Lattice Measurement of the $\Delta I=1/2$ Contribution to Standard Model Direct CP-Violation in $K\to\pi\pi$ Decays at Physical Kinematics: Part I

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Outline

- Motivation
- Obtaining physical kinematics
- G-parity boundary conditions
- Physical point ensemble
- Conclusions and Outlook

$\mathbf{K} \to \pi\pi$ Decays

- Direct CP-violation first observed in $K \to \pi\pi$ decays.
- Two types of decay:

$$\Delta I = 3/2 \;: K^+ \to (\pi^+\pi^0)_{I=2} \text{ with amplitude } A_2$$
$$\Delta I = 1/2 \;: K^0 \to (\pi^+\pi^-)_{I=0} \text{ with amplitude } A_0$$
$$K^0 \to (\pi^0\pi^0)_{I=0}$$

- Direct CP-violation: $$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$
  where

  $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$ and δ_I are strong scattering phase shifts.
- ϵ' is highly sensitive to BSM sources of CPV.
- Strong interactions very important – origin of the ΔI=1/2 rule: preference to decay to I=0 final state.

[arXiv:1212.1474]

Obtaining Physical Kinematics

- Correct description of physical decay requires energy to be conserved.
- Lattice I=0 pi-pi ground state is vacuum. Must perform explicit subtraction.
- First excited pi-pi ground state comprises pions at rest: $E_{\pi\pi} \sim 2m_\pi \ll m_K$. Require 2^{nd} or higher excited state with moving pions.
- Conventional approach requires multi-exponential fits, typically resolution of excited state contribution is poor.
- Instead modify spatial boundary conditions to impose momentum on the pion ground-state, after which only vacuum subtraction is required.

ΔI=1/2 Decay

- Must measure $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$ with I=0 final state. Boundary conditions must therefore:
  - ➔ Conserve isospin such that I=0 state can be isolated.
  - ➔ Give momentum to both charged and neutral pions.

- Conventional application of twisted BCs to the d-quark breaks both of these.
- Instead, we use G-parity:

$$\hat{G} = \hat{C} e^{i\pi \hat{I}_y} \quad : \qquad \begin{aligned} \hat{G}|\pi^\pm\rangle &= -|\pi^\pm\rangle \\ \hat{G}|\pi^0\rangle &= -|\pi^0\rangle \end{aligned}$$

- As a boundary condition:

$$\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x)$$

$\longrightarrow$ Discretized lattice momenta

$$|p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L \ldots)$$

Moving ground-state

G-parity BCs

- At quark level: $\hat{G}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}$ where $C = \gamma^2\gamma^4$. in our conventions
- Gauge invariance requires gauge field to obey charge conjugation (complex conjugate) boundary conditions.
- New ensembles needed (true for all modifications of BCs due to disconnected diagrams).
- For stationary kaon eigenstates we must introduce a fictional partner to the strange quark; s'

  $|\tilde{K}^0\rangle = \left(|\bar{s}d\rangle + |\bar{u}s'\rangle\right)/\sqrt{2}$ is G-parity even
- Must take root of s/s' determinant to remove it from action; introduces non-locality that vanishes exponentially in L.

Implementation

- Dirac operator applied simultaneously to two fermion fields that mix at the global lattice boundary.
- Naively expect factor of 2 in cost due to two flavors.

  However fields are intrinsically two-flavor; use of $M^\dagger M$ in HMC to ensure positive-definite matrix requires square-root of light determinant (fourth-root for s/s')

