Anisotropy of the quark anti-quark potential in a magnetic field

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Based on: PRD 89 (2014) 114502
C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, F. Sanfilippo.

Outline

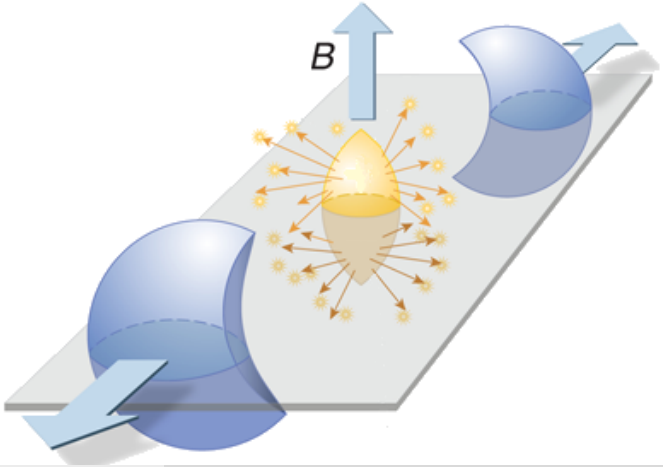
- Phenomenological Motivation
- Lattice QCD & Magnetic Fields (eB)
- Static $Q\overline{Q}$ potential in the presence of (eB)
- Summary and Perspectives

Phenomenological Motivation

ElectroWeak corrections are often small if compared to the Strong int.
But: what happens if we consider the presence of an external magnetic field, eB, large enough to be comparable with the scale Λ_{QCD}?

- Astrophysics - in a class of neutron stars, called **magnetars**: $eB \sim 10^{10}$ T
  [Duncan and Thompson, '92]
- Cosmology - during the **ElectroWeak phase transition**: $eB \sim 10^{16}$ T
  [Vachaspati, '91]
- Heavy ion collisions - at LHC in
  non-central HIC: $eB \sim 10^{15}$ T $\sim 15 m_\pi^2$
  [Skokov, Illarionov and Toneev, '09]

$1\ \mathrm{GeV}^2 \sim 5 \cdot 10^{15}$ T



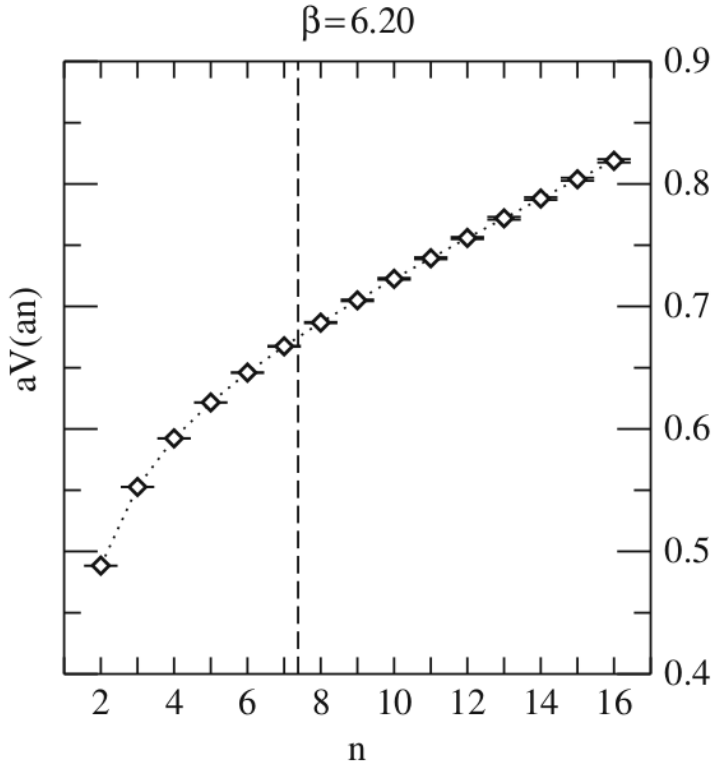
The static $Q\overline{Q}$-Potential

$V_{Q\overline{Q}}$ is a non-perturbative feature of QCD.
A property of gauge fields only.

