The leading hadronic contribution to (g-2) of the muon: The chiral behavior using the mixed representation method

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Overview

What is the leading order anomalous magnetic moment of the muon a_μ^{HLO} and what precision can be reached from lattice calculations?

The central quantity a_μ^{HLO} is accessible from the lattice by computing the hadronic vacuum polarization (HVP) function $\Pi(Q^2)$

$$a_\mu^{HLO} = \left(\frac{\alpha}{\pi}\right)^2 \int dQ^2 K_E(Q^2, m_\mu)\Big(\Pi(Q^2) - \Pi(0)\Big) \tag{1}$$

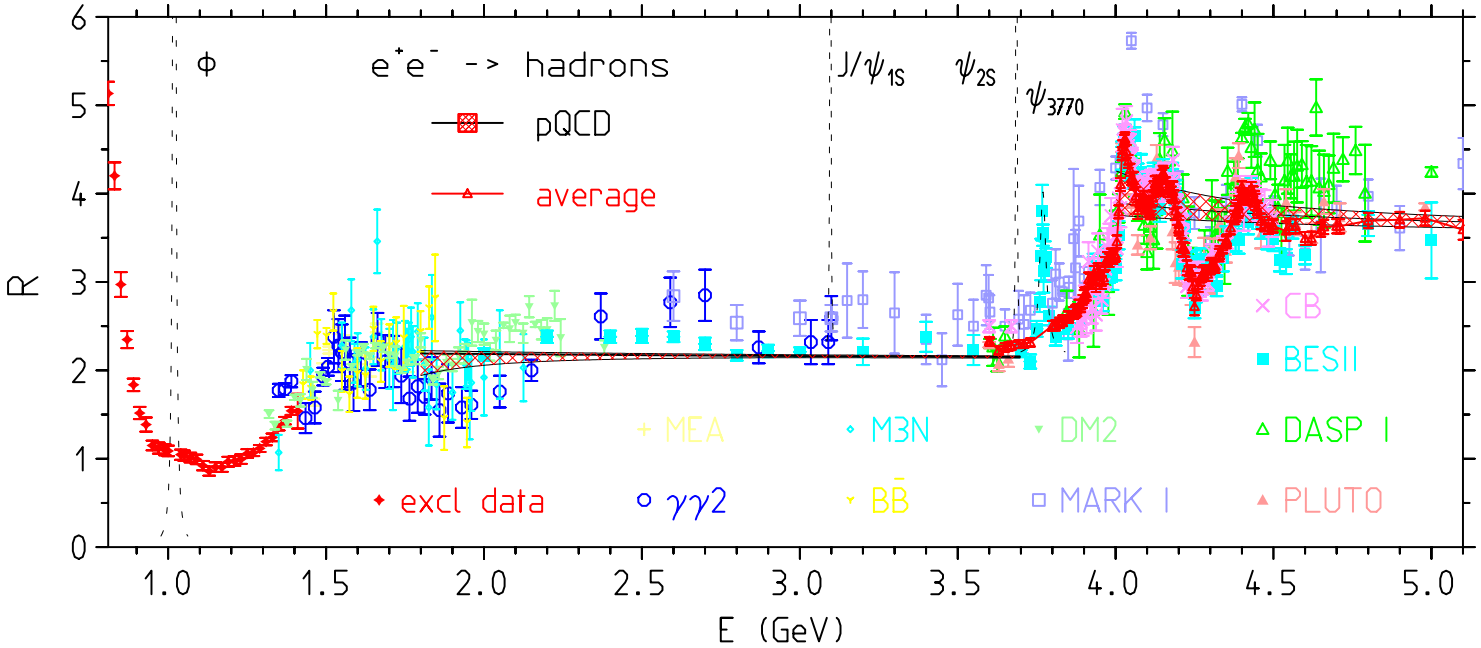
In the following and the talks by *V. Gülpers*, *H. Horch* and *G. Herdoiza*, we study different methods of obtaining $\Big(\Pi(Q^2) - \Pi(0)\Big)$ and discuss the uncertainties arising from their respective systematics, as well as the disconnected diagrams.

Hadronic vacuum polarization

In phenomenology the hadronic vacuum polarization can be computed via

$$\left(\Pi(Q^2) - \Pi(0)\right) = \frac{Q^2}{3} \int_0^\infty ds \frac{R(s)}{s(s+Q^2)} \tag{2}$$

where $R(s) \propto \sigma(e^+e^- \to \text{hadrons})$



[0902.3360]

On the lattice both sides of (2) can be used to compute the HVP

- **Lhs:**

$$\Big(\Pi(Q^2) - \Pi(0)\Big) = ...$$

Extract the $\Pi(Q^2 > Q^2_{latt,min}(L,a))$ by noting

$$\Pi_{\mu\nu}(Q) = \big(Q_\mu Q_\nu - \delta_{\mu\nu} Q^2\big)\Pi(Q^2)$$

where $\Pi_{\mu\nu}(Q)$ is given in terms of the vector meson current-current correlator $\langle j_\mu(x) j_\nu(0)\rangle$

$$\Pi_{\mu\nu}(Q) \equiv \int d^4x\, e^{iQ\cdot x} \langle j_\mu(x) j_\nu(0)\rangle$$

- We refer to this approach as the "standard method",

[0212018], [0608011], [1103.4818], [1011.5793]