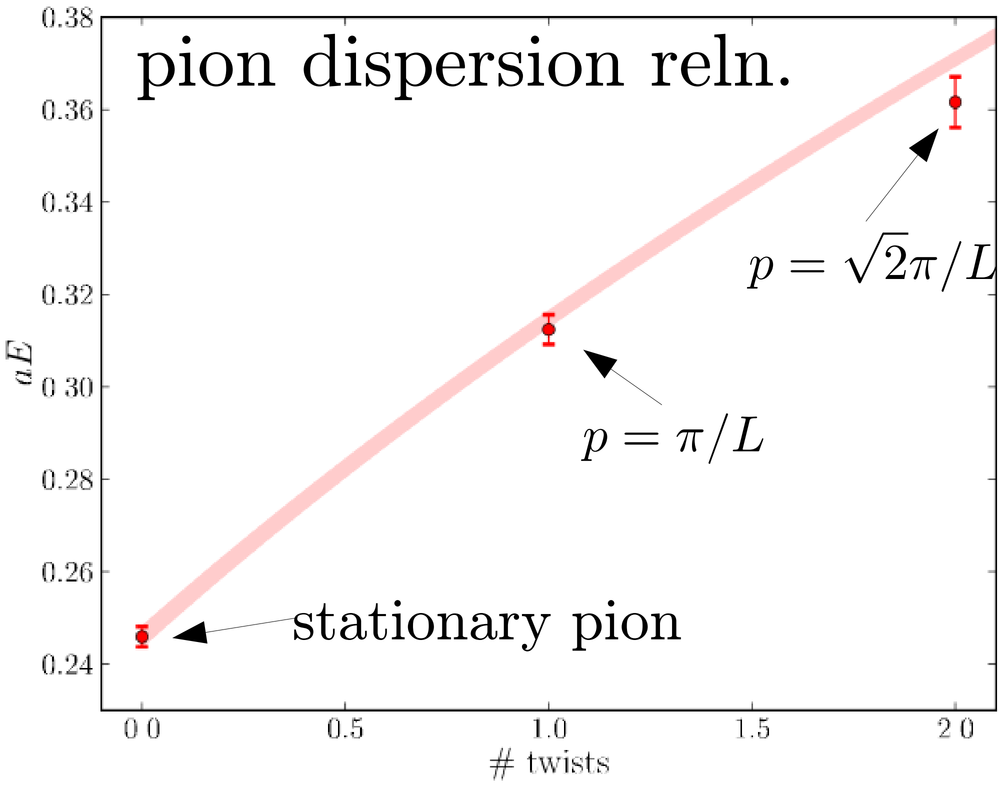
  RHMC needed throughout
- Standard double-precision multi-shift solver is quite slow due to linear algebra overheads coupled with finite BG/Q memory bandwidth.
- Developed optimized mixed-precision multi-shift inverter for RHMC in BFM/Bagel to optimize memory bandwidth usage on BG/Q.
- Alternative solution might be to use TWQCD's single-flavor action to avoid RHMC for light quarks.

