Leptonic B and D decay constants with 2+1 flavor asqtad fermions

Ethan T. Neil (University of Colorado/RIKEN BNL)
Lattice 2014
for the Fermilab/MILC Collaboration

Motivation

- Leptonic heavy meson decays are sensitive to both strong and weak physics:

$$\Gamma(H \to \ell\nu) \propto f_H^2 G_F^2 |V_{Qq}|^2$$

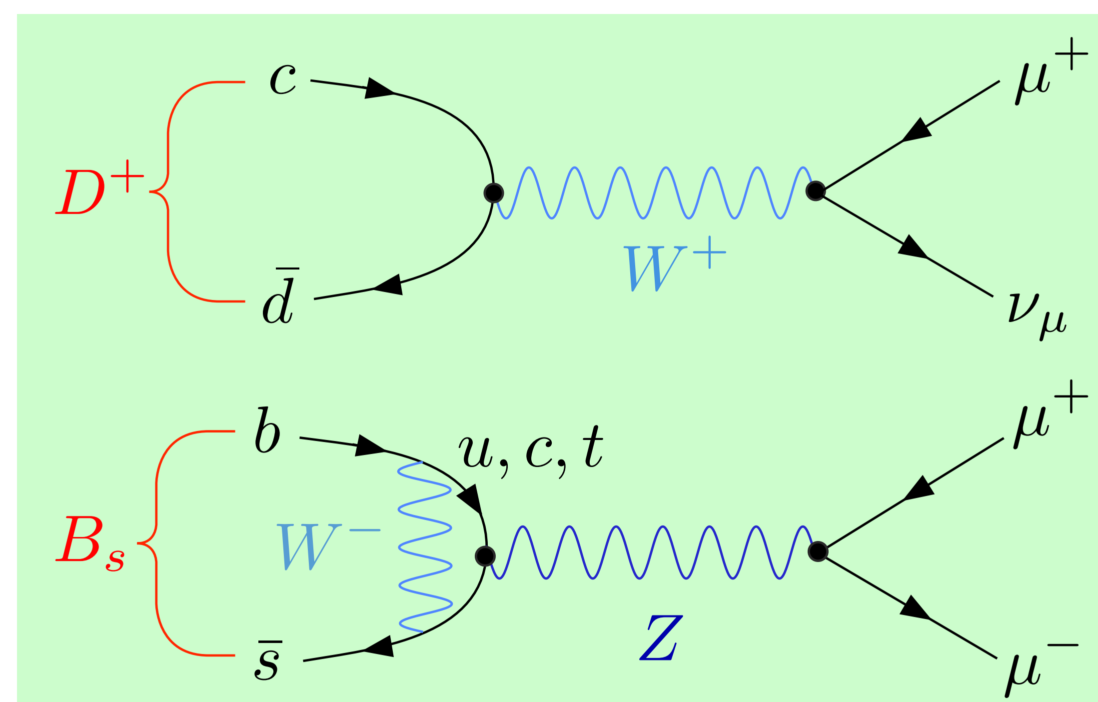
meson decay constant

weak coupling

CKM matrix element

- Accurate f_H (from lattice) is crucial for precise CKM matrix elements!

- Aside from determining CKM, decay constants are needed for rare leptonic decays - whether mediated by standard model or by new physics



(from talk by J. Serrano (LHCb), La Thuille 2014)

$B_{s/d} \to \mu^+\mu^-$

CMS PAS BPH-13-007, LHCb-CONF-2013-012

- CMS (25 fb^{-1}) and LHCb (3 fb^{-1}) both found evidence for the very rare decay $B_s \to \mu^+\mu^-$, in agreement with SM
- Combining CMS and LHCb: first observation of $B_s \to \mu^+\mu^-$

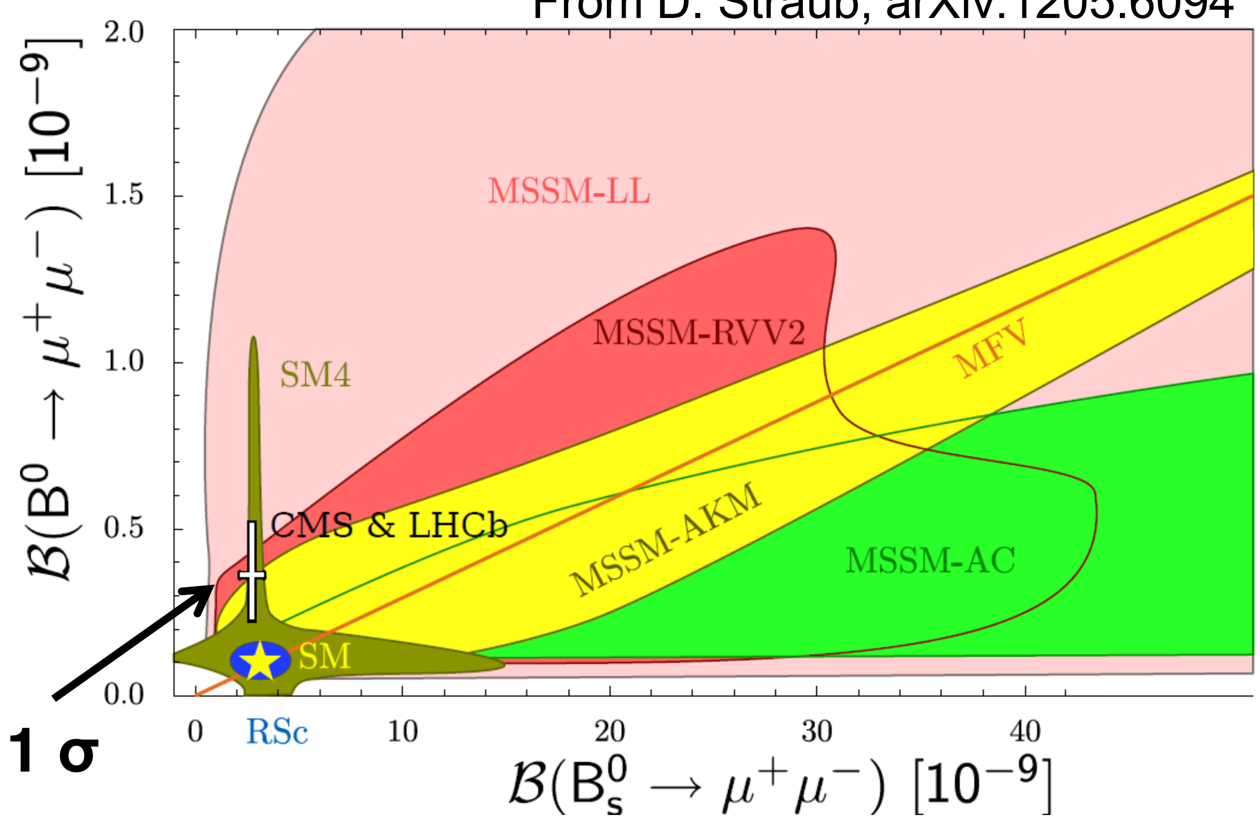
$$\boxed{BR(B_S^0 \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9}}$$

