Decay constants of the pion and its excitations of the lattice

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Lattice 2014, Columbia University, New York, USA

June 23 - 28, 2014

Objectives

- Within this project, we want to go beyond the spectrum to compute the properties of the excited states. In particular, we probe the structure of the pion and its excitations through the computation of the quark distribution amplitudes on improved anisotropic lattices.

Anisotropic lattices

- In Euclidean space-time, the excited-state spectrum can be computed by observing the behavior of correlation functions formed from appropriately constructed operators:

$$\langle \mathcal{O}_n(t)\mathcal{O}_n^\dagger(0)\rangle = \langle 0|\hat{\mathcal{O}}_n|n\rangle\langle n|\hat{\mathcal{O}}_n^\dagger|0\rangle e^{-m_n t} + \langle 0|\hat{\mathcal{O}}_n|n'\rangle\langle n'|\hat{\mathcal{O}}_n^\dagger|0\rangle e^{-m'_n t} + \ldots$$

  These correlation functions decay faster than those for ground state, and at large times propagation of noise swamps signals.

- To overcome this difficulty we use anisotropic lattices with finer temporal discretization.

The lattice action

We use dynamical anisotropic lattices generated by the Hadron Spectrum Collaboration [1, 2]:

- $N_f = 2 + 1$ flavor (2 dynamical light quarks and a dynamical strange quark) 'clover' action with stout-link smearing;
- Symanzik- and tadpole-improved gauge action.

N_s	N_t	$a_t m_l$	$a_t m_s$	$a_t m_\pi$	$N_{\rm cfg}$
16	128	-0.0743	-0.0743	0.1483(2)	535
16	128	-0.0808	-0.0743	0.0996(6)	470
16	128	-0.0840	-0.0743	0.0691(6)	480

Table I. Gauge configurations.

- Relation between spatial and temporal lattice spacing:

$$\xi = \frac{a_s}{a_t} \approx 3.5$$

[1] R. Edwards, B. Joo, H.-W. Lin, Phys. Rev. D78 (2008) 054501.

[2] H.-W. Lin et al., Phys. Rev. D79 (2009) 043502.

Meson spectroscopy on the lattice

To extract the spectrum of the excited states from the exponentially suppressed signals, we apply the variational method [3, 4]:

- It let us extract more information by analyzing a whole matrix of correlators for each irrep:

$$C_{ij}(t) = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle$$

- To determine the physical observables from this matrix, we solve generalized eigenvalue problem:

$$C_{ij}(t) v_j^{(n)} = \lambda(t)^{(n)} C_{ij}(t_0) v_j^{(n)}$$

[3] C. Michael, Nucl. Phys. B259 (1985) 58.

[4] M. Luscher, U. Wolf, Nucl. Phys. B339 (1990) 222.

Basis of interpolators and distillation technique

- It is essential to use a "good" basis of interpolators which would generate states from the vacuum that have large overlap with the physical state we are interested in.

  To achieve this, we use the distillation technique [5]. It defines a smearing function

$$\Box_{xy}(t) = \sum_{k=1}^{N_{vec}} F(\lambda^{(k)}) \xi_x^{(k)}(t) \xi_y^{(k)*}(t)$$

  and provides an efficient method which allow us to calculate correlation functions with large basis of operators.

- Smeared quark fields are constructed by applying this distillation operator to each quark field appearing in the interpolating operators.

[5] M. Peardon et al., Phys. Rev. D80 (2009) 054506.

“Ideal” operator

- Different interpolators one might use in the variational approach are just the basis one offers to the system. The relative weight of these basis elements come out of the variational procedure:

  once generalized eigenvalue problem

$$C_{ij}(t)v_j^{(n)} = \lambda(t)^{(n)} C_{ij}(t_0) v_j^{(n)}$$

  is solved, one can define new interpolators $\Omega^{(n)}$ as a linear combination of the original interpolators:

$$\Omega^{(n)} = \sum_{i=1}^{r} v_i^{(n)*} \mathcal{O}_i$$

  ➡ the variational method determines which linear combination of the basis interpolators best describe a physical state (an optimal operator).

