ETH
Eidgenössische Technische Hochschule Zürich
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QCD with Wilson fermions and isospin chemical potential at zero and finite temperature

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Motivation

- Isospin chemical potential

$\Rightarrow$ no sign problem: $\det(D(\mu))\det(D(-\mu)) = \det(D(\mu))\det(D(\mu))^* \in \mathbb{R}_{\geq 0}$

$\Rightarrow$ interesting phenomena:

- well known:
  - > π^+ condensation [Son & Stephanov, hep-ph/0011365]
  - > "rotation of condensate": $\langle\bar{\psi}\psi\rangle \longrightarrow \langle\bar{\psi}\gamma_5 i\tau_2\psi\rangle$ [Kogut & Sinclair, hep-lat/0202028]
- new:
  - > hadron mass-spectrum as function of μ_I
  - > condensation of additional mesons?
  - > saturation effects on the lattice

- Wilson fermions

$\Rightarrow$ hadron spectrum

$\Rightarrow$ not used in previous studies with an isospin chemical potential
[SU(2): S. Hands], [SU(3) Staggered: Kogut & Sinclair]

Isospin μ

- $N_f = 2$, $m_u = m_d$: isospin symmetry $\mathrm{SU}(2)_V$
- Isospin chemical potential $\mu_I = \mu\tau_3$

  explicitly breaks $\mathrm{SU}(2)_V \to \mathrm{U}(1)_V$

  $\mu_I = m_\pi/2 \quad \Rightarrow \quad \mathrm{U}(1)_V$ breaks spontaneously, π^+ condenses

- Son & Stephanov:

  Chiral Lagrangian: $\mathcal{L}_{eff} = \frac{f_\pi^2}{4} \mathrm{Tr}\left(\left(\mathrm{D}_\mu\Sigma\right)\left(\mathrm{D}_\mu\Sigma\right)^\dagger - 2m_\pi^2 \mathrm{Re}\left(\Sigma\right)\right)$

  Isospin chemical potential: $\mathrm{D}_\mu\Sigma = \partial_\mu\Sigma - \frac{\mu_I}{2}\delta_{0,\mu}\left(\tau_3\Sigma - \Sigma\tau_3\right)$

  Effective potential minimized for $\bar{\Sigma} = \cos(\alpha)\,\mathbb{1} + i\sin(\alpha)\left(\tau_1\cos(\phi) + \tau_2\sin(\phi)\right)$

  - $\mu < \mu_c = m_\pi/2 \quad \to \quad \alpha = 0$
  - $\mu \geq \mu_c = m_\pi/2 \quad \to \quad \cos(\alpha) = m_\pi^2/4\mu_I^2$

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quark condensate $\langle\bar{\psi}\psi\rangle$ rotates into pion condensate $\langle\bar{\psi}\gamma_5 i\tau_2\psi\rangle$

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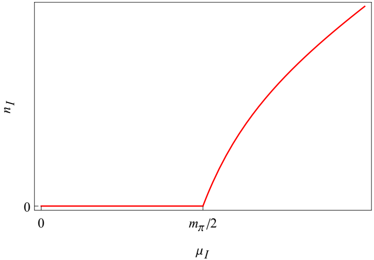
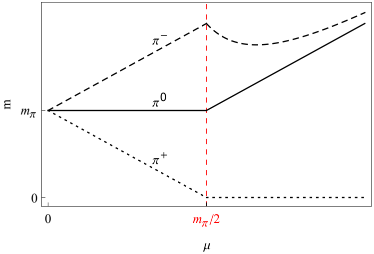


pion masses



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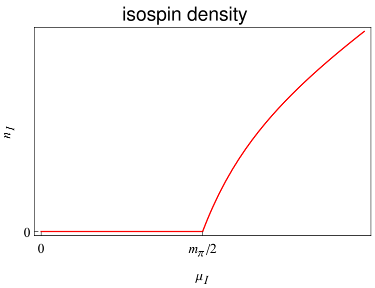
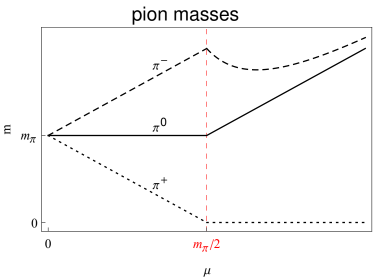
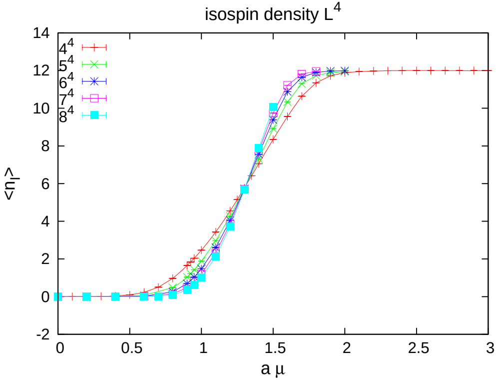
- Son & Stephanov: valid only for $\mu_I < m_\rho/2$





Isospin μ at Zero Temperature

- Fixed lattice spacing: $\beta = 5.0$, $\kappa = 0.150 \quad \rightarrow \quad am_\pi \approx 1.84, am_\rho \approx 1.86$
- Expect Bose condensation of π^+ at $a\mu_I = am_\pi/2 \approx 0.92$
- $U(1)$ symmetry breaking in $d = 4 \quad \rightarrow \quad$ mean-field
- Measured data:



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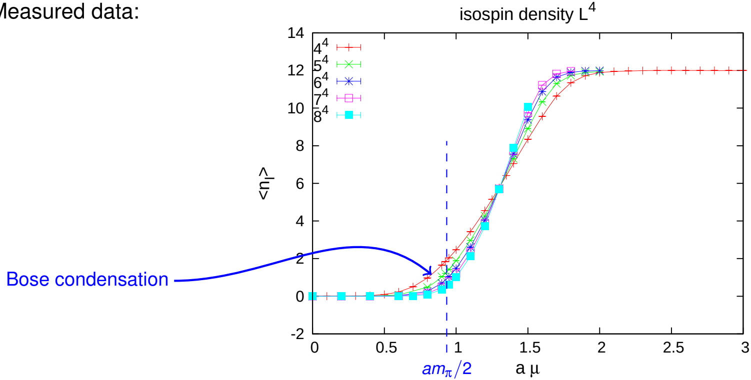

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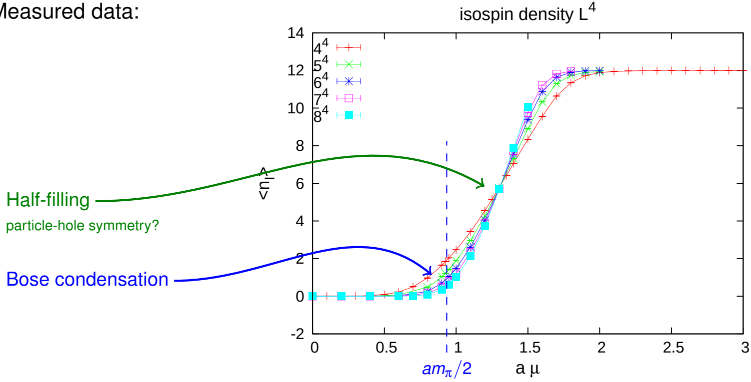

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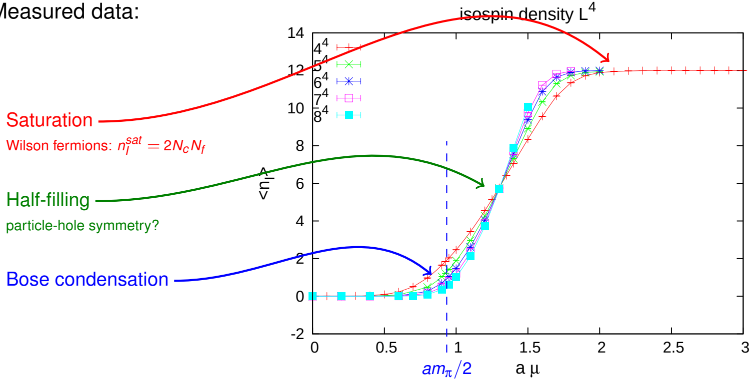

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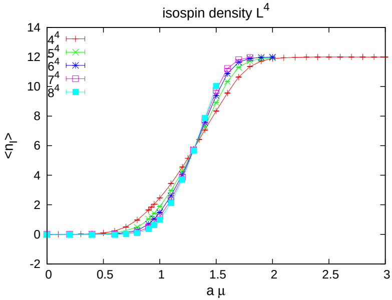
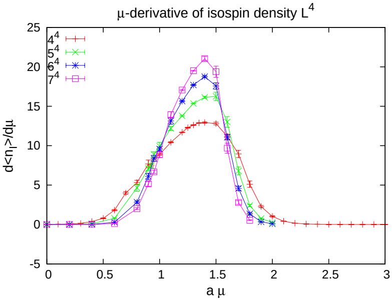
- isospin density and its derivative at zero T





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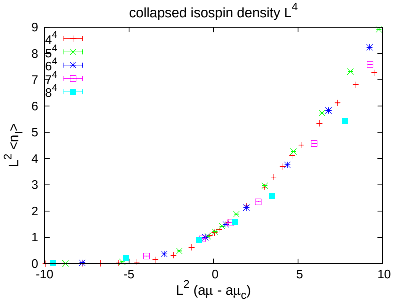
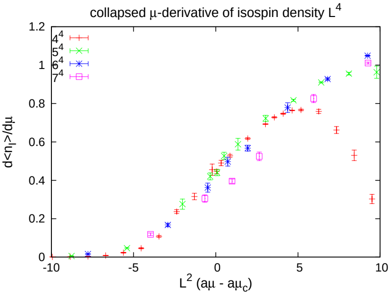
- scaling collapse: $\quad \xi \sim |\mu - \mu_c|^{-1/2}, Q(\mu_c) \sim V^{1/2}$





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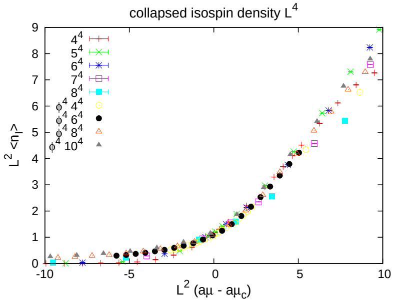
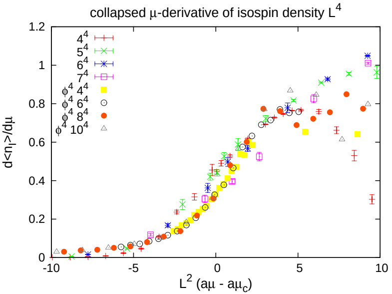
- $U(1)$ symmetry breaking in $d = 4 \quad \rightarrow \quad$ mean-field (log corrections)
- comparison with complex $\phi^4 \quad \rightarrow \quad$ same universality class