$$\boxed{BR(B^0 \to \mu^+\mu^-) = (3.6^{+1.6}_{-1.4}) \times 10^{-10}}$$

- We are entering the precision era
- The current SM BR($B_s \to \mu^+\mu^-$) has a 10% uncertainty ⇒ crucial to improve theoretical errors

dominant source of SM error is f_{Bs} and CKM elements - see plenary by C. Bouchard

From D. Straub, arXiv:1205.6094

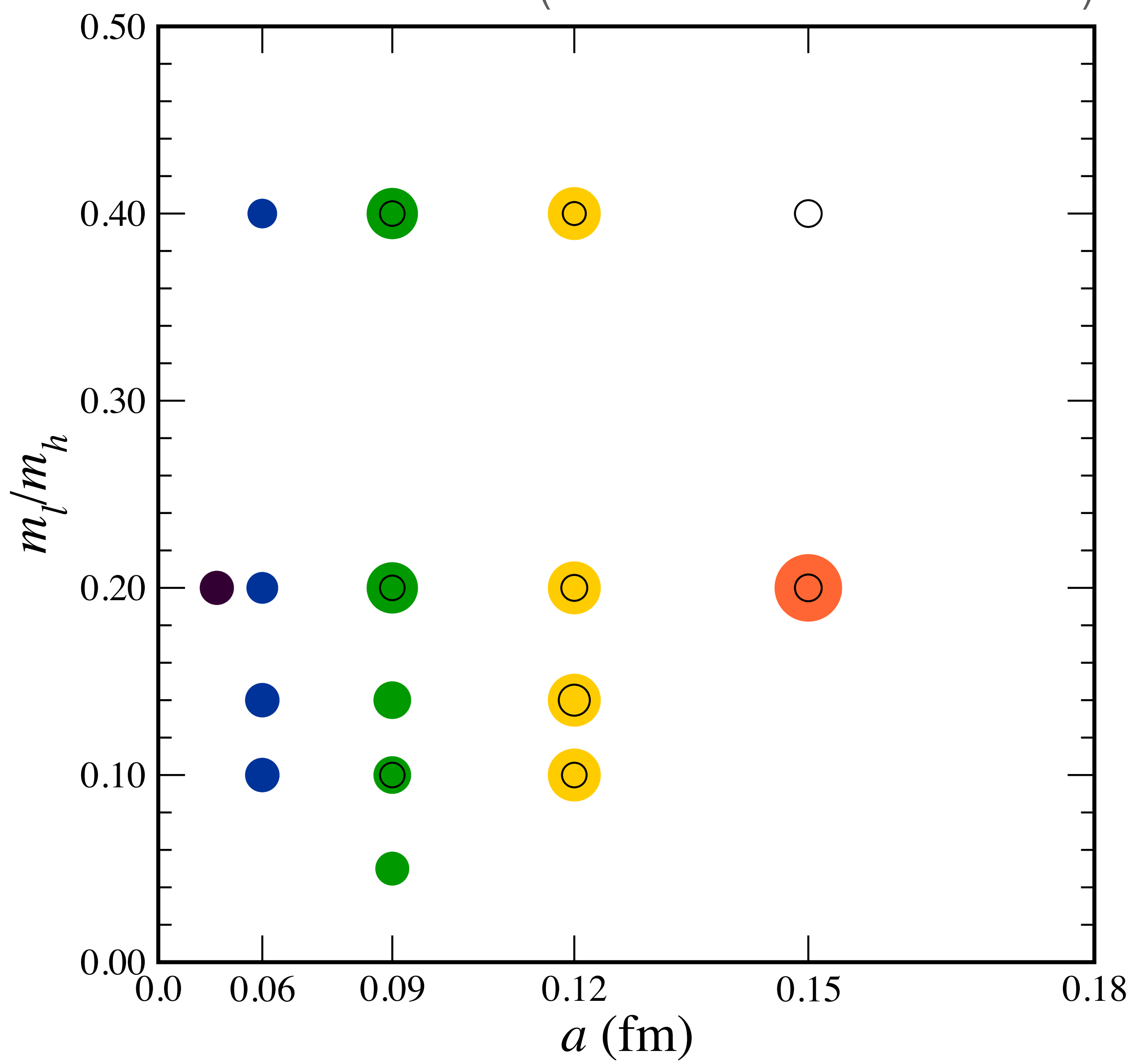


See A. Morda talk for more details!