Pion decay constant

We apply the combination of the variational method, operator constructions, and the distillation method to the anisotropic lattice ensembles to extract the vacuum-to-hadron matrix elements of excited states:

- The decay constant on the Nth excitation of the pion: $\langle 0|A_\mu(0)|\pi_N\rangle = p_\mu f_{\pi N}$

  (with $A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$ and $\pi = \bar{\psi}\gamma_5\psi$)

- We extract the lattice decay constant of the Nth excited state through the two-point SL correlation function constructed using the optimal operator at the source:

$$C_{A_4 N}(t) = \frac{1}{V_3}\sum_{\vec{x}\vec{y}}\langle 0|A_4^L(\vec{x},t)\Omega_N^{\dagger S}(\vec{y},0)|0\rangle \to e^{-m_N t} m_N \tilde{f}_{\pi N}$$

Axial-vector current: improved *vs*. unimproved

- On *isotropic* lattice, $A_\mu = Z_A A_\mu^{lat}$ ($Z_A = 1$ at tree level).

- On *anisotropic* lattice, mixing with higher dimension operators occurs at tree level

$$A_4^I = (1 + m a_t \Omega_m) \left[A_4^U - \frac{1}{4}(\xi - 1) a_t \partial_4 P \right]$$

- we consider the ratios of the decay constant of an excited state and that of the ground state for both unimproved and improved currents;

- we quote the absolute values of the decay constants in the subsequent analyses.

Calculations

- We form the combination $\frac{e^{m_N t}}{m_N} C_{A_4 N}(t) \longrightarrow \tilde{f}_{\pi N} + B_N e^{-\Delta m_N t}$

- A three-parameter fit in $\{\tilde{f}_{\pi N},\, B_N,\, \Delta m_N\}$ yields the value of the decay constant.

Calculations

- 2 ways to evaluate the partial derivative of the pseudoscalar current:

$$\partial_4 P \to E_N P \qquad\qquad \partial_4 P \to P(t+1) - P(t)$$

m_π (MeV)	$N=0$	$N=1$	$N=2$	$N=3$
702	0.0551(3)	0.0319(10)	0.0005(12)	0.0307(23)
	0.0716(6)	0.0556(52)	0.0041(23)	0.0565(54)
	0.0710(4)	0.0543(8)	0.0017(21)	0.0466(54)
524	0.0441(5)	0.0261(12)	0.0057(3)	0.0315(31)
	0.0565(18)	0.0465(27)	0.0065(43)	0.0493(132)
	0.0564(6)	0.0476(62)	0.0083(10)	0.0483(91)
391	0.0369(7)	0.0218(15)	0.0062(18)	0.0256(5)
	0.0476(8)	0.0429(113)	0.0138(28)	0.0508(11)
	0.0473(9)	0.0398(90)	0.0140(67)	0.0462(11)

Table II. Unrenormalized values of the pion decay constants for the ground state and first three excitations.