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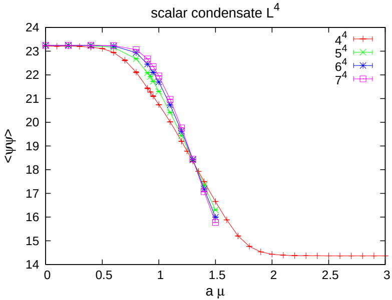
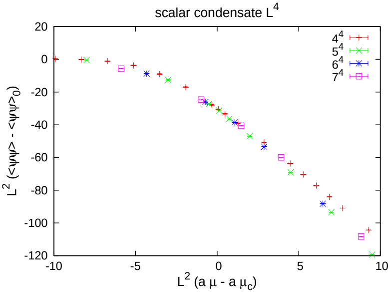
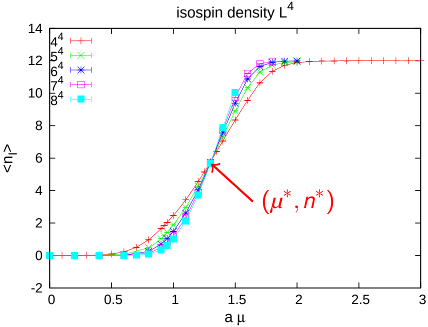
- quark condensate





Half-Filling

- Half filling at $\mu^* \approx 1.33$, $n^* = 6$ $(\beta = 5.0, \kappa = 0.15)$
  - > Volume independence



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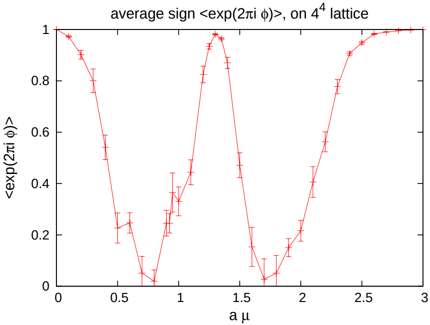
> **Average sign:** $\sim \langle \exp(2\mathrm{i}\phi) \rangle \approx 1$
with ϕ: phase of single flavour fermion determinant



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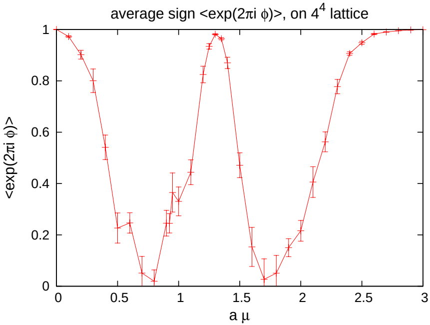

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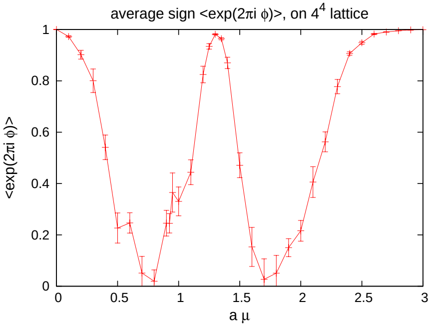

- sign-problem is absent:

  > interesting point for running simulations?

  > base point about which to expand?

Heavy-Dense Limit

- Heavy-dense limit

$\rightarrow$ turn off spatial hopping $\rightarrow \det(D(\mu)) = \prod_{n=1}^{n_x n_y n_z} \det(D_s(\mu))$, (single site determinants)

$\rightarrow$ temporal gauge $\rightarrow P = \mathrm{diag}\left(e^{i\Theta_1}, e^{i\Theta_2}, e^{-i(\Theta_1+\Theta_2)}\right)$

$$\Rightarrow \quad \det(D_s(\mu)) \rightarrow e^{-4i(\Theta_1+\Theta_2)-6n_t\mu}$$
$$\times \left(e^{i\Theta_1+n_t\mu}+(2\kappa)^{n_t}\right)^2 \left(e^{i\Theta_2+n_t\mu}+(2\kappa)^{n_t}\right)^2 \left(e^{n_t\mu}+(2\kappa)^{n_t} e^{i(\Theta_1+\Theta_2)}\right)^2$$
$$\times \left(1+(2\kappa)^{n_t} e^{i\Theta_1+n_t\mu}\right)^2 \left(1+(2\kappa)^{n_t} e^{i\Theta_2+n_t\mu}\right)^2 \left((2\kappa)^{n_t} e^{n_t\mu}+e^{i(\Theta_1+\Theta_2)}\right)^2$$

$\rightarrow$ Haar measure: $H(\Theta_1,\Theta_2) = \frac{1}{6\pi^2}\left(\sin(\Theta_1-\Theta_2)-\sin(2\Theta_1+\Theta_2)+\sin(\Theta_1+2\Theta_2)\right)^2$

$$\Rightarrow \quad Z_s(\mu) = \int_0^{2\pi} \mathrm{d}\Theta_1 \int_0^{2\pi} \mathrm{d}\Theta_2\, H(\Theta_1,\Theta_2) \det(D_s(\mu)) \det(D_s(\mu))^*,$$

$$Z(\mu) \approx \prod_{s=1}^{n_x n_y n_z} Z_s(\mu) \quad \text{(neglecting plaquettes)}$$

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$$\Rightarrow \qquad Z_s\left(\mu\right) = \int_0^{2\pi} d\,\Theta_1 \int_0^{2\pi} d\,\Theta_2\, H\left(\Theta_1,\Theta_2\right) \det\left(D_s\left(\mu\right)\right) \det\left(D_s\left(\mu\right)\right)^*,$$

$$Z\left(\mu\right) \approx \prod_{s=1}^{n_x n_y n_z} Z_s\left(\mu\right) \quad \text{(neglecting plaquettes)}$$

$\Rightarrow$ closed, analytic expressions for average Polyakov loop, isospin density, average sign, etc.