Simulation details

- MILC asqtad ensembles (see right), clover heavy quarks w/ Fermilab interpretation
- Open discs show the ensembles used in last iteration of this analysis, filled circles are new; circle area ~ statistics
- New analysis has more lattice spacings, more stats, lower accessible quark masses. 4 time-sources per config.

(from arXiv:1403.0635)



Two-point correlators

- Decay constant comes from two-point function of pseudoscalar (PS) source, axial-vector sink. Calculate PS-PS correlator as well to fix normalization.

$$C_{ij}(t) = \sum_{n=0}^{N_X} \left[A_{i,n} A_{j,n} \left(e^{-E_n t} + e^{-E_n (N_t - t)} \right) - (-1)^t A'_{i,n} A'_{j,n} \left(e^{-E'_n t} + e^{-E'_n (N_t - t)} \right) \right]$$

- Our procedure: fit uniformly down to small t (t_{min}=4) for all correlators, using as many excited states as necessary.

- N_x=3 (4+4 states) for most ensembles, with 1 more state pair used on finest lattices

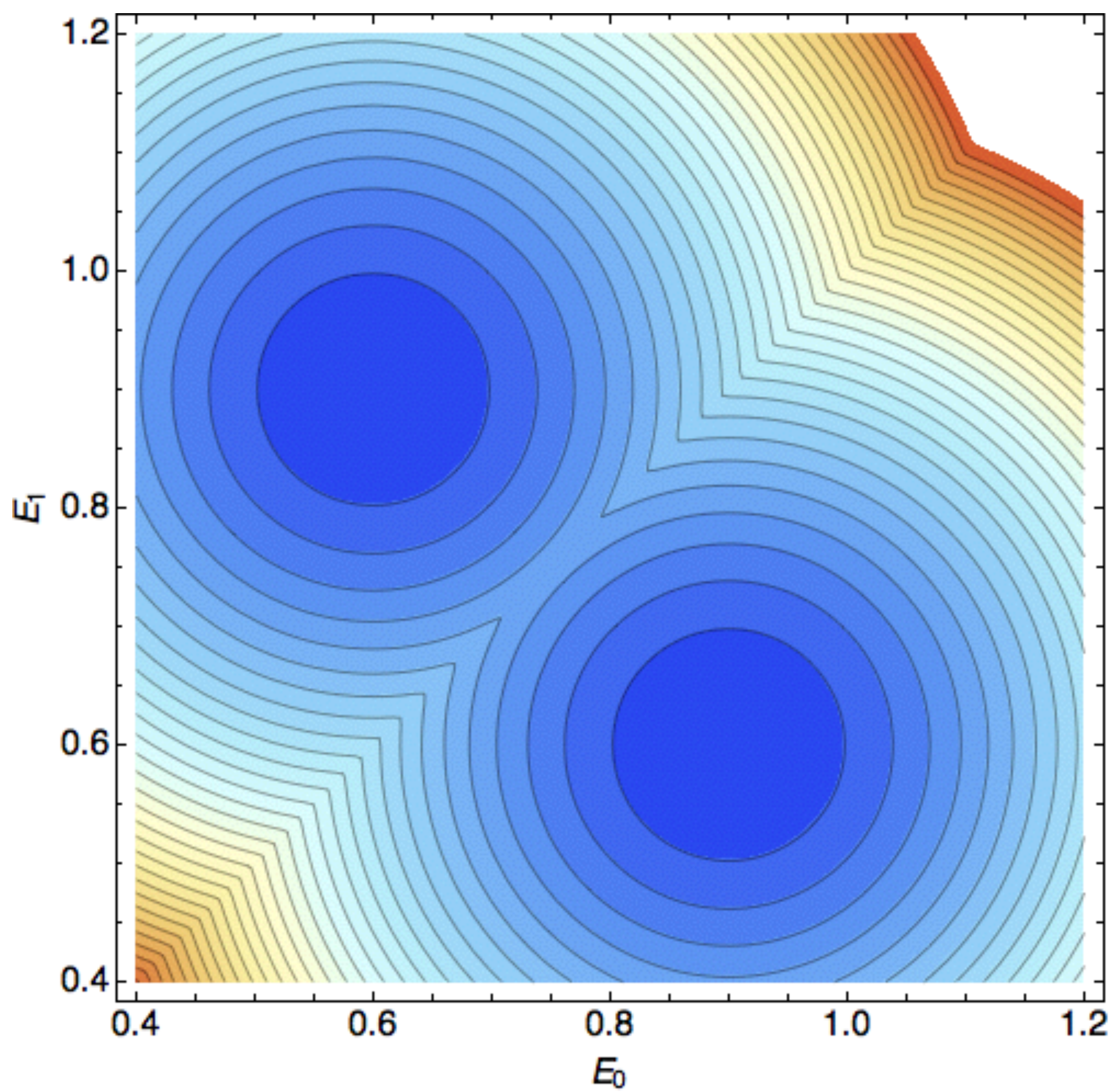
- This approach is relatively simple (no tuning fit ranges) and doesn't throw away statistics, but as a multi-exponential fit is highly prone to numerical instability!

