully analyzed.

Thank you !

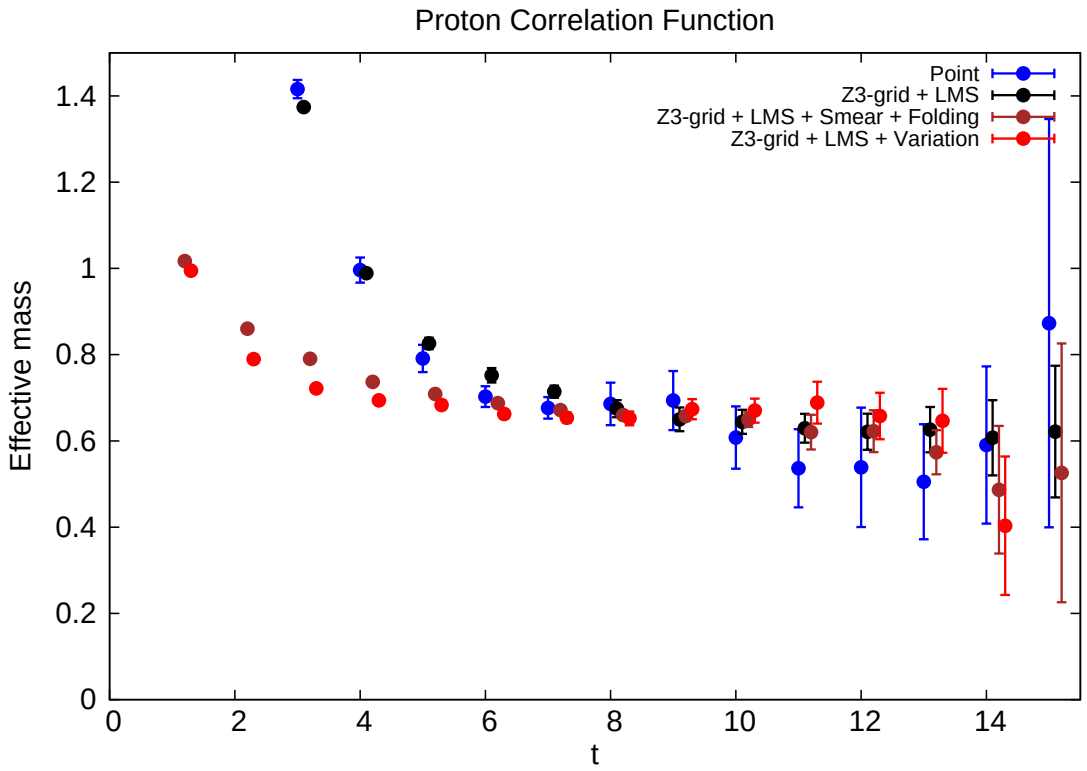
Backup pages



Backup pages

Lattice settings

Lattice size : $24^3 \times 64$, $m_{ud}^{(sea)} = 0.005$, $m_s^{(sea)} = 0.04$, $m_\pi \approx 305\text{MeV}$
47 configurations are used