On the lattice both sides of (2) can be used to compute the HVP

- **Rhs:**:

$$... = \frac{Q^2}{3} \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

Extract the *difference* $\Pi(Q^2) - \Pi(0)$ by noting

$$R(s) = 12\pi^2 \rho(s)$$

where $\rho(s)$ is the spectral function of the vector meson current-current correlator $\langle j_\mu(x) j_\nu(0) \rangle$

$$G(x_0, \vec{k}) \overset{\mu=\nu}{=} \int d^3x \, e^{i\vec{k}\vec{x}} \langle J_\mu(x_0, \vec{x}) J_\nu(0) \rangle = \int_0^\infty ds \sqrt{s} \rho(s) K(s, x_0).$$

One finds:

$$\Pi(Q^2) - \Pi(0) = \int_0^\infty dx_0 \, G(x_0) \Big[x_0^2 - \frac{4}{Q^2} \sin^2(\frac{1}{2} Q x_0) \Big]$$

- We refer to this approach as the "mixed representation method", [1305.5878], [1306.2532]

Caveats of the current methods

$$\Big(\Pi(Q^2) - \Pi(0)\Big) = \frac{Q^2}{3} \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

$$\Pi(Q^2 > Q^2_{min}) - \Pi(Q^2 \to 0) = \ldots \qquad \ldots = \int_0^\infty dx_0 G(x_0) K(x_0, Q^2)$$

Lhs: Forming in the standard method $\Big(\Pi(Q^2) - \Pi(0)\Big)$ and a_μ^{HLO} note ...

- ... lattice data is not available at $\Pi(0)$
- ... extrapolation from $Q^2 = 0$ to $Q^2_{min} = min(Q^2_{latt}(L, a))$ is required
- ... a_μ^{HLO} depends crucially on precise data/interpolation at low Q^2

Rhs: Integrating using the mixed rep. method for $\Big(\Pi(Q^2) - \Pi(0)\Big)$ and a_μ^{HLO} note ...

- ... the correlator has to be known for all times $t \to \infty$
- ... lattice data has to extrapolated to its asymptotic behavior
- ... a_μ^{HLO} depends crucially on precise knowledge of the correlator/spectrum

Towards a precision determination of a_μ^{HLO}

Both the standard and the mixed rep. method rely on the same data and ultimately process/display equivalent information.

However, what is low $Q^2 \to 0$ in one is large Euclidean times $t \to \infty$ in the other. Can we use this to our advantage?

Ad-/disadvantages of the standard method $\to$ talks at this conference.

In the mixed rep. method the key observable, $G(x_0, \vec{k} = 0)$, ...

- ... has a well established machinery to study signal/noise behavior and finite size/mass/lattice spacing effects.
- ... can draw on a large body of experience/methods to systematically improve the results.
- ... enables a systematic study of the spectrum of QCD.
- ... opens the possibility of a straight forward inclusion of the disconnected diagrams (see talk by *V. Gülpers*).

What advantages can we expect to exploit?

- The results for $G(x_0, \vec{k} = 0)$ can be extracted at runtime from a program computing $\Pi(Q^2)$ at negligble cost.

- The difference of the standard method and mixed rep. method can be monitored

$$\Pi_{STD}(Q^2) - \Big(\Pi(Q^2) - \Pi(0)\Big)_{MRM} = \Pi(0) \tag{3}$$

  iff both analysis are in fact equivalent,
  - difference should be approx. constant
  - gives a measure of $\Pi(0)$

- The different systematics should become visible in quantities like (3)