Results

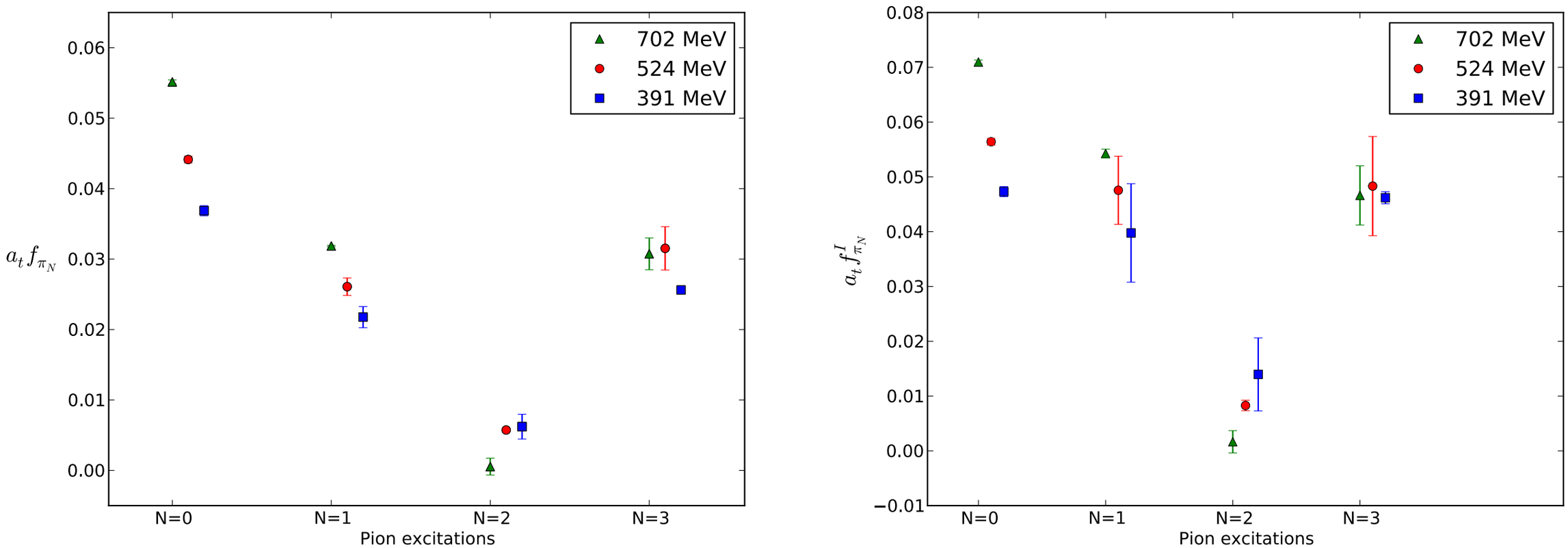

Fig. I. The unrenormalized values of the pion decay constants for the ground and first three excitations: using unimproved (left) and improved (right) axial-vector current [6].

[6] E. Mastropas, D. Richards, arXiv:1403.5575[hep-lat].