Parameterization $\rightarrow$ **Cornell Potential**

$$V_{Q\overline{Q}}(\vec{r}) = C + \sigma|\vec{r}| + \frac{\alpha}{|\vec{r}|}$$

- $\sigma \equiv$ String Tension
- $\alpha \equiv$ Coulomb Parameter



- A description of confinement
- Spectrum for heavy mesons $\rightarrow$ NR bound states of heavy quarks ($c\overline{c}$,$b\overline{b}$)
- Sommer parameter r_0 for scale setting

POSSIBLE DEPENDENCE ON eB?
PRESENCE OF ANISOTROPIES?

Lattice QCD & magnetic field

- A background QED field enters the Lagrangian by modifying the covariant derivative:

$$D_\mu = \partial_\mu + igA^a_\mu T^a \qquad \longrightarrow \qquad \partial_\mu + igA^a_\mu T^a + iqa_\mu$$

On the lattice:

- Gluon field $A^a_\mu(x)T^a \longrightarrow U_\mu(n)$, SU(3) link variables (integration variables)
- Photon field $a_\mu(x) \longrightarrow u_\mu(n)$, U(1) link variables (fixed)

- Quantization of (eB) $\longrightarrow$ IR Effect due to periodic b.c.

$$u_y(n) = e^{ia^2 qBn_x} \qquad u_x(n)|_{n_x=N_x} = e^{-ia^2 qBN_x n_y}$$

- The lattice discrete derivative will read:

$$D_\mu \overline{\psi} \longrightarrow \frac{1}{2a}\left(U_\mu(n)u_\mu(n)\psi(n+\hat{\mu}) - U^\dagger_\mu(n-\hat{\mu})u^*_\mu(n-\hat{\mu})\psi(n-\hat{\mu})\right)$$

- The magnetic field can influence the gluon field through quark loops!

HOW MUCH?

Effects of (eB) on the gluon fields

- Quark condensate vs (eB) = valence + sea
  [D'Elia and N, PRD 83 (2011) 114028]
- Effective θ term induced by CP-odd e.m. fields
  [D'Elia, Mariti and N, PRL '13]
- Topological charge correlators
  [Bali, Bruckmann et al., JHEP 1304 (2013) 130]
- Polyakov loop dependence on (eB)
  [D'Elia, Mukherjee, Sanfilippo, PRD 82 051501 2010]
- Inverse catalysis
  [Bruckmann, Endrődi and Kovács, JHEP 1304 (2013) 112]
- The Pseudo Critical Temperature decreases
  [Bali et al., JHEP 1202 (2012) 044]

Moreover, the magnetic field direction can induce anisotropies:

- Anisotropy of the plaquettes [Ilgenfritz, Muller-Preussker et al., PRD 89 (2014); Bali, Bruckmann et al., JHEP 1304 (2013) 130]

... what about the static potential?

The static potential can be extracted from the Wilson Loop observable:

$$aV(an_s) = \lim_{n_t \to \infty} \log \left(\frac{\langle W(an_s, a(n_t+1)) \rangle}{\langle W(an_s, an_t) \rangle} \right)$$

Since we expect to see anisotropies, we cannot average all the possible Wilson Loops. We build two classes of Wilson Loops:

$$\begin{aligned} W_\perp &= W_{XY} = (W(an_x, an_t) + W(an_y, an_t))/2 \\ W_{||} &= W_Z = W(an_z, an_t) \end{aligned}$$

The potentials obtained from the two classes ($V_\perp$ and $V_{||}$) at $eB \neq 0$ are different.
We fitted the potentials separately, according to the standard Cornell potential:

$$aV_\perp(an_\perp) = aV_{XY}(an_{XY}) = ac_{XY} + \frac{\alpha_{XY}}{n_{XY}} + \sigma_{XY} a^2 n_{XY}$$

$$aV_{||}(an_Z) = aV_Z(na_Z) = ac_Z + \frac{\alpha_Z}{n_Z} + \sigma_Z a^2 n_Z$$

Then we compute ratios $\mathcal{O}(B)/\mathcal{O}(B=0)$.