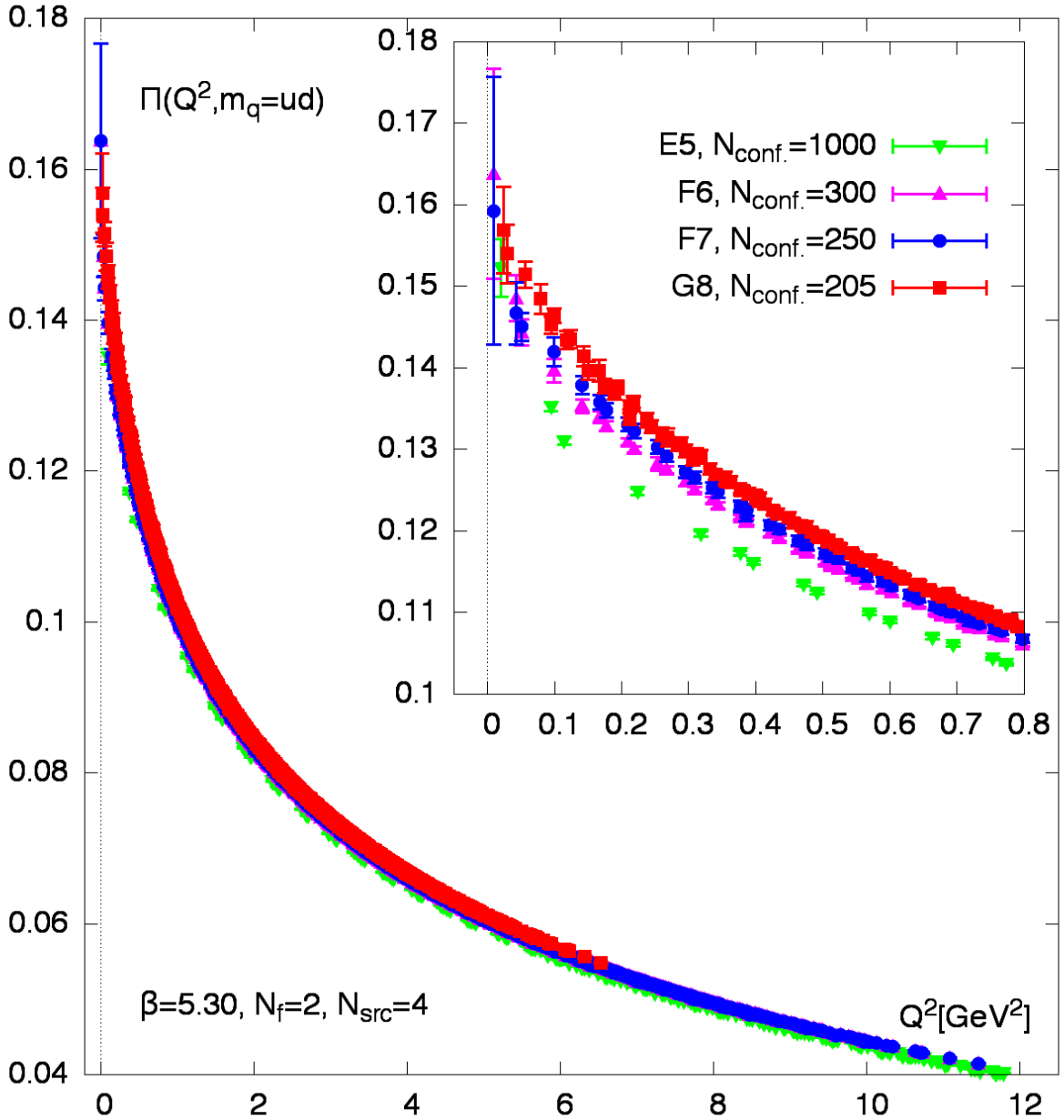
Towards chiral behaviour of a_μ^{HLO}

- We explore the chiral behaviour of a_μ^{HLO} at fixed lattice spacing $a = 0.063$ and $\beta = 5.30$

lattice	L [fm]	m_π [MeV]	$m_\pi L$	$N_{meas}(N_{conf})$	Label
64×32^3	2.0	451	4.7	4000(1000)	E5
96×48^3	3.0	324	5.0	1200(300)	F6
96×48^3	3.0	277	4.2	1000(250)	F7
128×64^3	4.0	190	4.0	820(205)	G8

- All ensembles were generated within the CLS effort with two flavors of $\mathcal{O}(a)$ improved Wilson-Clover fermions
- Correlation functions for strange and charm (not shown here) quark masses are available as quenched, valence observables

