Lattice Measurement of the Delta $I=\frac{1}{2}$ Contribution to Standard Model Direct CP-Violation in $K \rightarrow \pi\pi$ Decays at Physical Kinematics: Part II

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<td>Shane Drury (Southampton)</td>
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Outline

1. Motivation
2. Method
   ▶ Weak matrix elements.
   ▶ Decay amplitude.
3. Current results.
   ▶ $\pi\pi$ phase shift.
   ▶ $K \rightarrow \pi\pi (I = 0)$ weak matrix elements, decay amplitude $A_0$.
4. Conclusion
First ab initio calculation of direct CP-violation (in $K \rightarrow \pi\pi$).

Current experiment result: $Re(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}$

\[ \epsilon' = \frac{ie^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \frac{ReA_2}{ReA_0} \left[ \frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0} \right] \] (1)

Current lattice result: Only has $Re(A_2)$ and $Im(A_2)$, both with $< 10\%$ error. (mainly from stat and Wilson coefficients)

Once we obtain $A_0$ with $\approx 20\%$ error, could compare $\epsilon'$ with experiments.
Weak matrix elements $\langle \pi \pi | Q_i | K \rangle$

- G-parity Boundary introduces even larger numbers of contractions.

\[
\begin{align*}
\mu &\rightarrow -\gamma_4 \gamma_2 \bar{d}^T \\
\bar{d} &\rightarrow \gamma_4 \gamma_2 \bar{u}^T
\end{align*}
\]

\[
\langle \pi \pi \rangle = \bar{u} \gamma_5 d \quad \bar{d} \gamma_5 u
\]

\[+ \bar{u} \gamma_5 \bar{d} \quad \bar{d} \gamma_5 \bar{u} \]

Not like single pion, the 10 matrix elements $\langle \pi \pi | Q_i | K \rangle$ each contains 256 possible contractions. One has to figure out the linear combination:

\[
\langle \pi \pi | Q_i | K \rangle = \sum_{j=1}^{256} c_{ij} [\text{Contraction}_j]
\]
Weak matrix elements $\langle \pi\pi | Q_i | K \rangle$

- G-parity boundary introduces subtlety in momentum directions.

Under G-parity boundary condition, the degrees of freedom doubles in momentum space. Allowed quark momentum are in 'diagonal' direction.
Weak matrix elements $\langle \pi \pi | Q_i | K \rangle$

- Reducing errors from 'disconnected' diagrams.

Since the $\pi \pi (I = 0)$ state couples with vacuum, the amplitude doesn't decay as separation increases, small fluctuation could result in huge error.

In order to reduce the $\pi \pi (I = 0)$ to vacuum coupling, we chose to use the localized meson source, and separate the two pions in time direction.
Weak matrix elements \( \langle \pi \pi \mid Q_i \mid K \rangle \)

- Using localized source (all-to-all propagators).

Shaded boxes are where the random sources have been used.

\[
\sum \overleftarrow{\gamma}_\mu (1 - \gamma_5) L(\vec{x}_{op}, t_{op}; t_\pi) \gamma_5 L_w(t_\pi; t_{\pi'}) \gamma_5 L_w(t_{\pi'}; t_K) \gamma_5 S(t_K; \vec{x}_{op}, t_{op}) \cdot \overrightarrow{\gamma}_\mu (1 - \gamma_5) L(\vec{x}_{op}, t_{op}; \vec{x}_{op}, t_{op})
\]

\[
= \sum \\overleftarrow{w}^m_{x_{op}} \overrightarrow{\gamma}_\mu (1 - \gamma_5) \overrightarrow{v}_{x_{op}} \cdot \{ \overleftarrow{w}^j_{x_{op}} \overrightarrow{\gamma}_\mu (1 - \gamma_5) \overrightarrow{v}_{x_{op}} \} \cdot \pi^{i_k}_{t_\pi} \pi^{k_l}_{t_{\pi'}} K^{l_m}_{t_K}
\]

The complexity is \((\text{Mode Number})^2 \times (\text{Volume}) \times (T \text{ size}) \times 144\)

Mode number for light quark is 2436, volume is \(32^3 \times 64\), T is 64.
From $M_i = \langle \pi\pi | Q_i | K \rangle$ to decay amplitude

Bare $M_i$ on Lattice

$\downarrow$

$M_i$ in infinite volume

$\downarrow$

$M_i$ in RI/SMOM scheme

$\downarrow$

$M_i$ in $\overline{MS}$ scheme

$\downarrow$

Decay amplitude $A_0$

Finite volume correction$^1$

Lat$\rightarrow$RI/SMOM matching at 1.52GeV$^2$

RI/SMOM$\rightarrow$ $\overline{MS}$ matching at 1.52GeV$^3$

times $\overline{MS}$ Wilson coefficients at 1.52GeV$^4$

$^1$Laurent Lellouch et al. HEP-LAT/0003023;
$^2$C.Sturm et al. ARXIV:0901.2599
$^3$Christoph Lehner et al. ARXIV:1104.4948;
$^4$Buchalla et al. HEP-PH/9512380;
Lattice setup and measurement time

- Used $32^3 \times 64$ lattice, DWF+IDSDR action, $a^{-1} \approx 1.38\text{GeV}$, $(4.6\text{fm})^3$ box, physical pion and kaon. With G-parity boundary in X,Y,Z directions.

- Measurement time on IBM BG/Q 512-node machine:

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<tr>
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<th>time</th>
<th>flops</th>
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<tr>
<td>Generating eigenmodes</td>
<td>3.6h</td>
<td>22 Gflops/Node</td>
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<tr>
<td>Quark propagator (CG)</td>
<td>7.5h</td>
<td>38 Gflops/Node</td>
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<tr>
<td>Meson field contraction</td>
<td>5h</td>
<td>$\sim20$ Gflops/Node</td>
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<tr>
<td>Total</td>
<td>$\sim17$h</td>
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Result: Meson spectrum

<table>
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<tr>
<th>Lat</th>
<th>$E_\pi$</th>
<th>$\sqrt{E_\pi^2 - p_\pi^2}$</th>
<th>$m_K$</th>
<th>$E_{\pi\pi}(l=0)$</th>
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<tr>
<td>MeV</td>
<td>0.19834(67)</td>
<td>0.1021(12)</td>
<td>0.35490(32)</td>
<td>0.3888(86)</td>
</tr>
<tr>
<td>MeV</td>
<td>273.71(92)</td>
<td>140.9(17)</td>
<td>489.76(44)</td>
<td>537(12)</td>
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![Figure: Phase shift](image_url)

- $l=2$, 3 G.B.C.
- $l=0$, 3 G.B.C.
Result: Weak matrix elements and decay amplitude

\[ \langle \pi \pi | Q_2 | K \rangle = (1.30 \pm 0.96) \times 10^{-3}, \text{ using 50 configurations, fitting from 4 to 8:} \]
Result: Weak matrix elements and decay amplitude

\[ \langle \pi \pi | Q_6 | K \rangle = (-1.35 \pm 0.37) \times 10^{-2}, \] using 50 configurations, fitting from 4 to 8:
Conclusion

- $K \to \pi\pi (I = 0)$ decay amplitude is underway, with physical $\pi$, $K$, and physical kinematics. Estimate 100 more measurements in order to get 50% error for $A_0$. The measurement will take a few months.

- Future work:
  - estimate lattice artefacts / do the same computation on a finer lattice.
  - Match at higher scale in $\overline{MS}$ scheme / use dynamic charm quark.
Thank you!