Point source

$m_{proton} = 1.13(14)\text{GeV}$

Grid with LMS

$m_{proton} = 1.08(5)\text{GeV}$

Smeared Grid with LMS

$m_{proton} = 1.14(2)\text{GeV}$

Variation

$m_{proton} = 1.12(1)\text{GeV}$

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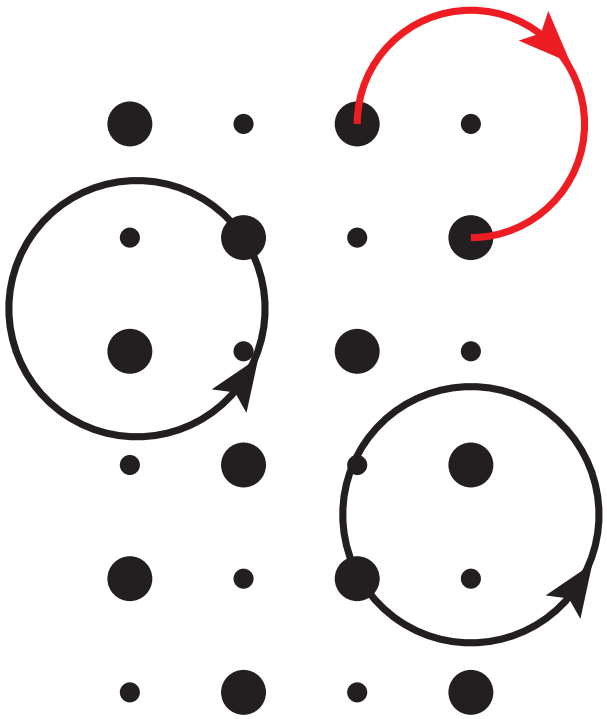
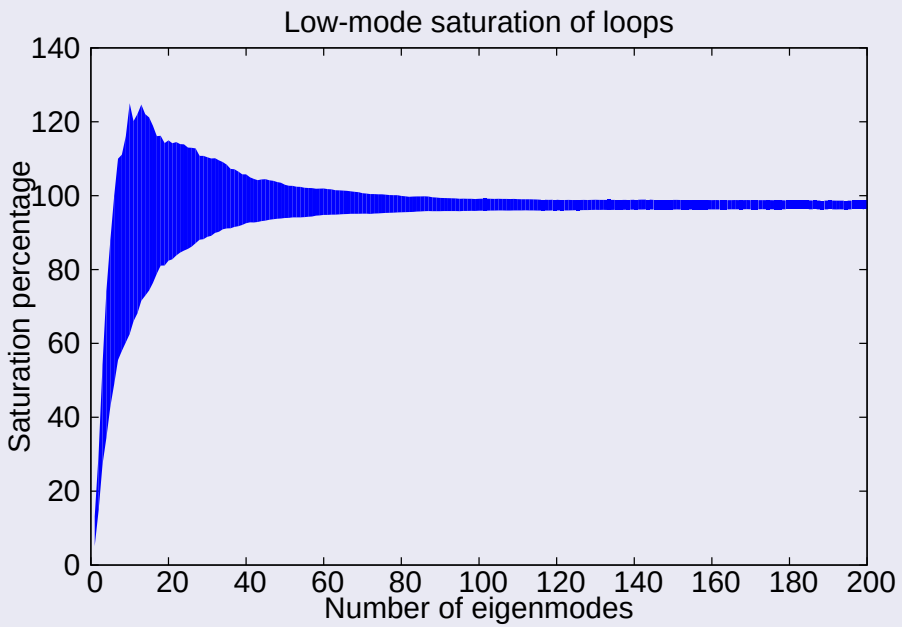


Backup pages

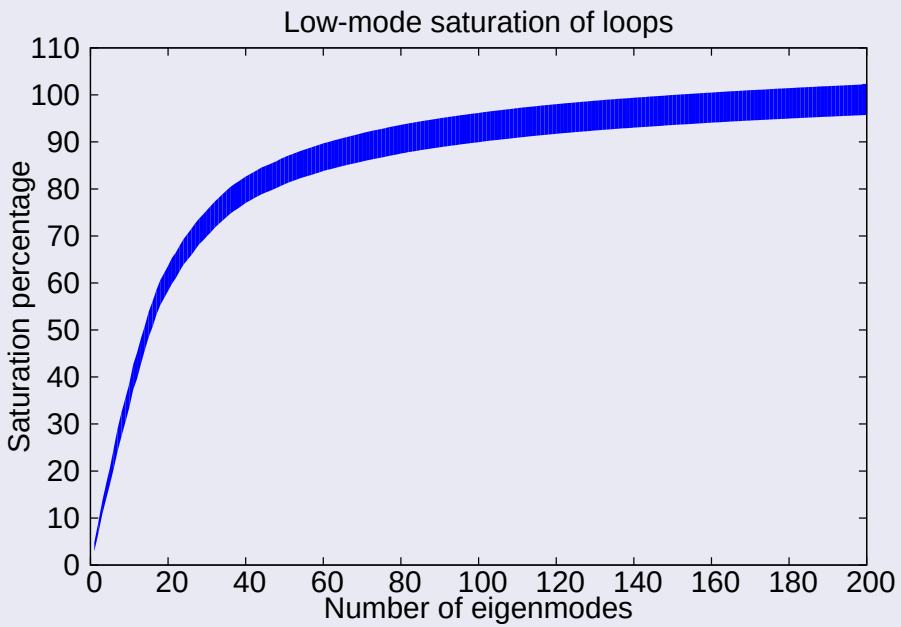
Lattice settings

Lattice size : $24^3 \times 64$, $m_{ud}^{(sea)} = 0.005$, $m_s^{(sea)} = 0.04$, $m_\pi \approx 305$MeV
11 configurations are used
The lowest 200 pair eigenmodes of each configuration are extracted