Results

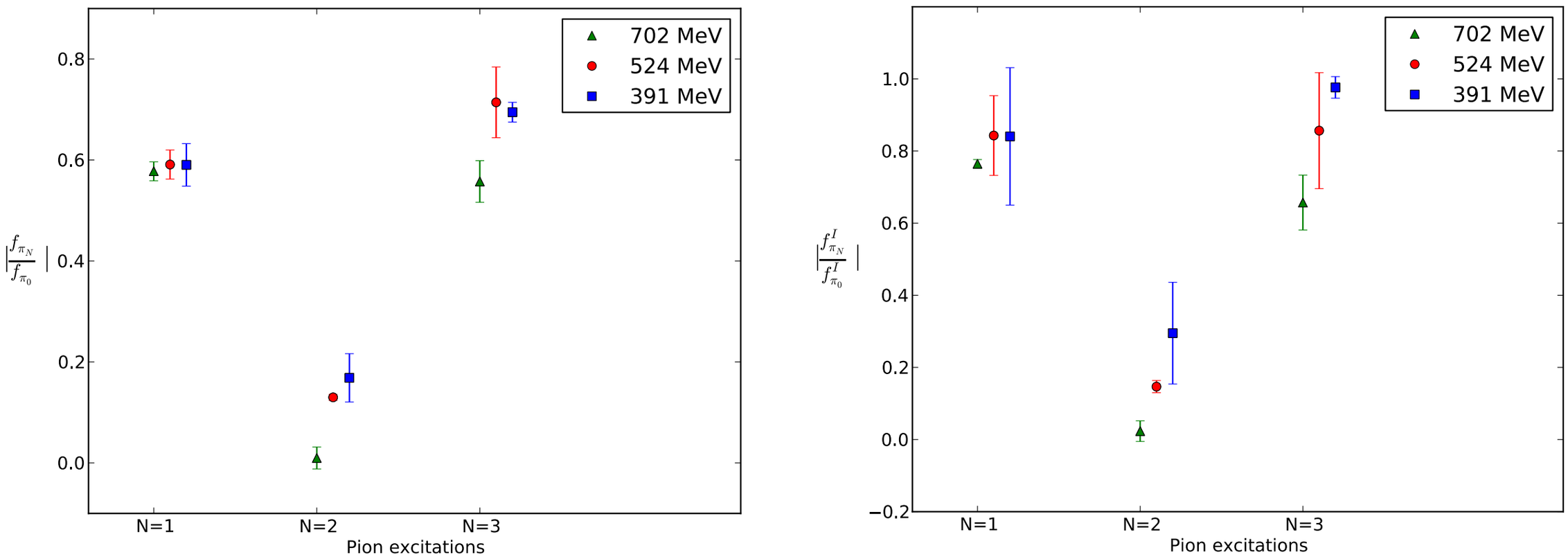

Fig. II. Ratios of the excited-state decay constants to the ground-state decay constant for the first three pion excitations, using unimproved (left) and improved (right) axial-vector current [6].