Numerical Setup

The Lattice approach consists in a discretization of the Euclidean Feynman path integral:

$$\mathcal{Z} = \int \mathcal{D}U\, \mathcal{D}\overline{\psi}\, \mathcal{D}\psi \;\; e^{-S_{YM}[U] - \overline{\psi}_f M_f^D \psi_f} = \int \mathcal{D}U \;\; e^{-S_{YM}[U]} \prod_f (\det M_f^D[U])^{1/4}$$

We adopt a state-of-art discretization for QCD with $N_f = 2 + 1$:

- Gauge sector: tree level improved Symanzik action
  [Weisz, Nucl Phys B '83; Curci, Menotti and Paffuti, Phys Lett B '83]
- Fermionic sector: rooted staggered fermions $\oplus$ stout smearing improvement
  [Morningstar and Peardon, PRD '04]

The bare parameters we adopted in our simulations have been taken from [Borsanyi, Endrodi, Fodor et al., JHEP '10].
They correspond to the "physical" line of constant physics ($m_\pi^{LAT} = m_\pi^{PHYS}$).

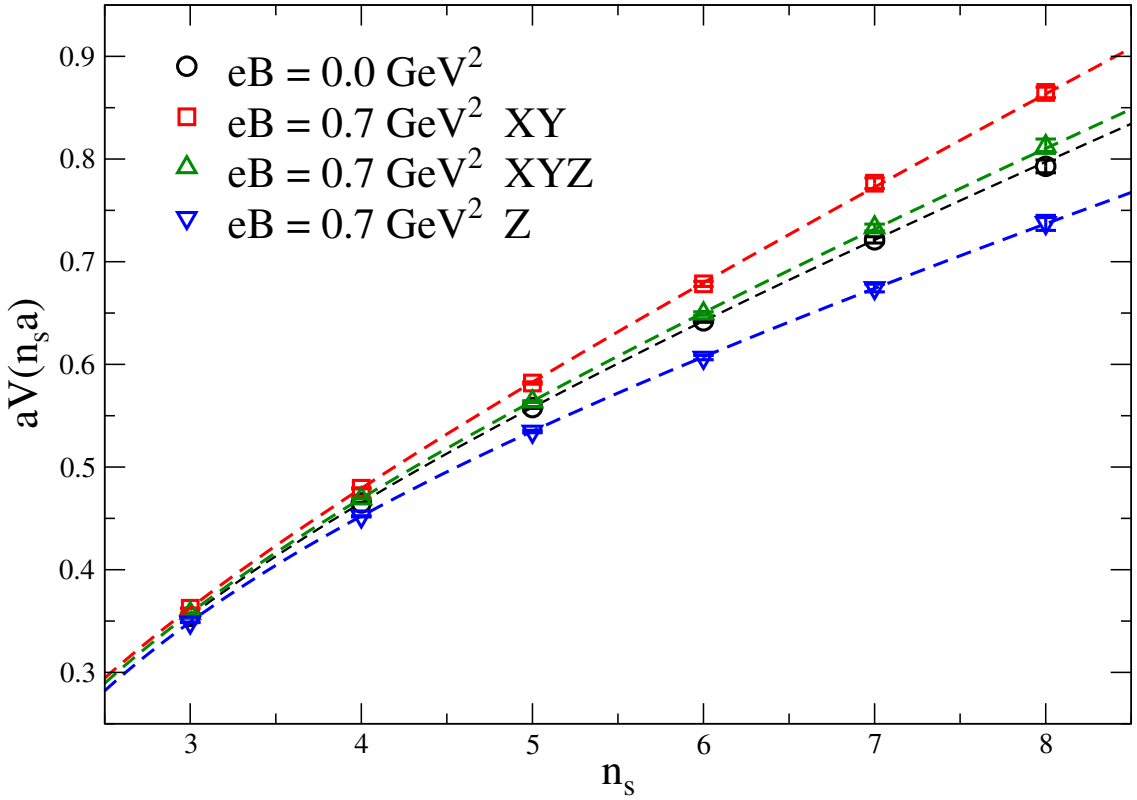
Simulations have been performed on the BlueGene/Q machine at CINECA, Italy.