Multi-exponential fits and instability

- Without additional constraints, multi-exponential fits suffer from ordering ambiguity: who says "E_0" is the smallest energy? N! minima in probability space!
- Variable re-mapping can force a particular ordering, e.g.

$$E_0, \log(E_1 - E_0), \log(E_2 - E_1), \ldots$$

- Still numerically challenging, many flat directions even if we lift the spurious minima
- Getting the ground state right tends to stabilize the rest of the fit…

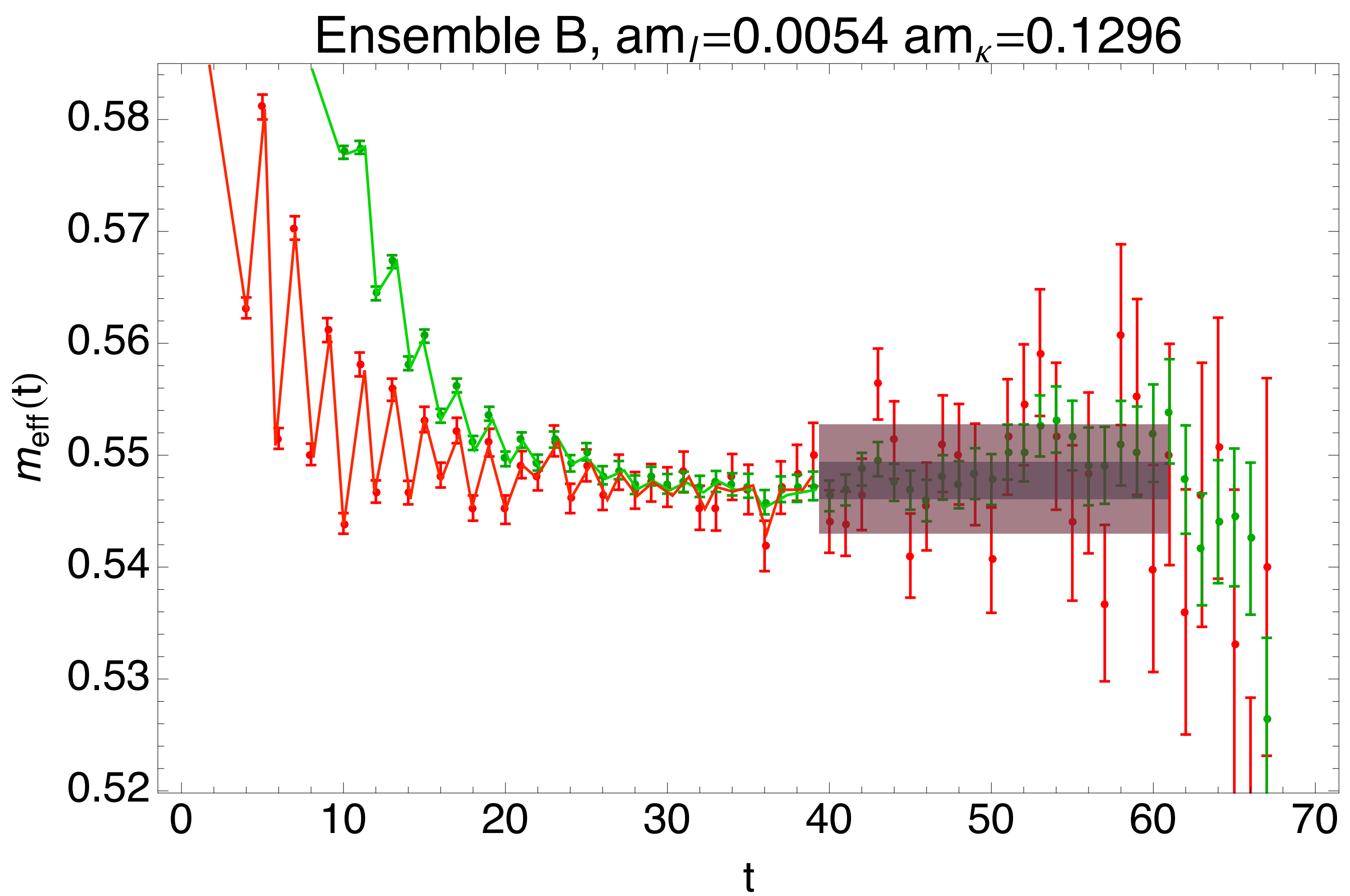


Two-stage constrained fits

- Lots of different methods out there for dealing with this problem; this isn't a review talk so I'll just focus on my approach

- Constrained curve fitting (Lepage et al, arXiv:hep-lat/0110175) eliminates flat directions by use of Gaussian "prior" constraints on fit parameters

- "Priors" tend to be set in a data-driven way in many analyses, e.g. by looking at a subset of the data. So why not use the same correlator to set up the full fits?