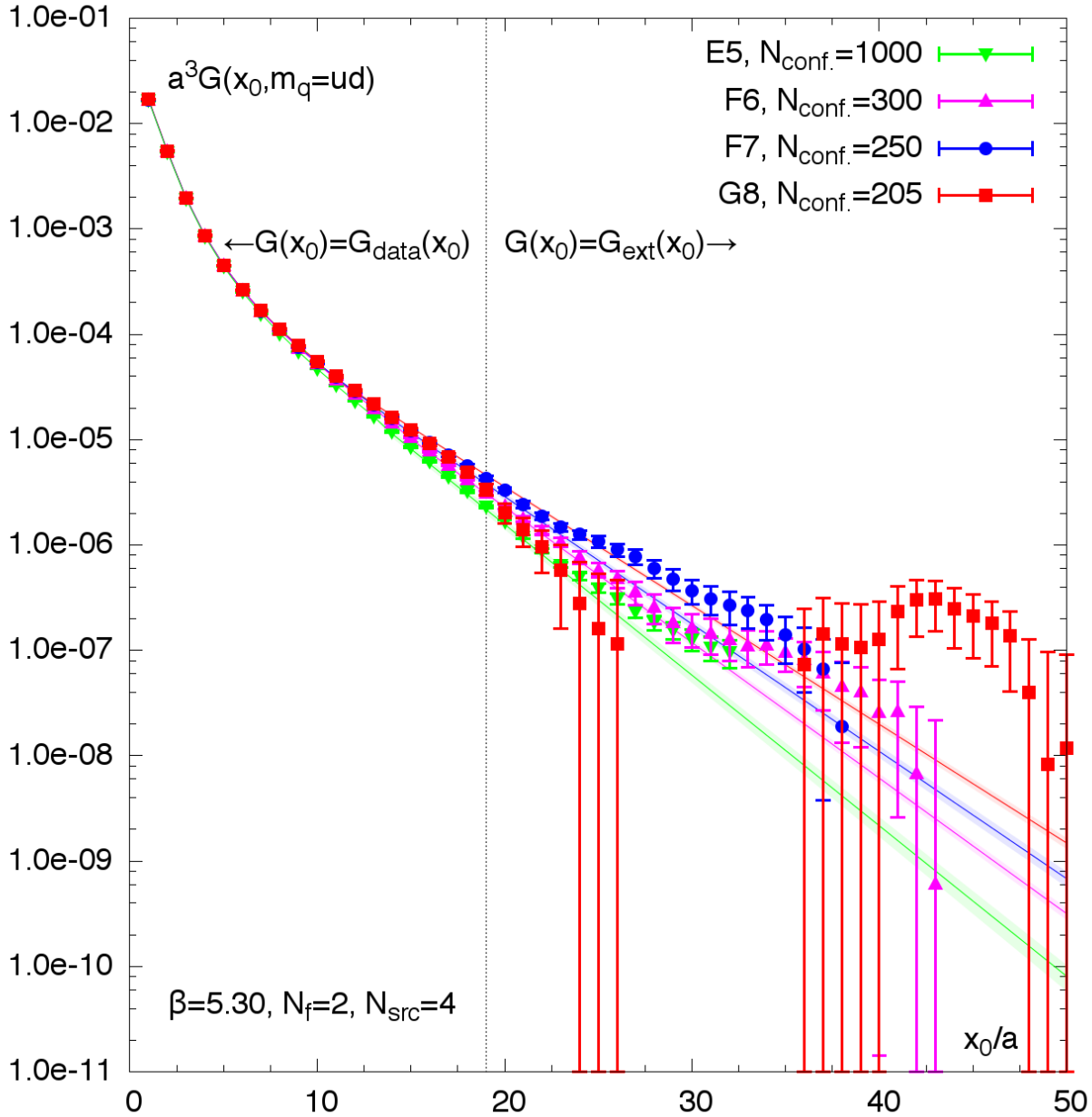
Standard method



To do:

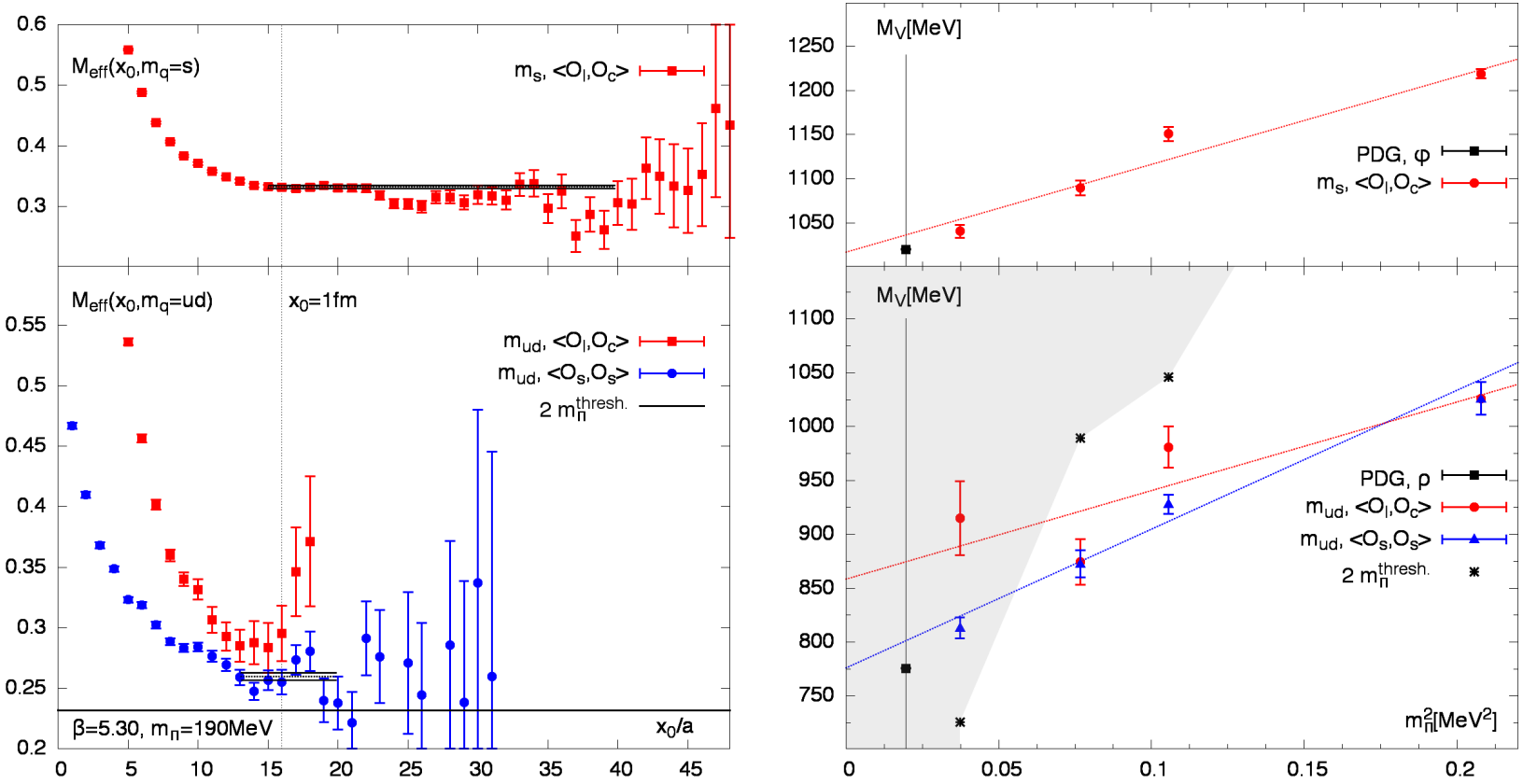
- Extrapolation to $Q^2 = 0$, via Padé-fit [1112.2894], [1205.3695], [1309.2153]
- Integration of $\Pi_{fit}(Q^2) - \Pi_{fit}(0)$ to obtain a_μ^{HLO}

Mixed rep. method



To do:

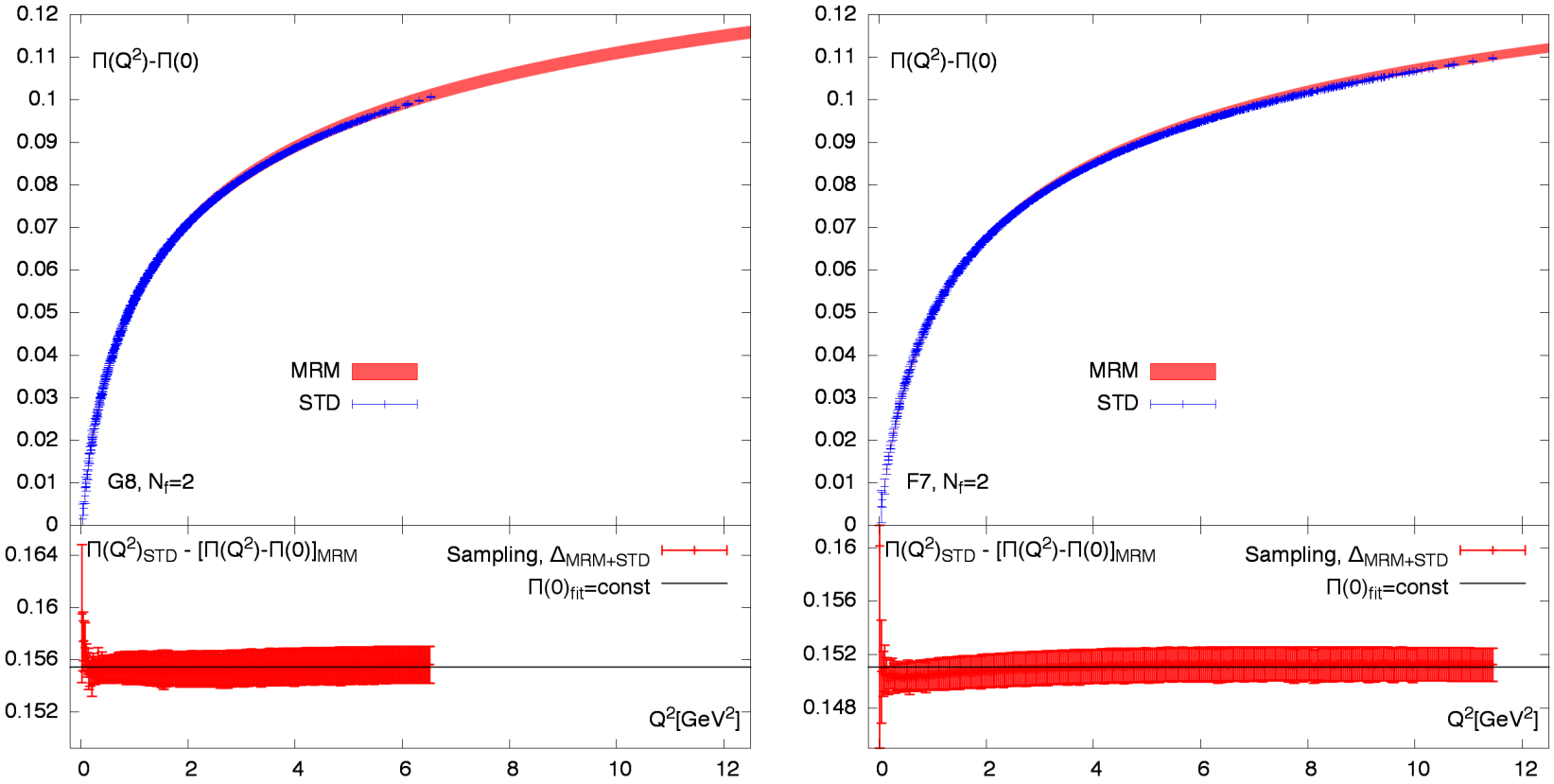
- Asymptotic extrapolation, via single exponential-fit [1306.2532]
- Integration of $G(x_0)$ for $\left(\Pi(Q^2) - \Pi(0)\right)$
- **or** direct integration of $G(x_0)$ to obtain a_μ^{HLO}