Heavy-Dense Limit

- Heavy-dense limit

half filling: $\mu_{cr} = -\log(2\kappa)$, for $(2\kappa)^{2n_t} << 1$, i.e. $T \approx 0$

fermion determinant symmetric about μ_{cr} up to terms of order $\mathcal{O}\left((2\kappa)^{2n_t}\right)$:

$$\det(D(\mu,U))\det(D(\mu,U))^* \approx (2\kappa\, e^{\mu})^{12\,n_t}\det(D(2\mu_{cr}-\mu,U))\det(D(2\mu_{cr}-\mu,U))^*$$

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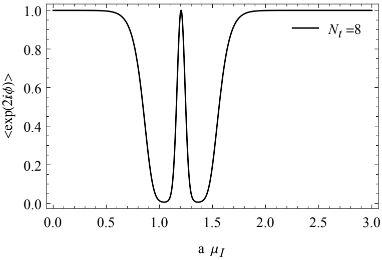
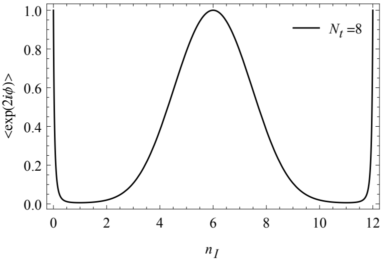


average sign vs. n_I



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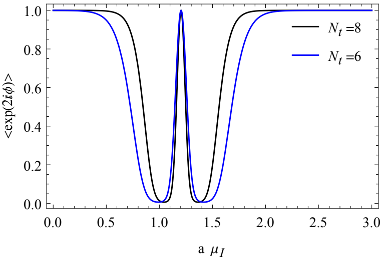

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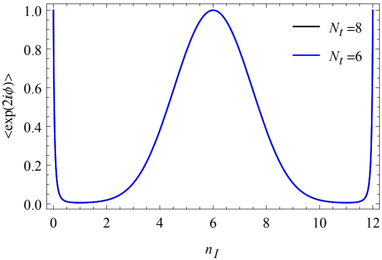

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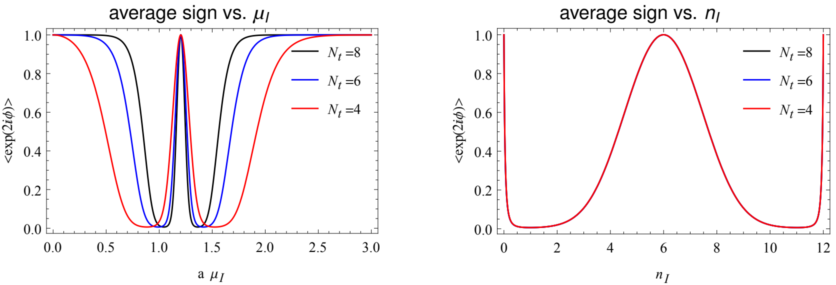

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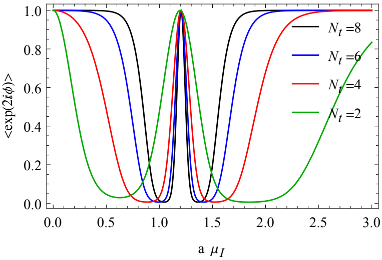

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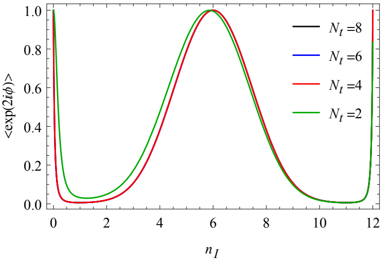
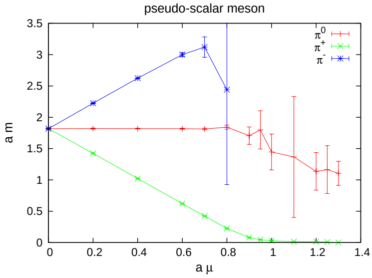


$n_t = 2$ too small $\Rightarrow$ asymmetric

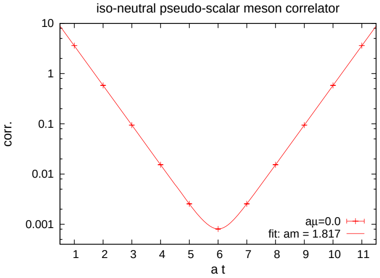
Meson Masses

- Meson masses – $4^3 \times 12$ lattice, $\beta = 5.0$, $\kappa = 0.15$ $\Rightarrow$ $am_\pi \approx 1.84$, $am_\rho \approx 1.86$
- Source and sink smearing, $\kappa_2 = 0.12$ (optimized)
- Pion:

π^0 and $\pi^{+/-}$ masses



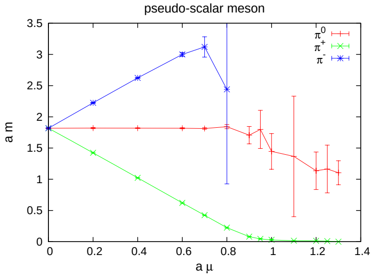
π^0 correlators (incl. disconnected part)