Lattices

$a = 0.2173$ fm $\longrightarrow 24^4$
$a = 0.1535$ fm $\longrightarrow 32^4$
$a = 0.1249$ fm $\longrightarrow 40^4$
The physical volume is kept $\sim$fixed at $V_4 = (5\text{ fm})^4$

Anisotropic potential

An example: 40^4 with a lattice spacing $a = 0.1249$ fm
The modification can be ascribed to a modification of both σ and α.



PS: If we do not distinguish between the XY and the Z directions, we almost loose any dependence of the potential on (eB).

Isotropic smearing for noise reduction

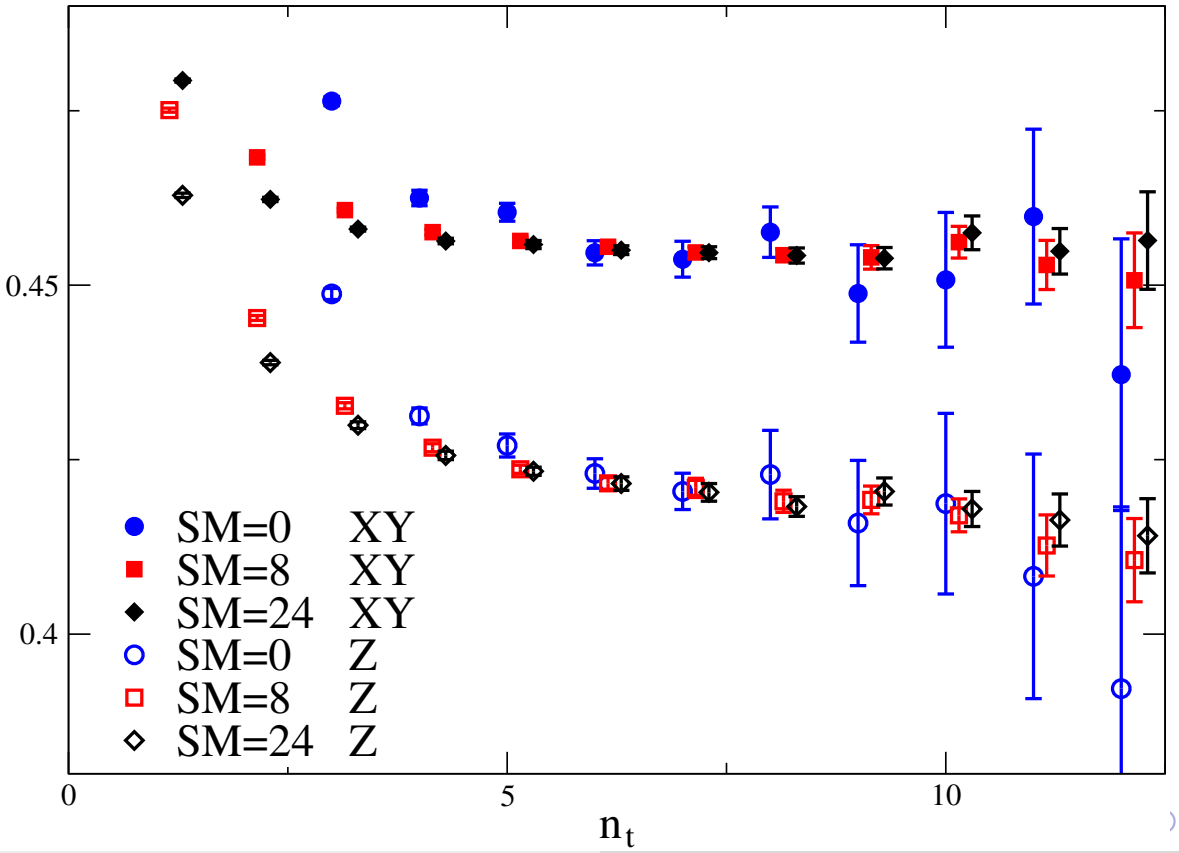
Typically, Wilson Loops (ratios) are noisy observables