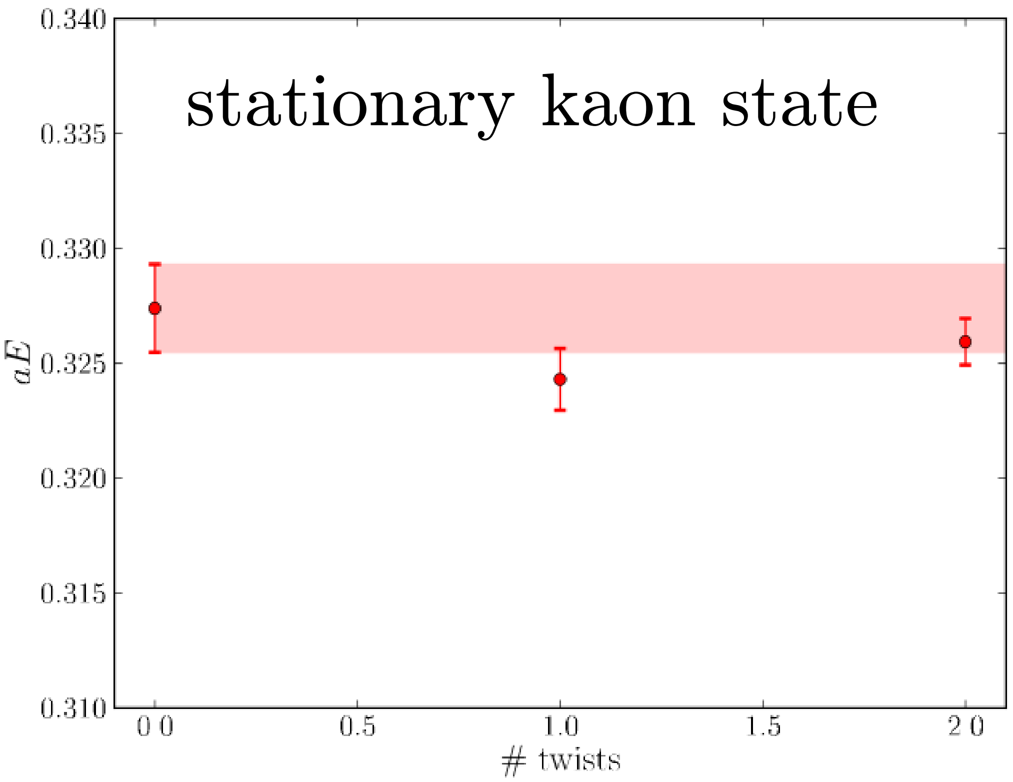
Summary of code changes

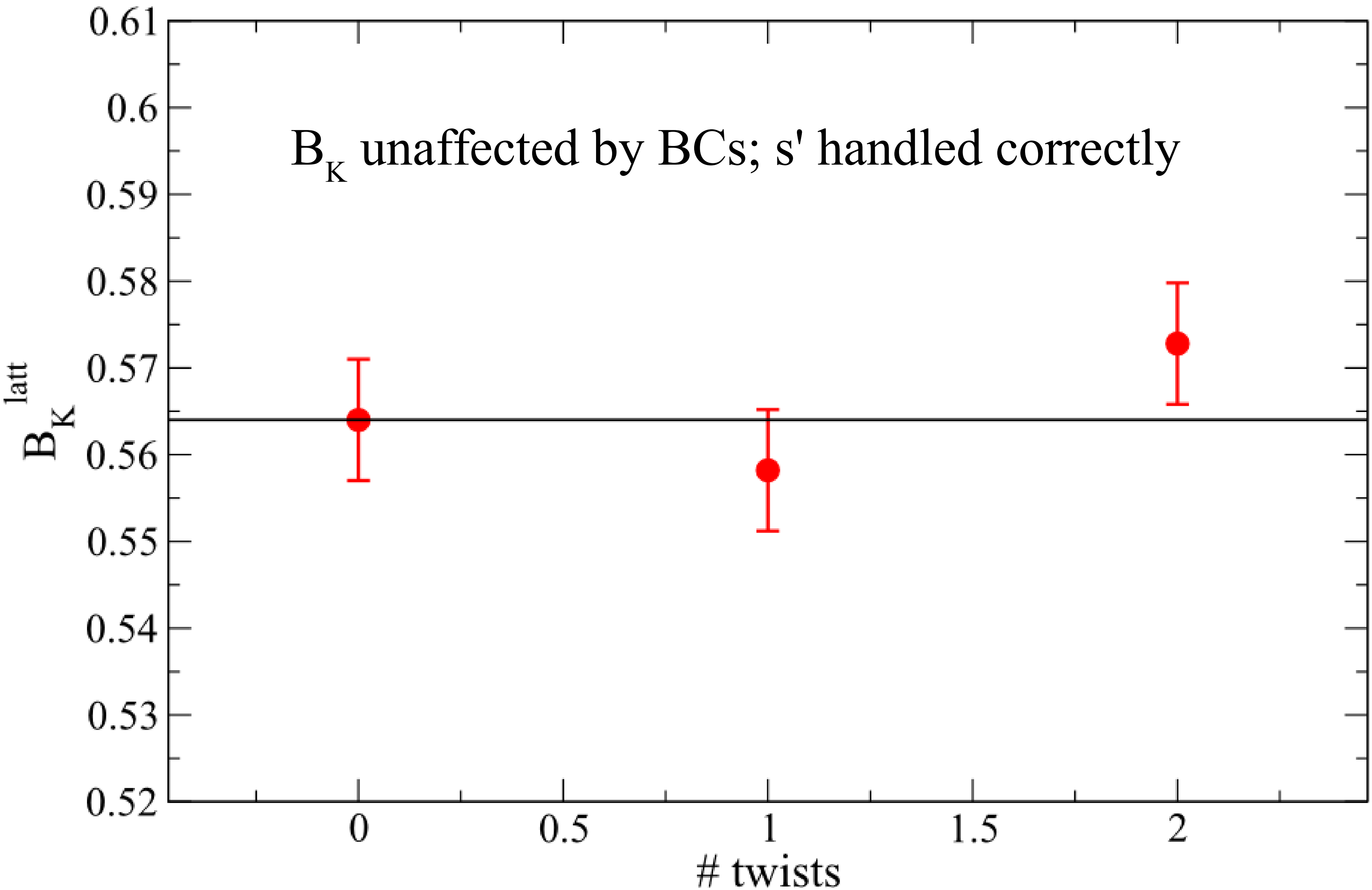
- HMC and basic measurement code written in CPS, with modified BFM/Bagel solvers for BG/Q.
- Complex conjugate BCs on gauge fields required changes to virtually all aspects of the codebase:
  - Gauge fixing algorithms
  - Plaquette and rectangle, plus staples.
  - Momentum field CC BCs, gauge force.
  - Memory layout reordering code.
  - Modified CPS+BFM/Bagel Dirac ops: Shamir DWF, Mobius DWF, twisted mass (for DSDR).
  - Fermion forces.
  - Eigenvalue algorithms: Ritz, Lanczos.
  - CPS propagator code.
  - Standard measurements: two-point correlators, B_K, Wilson flow, residual mass.
  - Multi-shift optimization