Meson Masses

- Meson masses – $4^3 \times 12$ lattice, $\beta = 5.0$, $\kappa = 0.15$ $\Rightarrow$ $am_\pi \approx 1.84$, $am_\rho \approx 1.86$
- Source and sink smearing, $\kappa_2 = 0.12$ (optimized)
- Pion:

π^0 and $\pi^{+/-}$ masses



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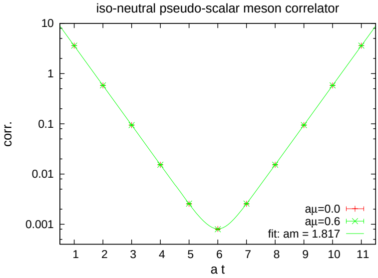

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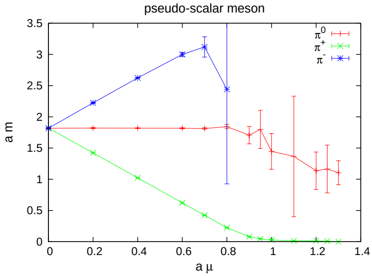

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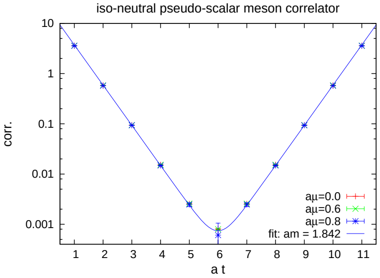

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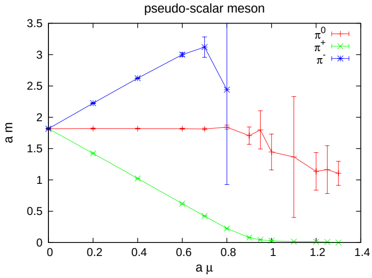

$\mu > \mu_{cr}$: π^0 mass decreasing

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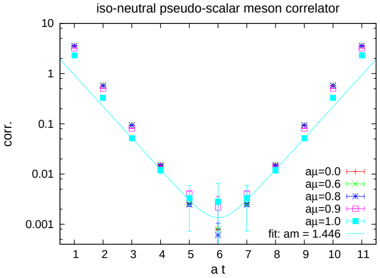

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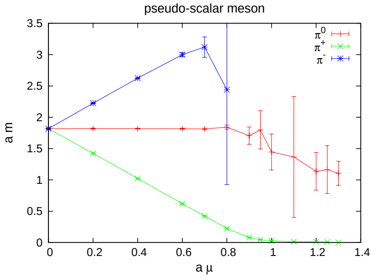

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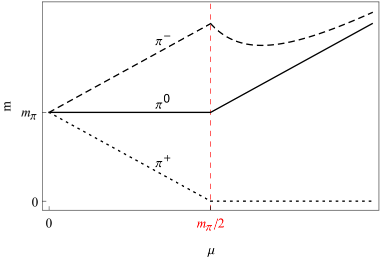
π^0 and $\pi^{+/-}$ masses from Son & Stephanov



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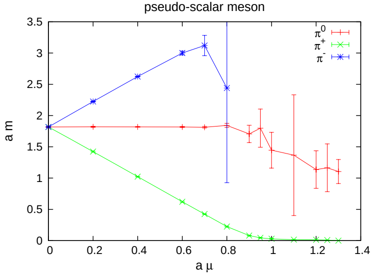

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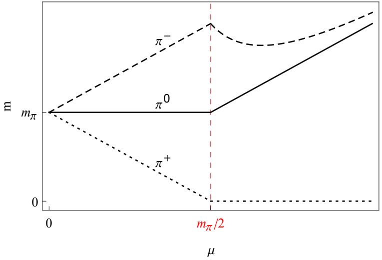

π^0 mass increasing
(valid only for $\mu_I < m_\rho/2$)

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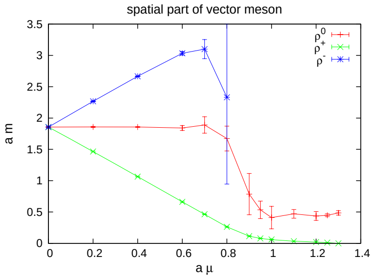
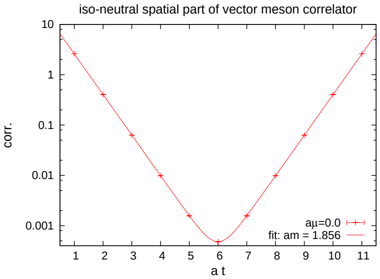
- Rho:

ρ^0 and $\rho^{+/-}$ masses

ρ^0 correlators (incl. disconnected part)





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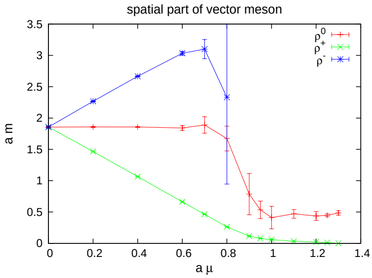

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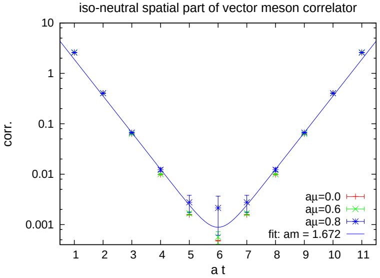