The extrapolation of $G(x_0)$ depends on the low mass spectrum

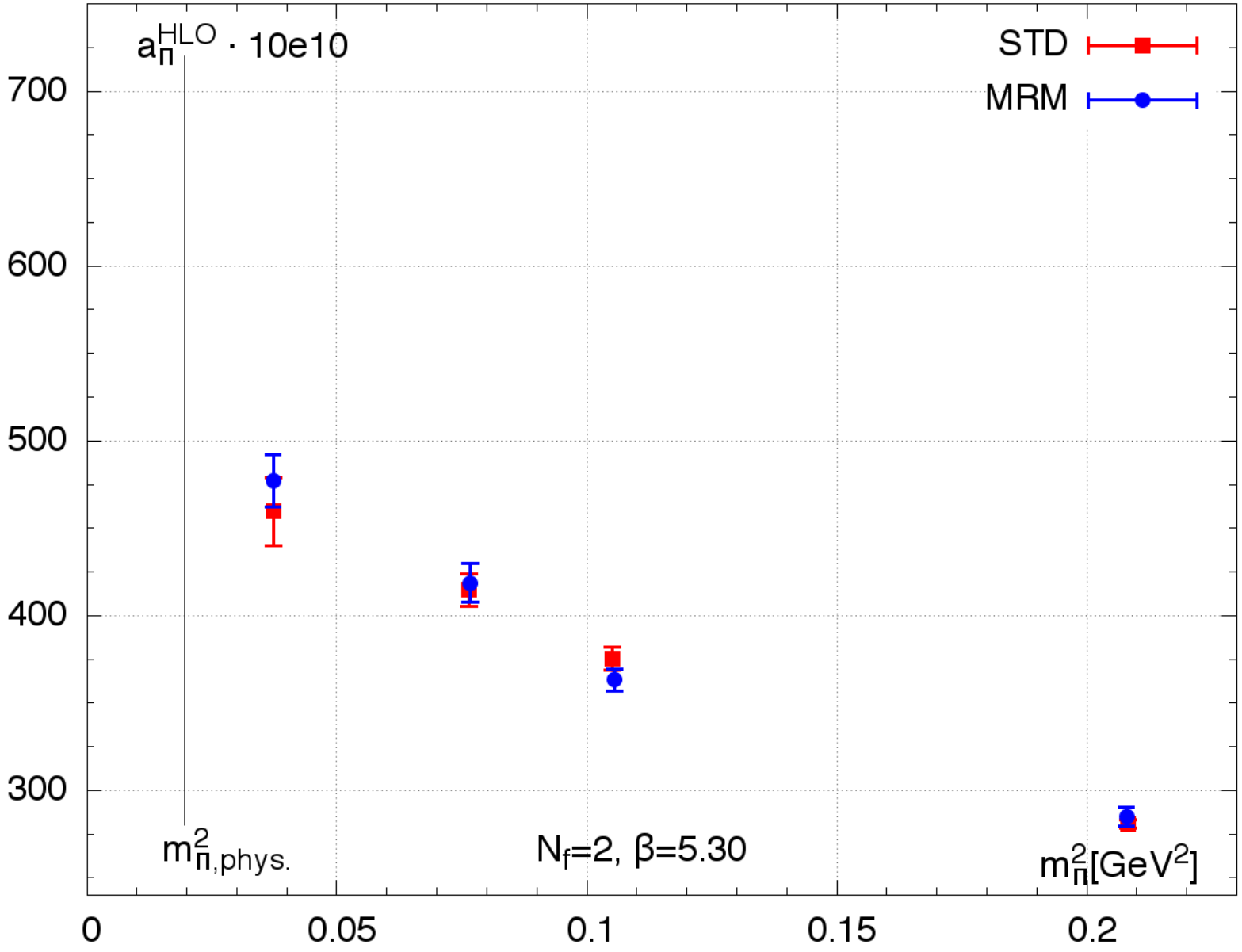
- We assume a single ground-state exponential contributes beyond $x_0 \simeq 1.2$fm (approx. $x_0/a \simeq 18 - 20$)
- Further contributions cannot be excluded/included from the current data
- We use smeared-smeared interpolating operators in the light case