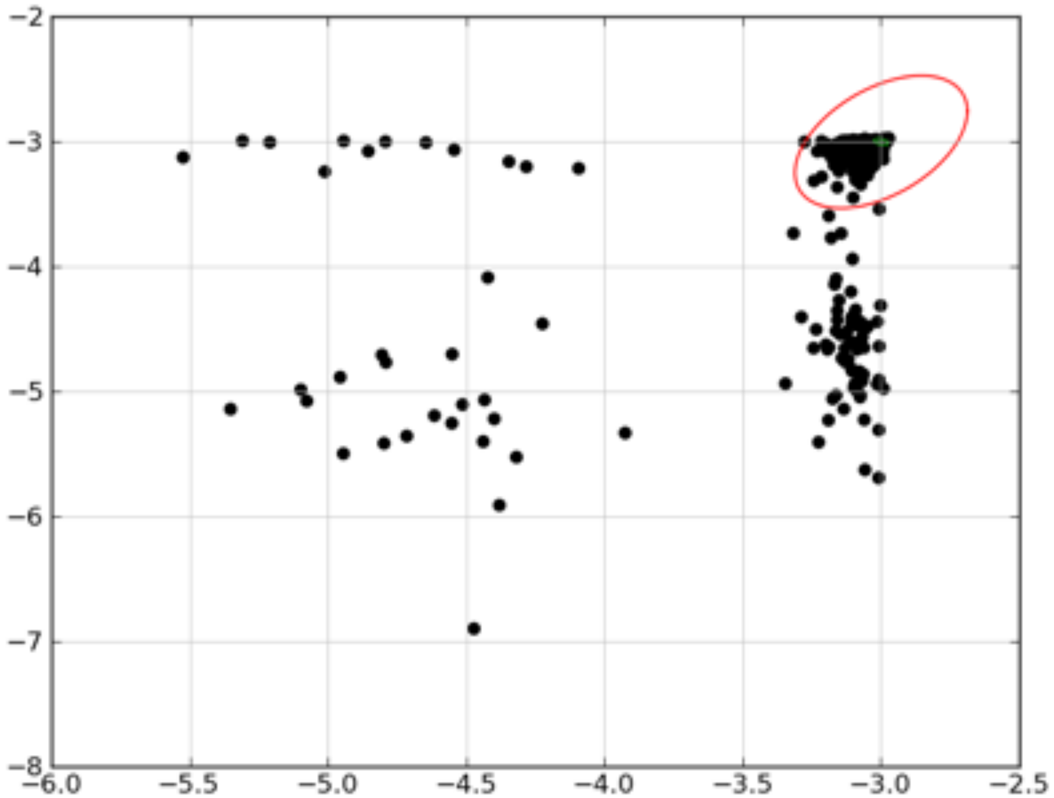
1) Fit ground state in plateau region

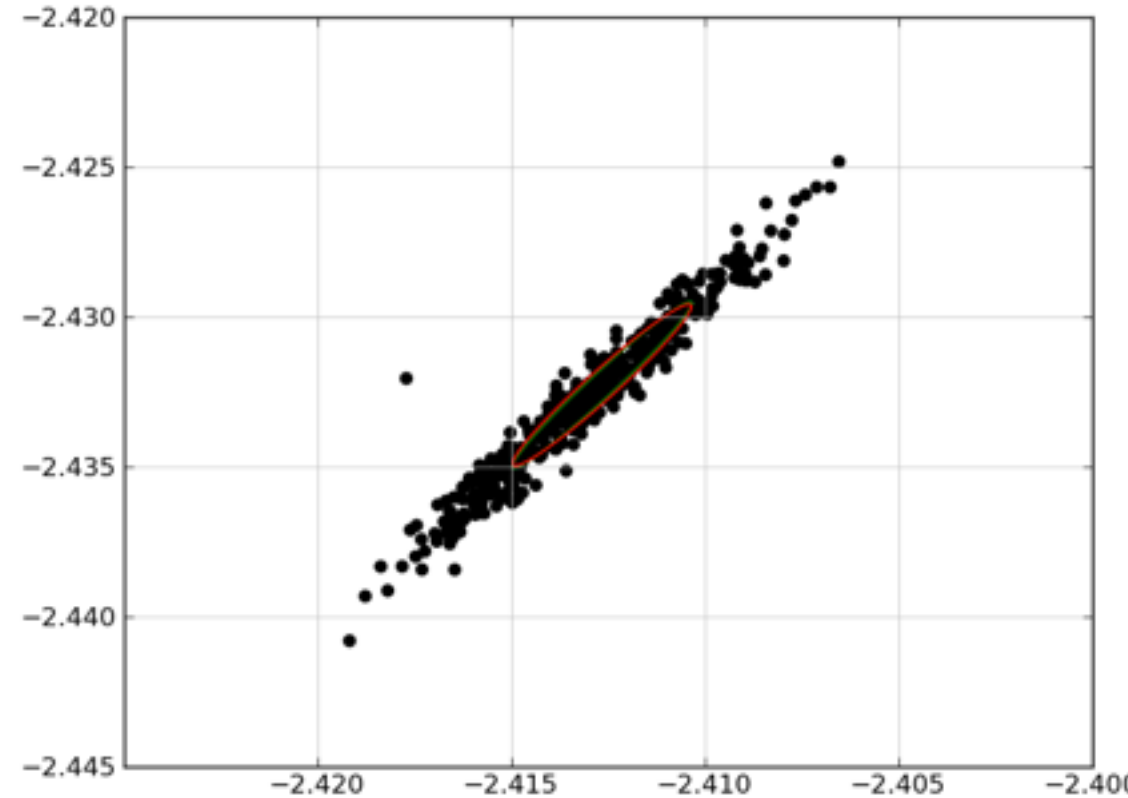
2) Use fit mean values and errors (scaled up) to set "priors"