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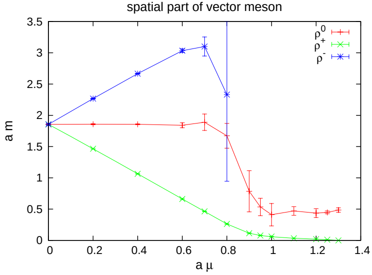

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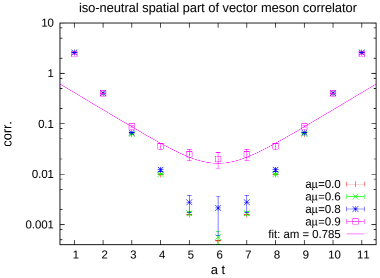

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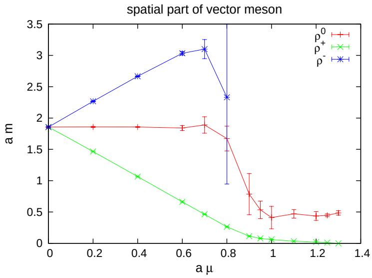

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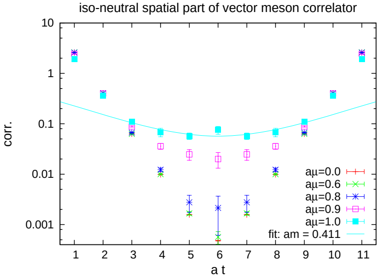
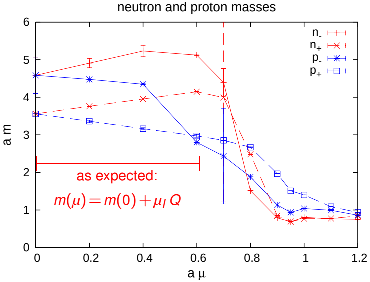


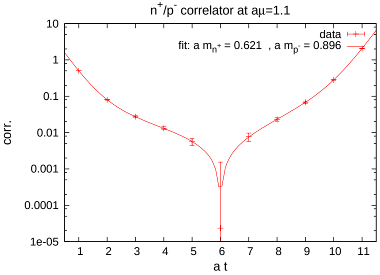
Baryon Masses

- Baryon masses – $4^3 \times 12$ lattice, $\beta = 5.0$, $\kappa = 0.15$
- Protons and neutrons:

$p^{+/-}$, $n^{+/-}$ masses:



correlators n^+ and p^-:



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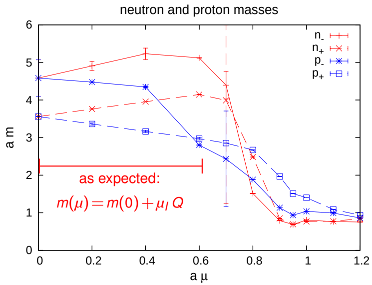

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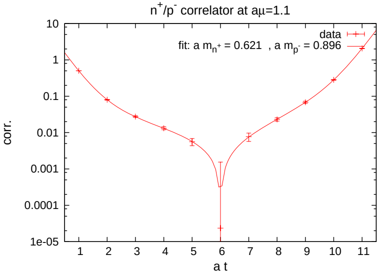
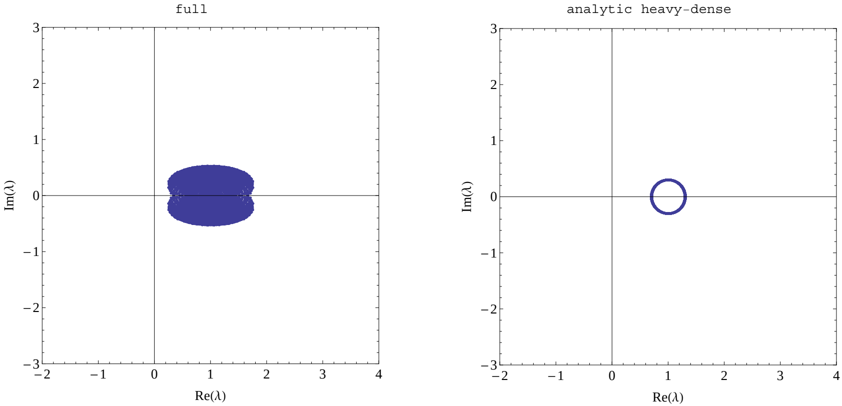