Demonstration

- 16^3x32 DWF+Iwasaki a^{-1} = 1.73(3) GeV with 420 MeV pion test configurations with GPBC in 0,1,2 directions.



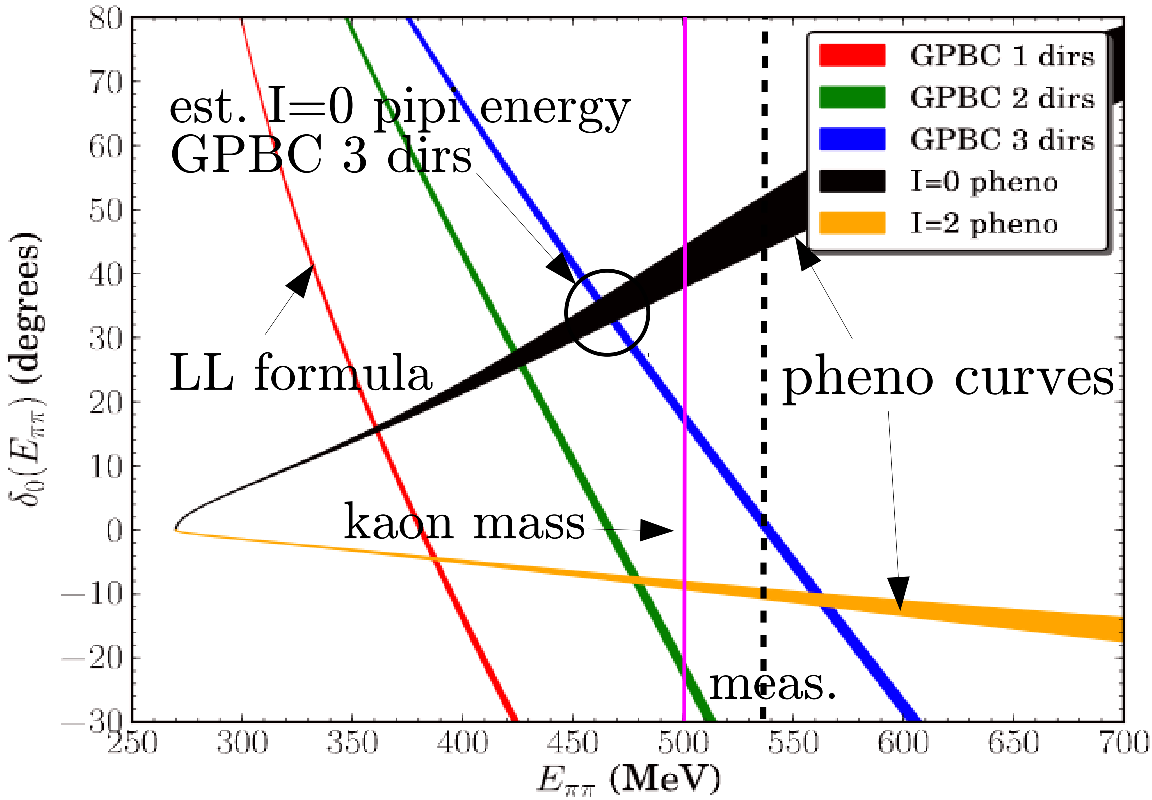




B_K unaffected by BCs; s' handled correctly
B_K^{latt}
twists
0.61
0.6
0.59
0.58
0.57
0.56
0.55
0.54
0.53
0.52
0
0.5
1
1.5