Previous lattice calculations

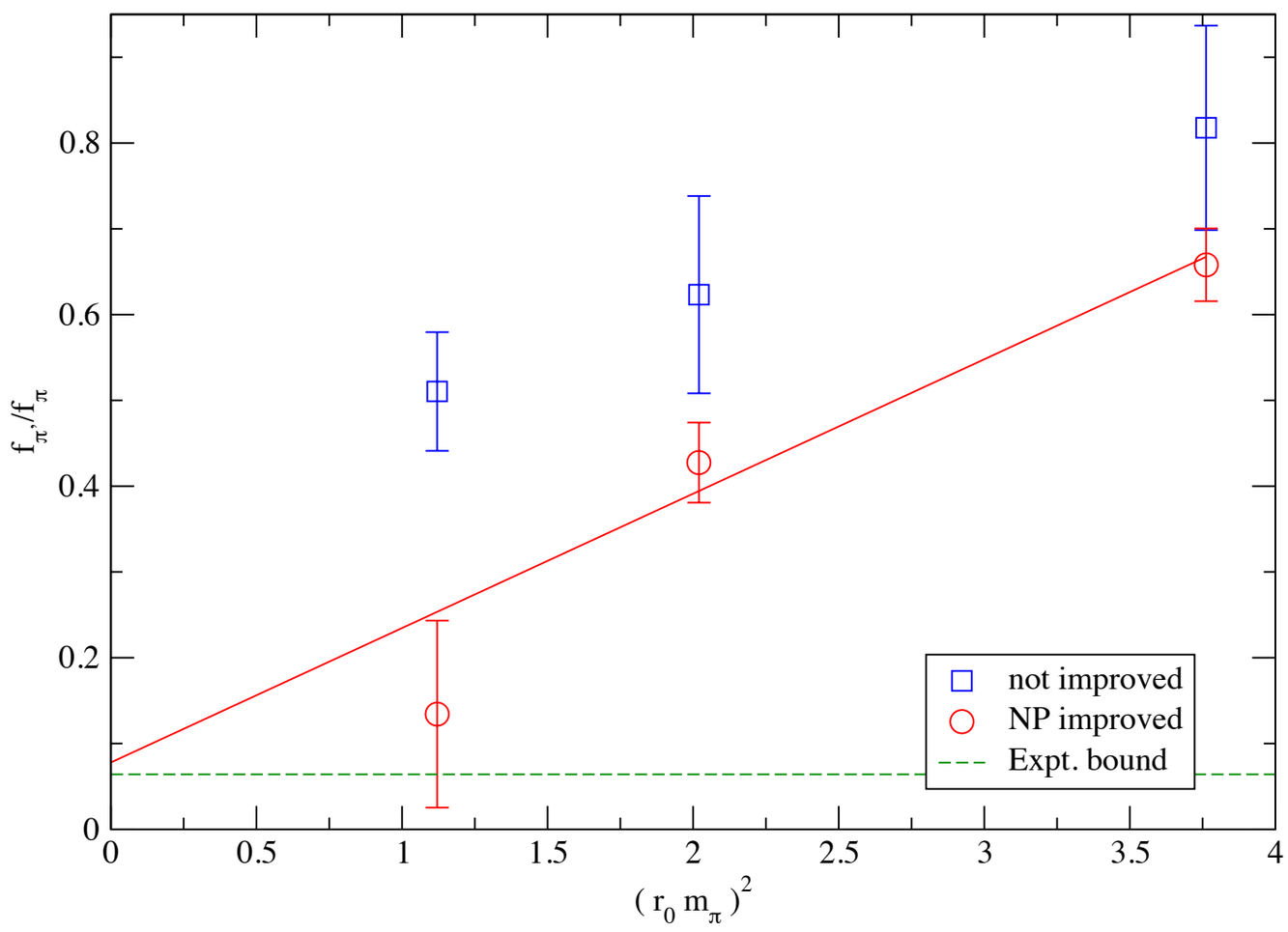

Fig. 3. Ratio of the decay constant of the first excited to ground state light pseudo-scalar meson as a function of the pion mass squared [7].

[7] C. McNeile,C. Michael, Phys. Lett. B 642 (2006) 244.