good fits with small masses at large isospin chemical potential

Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

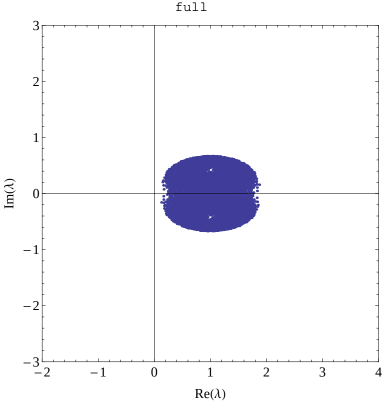
$\mu = 0.00$

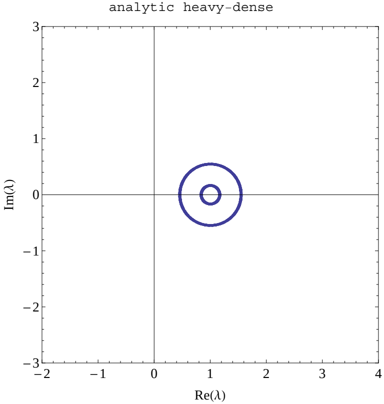


Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

$\mu = 0.60$





Dirac Operator Spectrum

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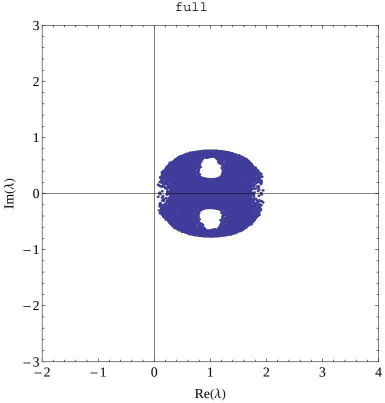
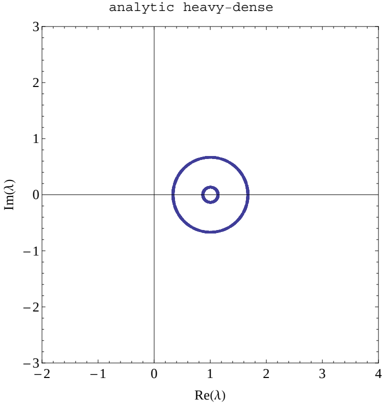
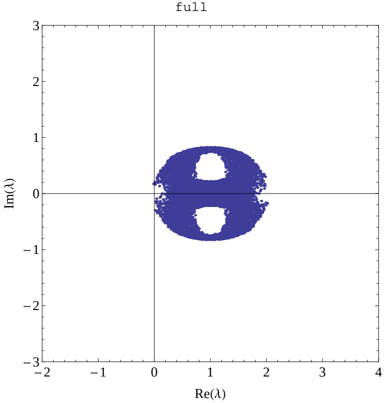