Comparing both methods

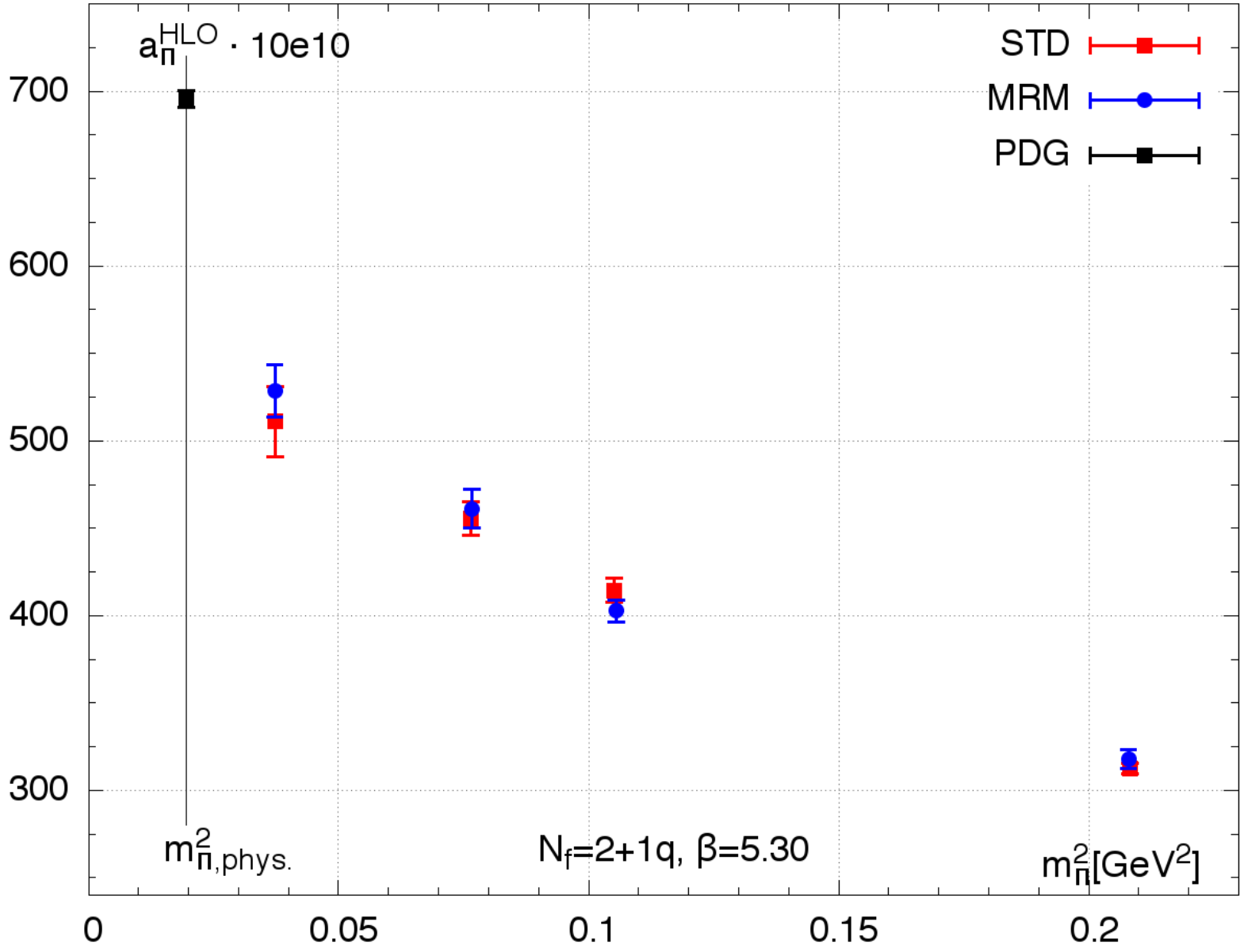


- **The difference** $\Pi_{STD}(Q^2) - \Big(\Pi(Q^2) - \Pi(0)\Big)_{MRM}$ **shows an almost constant behavior**
- In principle $\Pi(0)$ can be extracted from the difference ...
- ... here we will extract $\Pi(0)$ for the standard method from Padé-fit

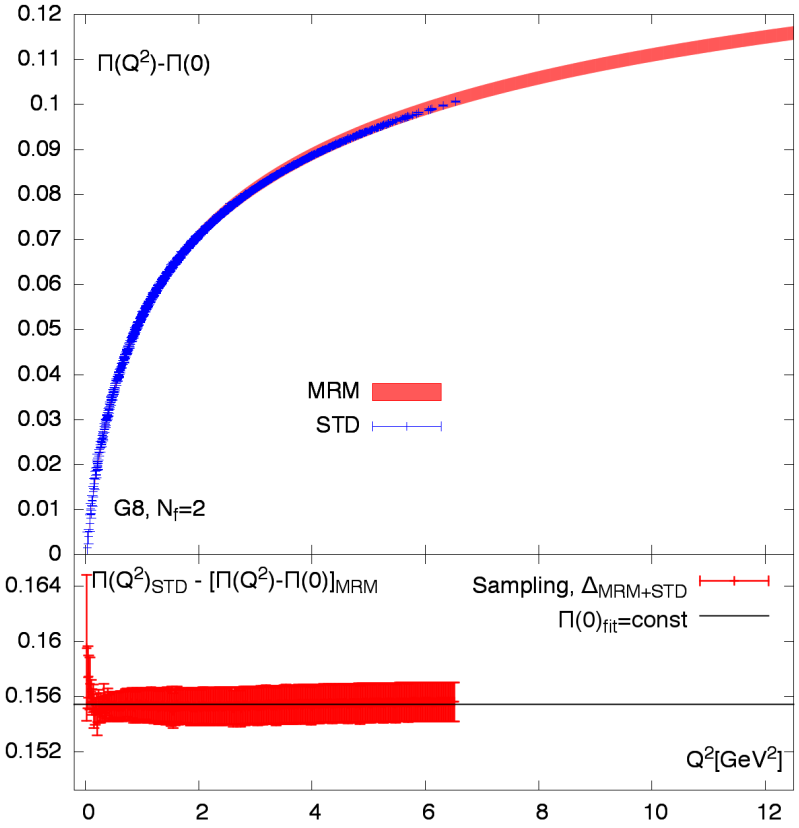
The anomalous magnetic moment of the muon for $N_f = 2$

