Phase diagram of QCD at finite isospin chemical potential with Wilson fermions

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• Finite density lattice QCD - sign problem

high $T$, low $\mu$: reweighting method, Taylor expansion...
phase boundary, QCD critical point

low $T$, high $\mu$: QCD like theories
no sign problem
• 2 color QCD
• imaginary chemical potential
• isospin chemical potential

new development: complex langevin etc.
Finite Isospin Chemical Potential

• Core of neutron stars?
  \[ \mu_u = \mu + \mu_I \]
  \[ \mu_d = \mu - \mu_I \]
  \[ \mu_I > 0 : \mu_u > \mu_d, \text{positive charge} \]
  \[ \mu_I < 0 : \mu_u < \mu_d, \text{negative charge} \]

  Pion condensation occurs at \( \mu_{IC} = \frac{1}{2} m_\pi \) (lowest meson mass)
  Rho condensation?
  Strangeness: kaon condensation? hyperons?
  – radius and mass?

• Insight of finite chemical potential
  – Phase diagram as a function of \( T, \mu \) and \( \mu_I \)
Pion Condensation

Chiral perturbation theory
Son, Stephanov
hep-ph/0011365

\( \mu_I \ll m_\rho \)

KS fermion
Kogut, Sinclair

Condensation happens in other channels?
Phase Diagram of QCD at $\mu_I \neq 0$

- Condensation and confinement

Chiral perturbation theory
Son, Stephanov, hep-ph/0011365

KS fermion
Proposed phase diagram in
Introduction of $\mu_i$

- 2 flavor fermion action (Wilson fermion)

\[
S_F = \langle \bar{\Psi} [\gamma_\mu D_\mu + m_q + \mu \gamma_4 \frac{\tau^3}{2} + i \lambda \gamma_5 \frac{\tau^2}{2}] \Psi \rangle
\]

\[
= \bar{\Psi} \left( \begin{array}{cc} D(\mu) & \lambda \gamma_5 \\ -\lambda \gamma_5 & D(-\mu) \end{array} \right) \Psi
\]

\[
= \bar{\Psi} D(U) \Psi
\]

$D(\mu) = \gamma_\mu D_\mu + m_q + \frac{\mu}{2} \gamma_4$

$\tau^2, \tau^3$: Pauli matrix

- $\lambda$: explicit $I_3$ breaking parameter
- positivity of $\det D(U) \iff$ sign problem at finite $\mu$

\[
\det D(U) = \det \left[ D(\mu) D(\mu) + \lambda^2 \right]
\]

Hybrid Monte Carlo method

- observables: $\lambda \rightarrow 0$
**Parameters**

- Wilson fermion

<table>
<thead>
<tr>
<th>T=0</th>
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</thead>
<tbody>
<tr>
<td>· lattice size: 4³×8</td>
</tr>
<tr>
<td>· β = 5.0</td>
</tr>
<tr>
<td>· κ = 1.50</td>
</tr>
<tr>
<td>· μ_I = 0 ~ 1.3</td>
</tr>
<tr>
<td>· λ = 0</td>
</tr>
<tr>
<td>· m_π = 1.869(8), m_ρ = 1.916(9)</td>
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</tbody>
</table>
Correlators

- $4^3 \times 8$

\[ f(t) = a \exp(-m_1 t) + a \exp(-m_2(8 - t)) \]

$\mu_i = 0$

$\pi^+, \pi^0, \pi^-$ degenerate

$\mu_i \neq 0$

$\mu_i$ affects $\pi^+, \pi^0, \pi^-$
Correlators ($\pi$)

$\mu_i=0 \sim 1.0$

$\pi^-$
- The behavior of correlator changes dramatically.
- Mass becomes lighter.

$\pi^0$
- The behavior of correlator does not change.
- $\mu_i$ does not affect $\pi^0$
Pion Masses vs $\mu_I$

- Consistent with chiral perturbation theory
  - $\pi^-$: condensates
  - $\pi^0$: constant
  - $\pi^+$: mass increases.
• Charged $\pi$ and $\rho$
  The behavior of mass is the same.
• Neutral $\pi$ and $\rho$
  $\pi^0$: mass does not change.
  $\rho^0$: mass becomes lighter.

Caveat
The quark mass is heavy.
The interesting window is very small.

C. NONAKA
Finite Temperature

They observe finite pion condensation.

SU(3) $N_f=2$ $\mu=0.8$ $m=0.05$ $8^3\times4$ lattice

$T\neq 0$

KS fermion
Kogut,Sinclair
arXiv:hap-lat/0202028

• $T\neq 0$

They observe finite pion condensation.
Parameters for $T \neq 0$

- Wilson fermion

- We changed $N_T$, but we do not observe the phase transition on the small lattice.
- $\pi$ condensation seems to evaporate.
- Parameter choices are important

Larger lattice volume, $\beta$

- lattice size: $4^3 \times N_t$
- $\beta = 5.0$
- $\kappa = 1.50$
- $\mu_I = 0 \sim 1.3$
- $m_\pi = 1.869(8), m_\rho = 1.916(9)$
Summary

- Revisit finite isospin chemical potential with Wilson fermion
  - We observe pion condensation as well as rho condensation
  - $\pi$: Consistent with chiral perturbation theory
    - $\pi^-$ condensate, $\pi^0$: mass is constant, $\pi^+$: mass increases
  - $\rho$: mass behavior is almost the same as that of
    - $\rho^0$: mass decreases.
  - Finite temperature
    - We do not observe the phase transition
      -> lager volume? $\beta$ should be changed.