Targeting the Conformal Window: Determining the Running Coupling

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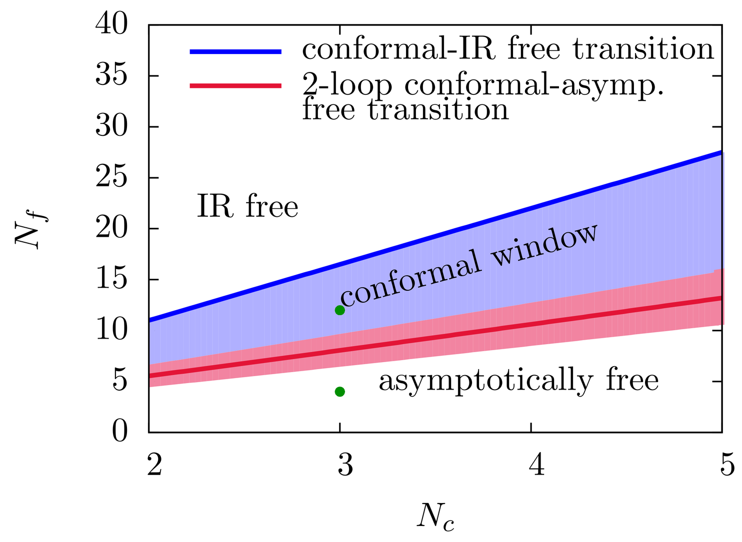
Can the Higgs be a composite resonance?

- A composite resonance is a natural mechanism, as e.g. in superconductivity
- Avoids fine-tuning of the scalar mass
- Likely requires a "walking" theory near a conformal infrared fixed point (IRFP)
  - → Light Higgs could be the dilaton of broken conformal symmetry
  - → Walking coupling leads to enhanced chiral condensate needed for precision EW constraints
- Strongly coupled model requires non-perturbative studies
  - → exploratory lattice results [1]

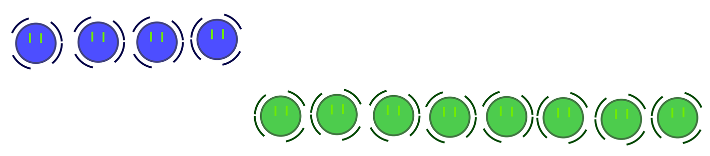
The conformal window

- Seek a model with "walking" behavior
  - → close to the conformal window
  - → still in chirally broken phase
- Does such a model with integer flavor number exist?
  - → Even if so, hard to study with typical lattice methods