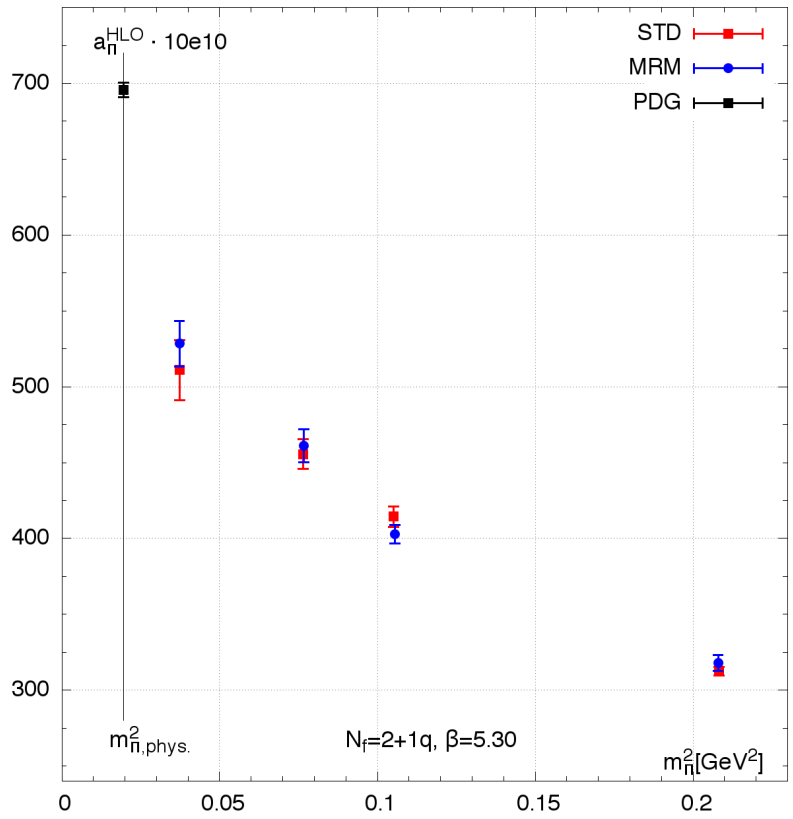


The anomalous magnetic moment of the muon for $N_f = 2 + 1q$



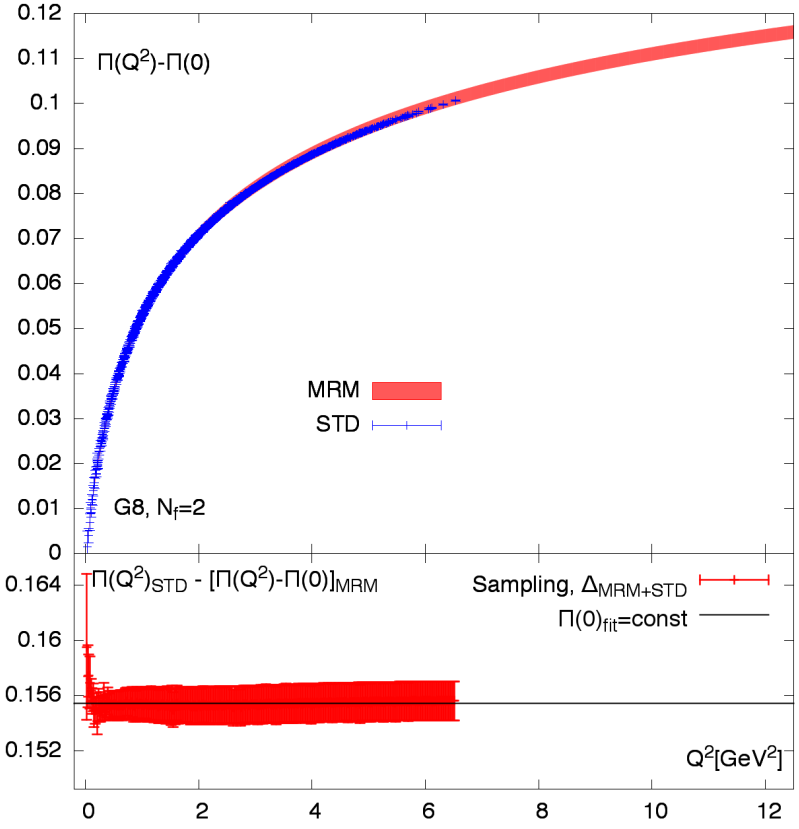
Overview

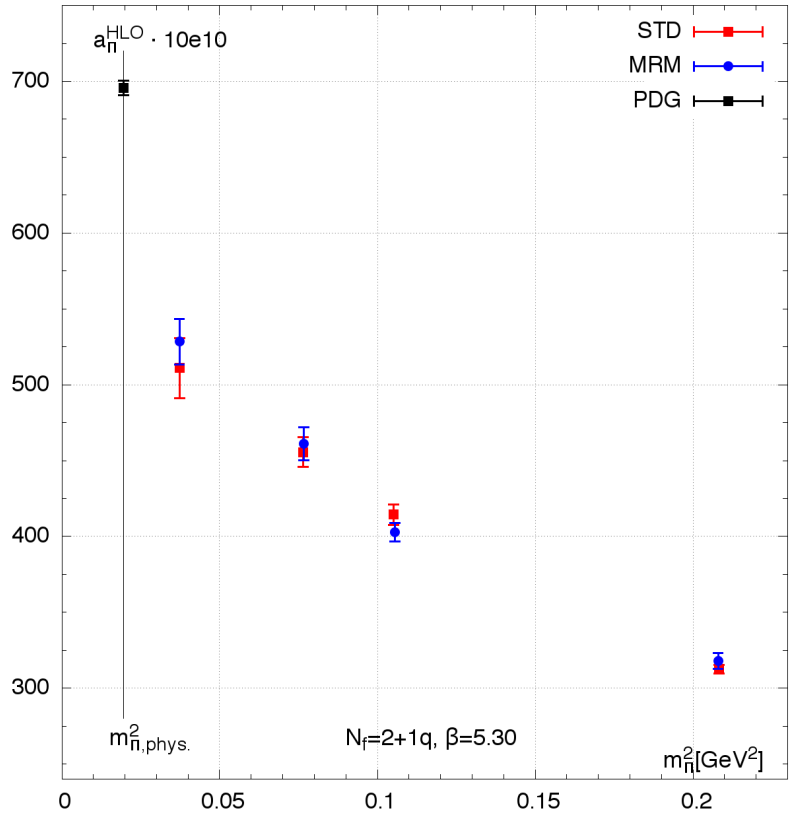




- We explored the chiral behavior of a_μ^{HLO}
- We extended our analysis of the standard method to $m_\pi = 190$MeV
- We included the mixed rep. method and compared systematically

Overview





- The standard method and the MRM are seen to be highly compatible
- The MRM gives a new handle to study the systematic effects and their induced errors on a_μ^{HLO}
- In the future, we will give an estimate of a_μ^{HLO} at the physical point including also disconnected diagrams (see talk by *V. Gülpers*)