2

Physical Ensemble

- Physical volume must be tuned to match kaon and pi-pi energy as closely as possible.



- For 32^3x64 a^{-1}~1.38 GeV DWF+IDSDR β=1.75 close match with GPBC in 3 dirn.
- 4.6 fm^3 box.
- Measure:

E_π=0.1983(7) ~ 274 MeV

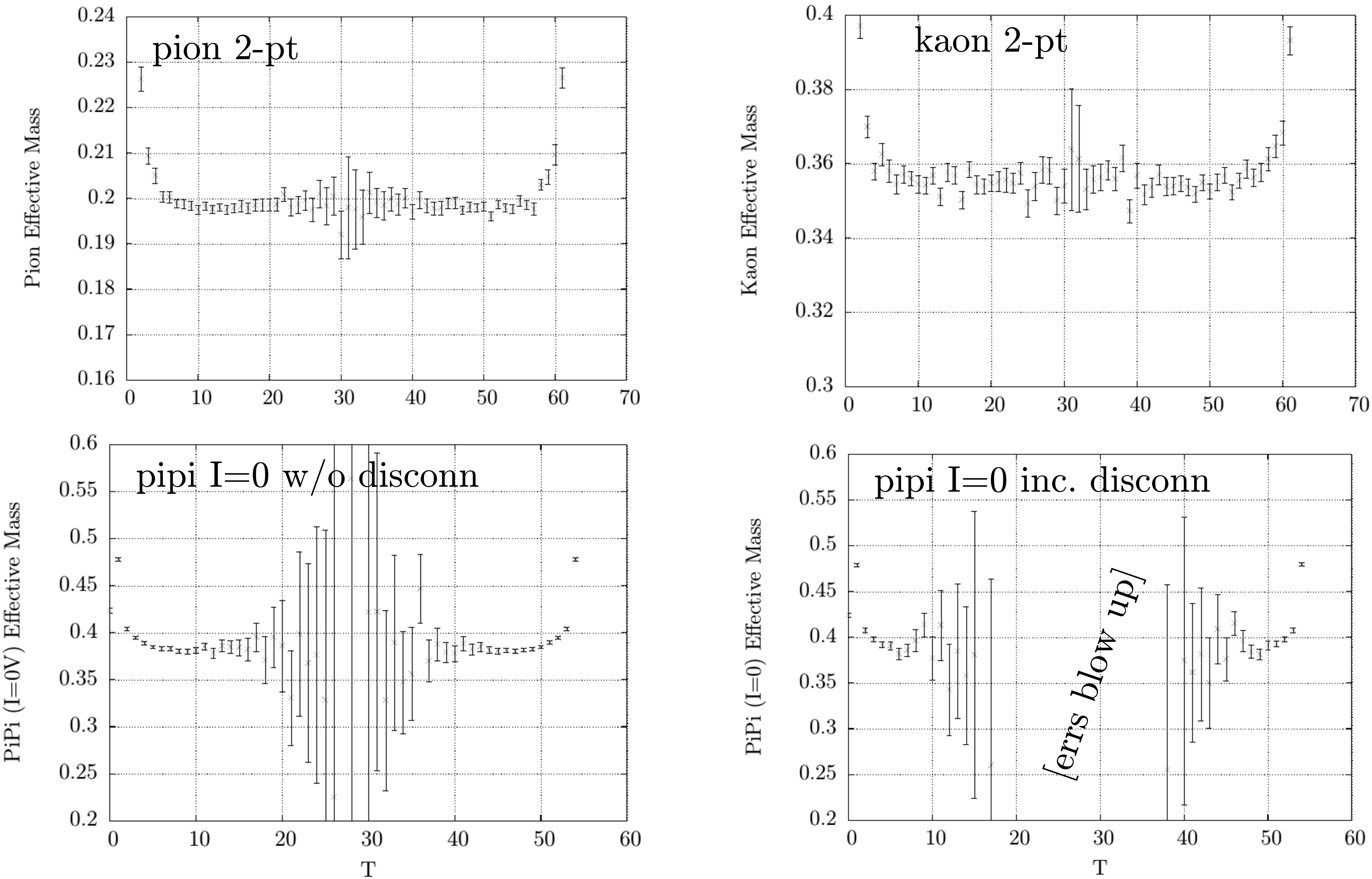
(m_π~141 MeV)