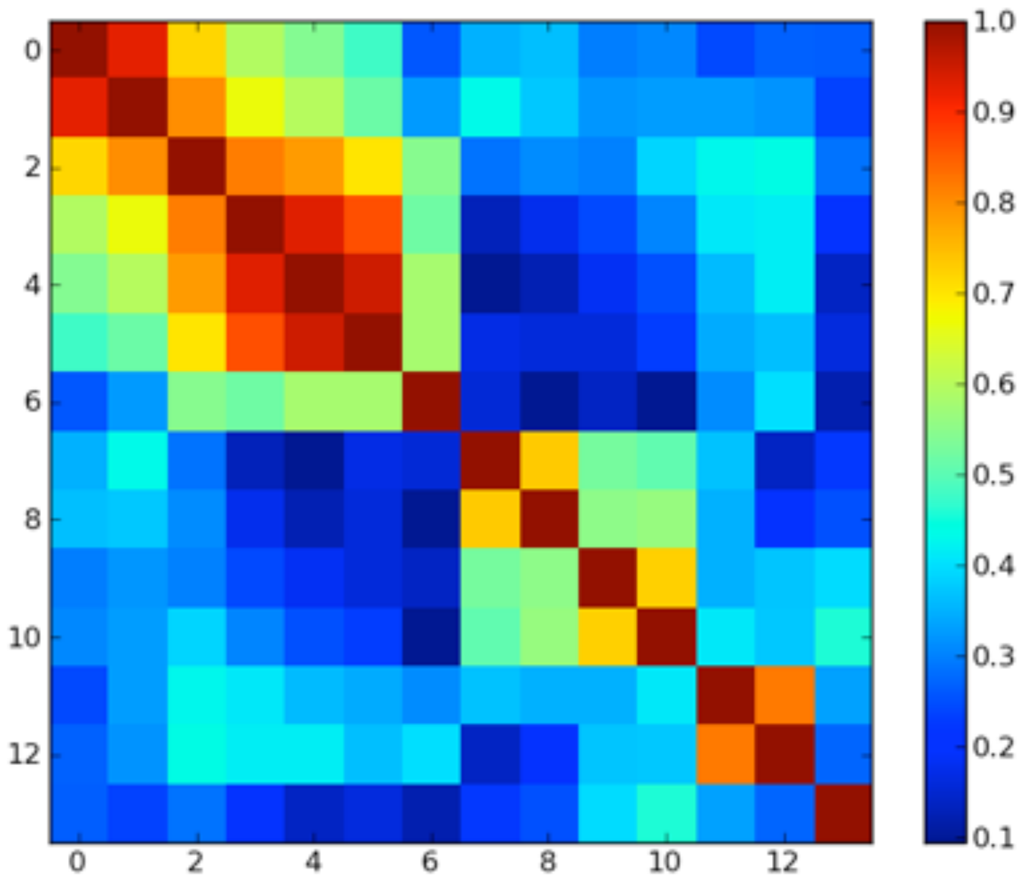
3) Fit full correlator with N_x excited states

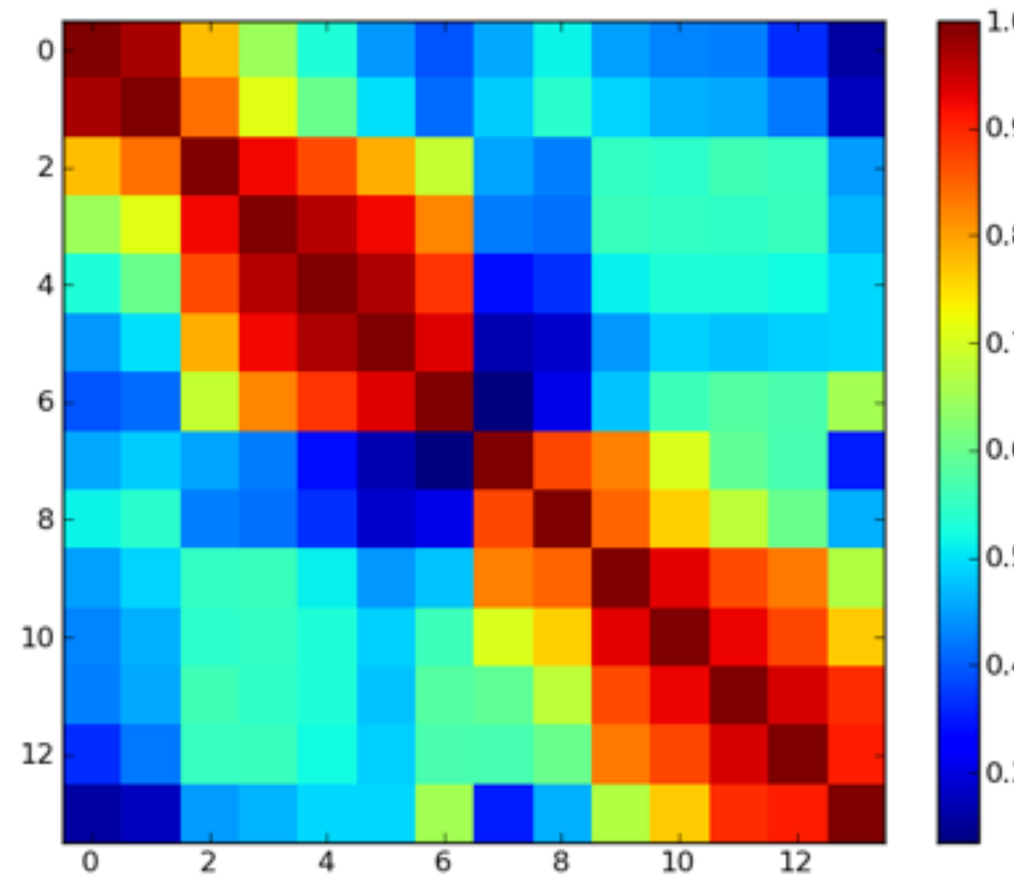
"Two-stage" constrained fits

Stability for replication methods (jackknife/bootstrap)!