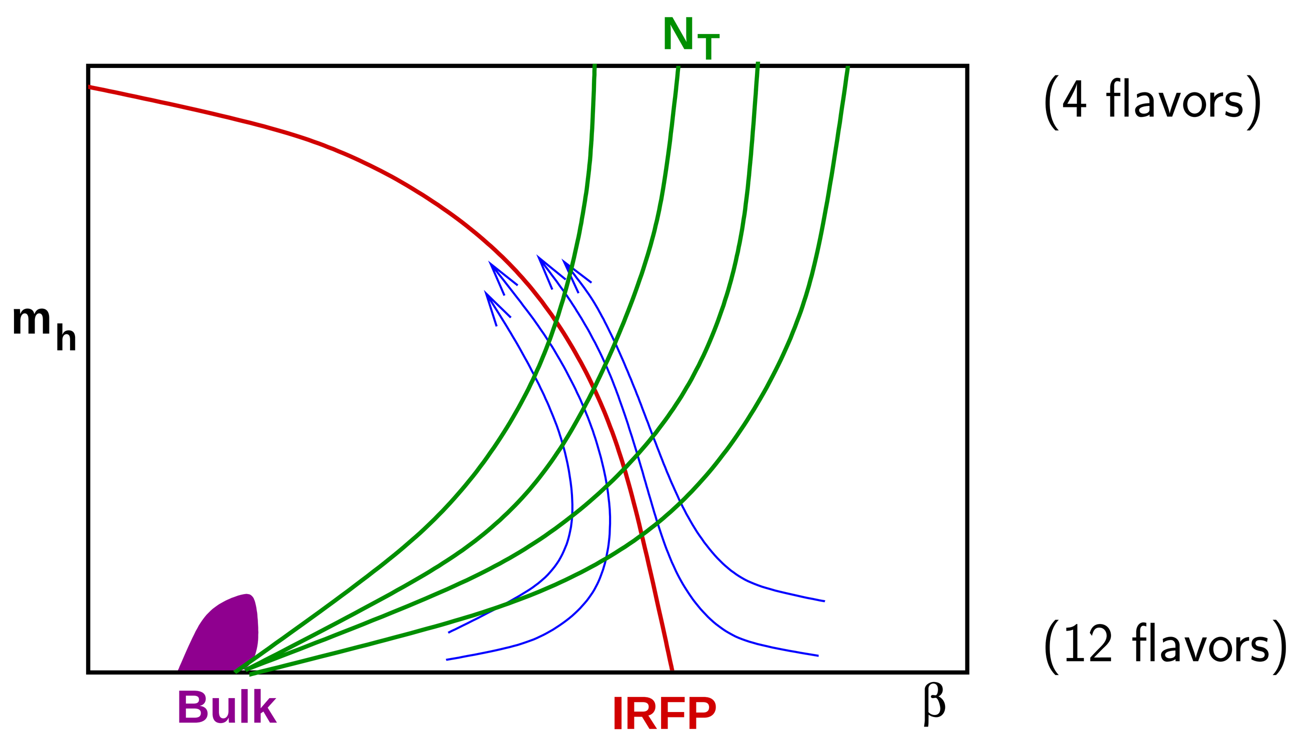


Alternative model: 4+8 flavors

- Study SU(3) with $N_l + N_h$ flavors
  - → N_l massless (light) flavors
  - → N_h heavy flavors of mass m_h
- Infrared: system is chirally broken for $am_h = \mathcal{O}(1)$ (4 light flavors); system is chirally symmetric for $am_h \to 0$ (12 light flavors) [2,3]
- Tuning mass m_h allows interpolation between chirally symmetric and broken phases