$\longrightarrow$ we need to smooth the configurations

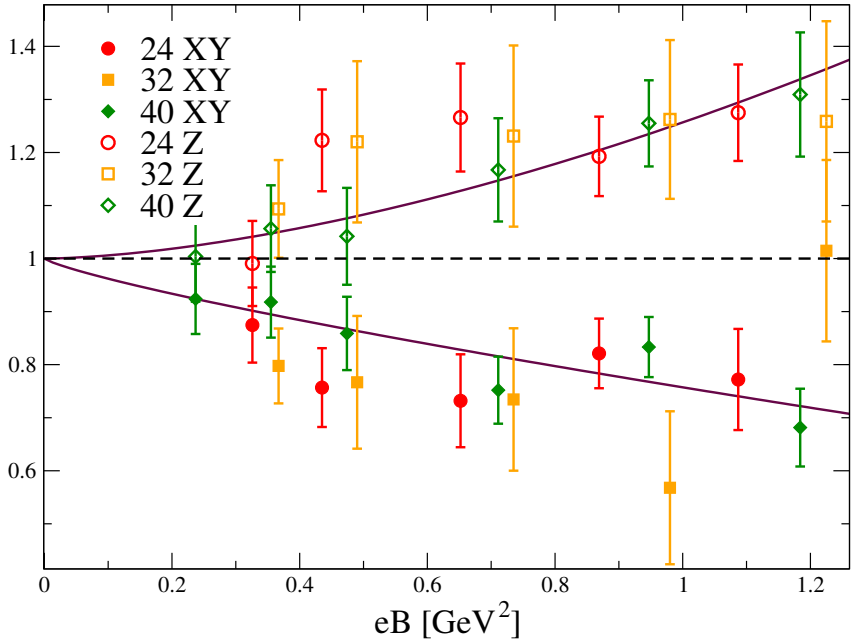
- Temporal links: 1 single HYP smearing.
- Spatial links: n levels of spatial APE smearing ($n = 8,\ 16,\ 24,\ 32,\ 40$).

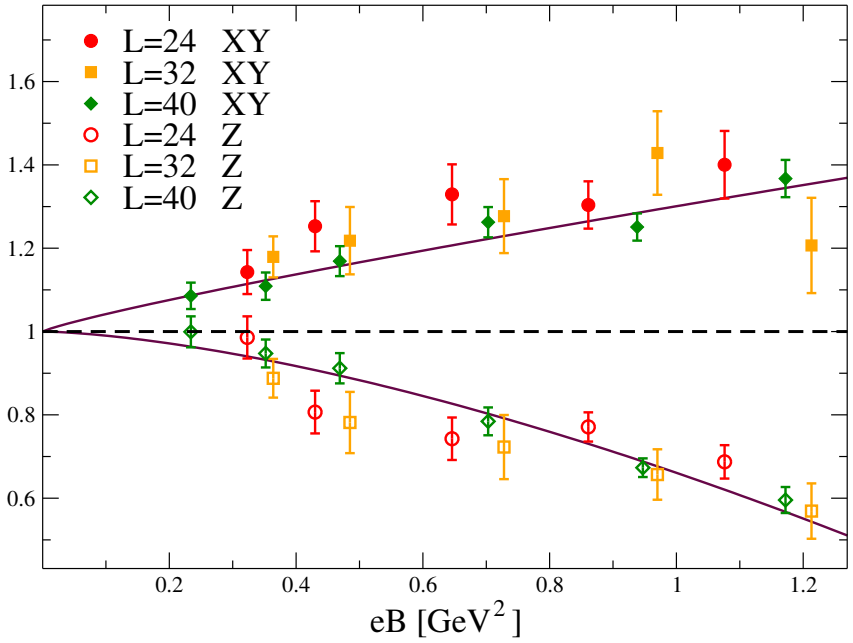
Example: 32^4 lattice at $(eB) \simeq 0.97$ GeV^2, $R = 3a \simeq 0.46$ fm

$$\log\left(\frac{\langle W(an_s, a(n_t+1))\rangle}{\langle W(an_s, an_t)\rangle}\right)$$