Saturation of pseudo-scalar loops



Saturation of scalar loops



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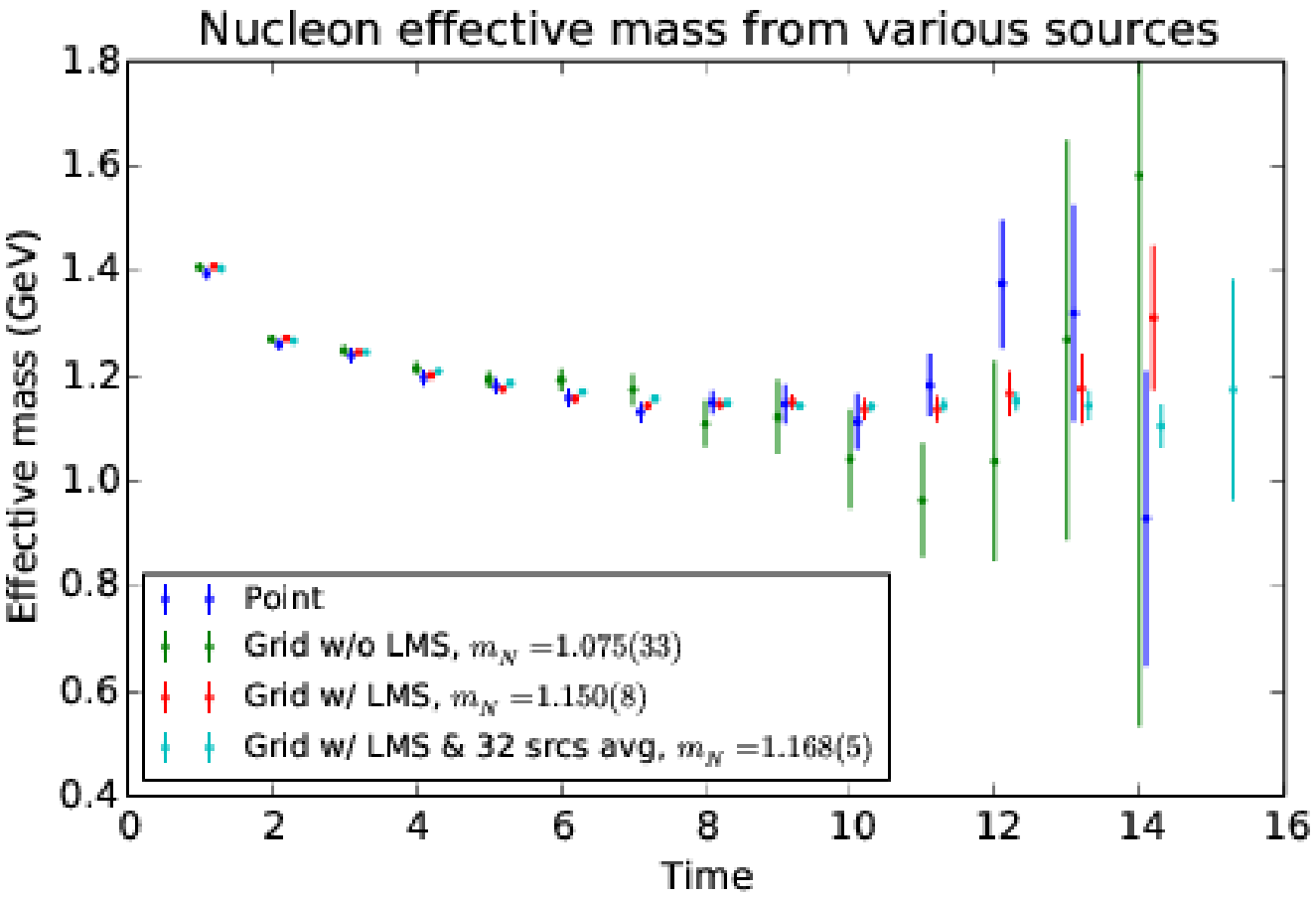