The phase diagram

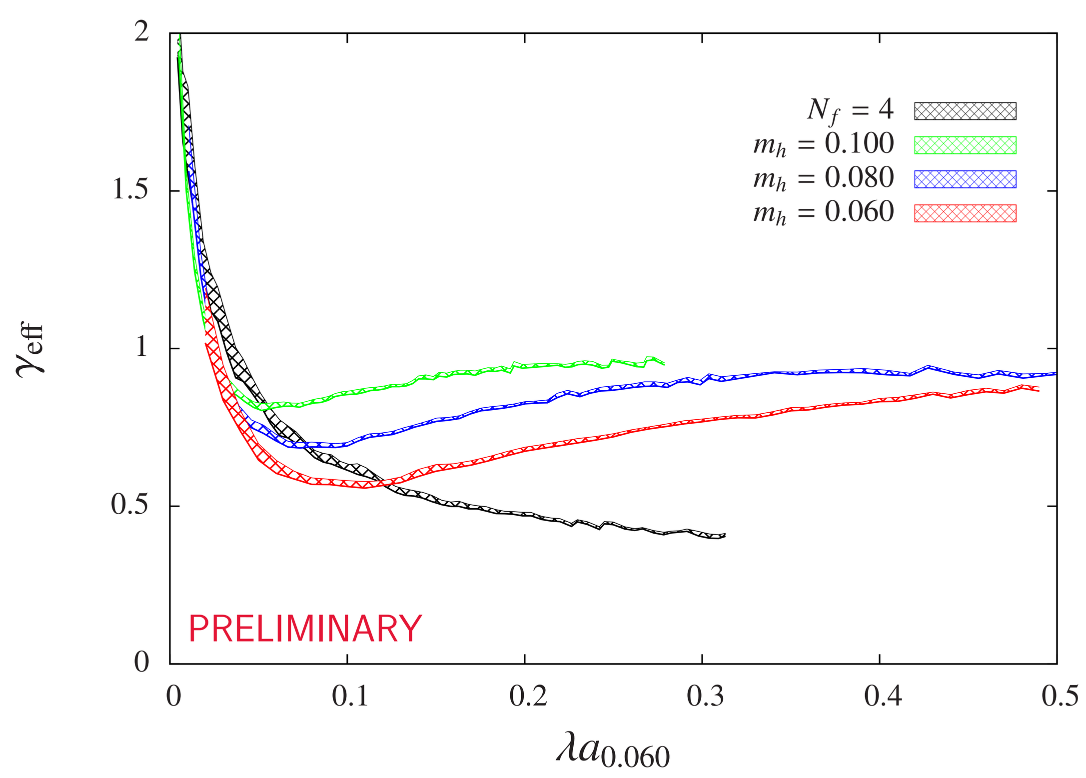


- Renormalized trajectory (RT) emerges from the IRFP of 12-flavor system ($m_h = 0$)
  - → runs to the trivial $\beta = 0$ point at $m_h = \infty$
- For $am_h \ll 1$ the RG flow lines approach this IRFP
  - − hover around it, then run to trivial FP along the renormalized trajectory
- If original gauge coupling is close to RT, IR behavior of the system is characterized by m_h
  - → investigate the system as a function of m_h with fixed β
- At finite temperature the chiral condensate $\langle\psi\bar{\psi}\rangle_l$ serves as order parameter

Numerical setup

- nHYP smeared staggered fermions, fundamental-adjoint gauge action [4]
- Code based on FUEL [5]

Anomalous Dimension from the mode number



Energy scale $\mu = Z_m\lambda$

→ UV: $\lambda a \gtrsim \mathcal{O}(1)$, perturbative

→ IR: $\lambda a \ll 1$, $\gamma_{\rm eff} \gtrsim 1$ chiral broken

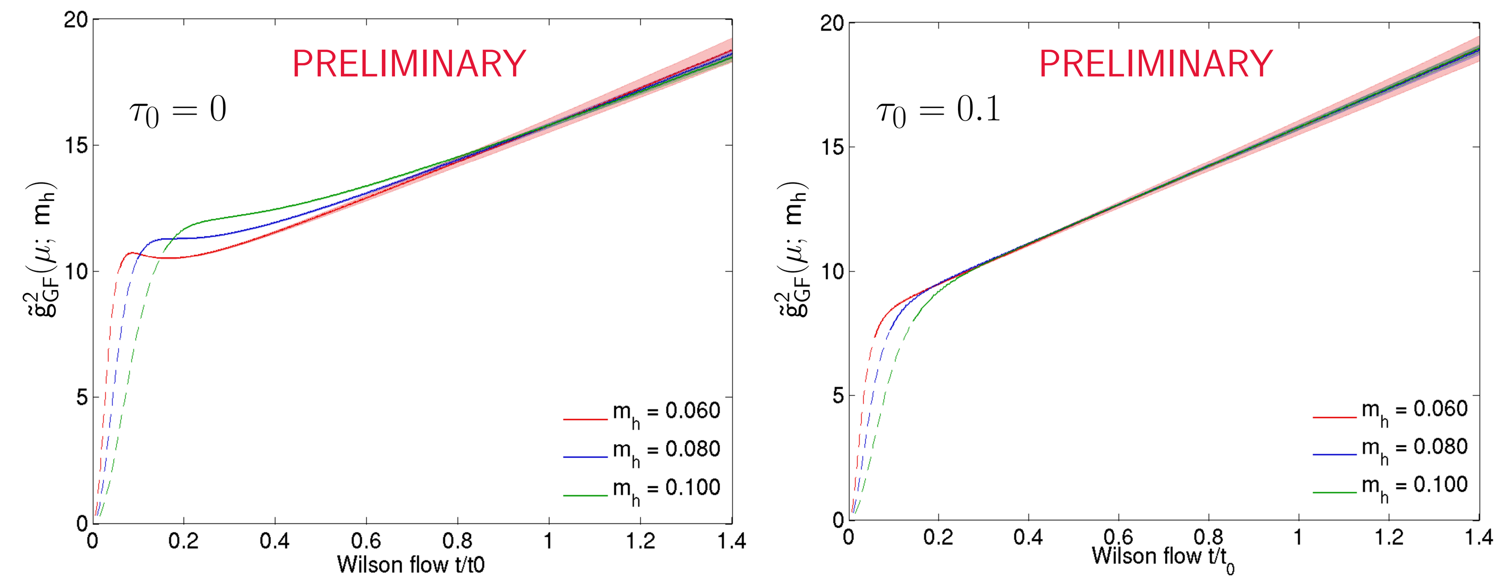
- Large anomalous dimension with walking coupling leads to enhancement of the condensate, important for phenomenological applications
- The scale dependent anomalous dimension can be determined from the mode number [2]
  - → $N_f = 4$: $\gamma_{\rm eff} = 0$ in the UV (perturbative), increases to $\gamma_{\rm eff} = \mathcal{O}(1)$ when chiral symmetry breaks
  - → $N_f = 4+8$: $\gamma_{\rm eff}$ is large in the investigated energy range
  - → As $m_h \to 0$ $\gamma_{\rm eff} \to \gamma^\star(\approx 0.28)$ before chiral symmetry breaking sets in