m_K=0.3549(3) ~ 490 MeV

$E_{\pi\pi}$(I=0) = 0.389(9) ~ 537 MeV

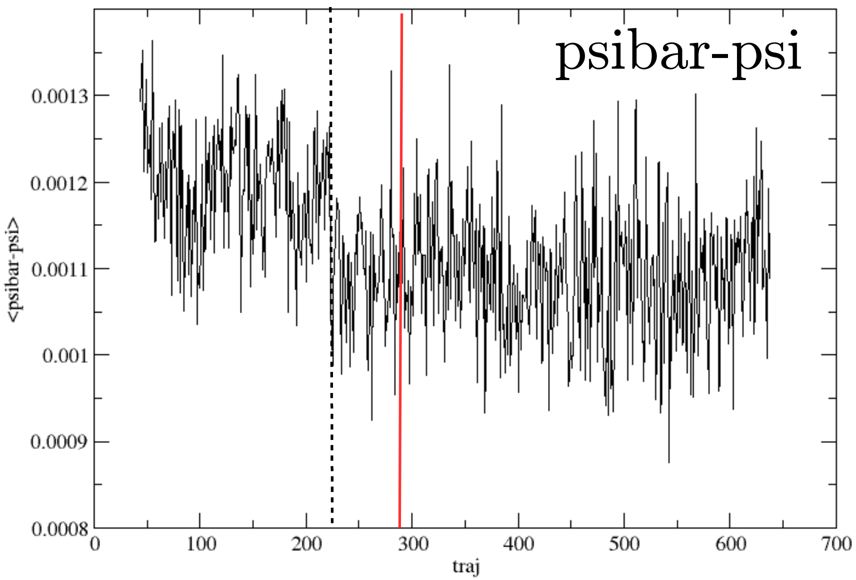
Effective Energies

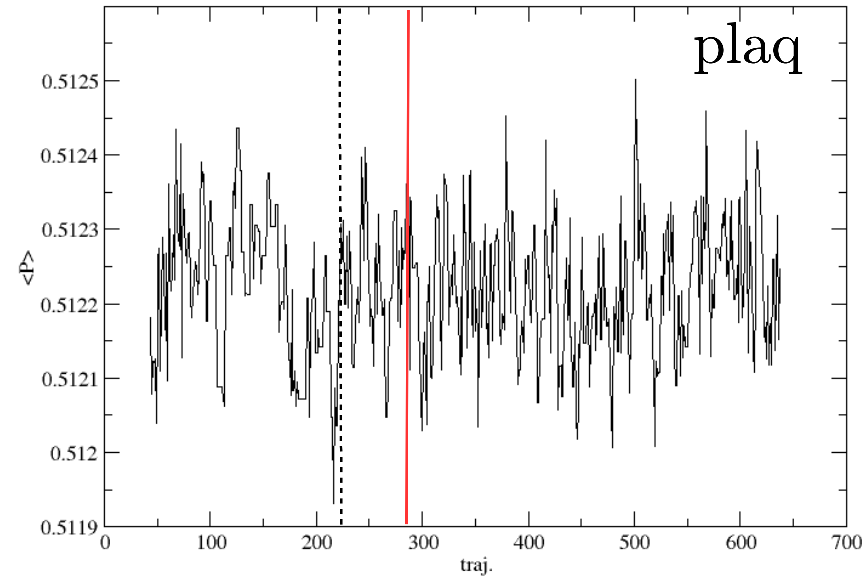
- Results on previous slide obtained using A2A props. on 41 configs. cf. Daiqian's talk for details



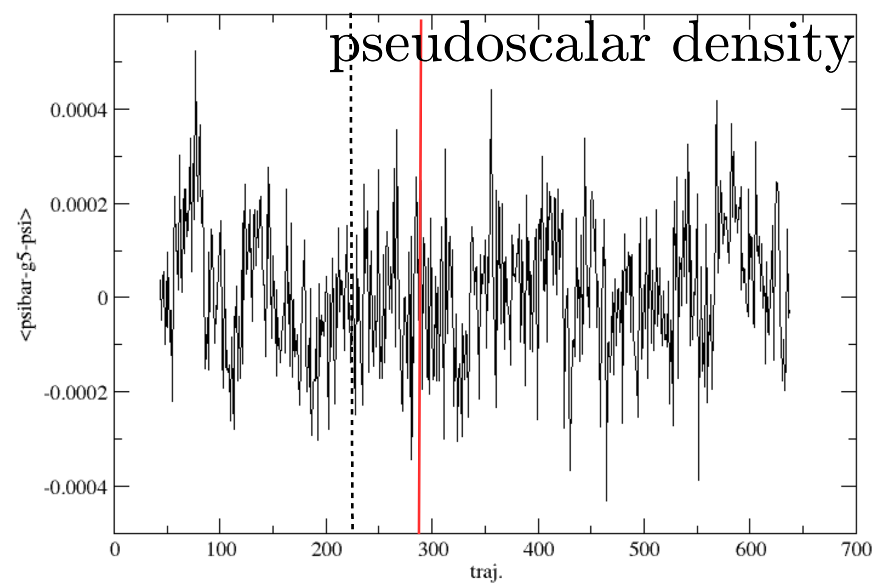
Ensemble Generation

- Ensemble generated on USQCD 512-node BGQ machine at BNL.
  - ~660 configurations to date.
  - Approx 400 thermalized.
  - 6.8 hours per configuration.
  - 89% Metropolis acceptance (88% theor.)





Dashed line: reduced quark mass
Red line: measurements begin (286)



Conclusions and Outlook

- Energy conserving kinematics necessary to describe physical decay.
- For I=0, G-parity BCs impose momentum on ground state pion without breaking isospin symmetry.
- Stationary kaon requires new strange-quark partner field. Subsequent removal from gauge action introduces non-locality that vanishes exponentially in volume.
- Highly optimized implementation and thorough testing / demonstration.
- Thermalized dynamical ensemble with large volume and physical pion mass.
- Kaon and pi-pi energies closely matched:

D. Zhang will follow this talk with his discussion of the $K \rightarrow \pi\pi$ measurement!