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- the standard way to get matrix element from 3pt and 2pt (with square root trick) is

$$\langle O_2(p')|O|O_1(p)\rangle = C^3(p', t_2, p-p', t_1, p, 0) * f(E(p'), E(p), m) * \sqrt{\frac{C^2_{O_1O_1}(p, t_2-t_1, 0)}{C^2_{O_1O_1}(p, t_1, 0)C^2_{O_1O_1}(p, t_2, 0)}}\sqrt{\frac{C^2_{O_2O_2}(p', t_1, 0)}{C^2_{O_2O_2}(p', t_2-t_1, 0)C^2_{O_2O_2}(p', t_2, 0)}}$$

- The benefit of this trick is that it avoids the work to pick out the matrix element of the 2pt function with jackknife, just need to play with the 2pt function itself.

- If we don't have O2 as source operator? No problem!

$$\sqrt{\frac{C^2_{O_2O_2}(p', t_1, 0)}{C^2_{O_2O_2}(p', t_2-t_1, 0)C^2_{O_2O_2}(p', t_2, 0)}} \rightarrow \sqrt{\frac{C^2_{O_1O_1}(p', t_1, 0)}{C^2_{O_2O_1}(p', t_2-t_1, 0)C^2_{O_2O_1}(p', t_2, 0)}}$$

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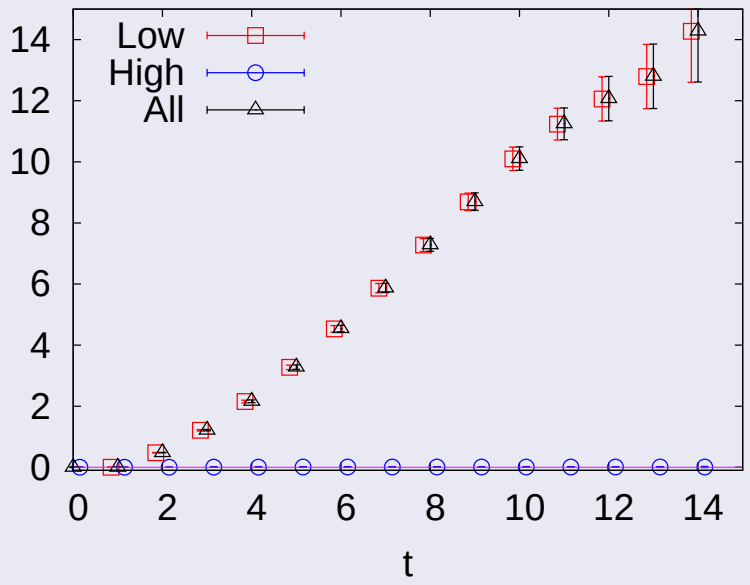
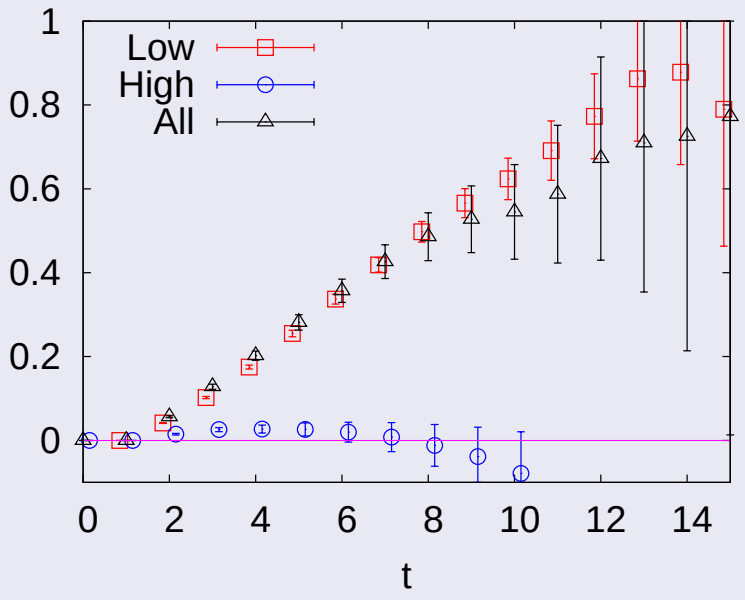
Saturation of 2mp for u/d quarks



Saturation of 2mp for c quark



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