Determination of the running coupling using Wilson flow

- Extrapolate Wilson flow data to the chiral limit
- Define an improved renormalized coupling using the the gradient flow [6,3]

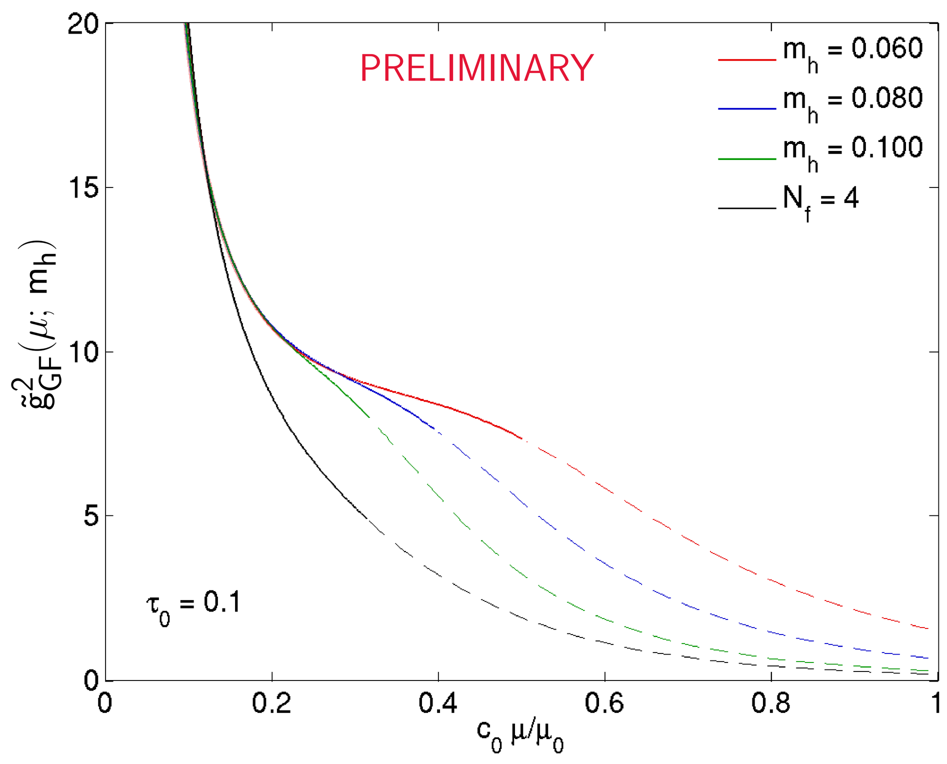
$$\tilde{g}^2_{GF}\left(\mu = 1/\sqrt{8t}; m_h\right) = \frac{3(N_c^2-1)}{128\pi^2}\, t^2\langle E(t+\tau_0)\rangle \quad \text{with} \quad E(t) = \frac{1}{4}F^a_{\mu\nu}(t)F^a_{\mu\nu}(t)$$