“Two-stage” features and plans

- Very stable, insensitive to detailed choice of plateau or factor used to scale errors up. (We picked one plateau on one correlator, then rescaled in terms of r_1/a and got stable results almost everywhere.)

- Method write-up to be included in finished paper; code to be publicly released

- Fitter is implemented in Python, with the following features:

    - Bayesian constrained fits or unconstrained, interface to standard Python optimizers, MINUIT, and MCMC packages

    - Symbolic manipulation of model functions (no need to input derivatives by hand)

    - On-the-fly compilation to C or Fortran for quick fitting - useful for bootstrap and jackknife loops

    - Black-box implementation of two-stage fit procedure with “plateau finder” (user controllable)

Renormalization and tuning

- "Mostly non-perturbative" renormalization of results: Z_V determined non-perturbatively, leftover piece computed in lattice PT.

$$\phi_Q = \sqrt{2} Z_{A^4_{Qq}} A_{A^4_{Qq}} = \sqrt{2} \left(\rho_{A^4_{Qq}} \sqrt{Z_{V^4_{qq}} Z_{V^4_{QQ}}} \right) A_{A^4_{Qq}}$$

- Simulation values of heavy-quark mass parameter κ do not exactly match, so we have to re-tune:

$$\phi_Q \to \phi_Q + \Delta\phi_Q = \phi_Q + \left(\frac{d\phi_Q}{d\kappa} \right) (\kappa_{\rm sim} - \kappa_{\rm tune})$$

- Tuning factor calculated non-perturbatively from runs at various κ on same ensembles; numerically small (mistunings are $O(10^{-3})$.)

Chiral/continuum extrapolation

- Fit to rooted staggered chiPT (rSxPT) to extrapolate to continuum and physical quark masses:

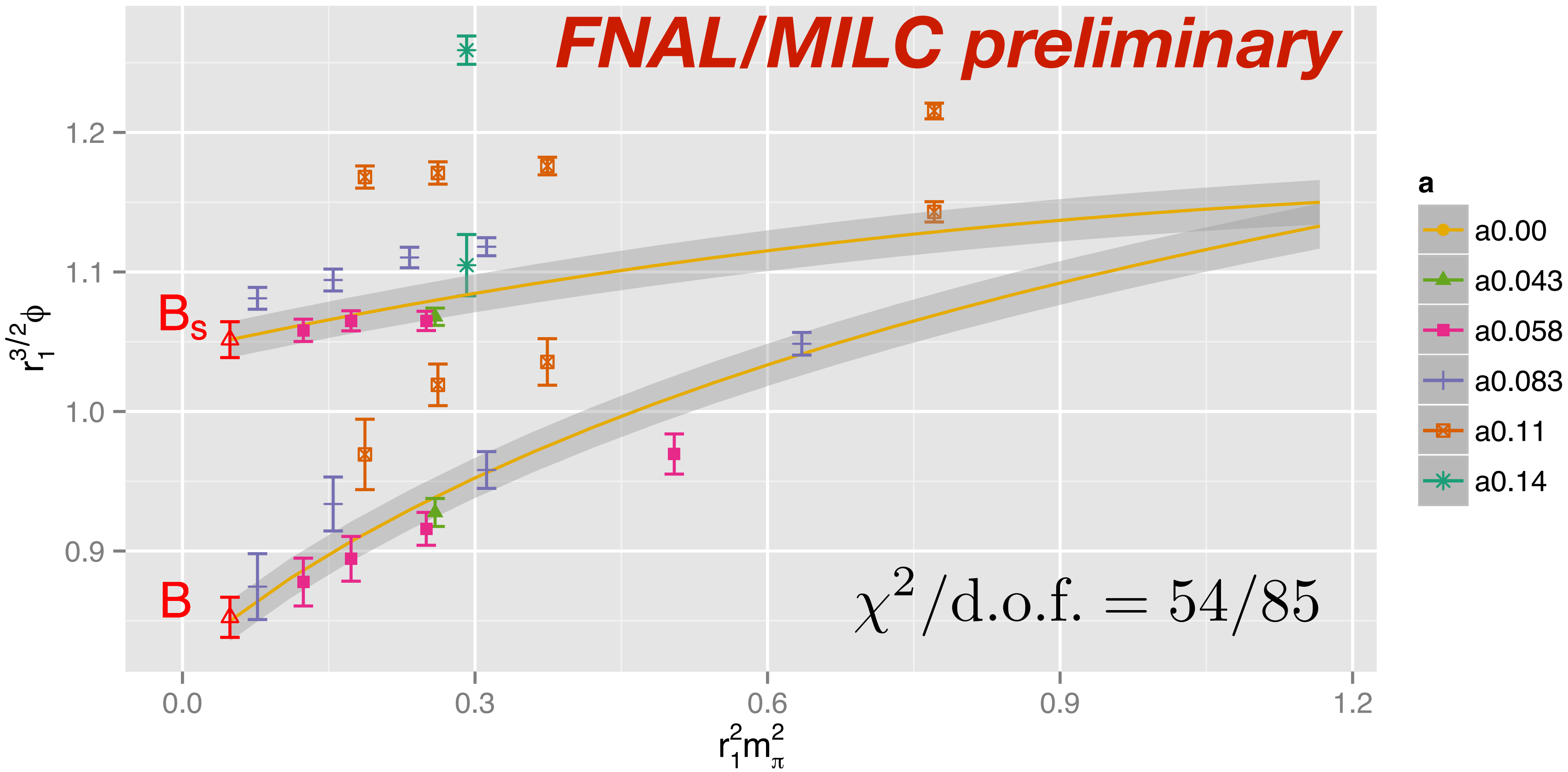
$$\phi = \phi^0[1 + (\text{chiral logs}) + (\text{NLO analytic}) + (\text{NNLO analytic}) \\ + (\text{LQ discretization}) + (\text{HQ discretization})]$$

