Anisotropy of the quark anti-quark potential in a magnetic field

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Based on: PRD 89 (2014) 114502
C. Bonati, M. D’Elia, M. Mariti, M. Mesiti, F. Negro, F. Sanfilippo.
Outline

- Phenomenological Motivation
- Lattice QCD & Magnetic Fields \((eB)\)
- Static \(Q\bar{Q}\) potential in the presence of \((eB)\)
- Summary and Perspectives
ElectroWeak corrections are often small if compared to the Strong int. But: what happens if we consider the presence of an external magnetic field, $eB$, large enough to be comparable with the scale $\Lambda_{QCD}$?

- **Astrophysics** - in a class of neutron stars, called **magnetars**: $eB \sim 10^{10} \ T$  
  [Duncan and Thompson, ’92]

- **Cosmology** - during the **ElectroWeak phase transition**: $eB \sim 10^{16} \ T$  
  [Vachaspati, ’91]

- **Heavy ion collisions** - at LHC in **non-central HIC**: $eB \sim 10^{15} \ T \sim 15m_{\pi}^2$  
  [Skokov, Illarionov and Toneev, ’09]

$$1 \ \text{GeV}^2 \sim 5 \cdot 10^{15} \ T$$
The static $Q\bar{Q}$-Potential

$V_{Q\bar{Q}}$ is a non-perturbative feature of QCD. A property of gauge fields only.

Parameterization $\rightarrow$ Cornell Potential

$$V_{Q\bar{Q}}(\vec{r}) = C + \sigma |\vec{r}| + \frac{\alpha}{|\vec{r}|}$$

- $\sigma \equiv$ String Tension
- $\alpha \equiv$ Coulomb Parameter

- A description of confinement
- Spectrum for heavy mesons $\rightarrow$ NR bound states of heavy quarks ($c\bar{c}, b\bar{b}$)
- Sommer parameter $r_0$ for scale setting

POSSIBLE DEPENDENCE ON $eB$? PRESENCE OF ANISOTROPIES?
A background QED field enters the Lagrangian by modifying the covariant derivative:

\[
D_\mu = \partial_\mu + igA_\mu^a T^a \quad \rightarrow \quad \partial_\mu + igA_\mu^a T^a + iqa_\mu
\]

On the lattice:

- **Gluon field**  \( A^a_\mu(x) T^a \rightarrow U_\mu(n), \text{SU(3) link variables (integration variables)} \)
- **Photon field**  \( a_\mu(x) \rightarrow u_\mu(n), \text{U(1) link variables (fixed)} \)

- **Quantization of** \((eB)\)  \( \rightarrow \)  **IR Effect due to periodic b.c.**

\[
u_y(n) = e^{ia^2 qBn_x}, \quad u_x(n)|_{n_x=N_x} = e^{-ia^2 qBN_x n_y}
\]

- The lattice discrete derivative will read:

\[
D_\mu \bar{\psi} \rightarrow \frac{1}{2a} \left( U_\mu(n)u_\mu(n)\psi(n + \hat{\mu}) - U_\mu^\dagger(n - \hat{\mu})u_\mu^*(n - \hat{\mu})\psi(n - \hat{\mu}) \right)
\]

- The magnetic field can influence the gluon field through quark loops!

**HOW MUCH?**
Effects of \((eB)\) on the gluon fields

- Quark condensate vs \((eB) = \text{valence} + \text{sea} \)  
  [D’Elia and N, PRD 83 (2011) 114028]

- Effective \(\theta\) term induced by \(CP\)-odd e.m. fields  
  [D’Elia, Mariti and N, PRL ’13]

- Topological charge correlators  
  [Bali, Bruckmann et al., JHEP 1304 (2013) 130]

- Polyakov loop dependence on \((eB)\)  
  [D’Elia, Mukherjee, Sanfilippo, PRD 82 051501 2010]

- Inverse catalysis  
  [Bruckmann, Endrődi and Kovács, JHEP 1304 (2013) 112]

- The Pseudo Critical Temperature decreases  
  [Bali et al., JHEP 1202 (2012) 044]

Moreover, the magnetic field direction can induce anisotropies:

- Anisotropy of the plaquettes [Ilgenfritz, Muller-Preussker et al., PRD 89 (2014); Bali, Bruckmann et al., JHEP 1304 (2013) 130]
... what about the static potential?

The static potential can be extracted from the Wilson Loop observable:

\[
aV(an_s) = \lim_{n_t \to \infty} \log \left( \frac{\langle W(an_s, a(n_t + 1)) \rangle}{\langle W(an_s, an_t) \rangle} \right)
\]

Since we expect to see anisotropies, we cannot average all the possible Wilson Loops. We build two classes of Wilson Loops:

\[
W_\perp = W_XY = (W(an_x, an_t) + W(an_y, an_t))/2
\]
\[
W_\parallel = W_Z = W(an_z, an_t)
\]

The potentials obtained from the two classes (\(V_\perp\) and \(V_\parallel\)) at \(eB \neq 0\) are different. We fitted the potentials separately, according to the standard Cornell potential:

\[
aV_\perp(an_\perp) = aV_XY(an_{XY}) = ac_{XY} + \frac{\alpha_{XY}}{n_{XY}} + \sigma_{XY} a^2 n_{XY}
\]
\[
aV_\parallel(an_Z) = aV_Z(na_Z) = ac_Z + \frac{\alpha_Z}{n_Z} + \sigma_Z a^2 n_Z
\]