String Tension and Coulomb Term





$\alpha_Z > \alpha_{XY}$

$\sigma_{XY} > \sigma_Z$

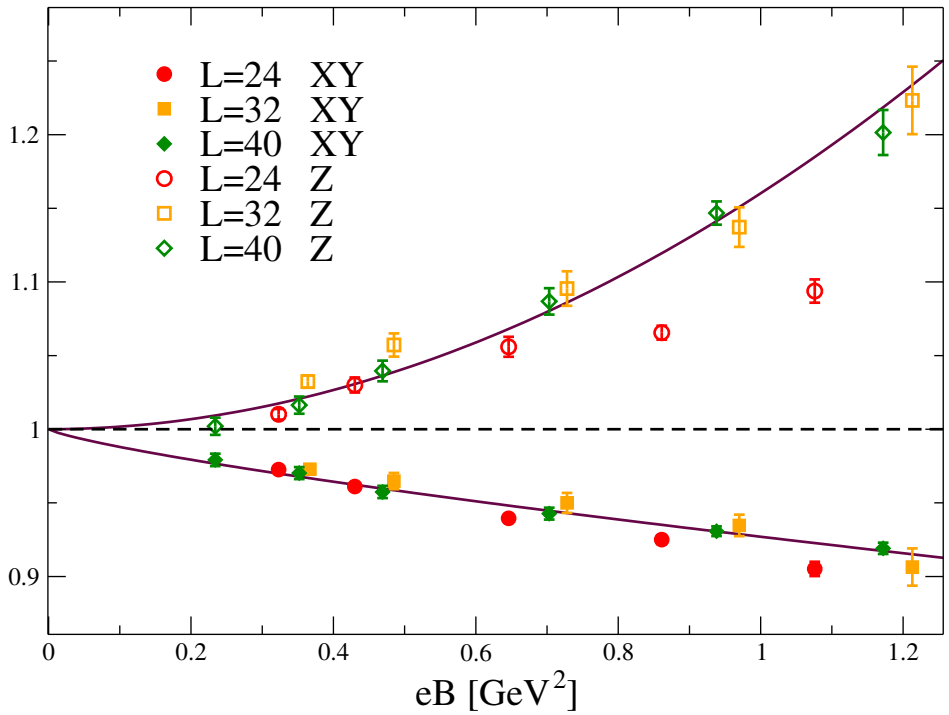
We fit our data at the finest lattice $\longrightarrow$ $Ratio = 1 + A(eB)^C$

Obs	A	C	χ^2/dof
α_{XY}	-0.24(3)	0.7(2)	1.5
α_Z	0.24(3)	1.7(4)	0.3

Obs	A	C	χ^2/dof
σ_{XY}	0.29(2)	0.9(1)	1.1
σ_Z	-0.34(1)	1.5(1)	0.9

Sommer Parameter

We evalute the Sommer parameter by means of its definition



$$\frac{r_0}{a} = \sqrt{\frac{\alpha + 1.65}{a^2 \sigma}}$$

We get: $r_{0Z} > r_{0XY}$

Again fitting with: $Ratio = 1 + A(eB)^C$

Obs	A	C	χ^2/dof
r_{0XY}	-0.072(2)	0.79(5)	0.6
r_{0Z}	0.161(6)	1.9(1)	1.3

- Emergence of a possible anisotropy of the lattice spacings? $\longrightarrow$ $a_{ZT} < a_{XY}$
- It can be excluded by means of the determination of the pion mass at finite magnetic field. [Bali et al., JHEP 1202 (2012) 044]
- We expect nothing weird to happen up to $(eB) \sim 0.4\text{GeV}^2$

Summary

- Simulation of QCD at the physical point at non-zero (eB)
- Gauge fields gets modified by the magnetic field
- Determination of $V_{Q\overline{Q}} \longrightarrow$ anisotropy
- Determination of σ, α and r_0 ratios at 3 lattice spacings

Perspectives and open questions

- Finer lattice spacing ($N_t = 10$) to go towards the continuum limit
- Complete angular dependence of $V_{Q\overline{Q}}$: still missing
- Vanishing string tension along Z for large enough eB?
- What happens at finite temperature T?
- Heavy meson spectrum in the presence of (eB)
  - In the NR limit one can solve the Schroedinger equation
  - Spin-Spin interaction + Cornell [Alford and Strickland, '13]
    Mass modification $\oplus$ triplet and singlet mixings
  - Further effect due to the anisotropic static potential
    Work in progress with A. Rucci
- Need for a direct lattice computation of the meson spectrum
- Possible influence on the physics of HIC.

Thank you for the attention!