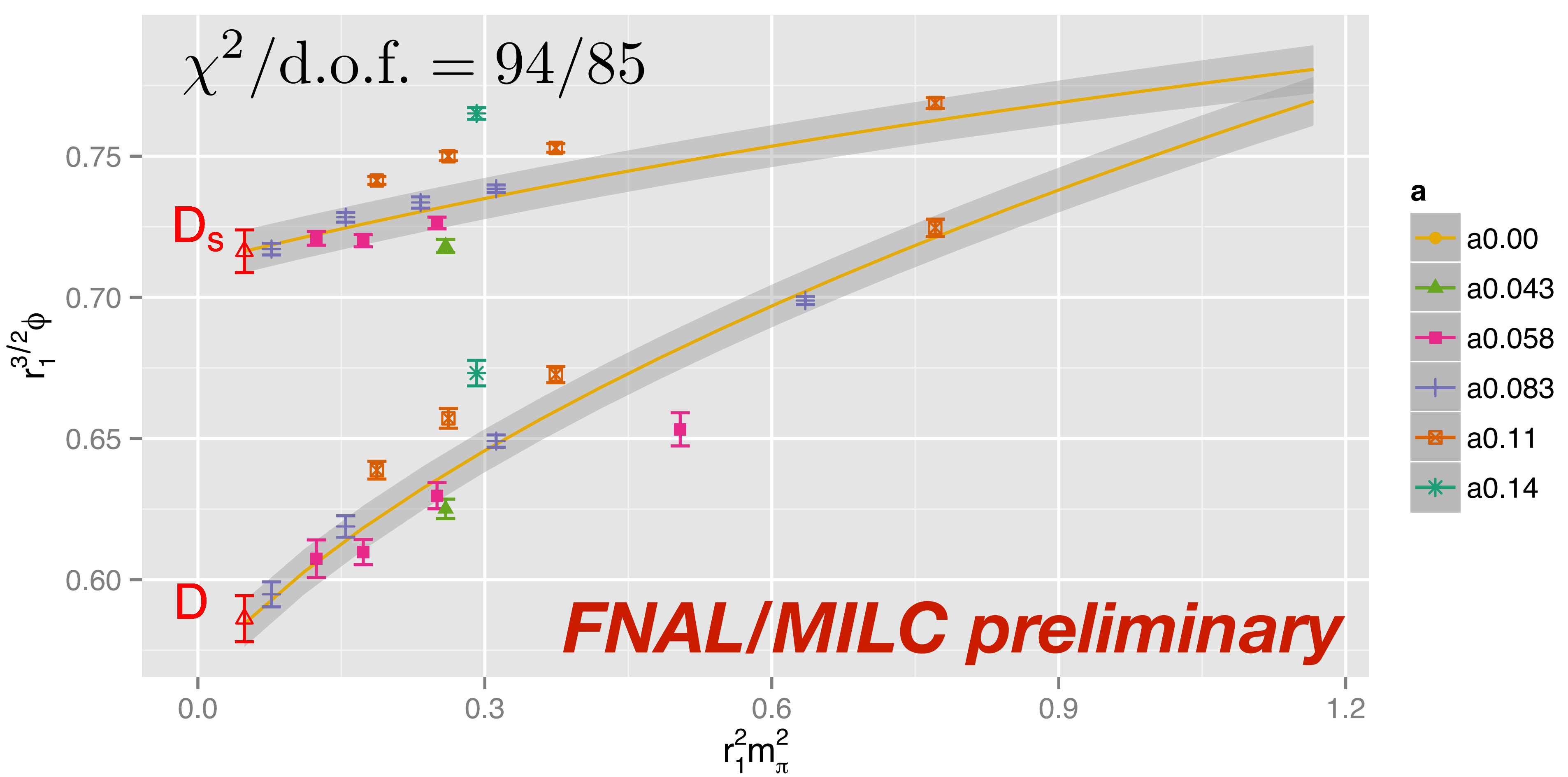
- Terms included for taste-breaking effects, finite-volume corrections in chiral logs, hyperfine/LQ flavor splitting of heavy-light mesons (details in arXiv: 1112.3051.) Hyperfine splittings found to be important for good fits!

- Fits can be used to split out and quantify each discretization error. Remaining systematic effects (mainly from exact form of chiral fit and scale-setting) are estimated by comparison with variant chiral fits.

- NNLO terms found necessary to fit points with valence mass near strange.

Warning: results include an unknown blinding normalization!





Conclusion

- Correlator fits and central chiral fits are done, unblinding after systematic error estimation is in place (explicit fit parameters and variant chiral fits)

- Details on “two-stage” fitting method and Python fitting code to be released with paper

- Competitive results for decay-constant ratios (*preliminary*: **0.5%** stats+discretization for f_{Ds}/f_D, **0.8%** for f_{Bs}/f_B)

- Combination of f_B/f_D with MILC HISQ results for D decay constants planned, may allow for more precise f_B estimate