Then we compute ratios \(\mathcal{O}(B)/\mathcal{O}(B = 0)\).
Numerical Setup

The Lattice approach consists in a discretization of the Euclidean Feynman path integral:

$$Z = \int D\!U \: D\bar{\psi} \: D\psi \: e^{-YM[U] - \bar{\psi}_f M_f^D \psi_f} = \int D\!U \: e^{-YM[U]} \prod_f \left( \det M_f^D [U] \right)^{1/4}$$

We adopt a state-of-art discretization for QCD with $N_f = 2 + 1$:

- **Gauge sector:** tree level improved Symanzik action
  [Weisz, Nucl Phys B ’83; Curci, Menotti and Paffuti, Phys Lett B ’83]
- **Fermionic sector:** rooted staggered fermions $\oplus$ stout smearing improvement
  [Morningstar and Peardon, PRD ’04]

The bare parameters we adopted in our simulations have been taken from
[Borsanyi, Endrodi, Fodor et al., JHEP ’10].
They correspond to the “physical” line of constant physics ($m_{\pi}^{LAT} = m_{\pi}^{PHYS}$).

Simulations have been performed on the BlueGene/Q machine at CINECA, Italy.

Lattices

- $a = 0.2173 \text{ fm} \rightarrow 24^4$
- $a = 0.1535 \text{ fm} \rightarrow 32^4$
- $a = 0.1249 \text{ fm} \rightarrow 40^4$

The physical volume is kept $\sim$fixed at $V_4 = (5 \text{ fm})^4$
Anisotropic potential

An example: $40^4$ with a lattice spacing $a = 0.1249$ fm
The modification can be ascribed to a modification of both $\sigma$ and $\alpha$.

PS: If we do not distinguish between the $XY$ and the $Z$ directions, we almost lose any dependence of the potential on $(eB)$. 
Isotropic smearing for noise reduction

Typically, Wilson Loops (ratios) are noisy observables

\[ \rightarrow \text{we need to smooth the configurations} \]

- Temporal links: 1 single HYP smearing.
- Spatial links: \( n \) levels of spatial APE smearing (\( n = 8, 16, 24, 32, 40 \)).

Example: \( 32^4 \) lattice at \((eB) \approx 0.97 \text{ GeV}^2, R = 3a \approx 0.46 \text{ fm}\)

\[
\log \left( \frac{\langle W(an_s, a(n_t + 1)) \rangle}{\langle W(an_s, an_t) \rangle} \right)
\]
String Tension and Coulomb Term

\[ \alpha_Z > \alpha_{XY} \]

\[ \sigma_{XY} > \sigma_Z \]

We fit our data at the finest lattice \( \rightarrow \text{Ratio} = 1 + A(eB)^C \)

<table>
<thead>
<tr>
<th>Obs</th>
<th>A</th>
<th>C</th>
<th>( \chi^2 / \text{dof} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{XY} )</td>
<td>-0.24(3)</td>
<td>0.7(2)</td>
<td>1.5</td>
</tr>
<tr>
<td>( \alpha_Z )</td>
<td>0.24(3)</td>
<td>1.7(4)</td>
<td>0.3</td>
</tr>
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<tr>
<td>( \sigma_{XY} )</td>
<td>0.29(2)</td>
<td>0.9(1)</td>
<td>1.1</td>
</tr>
<tr>
<td>( \sigma_Z )</td>
<td>-0.34(1)</td>
<td>1.5(1)</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Sommer Parameter

We evaluate the Sommer parameter by means of its definition

\[ \frac{r_0}{a} = \sqrt{\frac{\alpha + 1.65}{a^2 \sigma}} \]

We get: \( r_{0Z} > r_{0XY} \)

Again fitting with: \( \text{Ratio} = 1 + A(eB)^C \)

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<th>C</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( r_{0XY} )</td>
<td>-0.072(2)</td>
<td>0.79(5)</td>
<td>0.6</td>
</tr>
<tr>
<td>( r_{0Z} )</td>
<td>0.161(6)</td>
<td>1.9(1)</td>
<td>1.3</td>
</tr>
</tbody>
</table>

- Emergence of a possible anisotropy of the lattice spacings? \( \rightarrow a_{ZT} < a_{XY} \)
- It can be excluded by means of the determination of the pion mass at finite magnetic field. [Bali et al., JHEP 1202 (2012) 044]
- We expect nothing weird to happen up to \( (eB) \sim 0.4 \text{GeV}^2 \)
Summary

- Simulation of QCD at the physical point at non-zero ($eB$)
- Gauge fields gets modified by the magnetic field
- Determination of $V_{QQ} \rightarrow$ anisotropy
- Determination of $\sigma$, $\alpha$ and $r_0$ ratios at 3 lattice spacings
Perspectives and open questions

- Finer lattice spacing \((N_t = 10)\) to go towards the continuum limit
- Complete angular dependence of \(V_{Q\bar{Q}}\): still missing
- Vanishing string tension along \(Z\) for large enough \(eB\)?
- What happens at finite temperature \(T\)?
- Heavy meson spectrum in the presence of \((eB)\)
  - In the NR limit one can solve the Schroedinger equation
  - Spin-Spin interaction + Cornell [Alford and Strickland, ’13]
    Mass modification ⊕ triplet and singlet mixings
  - Further effect due to the anisotropic static potential
    Work in progress with A. Rucci
- Need for a direct lattice computation of the meson spectrum
- Possible influence on the physics of HIC.
Thank you for the attention!