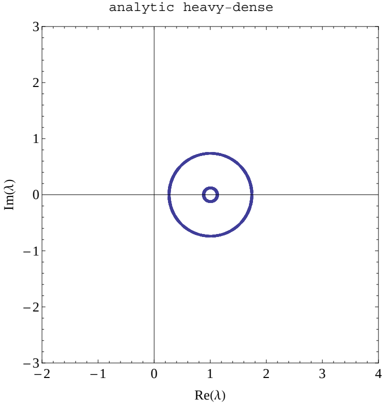


Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

$\mu = 0.90$ $\quad$ $(\mu_{cr} = 0.92)$

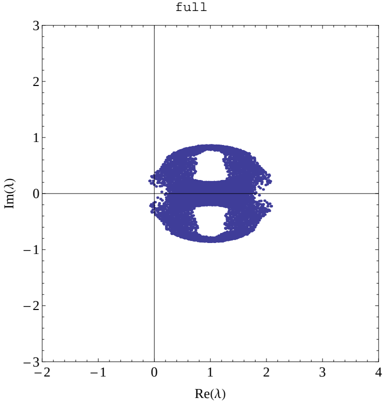


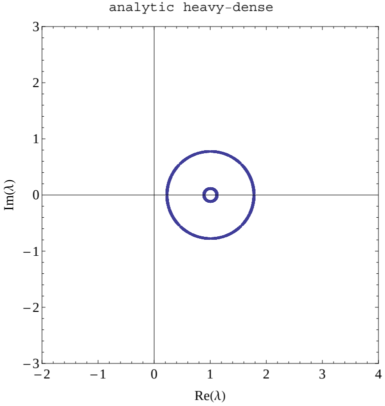


Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

$$\mu = 0.95 \qquad (\mu_{cr} = 0.92)$$

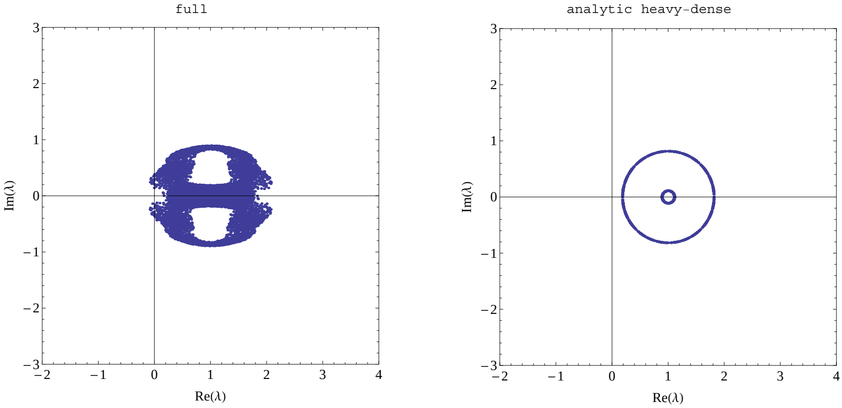




Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

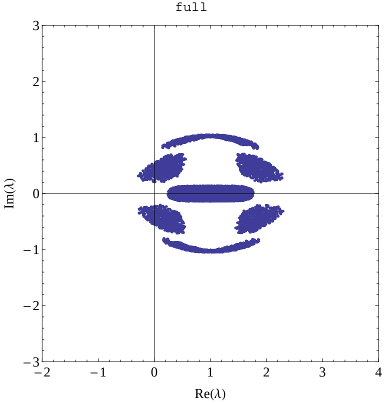
$\mu = 1.00$

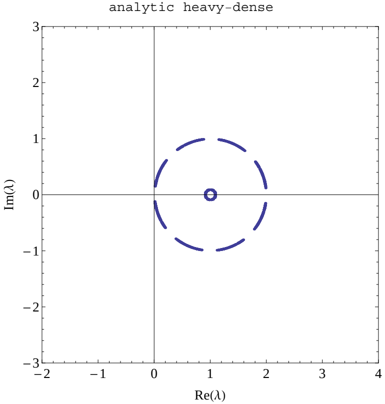


Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

$\mu = 1.20$

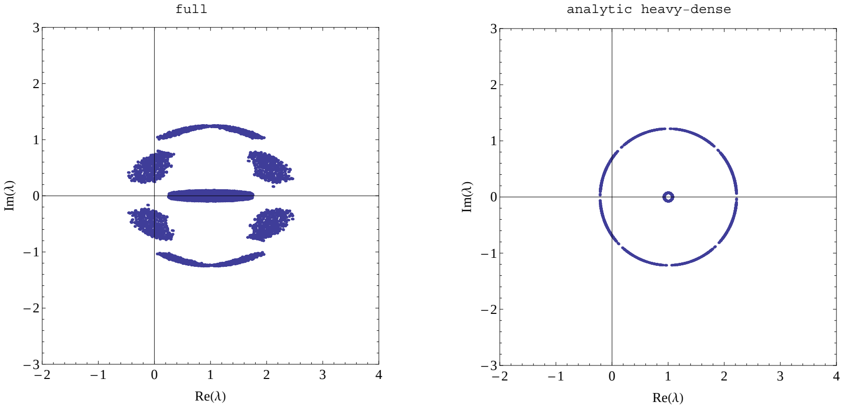




Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

$$\mu = 1.40$$



Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

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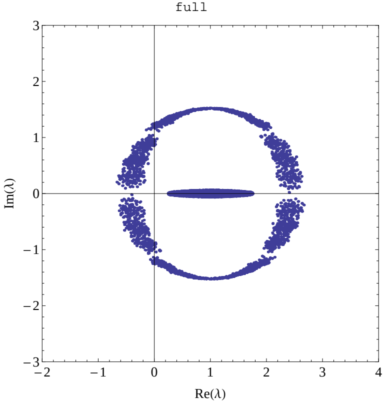

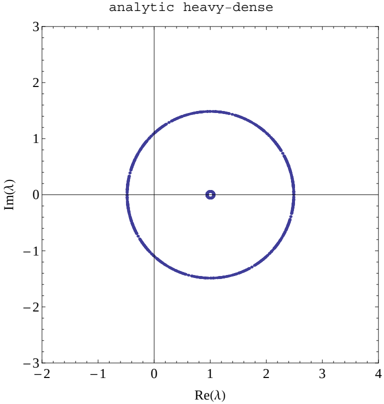

Dirac Operator Spectrum

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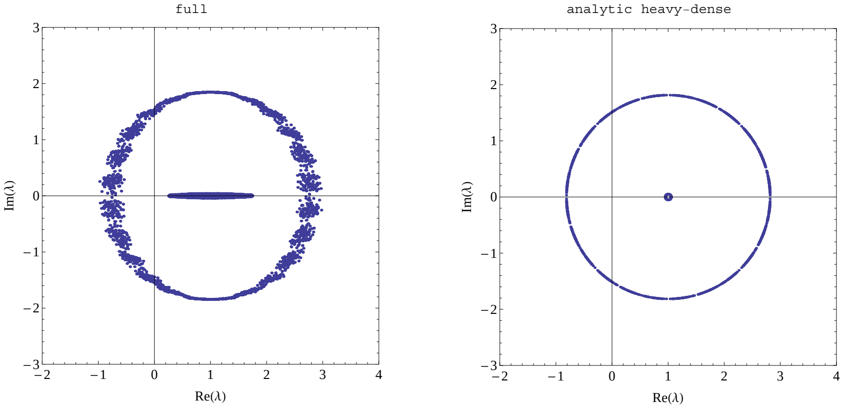
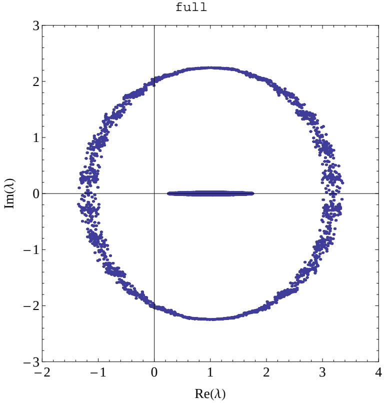
$\mu = 1.80$

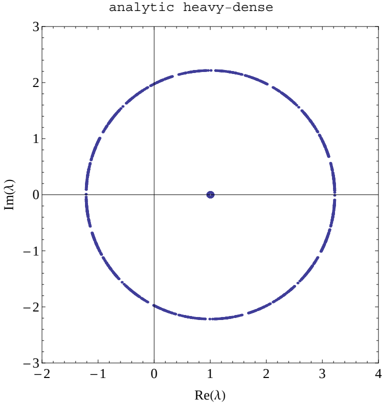


Dirac Operator Spectrum

- Typical eigenvalue distribution of single flavor Wilson Dirac operator $D(\mu)$

$$\mu = 2.00$$





Conclusion & Outlook

- Conclusion:
  - $T = 0$: Bose condensation for $\mu_I \to m_\pi/2$ as expected
  - Half-filling point $n_I = 6$:
    - particle-hole symmetry
    - no sign-problem! (interesting for simulations with Baryon chemical potential?)
  - Mass of iso-neutral mesons rather decrease than increase for $\mu_I > m_\pi/2$
  - Analytic results from heavy-dense limit
- In progress:
  - Larger $\kappa = 0.175 \quad \Rightarrow \quad$ larger mass separation: $m_\pi \approx 1.48$, $m_\rho \approx 1.70$
  - Finite temperature
  - Condensation of ρ^+? Heavier mesons?
  - Baryon spectrum