Results

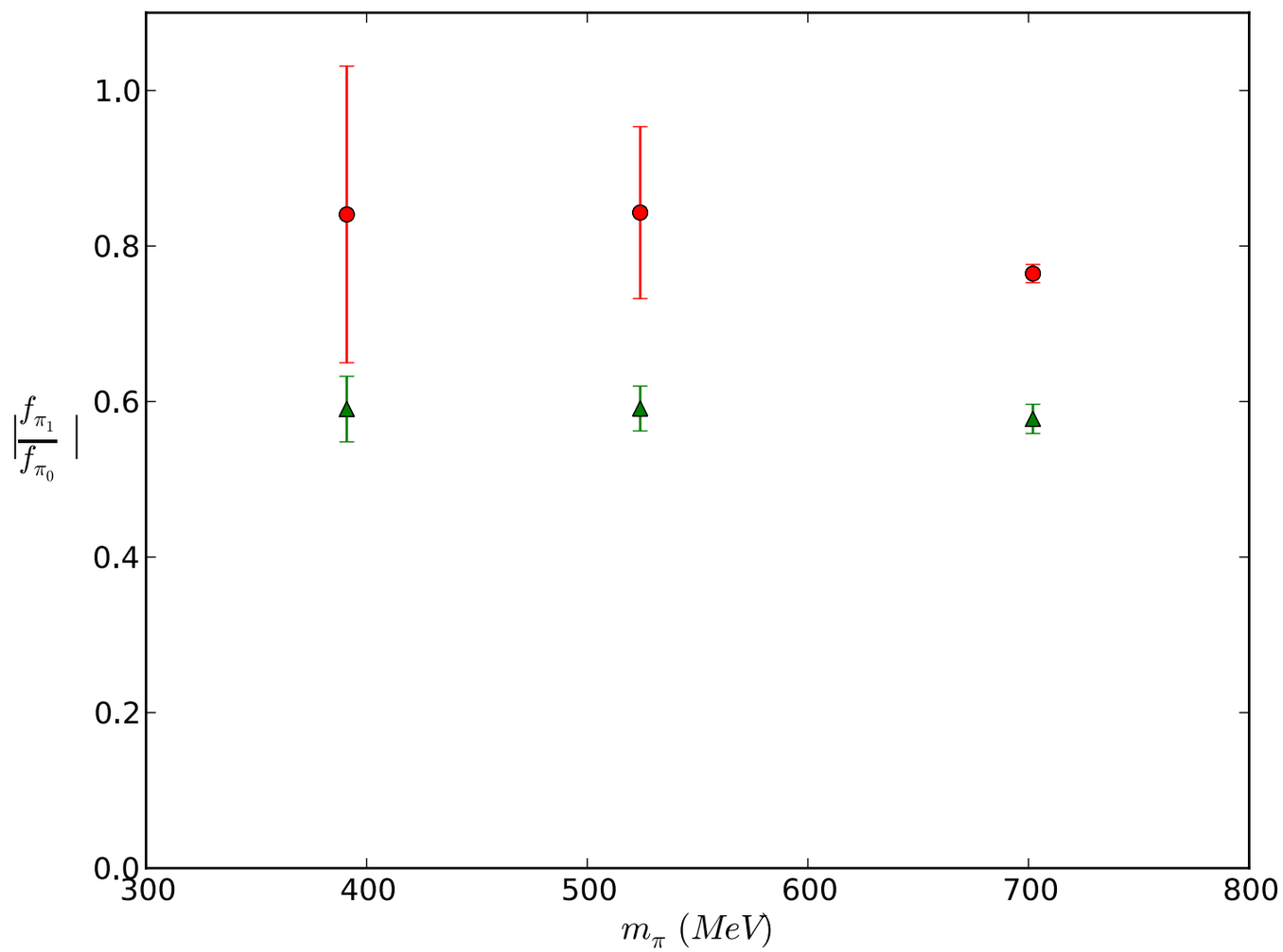

Fig. 4. Ratios of the decay constant of the first excited and ground-state pion as a function of the pion mass [6].

Results

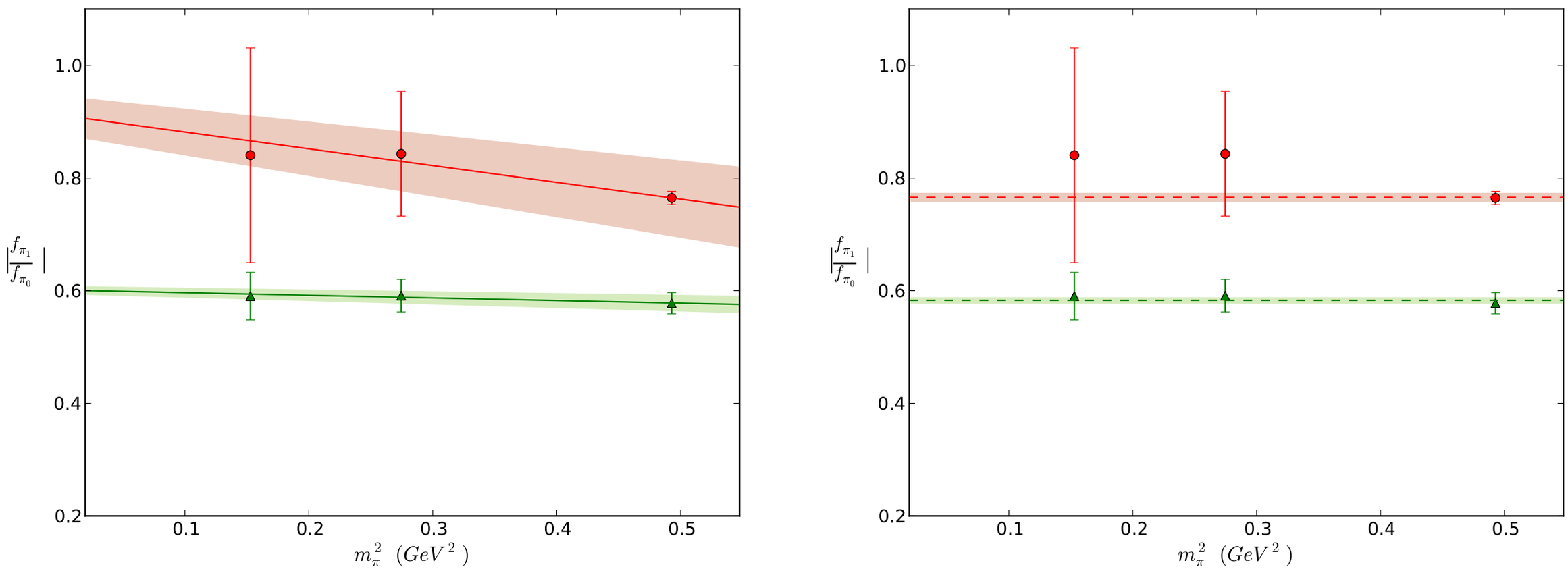

Fig. 5. The linear (left) vs. constant (right) fits in pion mass squared to the ratio of decay constant of the first excited state to that of the ground-state [6].