- The t-shift in $\langle E(t+\tau_0)\rangle$ reduces the $\mathcal{O}(a^2)$ cut-off errors of $\tilde{g}^2_{GF}(\mu)$
- Determine Wilson flow scale t_0 for shifted data: $t^2\langle E(t+\tau_0)\rangle\Big|_{t=t_0} = 0.3$
- Optimize τ_0 by requiring consistency of $\tilde{g}^2_{GF}(t)$ near $t \approx t_0$ between different m_h (talk by A. Hasenfratz Wednesday, 9:00am)
  - → Control finite volume effects by restricting $\sqrt{8t}/a \le 0.2L$, $L = 32$
  - → Control cut-off effects by restricting $\sqrt{8t}/a > 2$ (indicated by solid lines) (Data at $\beta = 4.0$ on $32^3 \times 64$ lattices)



- In the infrared this yields agreement of $\tilde{g}^2_{GF}(\mu; m_h)$ for all m_h

Running coupling for different masses m_h



Gradient flow scale:

$m_h = 0.060$: $\sqrt{t_0}$ =2.963(51)

$m_h = 0.080$: $\sqrt{t_0}$ =2.339(15)

$m_h = 0.100$: $\sqrt{t_0}$ =1.875(10)

$N_f = 4$: $\sqrt{t_0}$ =1.853(12)

- Plot $\tilde{g}^2_{GF}$ vs. $\mu/\mu_0 = \sqrt{8t_0}/\sqrt{8t}$ and normalize x-axis by $c_0 = 1/\sqrt{8t_0}|_{m_h=0.060}$
- Dashed lines indicate $\sqrt{8t} < 2$ (cut-off effects)
- Statistical errors are smaller or comparable to the line width
- We show $am_h = 0.060$, 0.080, and 0.100
  - → $am_h = \infty$ ($N_f = 4$): QCD-like running coupling
  - → $am_h = 0.100$ shows very little "walking" (almost QCD-like)
  - → $am_h = 0.080$ shows the emergence of "walking"
  - → $am_h = 0.060$ and below has extended "walking" range
- Tuning m_h controls the energy dependence of the gauge coupling

Summary and outlook

- The $N_f = 4+8$ flavor system allows controlled study of the emergence of the conformal window
- First results are promising and follow expectations:
  - → The coupling shows signs of "walking" as $m_h \to 0$
  - → The anomalous dimension is large across a wide energy range
  - → The 0^{++} scalar M_σ decreases as $m_h \to 0$ (talk by E. Weinberg Monday, 5:30pm)
- The $4+8$ flavor system presents new challenges:
  - → The phase diagram is complicated and the continuum limit requires $m_h \to 0$ in addition to $\beta \to \infty$
  - → Heavy and light flavors mix, complicating spectrum studies
- Future plans:
  - → Numerical exploration of the finite temperature phase diagram
  - → Study of the fermion condensate and the ratio Σ/f_π^3
  - → Spectrum studies, including the disconnected scalar, with smaller m_h, larger volumes

References

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Acknowledgment Computations for this work were carried out in part on facilities of the USQCD Collaboration, which are funded by the Office of Science of the U.S. Department of Energy, and on computers at the MGHPCC in part founded by the National Science Foundation. We thank BNL, Boston University, Fermilab, the NSF and the U.S. DOE for providing the facilities essential for the completion of this work.